Sensor and Simulation Notes

Note 554

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Maximizing Energy in Terahertz Pulse Radiation from a Switched Oscillator

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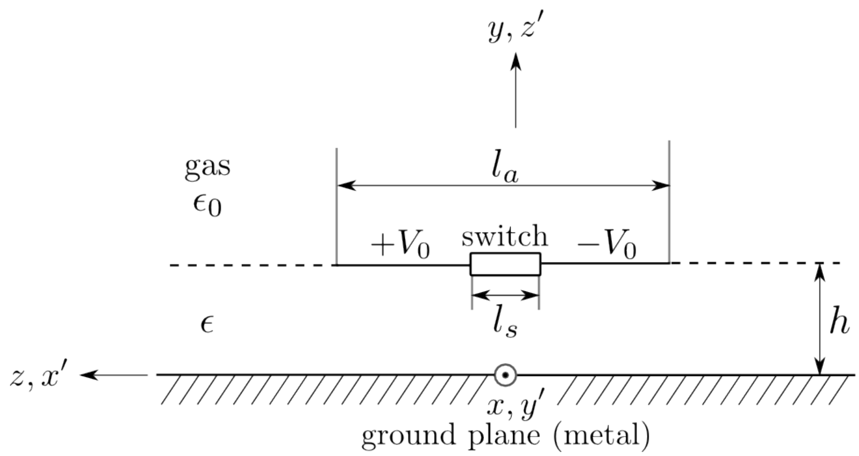
Abstract

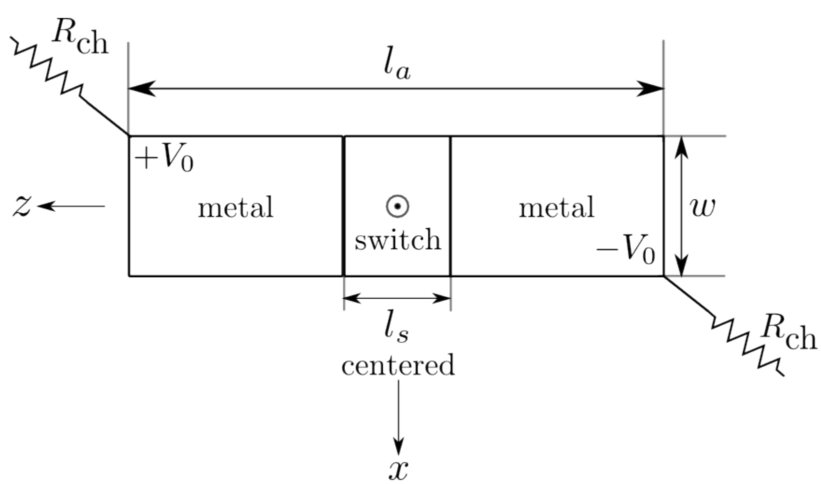
For a THz pulse (damped sinusoid) radiator consisting of a dipole-like antenna separated by a dielectric from a ground plane, there are many design considerations. These include the stored energy, radiation efficiency, and resistive losses.

1 Introduction

Pursuing the design of a pulsed (damped sinusoid) THz radiator we need to find some optimization conditions. Given some frequency, f_0, how much energy can we radiate in a damped sinusoidal pulse? This depends on the various parameters of the antenna and the source. For present purposes let us consider the configuration in Fig. 1.1.



(a) Side view



(b) Front view

Figure 1.1: THz radiator.

Here we have a conducting ground plane (typically copper). This is separated from the antenna by a dielectric of permittivity, ϵ, and thickness, h. The antenna is a half-wavelength radiator of length, l_a. Each half is charged to $\pm V_0$. These halves are separated by a switch medium (say semi-insulating gallium arsenide) of length, l_s. The width of the antenna is w. The switch is illuminated by a femto second laser to cause it to conduct (close).

2 Maximizing the Stored Energy

The stored energy is

$$U_0 = \frac{1}{2}C[2V_0]^2 = 2CV_0^2. \tag{2.1}$$

The energy available at f_0, the dominant half-wave resonant frequency, is estimated as [1]

$$U_1 = \frac{8}{\pi^2}U_0 \approx 0.81U_0, \tag{2.2}$$

the remaining energy being for higher order resonant frequencies, which we neglect. We need to maximize this energy and radiate as much of this as possible. Thus we need to maximize $2CV_0^2$.

Consider the capacitance, $C \approx \epsilon w l_a/h$. For a given h, this is maximized by large w, the length, l_a, being given by

$$\begin{aligned} &\lambda_{\rm os} \approx 2l_a = \frac{v}{f_0}, \quad (\text{“os”} = \text{oscillator}), \\ &v \approx c\epsilon_r^{-1/2}, \quad \epsilon_r = \frac{\epsilon}{\epsilon_0}, \end{aligned} \tag{2.3}$$

or more accurately

$$c > v > c\epsilon_r^{-1/2}, \tag{2.4}$$

with v approaching $c\epsilon_r^{-1/2}$ for large w, since most of the capacitive energy lies between the antenna and the ground plane. However, we do not want w to be so large that higher order oscillation modes are supported. So we might limit the width as

$$w \approx \frac{l_a - l_s}{2}, \tag{2.5}$$

for obtaining a large capacitance. This capacitance can be estimated as

$$C \approx \frac{\epsilon w[l_a - l_s]}{4h}, \tag{2.6}$$

as two capacitors in series.

