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Electromagnetic Implosion Using an Array

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Abstract

This paper considers the use of a spherical array of sources producing a large fast-transient electromagnetic wave near the center of the sphere. The results here complement the results for an electromagnetic implosion due to reflector and lens types of implosion IRAs.

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1. Introduction

As discussed in [3] there are at least three general ways to construct an IRA (impulse radiating antenna). The basic idea is to make a fast-rising pulse in the form of a plane wave on an antenna aperture. This can be done via a lens at the end of a TEM horn (lens IRA), a paraboloidal reflector driven by a TEM feed, typically four arms (a reflector IRA), and an array of very many elements at or near the antenna aperture (an array IRA).

Lately, IRA technology is being applied to focus a fast pulse on a small target (an IIRA, or implosion IRA). Much attention has been given to the reflector form [6-9, 13, 14]. A similar effect can be achieved using a lens (which can be massive in a large structure) [11]. For completeness, let us consider an array as another alternative. Such is the subject of this paper.

2. Spherical Array

There are various forms that an array IRA can take. The array can be planar with the elements triggered in a sequence so as to produce a converging spherical wave. For convenience let us consider a spherical array of radius *a* as in Fig. 2.1. Let us impose a tangential electric field on the array surface S_a of

$$\vec{E}_s = -E_0 \sin(\theta) \vec{1}_\theta f\left(t + \frac{a}{c}\right) \tag{2.1}$$

so that the field arrives at the origin at $t = 0$. Here our coordinates are

$$\begin{aligned} &(x, y, z) \equiv \text{Cartesian coordinates} \\ &(\Psi, \phi, z) \equiv \text{cylindrical coordinates} \\ &x = \Psi \cos(\phi) \ , \ \ y = \Psi \sin(\phi) \\ &(r, \theta, \phi) \equiv \text{spherical coordinates} \\ &\Psi = r \sin(\theta) \ , \ \ z = r \cos(\theta) \end{aligned} \tag{2.2}$$

and our array surface S_a is defined by $r = a$.

In Laplace form we have

$$\begin{aligned} \tilde{\vec{E}}_s &= -E_0 \sin(\theta) \vec{1}_\theta \tilde{f}(s) e^{s\frac{a}{c}} \\ s &= \Omega + j\omega = \text{Laplace-transform variable or complex frequency} \\ c &= [\mu \varepsilon]^{-1/2} = \text{speed of propagation in medium} \end{aligned}$$

The spatial variation of sin(θ) in (2.1) is chosen to match the lowest-order spherical vector wave function.`

Here we do not go into the details of the design of the individual array elements. Note that on S_a one can place conductors on contours of constant θ. Array elements can be set in bands around the sphere connecting two adjacent such conductors. The element voltages can vary with θ, and/or the spacing of the constant$-\theta$ conductors can vary with θ. Consult [4, 5] and the references therein for various details of the array design.

