Sensor and Simulation Notes
Note 105

August 1969

Some Theoretical-Numerical Procedures for the Study of the Impedance Loaded Dipole Antenna

by

Thomas H. Shumpert
Mississippi State University
State College, Mississippi

ABSTRACT

The pulse generation of the impedance loaded dipole antenna is considered. This antenna consists of a cylindrical dipole having a finite number of lumped impedance loadings symmetrically placed at distinct points along each of its two elements. The object of the problem is to determine the radiated pulse resulting from the application of a particular voltage pulse to the driving terminals of the antenna. This is accomplished by first solving for the steady-state radiated fields and then obtaining the inverse Fourier transform for the radiated pulse by numerical techniques. Theoretical results are presented and discussed.

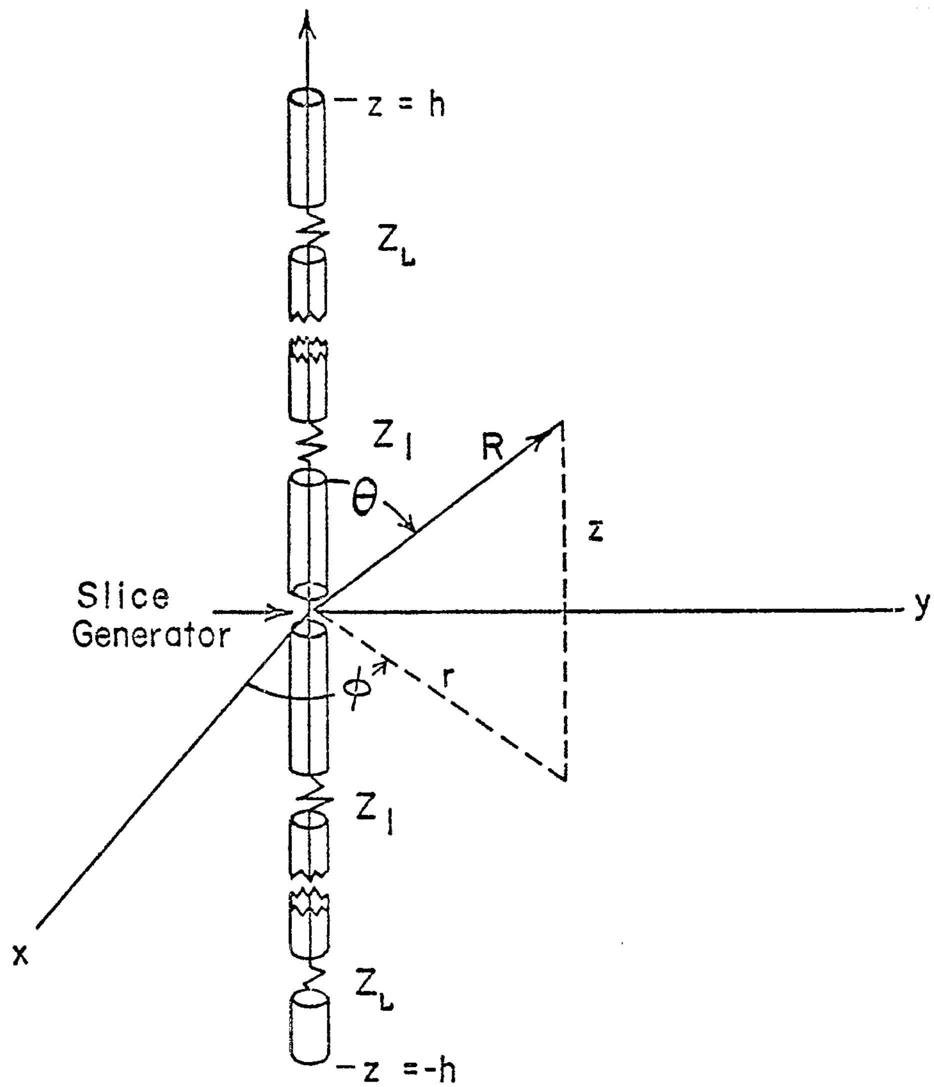

Figure 1: Impedance loaded dipole antenna with cartesian, cylindrical and spherical coordinates.

loaded with impedances Z_ℓ at points $\pm z_\ell$ along the antenna axis. If a slice generator driving mechanism is used, the integral equation solved to obtain the current distribution is Pocklington's equation (1).