One can maximize C by minimizing h, but it is the energy we wish to maximize. So consider maximizing V_0. As an approximation we can estimate this via

$$\begin{aligned} &V_1 = E_d h, \\ &E_d \equiv \text{ average breakdown electric field through dielectric to ground plane.} \end{aligned} \tag{2.7}$$

This includes the edge effects on the antenna which can be mitigated to some extent by roll-ups on the edges.

Another voltage limitation concerns the switch which needs to be highly insulating before the arrival of the fs laser pulse. Let us estimate the voltage stand-off as

$$\begin{aligned} &V_2 = E_s l_s, \\ &E_s \equiv \text{ average breakdown electric field through switch between two antenna halves.} \end{aligned} \tag{2.8}$$

As we increase V_0 and accordingly increase h we have

$$U_0 = 2CV_0^2 = \frac{\epsilon w[l_a - l_s]}{2h}[E_d h]^2 = \frac{\epsilon w[l_a - l_s]}{2} h E_d^2, \tag{2.9}$$

thereby increasing U_0. Similarly, as we increase V_0 and accordingly increase l_s we have

$$U_0 = 2CV_0^2 = \frac{\epsilon w[l_a - l_s]}{2h}[E_s l_s]^2. \tag{2.10}$$

Increasing h then (2.9) increases the stored energy up until the switch limits the voltage. Increasing h beyond this *decreases* the stored energy. Equating the two results gives

$$\frac{\epsilon w[l_a - l_s]}{2} h E_d^2 = \frac{\epsilon w[l_a - l_s]}{2h}[E_s l_s]^2 \Rightarrow \frac{h}{l_s} = \frac{E_s}{E_d}, \tag{2.11}$$

as an approximate optimum. However, while one may wish

$$h \approx \frac{\lambda}{4}, \tag{2.12}$$

for radiation characteristics, we need

$$l_s < l_a, \tag{2.13}$$

as a limiting factor which may alter the results of (2.11). For example, if

$$E_d > E_s, \tag{2.14}$$

then our limitation on the energy is given by (2.10) with the limit in (2.12). In that case we need to maximize (for fixed l_a)

$$X = [l_a - l_s] l_s^2. \tag{2.15}$$

Differentiating with respect to l_s gives

$$0 = 2 l_a l_s - 3 l_s^2 \Rightarrow \frac{l_s}{l_a} = \frac{2}{3}. \tag{2.16}$$

There are still other factors to consider such as switch losses and conductor losses.

3 Ideal Radiation Characteristics

3.1 Single electric dipole radiator

Let us make some simple estimate of the radiated fields. An electric dipole, in free space, has far fields [2]

$$\begin{aligned}\tilde{\vec{E}}_f(\vec{r},s) &= \frac{\mu_0}{4\pi r}s^2 e^{-\gamma r}\overleftrightarrow{1}_r \times [\vec{1}_r \times \tilde{\vec{p}}(s)] = -\frac{\mu_0 s^2}{4\pi r}\vec{1}_r \cdot \vec{p}(s)e^{-\gamma r}, \\ \tilde{\vec{H}}_f(\vec{r},s) &= Z_0^{-1}\vec{1}_r \times \vec{E}_f(\vec{r},s),\end{aligned} \tag{3.1}$$

$$\begin{aligned}\tilde{} &\equiv \text{Laplace transform (two sided)},\\ \gamma &\equiv s/c \equiv \text{propagation constant},\\ s &\equiv \Omega + j\omega \equiv \text{ Laplace-transform variable or complex frequency},\\ Z_0 &\equiv [\mu_0/\epsilon_0]^{1/2} \equiv \text{ wave impedance of free space},\\ \tilde{\vec{p}}(s) &\equiv \tilde{p}(s)\vec{1}_z \equiv \text{ electric dipole moment},\\ \overleftrightarrow{1}_r &\equiv \overleftrightarrow{1} - \vec{1}_r\vec{1}_r = \vec{1}_\theta\vec{1}_\theta - \vec{1}_\phi\vec{1}_\phi \equiv \text{ transverse dyad}.\end{aligned}$$

This gives a radiated power $s = j\omega$ as

$$P_1 = \frac{1}{2}\int_S [\tilde{\vec{E}}_f(\vec{r},j\omega) \times \tilde{\vec{H}}_f(\vec{r},-j\omega)]4\pi r^2 dS, \tag{3.2}$$

$S \equiv$ sphere of radius r. The factor 1/2 gives the average power over a cycle. Appropriately substituting gives

$$\begin{aligned}P_1 &= \frac{\mu_0\omega^4}{32\pi^2 r^2 c}\int_S \tilde{\vec{p}}(j\omega)\cdot\overleftrightarrow{1}_r\cdot\tilde{\vec{p}}(-j\omega)\ 4\pi r^2 dS\\ &= \frac{\mu_0\omega^4}{8\pi c}\int_0^\pi\int_0^{2\pi} |\vec{p}(j\omega)|^2\,\vec{1}_z\cdot\overleftrightarrow{1}_r\cdot\vec{1}_z\sin(\theta)d\phi d\theta\\ &= \frac{\mu_0\omega^4}{4c}|\vec{p}(j\omega)|^2\int_0^\pi [\vec{1}_z\cdot\vec{1}_\theta]^2\sin(\theta)d\theta\\ &= \frac{\mu_0\omega^4}{4c}|\vec{p}(j\omega)|^2\int_0^\pi \sin^3(\theta)d\theta\\ &= \frac{\mu_0\omega^4}{3c}|\vec{p}(j\omega)|^2 \approx 0.33\mu_0\omega^4|\vec{p}(j\omega)|^2,\\ c &= [\mu_0\epsilon_0]^{-1/2} \equiv \text{ speed of light}.\end{aligned} \tag{3.3}$$