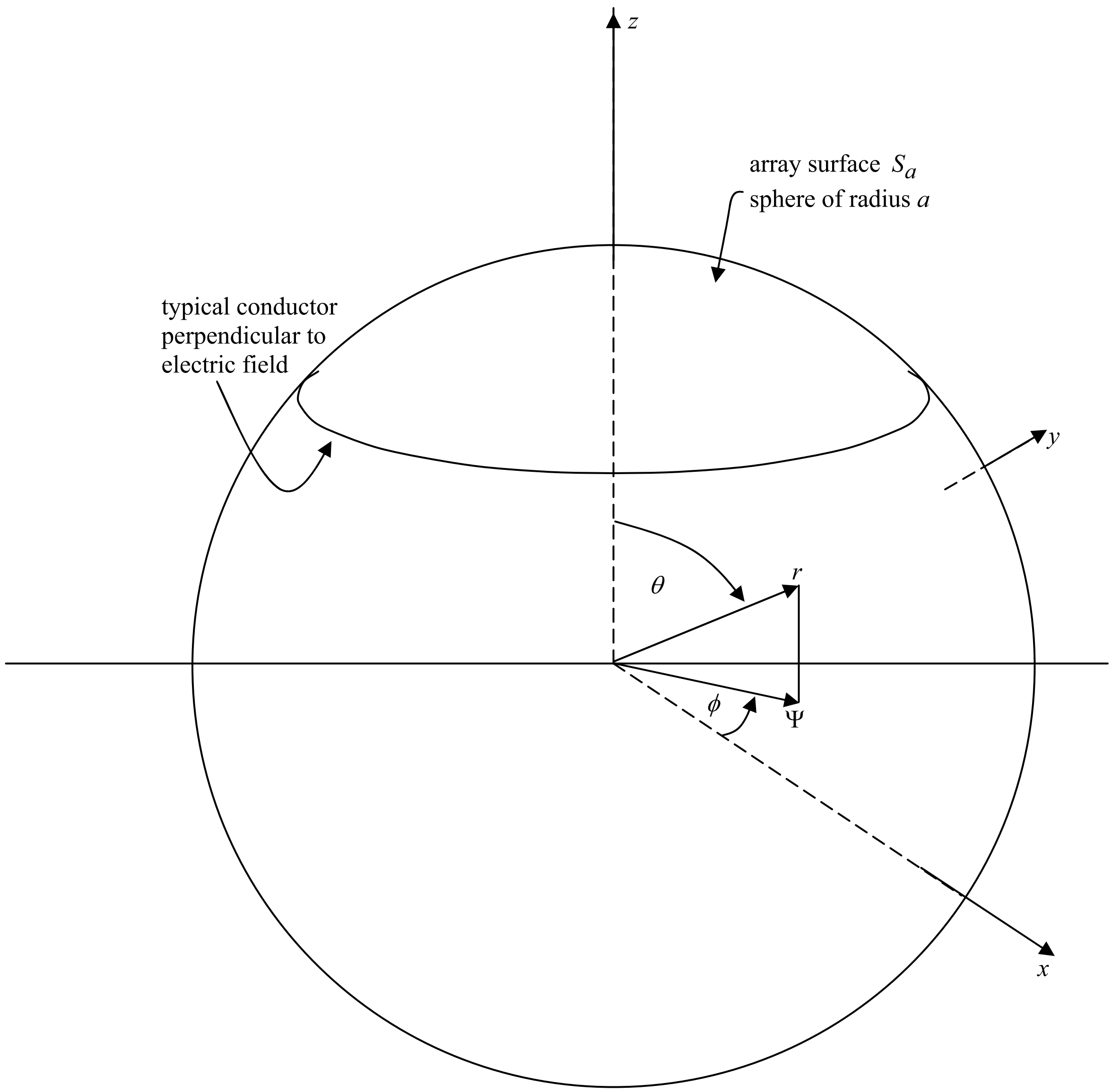

Fig. 2.1 Spherical Transient Array

3. Spherical Wave

Various papers and texts consider spherical electromagnetic waves. Here we cite a few [1, 2, 12] which place the functions in Laplace form.

For our present purposes we need the vector wave functions for the electric field. From [2(3.3)] we have the vector wave function

$$\begin{aligned}
\vec{N}_{1,0,e} &= 2\frac{f_1^{(\ell)}(\gamma r)}{\gamma r}\vec{P}_{1,0,e}(\theta,\phi) + \frac{\left[\gamma r\, f_1^{(\ell)}(\gamma r)\right]'}{\gamma r}\vec{Q}_{1,0,e} \\
\vec{P}_{1,0,e}(\theta,\phi) &= Y_{1,0,e}(\theta,\phi)\ \vec{1}_r = P_1^{(0)}(\cos(\theta))\vec{1}_r \\
&= \cos(\theta)\vec{1}_r \\
\vec{Q}_{1,0,e}(\theta,\phi) &= \nabla_s Y_{1,0,e}(\theta,\phi) = \vec{1}_\theta \frac{d\, P_n^{(0)}(\cos(\theta))}{d\theta} \\
&= -\sin(\theta)\vec{1}_\theta \\
f_1^{(\ell)}(\gamma r) &= \begin{cases} i_1(\gamma r) \text{ for } \ell = 1 \\ k_1(\gamma r) \text{ for } \ell = 2 \end{cases} \\
k_1(\gamma r) &= \frac{e^{-\gamma r}}{\gamma r}\left[1+[\gamma r]^{-1}\right] \\
i_1(\gamma r) &= \frac{1}{2}\left[k_1(\gamma r) - k_1(-\gamma r)\right] \\
k_1(\gamma r) &\equiv \text{outward propagating wave} \\
k_1(-\gamma r) &\equiv \text{inward propagating wave} \\
\gamma &= \frac{s}{c}
\end{aligned} \tag{3.1}$$

One can expand the field inside $'S_a$ using i_1 which is well-behaved at the origin, except at the resonance frequencies (on the $j\omega$ axis) for which the $i_1(jkr)$ is zero at $r = a$ with

$$\gamma = jk \ , \ k = \frac{\omega}{c} \tag{3.2}$$

Another approach involves thinking in time domain. Our interest concerns an incoming wave from S_a toward r = 0. This is described by $k_1(-\gamma r)$ for times before the wave reaches the origin. Let us then consider the wavefront on some $r = b$ for $0 < b < a$. This wave can be described (for negative times) by

$$
\begin{aligned}
\tilde{\vec{E}} &= \tilde{E}_1 \vec{N}_{1,0,e} \\
\frac{f_1^{(\ell)}(\gamma r)}{\gamma r} &= -\frac{k_1(-\gamma r)}{\gamma r} = e^{\gamma r}\left[[\gamma r]^{-2} - [\gamma r]^{-3}\right] \\
\frac{\left[\gamma r f_1^{(\ell)}(\gamma r)\right]'}{\gamma r} &= -\frac{\left[-\gamma r k_1(-\gamma r)\right]'}{\gamma r} \\
&= e^{\gamma r}\left[[\gamma r]^{-1} - [\gamma r]^{-2} + [\gamma r]^{-3}\right]
\end{aligned}
\tag{3.3}
$$