$$E_i(z) = -j \frac{\eta_o}{4\pi k_o} \int_{-h}^{h} dz' \; I(z') K_a(z-z') \tag{1.1}$$

where $E_i(z)$ is the z-component of the electric field at the surface of a thin antenna produced by the current distribution I(z), and

$$K_a(z-z') = \left(\frac{\partial^2}{\partial z^2} + k_o^{\;2}\right) K(z-z') \tag{1.2}$$

$$K(z-z') = \exp\left(-jk_o \sqrt{(z-z')^2 + a^2}\right) / \sqrt{(z-z')^2 + a^2} \tag{1.3}$$

where k_o and η_o are the free space propagation constant and characteristic wave impedance respectively. The current distribution for the impedance loaded dipole may be determined by following the procedure presented by Taylor [1].

For the resistive rod

$$E_i(z) = E_t(z) - E_a(z) \tag{1.4}$$

where $E_t(z)$ is the total tangential electric field parallel to the axis of the antenna, and $E_a(z)$ is the x-component of the applied (or incident) electric field at the surface of the antenna. The total electric field may be expressed as

$$E_t(z) = z^i(z) I(z) \tag{1.5}$$

where $z^i(z)$ is the internal impedance per unit length of the antenna.

A substitution of equations (1.4) and (1.5) into equation

(1.1) yields the integral equation for the current distribution.

$$\left[\int_{-h}^{h} dz'\tilde{I}(z',\omega)K_a(z-z')\right] -j\frac{4\pi k_o}{\eta_o} z^i(z)I(z) = -j\frac{4\pi k_o}{\eta_o} E_a(z) \tag{1.6}$$

For this particular antenna driven by a slice generator and having multiple impedance loadings along its elements

$$E_a(z) = \tilde{V}_o(\omega)\ \delta(z) \tag{1.7}$$

and

$$z^i(z) = \sum_{\ell=1}^{L} Z_\ell\ \delta(|z| - z_\ell) \tag{1.8}$$

where the antenna has 2L point loadings at points $\pm z_\ell$. Z_ℓ is the ℓ^{th} impedance load at points $\pm z_\ell$. Substituting equations (1.7) and (1.8) into equation (1.6) produces

$$\int_{-h}^{h} dz'\ \tilde{I}(z',\omega)K_a(z-z') - j\frac{4\pi k_o}{\eta_o}\sum_{\ell=1}^{L} Z_\ell\ \delta(|z| - z_\ell)\tilde{I}(z,\omega) = -j\frac{4\pi k_o}{\eta_o}\tilde{V}_o(\omega)\delta(z) \tag{1.9}$$

By representing the unknown current distribution with an expansion of orthogonal functions, a solution to equation (1.9) may be determined. The current distribution on a perfectly conducting thin antenna depends on z approximately as

$$M_{oz} \equiv \sin\left(k_o(h-|z|)\right) \tag{1.10}$$

Hence let the current take the form

$$I(z) = -j\frac{4\pi\ \tilde{V}_o(\omega)}{\eta_o}\left(f(z) + C\ M_{oz}\right) \tag{1.11}$$

Then a substitution of this current into equation (1.9) yields

$$\int_{-h}^{h} dz' f(z') K_a(z-z') - j \frac{4\pi k_o}{\eta_o} \sum_{\ell=1}^{L} Z_\ell \, \delta(|z| - z_\ell) f(z) = k_o g(z) \qquad (1.12)$$

where

$$g(z) = \delta(z) + C \Big(j \frac{4\pi}{\eta_o} \sum_{\ell=1}^{L} Z_\ell \, \delta(|z| - z_\ell) \, M_{oz}$$

$$-K(z-h) - K(z+h) + 2K(z) \cos k_o h \Big) \qquad (1.13)$$

If the internal impedance is considered to be symmetric with respect to z, then the function f(z) may be represented

$$f(z) = \sum_{n=0}^{N} f_n \cos \Big(\frac{(2n+1)\pi}{2h} z \Big) \qquad (1.14)$$

and K(z-z') as

$$K(z-z') = \sum_{m=0}^{\infty} a_m \cos \Big(\frac{m\pi}{2h} (z-z') \Big) \qquad (1.15)$$