3.2 Two dipole radiators $\lambda/2$ apart

Modifying this for our geometry let us approximate this as two dipoles separated by a half-wavelength, 180° out of phase, but only radiating into a half-space. Let one dipole be at $y = +\lambda/4$

with the second at $y = -\lambda/4$. Then we can superimpose the two dipole fields as

$$\begin{aligned}
\tilde{\vec{E}}_f(\vec{r}, s) &= \frac{-\mu_0}{4\pi r} s^2 \overleftrightarrow{1}_r \cdot \tilde{p}(s) \vec{1}_z [e^{-\gamma[r - \frac{\lambda}{4}\vec{1}_y \cdot \vec{1}_r]} - e^{-\gamma[r + \frac{\lambda}{4}\vec{1}_y \cdot \vec{1}_r]}] \\
&= \frac{-\mu_0}{4\pi r} \tilde{p}(s) \overleftrightarrow{1}_r \cdot \tilde{p}(s) e^{-\gamma r} 2 \sinh\left(\frac{\gamma\lambda}{4}\sin(\theta)\sin(\phi)\right), \qquad (3.4) \\
\tilde{\vec{H}}_f(\vec{r}, s) &= Z_0^{-1} \overleftrightarrow{1}_r \times \tilde{\vec{E}}_f(\vec{r}, s).
\end{aligned}$$

This is an odd function of ϕ (or $\sin(\phi)$). For $s = j\omega$ we also have

$$\gamma = jk = j\frac{\omega}{c}. \qquad (3.5)$$

Now it is convenient to change variables so that the argument of sinh is a function of only one coordinate. For this purpose choose

$$\begin{aligned}
(x', y', z') &= (z, x, y) \Rightarrow (\vec{1}_{x'}, \vec{1}_{y'}, \vec{1}_{z'}) = (\vec{1}_z, \vec{1}_x, \vec{1}_y), \\
r &= [x^2 + y^2 + z^2]^{1/2} = [x'^2 + y'^2 + z'^2]^{1/2}, \\
z' &= r\cos\theta', \\
\psi' &= [x'^2 + y'^2]^{1/2} = r\sin(\theta'), \\
x' &= \psi'\cos(\phi') = r\sin(\theta')\cos(\phi'), \qquad (3.6) \\
y' &= \psi'\sin(\phi') = r\sin(\theta')\sin(\phi'), \\
\vec{1}_r &= \vec{1}_{z'}\cos\theta' + \vec{1}_{\psi'}\sin(\theta') \\
&= \vec{1}_{z'}\cos(\theta') + \vec{1}_{x'}\sin(\theta')\cos(\phi') + \vec{1}_{y'}\sin(\theta')\sin(\phi') \\
&= \vec{1}_z\cos(\theta) + \vec{1}_\psi\sin(\theta) \\
&= \vec{1}_z\cos(\theta) + \vec{1}_x\sin(\theta)\cos(\phi) + \vec{1}_y\sin(\theta)\sin(\phi).
\end{aligned}$$

This implies

$$\begin{aligned}
&\sin(\theta)\sin(\phi) = \cos(\theta'), \qquad (3.7) \\
&\vec{1}_{x'} \cdot \overleftrightarrow{1}_r \cdot \vec{1}_{x'} = 1 - [\vec{1}_{x'} \cdot \vec{1}_r]^2 = 1 - \sin^2(\theta')\cos^2(\phi').
\end{aligned}$$

Then we have the radiated power

$$\begin{aligned}
P_2 &= \frac{1}{2}\int_S [\tilde{\vec{E}}_f(\vec{r}, j\omega) \times \tilde{\vec{H}}_f(\vec{r}, -j\omega)] \cdot \vec{1}_r \, dS \qquad (3.8) \\
&= \frac{\mu_0\omega^4}{32\pi^2 r^2 c} |\tilde{p}(j\omega)|^2 \int_S \vec{1}_{x'} \cdot \overleftrightarrow{1}_r \cdot \vec{1}_{x'} \; 4\sin^2\left(\frac{k\lambda}{4}\cos\theta'\right) dS \\
&= \frac{\mu_0\omega^4}{2\pi c} |\tilde{p}(j\omega)|^2 \int_0^\pi \int_0^{2\pi} \sin^2\left(\frac{k\lambda}{4}\cos\theta'\right) \left[1 - \sin^2(\theta')\cos^2(\phi')\right] \sin(\theta') d\phi' d\theta' \\
&= \frac{\mu_0\omega^4}{2c} |\tilde{p}(j\omega)|^2 \int_0^\pi \sin^2\left(\frac{k\lambda}{4}\cos\theta'\right) \left[2 - \sin^2(\theta')\right] \sin(\theta') d\theta'.
\end{aligned}$$

Substituting

$$\begin{aligned} \eta &= \cos(\theta'),\ d\eta = -\sin(\theta')d\theta', \\ \sin^2(\theta') &= 1 - \cos^2(\theta') = 1 - \eta^2, \end{aligned} \tag{3.9}$$

gives

$$P_2 = \frac{\mu_0\omega^4}{2c} \left|\widetilde{p}(j\omega)\right|^2 \int_0^1 \sin^2\left(\frac{k\lambda}{4}\eta\right)[1+\eta^2]d\eta. \tag{3.10}$$