The θ-directed electric field is described by the $\vec{Q}$ variation. Matching the tangential electric field on $r = a$ gives

$$
\begin{aligned}
\tilde{E}_1 e^{\gamma a}\left[[\gamma a]^{-1} - [\gamma a]^{-2} + [\gamma a]^{-3}\right] &= E_0 \tilde{f}(s) e^{\gamma a} \\
\tilde{E}_1 &= E_0 \tilde{f}(s)\left[[\gamma a]^{-1} - [\gamma a]^{-2} + [\gamma a]^{-3}\right]^{-1}
\end{aligned}
\tag{3.4}
$$

The electric field for $r < a$ is then described by

$$
\begin{aligned}
\vec{E} &= E_0 \tilde{f}(s)\left[[\gamma a]^{-1} - [\gamma a]^{-2} + [\gamma a]^{-3}\right]^{-1} \\
& e^{\gamma r}\left[2\left[[\gamma r]^{-2} - [\gamma r]^{-3}\right]\cos(\theta)\vec{1}_r - \left[[\gamma r]^{-1} - [\gamma r]^{-2} + [\gamma r]^{-3}\right]\sin(\theta)\vec{1}_\theta\right]
\end{aligned}
\tag{3.5}
$$

Comparing to (2.1) we see that it is the θ component that is of interest. For $\theta = \pi/2$ (the *xy* plane) we have

$$
E_z = E_0 \tilde{f}(s) e^{\gamma r} \frac{[\gamma r]^{-1} - [\gamma r]^{-2} + [\gamma r]^{-3}}{[\gamma a]^{-1} - [\gamma a]^{-2} + [\gamma a]^{-3}}
\tag{3.6}
$$

4. Transient Field Near Origin

For small r we can write

$$\tilde{E}_z = E_0\, f(s)\, e^{\gamma r} \frac{a}{r} \frac{1-[\gamma r]^{-1}+[\gamma r]^{-2}}{1-[\gamma a]^{-1}+[\gamma a]^{-3}} \tag{4.1}$$

showing that, as the wave approaches the origin, it increases as *a*/*r* (for high frequencies). Expanding (3.7) for early times gives

$$\tilde{E}_z = E_0\, f(s)\, e^{\gamma r} \frac{a}{r}\left[1+[\gamma a]^{-1}-[\gamma r]^{-1}+\mathrm{O}\left([\gamma r]^{-2}\right)\right] \tag{4.2}$$

Letting $\tilde{f}(s) = s^{-1}$ (step function with zero rise time) we have

$$E_z = E_0\left[\frac{a}{r}u\left(t+\frac{r}{c}\right)+\frac{c}{r}\left[1-\frac{a}{r}\right]t\,u\left(t+\frac{r}{c}\right)+\mathrm{O}\left(t^2 u\left(t+\frac{r}{c}\right)\right)\right] \tag{4.3}$$

For small *r*/*a* we see the amplitude rising like *a*/*r* initially. Then, as *t* increases to *r*/*c*, the next term (a ramp) cancels this term. So we can roughly say that the average pulse width is *r*/(2*c*). As $r \to 0$, the height times width is

$$E_\delta = E\frac{a}{r}\frac{r}{2c} = \frac{a}{2c}E_0 \quad Vs/m \tag{4.4}$$

which can be regarded as the area of a delta function incident upon *r* = 0. More detailed calculations can consider the effect of higher-order terms.

A realistic array will, of course, have a nonzero rise time, including both the switching time of the elements and the spread (or synchronization error) of these firing times. Convolving this with a delta function gives a height t_{mr} (rise time based on maximum rate of rise) times the δ-function area.

5. Concluding Remarks

While the present result is for a full spherical distributed source, it can be applied to a half-covered sphere (say $-\pi/2 \le \phi \le \pi/2$) using symmetry arguments. This will cut the delta function in half at the origin (when fields would otherwise be arriving from both sides of the target). This depents on the size of the target compared to ct_{mr}.

Note that we have not specified the characteristics of the target centered on $r = 0$. This will change the pulse amplitude based on reflection from the target. As discussed in [10] one can potentially improve matters with a special lens near the target.

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