The expansion coefficients f_n may be determined by substituting (1.14) and (1.15) into (1.12) . The result is a system of linear equations

$$\sum_{n=0}^{N} f_n \Pi_{mn} = \Gamma_m \quad m=0,1,\ldots,N \qquad (1.16)$$

where

$$\Pi_{mn} = (k_o h)^2 (\epsilon_m a_{2m} + a_{2n+1}) \gamma_{mn} - j \, \alpha_{mn} \qquad (1.17)$$

$$\Gamma_m = 1 + C\big(j 2\beta_m - 2(-1)^m a_{2m} + 4c_m \cos k_o h \big) \qquad (1.18)$$

$$\alpha_{mn} = \frac{4\pi}{\eta_o} \int_{-h}^{h} dz \sum_{\ell=1}^{L} Z_\ell \delta(|z| - z_\ell) \cos \Big(\frac{(2n+1)\pi}{2h} z \Big) \cos \Big(\frac{m\pi}{h} z \Big) \qquad (1.19)$$

$$\beta_m = \frac{4\pi}{\eta_o} \int_{-h}^{h} dz \sum_{\ell=1}^{L} Z_\ell \delta(|z| - z_\ell) \sin \big(k_o(h-z) \big) \cos \Big(\frac{m\pi}{h} z \Big) \qquad (1.20)$$

$$c_m = \int_0^h d\xi \; K(\xi) \; \cos \left(\frac{m\pi}{h}\xi\right) \qquad (1.21)$$

$$\gamma_{mn} = \frac{4}{\pi} \frac{(2n+1)(-1)^{m+n}}{(2n+1)^2 - 4m^2} \qquad (1.22)$$

$$\varepsilon_m = 2 \quad \text{for } m = 0 \qquad (1.23)$$

$$= 1 \quad \text{otherwise}$$

For convenience let the constant C shown in equation (1.11) be

$$C = -4 \left[\cos k_o h \int_0^h dz \;\; \mathrm{Re}\,\{K(z)\} \right]^{-1} \qquad (1.24)$$

Then the current distribution on the antenna driven by the pulsed voltage, $\tilde{V}_o(\omega)$, where $\tilde{V}_o(\omega)$ is the Fourier transform of the voltage pulse, is of the form $\tilde{I}(z,\omega)$. This may be interpreted as the Fourier transform of the current distribution. Hence the time history may be found by

$$I(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \; \tilde{I}(z,\omega) \; e^{j\omega t} \qquad (1.25)$$

Since I(z,t) must be real, then $\tilde{I}(z,-\omega) = \tilde{I}^*(z,\omega)$

Hence

$$I(z,t) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} d\omega \;\; \mathrm{Re}\,\{\tilde{I}(z,\omega) \; e^{j\omega t}\} \qquad (1.26)$$

2 <u>Radiated Field Components</u>

If the current distribution $\tilde{I}(z,\omega)$ on a dipole antenna is known, the radiated field components may be determined readily. These components were derived by Harrison et al. (3)

$$\tilde{H}_\phi(r,z,\omega) = -\frac{1}{4\pi}\int_{-h}^{h} dz'\ \tilde{I}(z',\omega)\frac{\partial}{\partial r}K(r,z-z') \qquad (2.1)$$

$$\tilde{E}_r(r,z,\omega) = -j\frac{\eta_o}{4\pi k_o}\int_{-h}^{h} dz'\ \tilde{I}(z,\omega)\frac{\partial^2}{\partial r\partial z}K(r,z-z') \qquad (2.2)$$

$$\tilde{E}_z(r,z,\omega) = -j\frac{\eta_o}{4\pi k_o}\int_{-h}^{h} dz'\ \tilde{I}(z',\omega)\left(\frac{\partial^2}{\partial z^2} + k^2\right)K(r,z-z') \qquad (2.3)$$

where

$$K(r,z-z') = \exp\left(-jk_o\sqrt{(z-z')^2 + r^2}\right) / \sqrt{(z-z')^2 + r^2} \qquad (2.4)$$

Equations (2.1) - (2.3) are valid in both the near zone and the far zone (radiation zone).