Substituting

$$\nu = \frac{k\lambda\eta}{4}, d\nu = \frac{k\lambda}{4}d\eta, \tag{3.11}$$

gives

$$P_2 = \frac{\mu_0\omega^4}{2c} \left|\widetilde{p}(j\omega)\right|^2 \frac{4}{k\lambda} \int_0^{\frac{k\lambda}{4}} \sin^2(\nu) \left[1 + \left[\frac{4\nu}{k\lambda}\right]^2\right]\ d\nu, \tag{3.12}$$

which is simplified, using *Mathematica*[1], as

$$P_2 = \frac{\mu_0\omega^4}{2c} \left|\widetilde{p}(j\omega)\right|^2 \left[\frac{-4k\lambda\cos\left(\frac{k\lambda}{2}\right) - 2\left(k^2\lambda^2 - 4\right)\sin\left(\frac{k\lambda}{2}\right)}{k^3\lambda^3} + \frac{2}{3}\right]. \tag{3.13}$$

For a resonant (half-wave) condition we have

$$\begin{aligned} \lambda_0 &= 2l_a, \\ f_0\lambda_0 &= c,\ f = \frac{\omega_0}{2\pi} = \frac{c}{\lambda_0} = \frac{c}{2l_a}, \\ k_0\lambda_0 &= \frac{\omega_0\lambda_0}{c} = 2\pi. \end{aligned} \tag{3.14}$$

In (3.13) we have

$$\sin\left(\frac{k\lambda}{2}\right) = 0,\ \ \cos\left(\frac{k\lambda}{2}\right) = -1,$$

giving

$$P_2 = \frac{\mu_0\omega^4}{2c} \left|\widetilde{p}(j\omega)\right|^2 \left[\frac{4}{k^2\lambda^2} + \frac{2}{3}\right] = \frac{\mu_0\omega^4}{2c} \left|\widetilde{p}(j\omega)\right|^2 \left[\frac{1}{\pi^2} + \frac{2}{3}\right] \approx 0.38\frac{\mu_0\omega^4}{c} \left|\widetilde{p}(j\omega)\right|^2. \tag{3.15}$$

Noting that, with the ground plane, the radiation is only in the z' direction we have

$$P_3 = \frac{P_2}{2} \approx 0.19\frac{\mu_0\omega^4}{c} \left|\widetilde{p}(j\omega)\right|^2, \tag{3.16}$$

as the forward radiated power.

[1] `http://www.wolfram.com/`

3.3 Applicability

The foregoing analysis applies strictly to an imaged dipole in free space. The permittivity of the medium above the ground plane alters these results somewhat. So ,they should be applied for relatively small ϵ_r.

As ϵ_r is increased the half-wave resonant length becomes less than $\lambda/2$ in free space, making the antenna a less efficient radiator. It sill radiates, but it takes a longer time to radiate the energy, U_1 giving a higher Q.

Also, as ϵ_r is increased there can be surface bound waves propagating along the dielectric surface. This takes some of the energy in an undesirable direction.

Even for $\epsilon = \epsilon_0$, the antenna is no longer electrically small, making the dipole characterization of the antenna only approximate.

3.4 Two closely spaced electric-dipole radiators

Now let the two dipoles be placed at

$$z' = \pm h, \quad h \ll \frac{\lambda}{4}. \tag{3.17}$$

Then (3.4) is replaced by

$$\begin{aligned} \tilde{\vec{E}}_f(\vec{r}, s) &= \frac{-\mu_0}{4\pi r} s^2 \tilde{p}(s) \overleftrightarrow{1}_r \cdot \vec{1}_z e^{-\gamma r} 2\sinh(\gamma D \sin(\theta)\sin(\phi)) \\ &\approx \frac{-\mu_0}{4\pi r} s^2 \tilde{p}(s) \overleftrightarrow{1}_r \cdot \vec{1}_z e^{-\gamma r} 2\gamma D \sin(\theta)\sin(\phi), \\ \tilde{\vec{H}}_f(\vec{r}, s) &= Z_0^{-1} \overleftrightarrow{1}_r \times \tilde{\vec{E}}_f(\vec{r}, s). \end{aligned} \tag{3.18}$$

Making the same coordinate transformation as in (3.6) and (3.7) gives a radiated power

$$\begin{aligned} P_2 &= \frac{1}{2} \int_s [\tilde{\vec{E}}_f(\vec{r}, j\omega) \times \tilde{\vec{H}}_f(\vec{r}, -j\omega)] \cdot \vec{1}_r \; dS \\ &= \frac{\mu_0 \omega^4}{8\pi^2 r^2 c} \left|\tilde{p}(j\omega)\right|^2 \int_S \vec{1}_{x'} \cdot \overleftrightarrow{1}_r \cdot \vec{1}_{x'} [kh]^2 \cos^2(\theta') \; dS \\ &= \frac{\mu_0 \omega^4}{2\pi c} \left|\tilde{p}(j\omega)\right|^2 [kh]^2 \int_0^{\pi} \int_0^{2\pi} \cos^2(\theta')[1 - \sin^2(\theta')\cos^2(\phi)] \sin(\theta') \; d\phi' d\theta' \\ &= \frac{\mu_0 \omega^4}{2c} \left|\tilde{p}(j\omega)\right|^2 [kh]^2 \int_0^{\pi} \cos^2(\theta')[2 - \sin^2(\theta')] \sin(\theta') \; d\theta'. \end{aligned} \tag{3.19}$$

$$\tag{3.20}$$

Substituting as in (3.9) gives

$$\begin{aligned} P_2 &= \frac{\mu_0 \omega^4}{c} \left|\tilde{p}(j\omega)\right|^2 [kh]^2 \int_0^1 \eta^2 [1 + \eta^2] \; d\eta \\ &= \frac{\mu_0 \omega^4}{c} \left|\tilde{p}(j\omega)\right|^2 [kh]^2 \left[\frac{1}{3} + \frac{1}{5}\right] \\ &= \frac{8}{15} \frac{\mu_0 \omega^4}{c} \left|\tilde{p}(j\omega)\right|^2 \left[\frac{\omega h}{c}\right]^2 \approx 0.53 \frac{\mu_0 \omega^4}{c} \left|\tilde{p}(j\omega)\right|^2 \left[\frac{\omega h}{c}\right]^2. \end{aligned} \tag{3.21}$$

Note the extra factor of $[kh]^2$, lowering the radiated power. This factor is the square of the fraction of a half wavelength given by l_a. The electrically small antenna is a less efficient radiator.

Noting that the radiation in only in the $+z'$ direction we have

$$P_3 = \frac{P_2}{2} = \frac{4}{15}\frac{\mu_0\omega^4}{c}\left|\widetilde{p}(j\omega)\right|^2\left[\frac{\omega h}{c}\right]^2 \approx 0.27\frac{\mu_0\omega^4}{c}\left|\widetilde{p}(j\omega)\right|^2\left[\frac{\omega h}{c}\right]^2. \tag{3.22}$$

4 Matching to Electric Dipoles

4.1 Dipole Characteristics

Our formulae are in terms of the dipole moment at the resonant frequency. We need to evaluate $\widetilde{p}(j\omega)$. When the switch is closed (ideally instantaneously) there is generated a square wave oscillation with frequency f_0. The current in this wave is

$$I = \frac{V_0}{Z_c},$$

$$Z_c \approx Z_w\frac{h}{w} \equiv \text{ transmission line characteristic impedance,} \tag{4.1}$$

$$Z_w \approx Z_0\epsilon_r^{-1/2} = \left[\frac{\mu_0}{\epsilon_0}\right]^{1/2} \equiv \text{ wave impedance.} \tag{4.2}$$

This is only approximate. Z_c is lowered by fringe fields if w is not $\gg h$. Furthermore, the impedance is influenced by ϵ_0 above the dielectric as well as ϵ in the dielectric.

In [1] it is shown that the peak current in the resonant mode is

$$I_{\max} = \frac{4}{\pi}I_0. \tag{4.3}$$

In this resonant mode the current is $I_{\max}$ at the antenna center, but zero at the ends, giving,

$$I(z) = I_{\max}\cos\left(\frac{\pi z}{l_a}\right), \tag{4.4}$$

as the spatial distribution of the current.

An alternate formula [2] (compared to charge times distance) for an electric dipole moment is

$$\begin{aligned}\widetilde{p}(j\omega_0) &= \frac{1}{s}\int_{-l_a/2}^{l_a/2}\widetilde{I}(z,s)\,dz = \frac{I_{\max}}{j\omega_0}\int_{-l_a/2}^{l_a/2}\cos\left(\frac{\pi z}{l_a}\right)dz = \frac{I_{\max}}{j\omega_0}\left(\frac{2l_a}{\pi}\right) \\ &= -j\frac{8}{\pi^2}\frac{l_a}{\omega_0}I_0 = -j\frac{8}{\pi^2}\frac{l_a}{\omega_0}\frac{V_0}{Z_c} \approx -j\frac{8}{\pi^2}\frac{l_a}{\omega_0}\frac{w}{h}\frac{V_0}{Z_w} \approx -j\frac{4}{\pi^2}\frac{\lambda_0}{\omega_0}\frac{w}{h}\frac{V_0}{Z_w} = -j\frac{8}{\pi}\frac{c}{\omega_0^2}\frac{w}{h}\frac{V_0}{Z_w}.\end{aligned} \tag{4.5}$$

Hence we see the advantage of a large (but not too large) width w. As discussed in Section 2 we may not use the $\lambda/4$ for h, but something less to increase the stored energy as in (2.10). This may also increase the Q of the resonance, except for possibly other losses.

4.2 Radiation Q

The radiation is, of course in the form of a damped sinusoid. The Q of this resonance can be estimated as

$$Q = \pi N, \tag{4.6}$$
$$N = \text{ number of cycled for field to fall to } e^{-1} \text{ or energy to } e^{-2}.$$

In one cycle the energy radiated is

$$U_3 \approx P_3 T,$$
$$T = \frac{1}{f_0} \approx \frac{\lambda_0}{c} \approx 2\frac{l_a}{c} \tag{4.7}$$
$$= \text{ period of resonance.}$$

The fractional energy radiated in one cycle is

$$\Delta U = \frac{U_3}{U_1} = \frac{\pi^2}{8}\frac{P_3 T}{U_0}. \tag{4.8}$$

The fractional field lost in one cycle is

$$\Delta F \approx \frac{\Delta U}{2} = \frac{\pi^2}{16}\frac{P_3 T}{U_0}, \tag{4.9}$$

giving

$$N = \frac{1}{\Delta F} = \frac{2}{\Delta U} = \frac{16}{\pi^2}\frac{U_0}{P_3 T}, \tag{4.10}$$
$$Q = \pi N = \frac{16}{\pi}\frac{U_0}{P_3 T}.$$

4.3 Combined results for $h = \lambda/4$

Combining (4.5) with (3.16) gives

$$P_3 = \frac{1}{2}\left[\frac{1}{3} + \frac{1}{2\pi^2}\right]\frac{\mu_0 \omega_0^4}{c}\left[\frac{8}{\pi}\frac{c}{\omega_0^2}\frac{V_0}{Z_w}\right]^2 \approx 1.25\frac{V_0^2}{Z_w}. \tag{4.11}$$

This result is for $h = \lambda/4$, which is also of the order of w. So radiated power is of the general order of V_0^2/Z_w as one might expect.

Assuming the dielectric as the basic limitation with

$$V_0 \approx h E_d, \tag{4.12}$$
$$U_0 \approx \frac{\epsilon w[l_a - l_s]}{h} V_0^2. \tag{4.13}$$

the Q is then

$$Q = \pi N \approx \frac{16}{\pi}\frac{\epsilon w[l_a - l_s]}{h} V_0^2 \frac{Z_w}{1.25 V_0^2}\frac{c}{2l_a} \approx 2\frac{w[l_a - l_s]}{h l_a}. \tag{4.14}$$

This is of the general order of 2.0 (since it is assumed that $l_a \gg l_s$ and $w \approx h$; $\therefore Q \approx 2w/h = 2.0$), indicating a highly damped oscillation, for which our approximations are inaccurate. This low Q is associated with our *very fat* dipoles. This would change significantly if w were $\ll h$.

4.4 Combined results for $h \ll \lambda/4$

Using the results of (3.22) instead, we have

$$P_3 = \frac{4}{15}\frac{\mu_0\omega_0^4}{c}\left[\frac{\omega_0 h}{c}\right]^2\left[\frac{8}{\pi}\frac{c}{\omega^2}\frac{V_0}{Z_w}\right]^2 = \frac{256}{15\pi^2}\left[\frac{\omega_0 h}{c}\right]^2\frac{V_0^2}{Z_w}. \tag{4.15}$$

Now we see the power reduction by the factor $[\omega_0 h/c]^2$. We see a power radiated proportional to $h^2V_0^2$, and a field proportional to hV_0.

Again in energy as in (4.12) we have

$$Q = \pi N \approx \frac{16}{\pi}\frac{\epsilon w[l_a - l_s]}{h}V_0^2\frac{15\pi^2}{256}\left[\frac{c}{\omega_0 h}\right]^2\frac{Z_w}{V_0^2}\frac{c}{2l_a} \approx \frac{15\pi}{32}\frac{w[l_a - l_s]}{hl_a}\left[\frac{c}{\omega_0 h}\right]^2. \tag{4.16}$$

With $h \ll l_a/2$ we might make l_s small since the switch needs to hold off less voltage. This gives,

$$Q \approx \frac{15\pi}{32}\frac{w}{h}\left[\frac{c}{\omega_0 h}\right]^2 = \frac{15}{128\pi}\frac{w}{h}\left[\frac{c}{f_0 h}\right]^2 = \frac{15}{128\pi}\frac{w}{h}\left[\frac{\lambda_0}{h}\right]^2 = \frac{15}{32\pi}\frac{w}{h}\left[\frac{l_a}{h}\right]^2. \tag{4.17}$$

With

$$w \approx \frac{l_a}{2}, \tag{4.18}$$

this becomes

$$Q \approx \frac{15}{64\pi}\left[\frac{l_a}{h}\right]^3, \tag{4.19}$$

allowing one to adjust to some desired Q.

4.5 Limitations

The foregoing is for the case of air, dielectric between the antenna and the ground plane. Practically, one needs a dielectric plane with relative dielectric constant of a few to support the antenna. This will shorten l_a for a given resonance frequency, thereby lowering the efficiency of radiating into air, thereby raising the Q. There may also be some thin dielectric to coat the switch and perhaps antenna to increase the voltage standoff to $> V_0$ as desired.

5 Effect of Dielectrics

Of necessity there needs to be a substrate to support the antenna above the ground plane as in Fig. 1.1. This complicates the electromagnetic analysis, introducing surface waves guided by the dielectric over the ground plane.

These surface waves have been studied by [3–6]. It is noted that the cutoff efficiency is obtained just below the cutoff thickness of the H_0 surface-wave mode. The E_0 surface wave mode does not have a cutoff frequency, but for small h is only weakly excited by the antenna. Maximum efficiency occurs just before the H_0 surface wave mode can be excited. The optimum substrate thickness satisfies [7]

$$h_{\text{opt}} \lessapprox \frac{\lambda_0}{4}[\epsilon_r - 1]^{-1/2}. \tag{5.1}$$

Furthermore, it would appear that the presence of a superstrate may further improve matters [7].

Note that the dielectrics decrease the size of the antenna for the resonant condition. This will significantly affect the Q.

6 Skin-Effect Losses

There are losses associated with the finite conductivity of the metal. The skin depth for a good conductor is [8]

$$\delta_s = \left[\frac{2}{\omega\mu_0\sigma}\right]^{1/2} = \left[\frac{1}{\pi f \mu_0 \sigma}\right]^{1/2}. \tag{6.1}$$

This gives a surface impedance

$$\begin{aligned} Z_s &= R_s + sL_s = [1+j][\sigma\delta_s]^{-1} = [1+j]\left[\frac{\omega\mu_0}{2\sigma}\right]^{1/2}, \\ R_s &= \left[\frac{\omega\mu_0}{2\sigma}\right]^{1/2} = 2\pi\left[\frac{10^{-7}f}{\sigma}\right]^{1/2} \equiv \text{ surface resistance.} \end{aligned} \tag{6.2}$$

At a frequency, f, of 0.3 THz we have for copper

$$R_s = 0.14\ \Omega. \tag{6.3}$$

To model this loss consider the antenna over the ground plane as a transmission line with

$$\begin{aligned} \widetilde{Z}_c(s) &= \left[\frac{sL' + R'}{sC'}\right]^{1/2} \equiv \text{ characteristic impedance,} \\ \widetilde{\gamma}(s) &= [[sL' + 2R_s]sC']^{1/2} \equiv \text{ propagation constant,} \\ L' &\approx \mu_0\frac{h}{w} \equiv \text{ inductance per unit length,} \\ C' &\approx \epsilon_0\epsilon_r\frac{w}{h} \equiv \text{ capacitance per unit length,} \\ R' &\approx \frac{R_s}{w} \equiv \text{ resistance per unit length.} \end{aligned} \tag{6.4}$$

There is also a small correction to L' from L_s, but we neglect this. Note the $2R_s$ to account for losses in both the antenna and the ground plane.

As a transmission line it propagates a wave with propagation approximated as

$$\begin{aligned} \widetilde{\gamma}(s)z &= s[L'C']^{1/2}\left[1 + \frac{2R'}{sL'}\right]^{1/2} z \\ &= \frac{s}{v}\left[1 + \frac{R'}{sL'} + O(s^{-2})\right] z \\ &= \frac{s}{v}z + \frac{R'z}{Z_{c0}} + O(s^{-1}), \end{aligned} \tag{6.5}$$

for high frequencies. The first term is delay and the second term is loss with

$$Z_{c0} = \left[\frac{L'}{C'}\right] = Z_w \frac{h}{w}, \tag{6.6}$$

$$Z_w = \left[\frac{\mu_0}{\epsilon}\right]^{1/2} = \epsilon_r^{1/2} Z_0. \tag{6.7}$$

The propagating wave goes like

$$e^{-\widetilde{\gamma}(s)} \approx \exp\left[-\frac{s}{v}z\right] \exp\left[-\frac{R'}{Z_{c0}}z\right] = \exp\left[-\frac{s}{v}z\right] \exp\left[-\frac{R_s}{Z_w}\frac{z}{h}\right]. \tag{6.8}$$

One period of oscillation corresponds to a round trip of distance $2l_a$, for which

$$\exp\left[-\frac{R_s}{Z_w}\frac{z}{h}\right] \approx 1 - \frac{R_s}{Z_w}\frac{z}{h} \approx 1 - \frac{2l_a}{h}\frac{R_s}{Z_w}, \qquad \text{for } \frac{2l_a}{h}\frac{R_s}{Z_w} \ll 1. \tag{6.9}$$

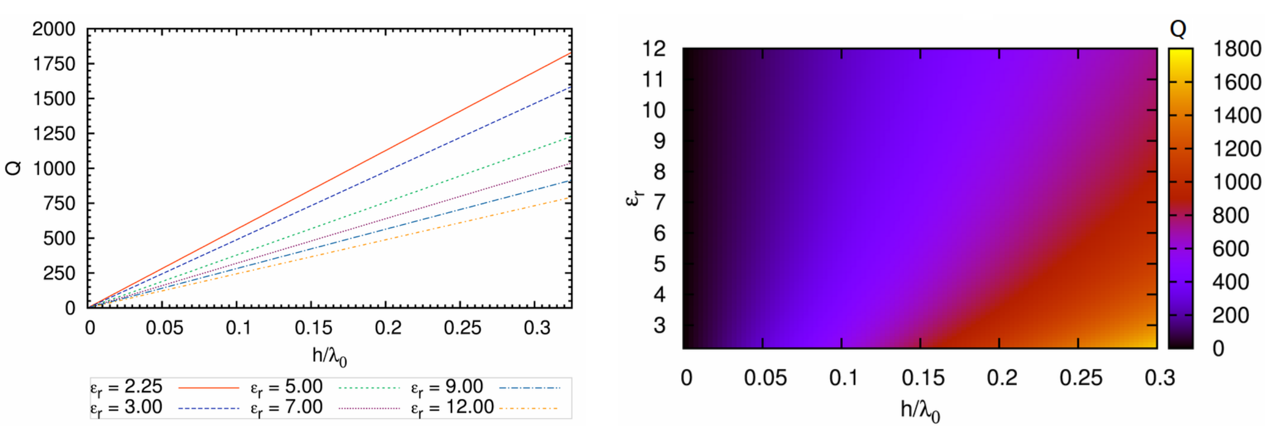
So that $\frac{2l_a}{h}\frac{R_s}{Z_w}$ is the fractional loss of field amplitude. The number of cycles to e^{-1} is then

$$N = \frac{hZ_w}{2l_a R_s} = \epsilon_r^{-1/2}\frac{h}{2l_a}\frac{Z_0}{R_s} = \epsilon_r^{-1/2}\frac{h}{\lambda_0}\frac{Z_0}{R_s}, \tag{6.10}$$

$$Q = \pi N = \pi\epsilon_r^{-1/2}\frac{h}{\lambda_0}\frac{Z_0}{R_s}. \tag{6.11}$$

Note that the switch length, l_s, is assumed negligible in this calculation.

As an example, the Q as a function of h/λ_0 and ϵ_r for $f_0 = 0.3$ THz ($R_s = 0.14\ \Omega$) is shown in Fig. 6.1. From this we can see that the copper skin losses do not appear to be a problem, as long as h/λ_0 is not too small.



(a) ϵ_r sweep. (b) Density plot of Q.

Figure 6.1: Quality factor (Q) as a function of h/λ_0 and ϵ_r at $f_0 = 0.3$ THz as per (6.11).

7 Switch Losses

The switch (1.1) has length l_s, width w, and some thickness to be determined ($\ll h$). Since we want the switch to conduct for a "long" time after the fs laser illumination, a good switch material is SI-GaAs (Cr doped) with characteristics [9]

$$\begin{aligned} 50 - 100 \text{ ps} &= \text{ carrier lifetime,} \\ 0.1 \ \frac{\text{m}^2}{\text{V s}} &= \text{ mobility,} \\ 10^4 \ \Omega \text{ m} &= \text{ resistivity,} \\ 50 \ \frac{\text{M V}}{\text{m}} &= \text{ breakdown field.} \end{aligned} \tag{7.1}$$

The switch resistance depends on the carrier density generated by the fs laser. Compared to the skin-effect resistance, the switch resistance can be somewhat larger and still achieve an acceptable Q.

Immediately we can that the carrier lifetime can be a limitation depending on the antenna Q. At 0.3 THz, the period is 3.3 ps which gives 30 cycles in 100 ps corresponding to a Q of about 94. So one may wish to choose h to get the number of cycles down to 30 or so (large h allows larger V_0).

Assume as in Fig. 1.1 that l_s is small, and can be modelled after closure by a resistance R_{sw} (time independent). As shown in Fig. 7.1 fold the antenna and ground plane.

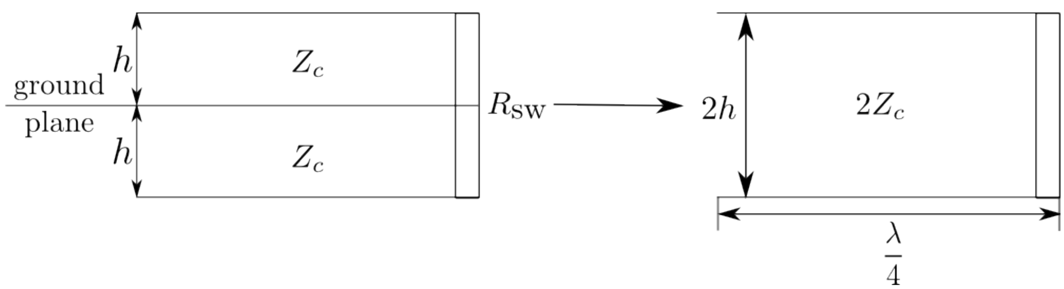


Figure 7.1: Equivalent transmission-line model of antenna with switch.

Here the reflection coefficient at R_{sw} is

$$\rho = \frac{2Z_c - R_{\text{sw}}}{2Z_c + R_{\text{sw}}} = \frac{1 - \dfrac{R_{\text{sw}}}{2Z_c}}{1 + \dfrac{R_{\text{sw}}}{2Z_c}} \approx 1 - \frac{R_{\text{sw}}}{Z_c}, \quad \text{for small } \frac{R_{\text{sw}}}{Z_c}. \tag{7.2}$$

In [10], the number of cycles to e^{-1} is

$$\begin{aligned} N &= -\frac{1}{2\ln(\rho)} \approx \frac{Z_c}{2R_{\text{sw}}}, \\ Q &= \pi N \approx \frac{\pi}{2} \frac{R_{\text{sw}}}{Z_c}. \end{aligned} \tag{7.3}$$

How small should R_{sw} be?

Choose the parameters

$$
\begin{aligned}
Z_c &\approx Z_w \frac{h}{w}, \\
h &< \frac{\lambda}{4} \text{ in dielectric}, \\
Z_w &\approx Z_0 \epsilon_r^{-1/2}, \\
\epsilon_r &\approx 3.
\end{aligned}
\tag{7.4}
$$

Let $w \approx \dfrac{\lambda}{4}, h \approx \dfrac{\lambda}{8}$,

$$
\begin{aligned}
\frac{h}{w} &\approx \frac{1}{2}, \\
2Z_c &\approx Z_w = \frac{377}{\sqrt{3}} \approx 218\ \Omega.
\end{aligned}
\tag{7.5}
$$

For $N = 30$ we have

$$
R_{\text{sw}} = \frac{Z_c}{2N} = \frac{Z_w}{4N} \approx \frac{377}{4\sqrt{3}} \frac{1}{30} \approx 1.8\ \Omega. \tag{7.6}
$$

This can be used to estimate the SI-GaAs doping and fs laser parameters.

8 Concluding Remarks

A THz pulse (damped sinusoid) radiator as in Fig. 1.1 has many design optimization questions. One needs to maximize the stored energy depending on the dielectric thickness and switch dimensions. The antenna needs to radiate most of this energy, and there is a tradeoff between amplitude and number of cycles (Q). The skin-effect losses are not significant, but the switch losses need to be quantified and made acceptably small.

As usual one can estimate the combined effect of the various previously discussed factors via

$$
\begin{aligned}
Q^{-1} &= \sum_n Q_n^{-1}\ , \quad \text{(quality factor)}, \\
N^{-1} &= \sum_n N_n^{-1}\ , \quad \text{(number of cycles to } e^{-1}\text{)}.
\end{aligned}
\tag{8.1}
$$

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