Consider the far zone, $k_o r \gg 1$, then equation (2.1) reduces to

$$\tilde{H}_\phi(r,z,\omega) \simeq +j\frac{k_o \sin\theta}{4\pi}\frac{e^{-jk_oR}}{R}\int_{-h}^{h} dz'\ \tilde{I}(z',\omega)e^{+jk_o z'\cos\theta} \qquad (2.5)$$

where $\theta = \tan^{-1}(\frac{r}{z})$ and $R = \sqrt{r^2 + z^2}$ are the usual spherical coordinates. The electric field component in the far zone is

$$\tilde{E}_\theta(r,z,\omega) = \eta_o\tilde{H}_\phi(r,z,\omega) \qquad (2.6)$$

The evaluation of equations (2.1) - (2.3) requires that the current distribution be an accurate one. However to obtain the far zone components given by equations (2.5) and (2.6), it has been found that a crude approximation to the current distribution on the antenna is sufficient to obtain reasonably accurate radiation zone field components.

3 Pulse Radiation

From equations (2.1) - (2.3) it is apparent that the resulting components are given in the frequency domain. The field components of the radiated pulse are desired in the real time domain, therefore taking the inverse Fourier transform yields

$$H_\phi(r,z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \, \tilde{H}_\phi(r,z,\omega) \, e^{j\omega t} \tag{3.1}$$

$$E_r(r,z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \, \tilde{E}_r(r,z,\omega) \, e^{j\omega t} \tag{3.2}$$

$$E_z(r,z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \, \tilde{E}_z(r,z,\omega) \, e^{j\omega t} \tag{3.3}$$

where $\tilde{H}_\phi$, $\tilde{E}_r$, and $\tilde{E}_z$ were determined from equations (2.1) - (2.3) using $\tilde{I}(z,\omega)$ which was obtained by solving the integral equation (1.6) where $\tilde{V}_o(\omega)$ is the Fourier transform of the exciting voltage pulse.

4 Numerical Results

For convenience the antenna is considered to be charged slowly (time constant T_2). Analytically this voltage pulse may be expressed as

$$V_o(t) = \begin{cases} V_o e^{T_1 t} & t<0 \\ V_o e^{-T_2 t} & t>0 \end{cases} \tag{4.1}$$

and for $T_1 \ll 1$

$$\tilde{V}_o(\omega) = \frac{V_o}{\sqrt{2\pi}}\left[\frac{T_2}{\omega^2 + T_2^{\,2}} + j\,\frac{T_2^{\,2}}{\omega(\omega^2 + T_2^{\,2})}\right] \qquad (4.2)$$

The impedance loadings of the "long-wire" antenna of Sandia Laboratory are used in the numerical computations. These are shown in Table 1.

The solution of equation (1.6) is obtained using the representation for $\tilde{I}(z,\omega)$ given in equation (1.11) and (1.14) where N = 15 and C is chosen as in Section 2.1 . Then $\tilde{I}(z,\omega)$ is substituted into equations (2.1) - (2.3) to compute the steady-state values for the field components which are used in (3.1) - (3.3) to compute the radiated pulse. Because of the smoothness of the functions that are to be integrated, it is most convenient to use the Simpson's Quadrature formula. However the inverse Fourier transforms (3.1) - (3.3) require a special technique. In Figures (2a) and (2b) plots of $\tilde{E}_z e^{jkR}/k_o\tilde{V}_o(\omega)$ and $2\pi r\ \tilde{H}_\phi e^{jkR}/\tilde{V}_o(\omega)$, respectively, versus kh show generally slowly varying functions of frequency. Because of this, in the evaluation of the inverse Fourier transforms the ranges of integration are divided into segments where $\tilde{E}_z e^{jkR}/k_o\tilde{V}_o(\omega)$ and $2\pi r\ \tilde{H}_\phi e^{jkR}/\tilde{V}_o(\omega)$ are approximated by straight lines and the integrals over the same segments then are evaluated analytically. This technique is found to be much superior to a "brute force" method such as using Simpson's rule. Because of the ease in obtaining the far zone field components, the far electric field is investigated and is compared to

Table 1: Impedance Loadings

Position, z_ℓ	Impedance Loading, Z_ℓ (in ohms)
0.04 h	6.0
0.08 h	9.0
0.12 h	10.5
0.16 h	12.0
0.20 h	15.0
0.24 h	21.0
0.28 h	29.0
0.32 h	32.0
0.36 h	43.0
0.40 h	44.0
0.44 h	44.0
0.48 h	49.0
0.52 h	54.0
0.56 h	71.0
0.60 h	71.0
0.64 h	71.0
0.68 h	92.0
0.72 h	100.0
0.76 h	105.0
0.80 h	120.0
0.84 h	125.0
0.88 h	150.0
0.92 h	205.0
0.96 h	250.0

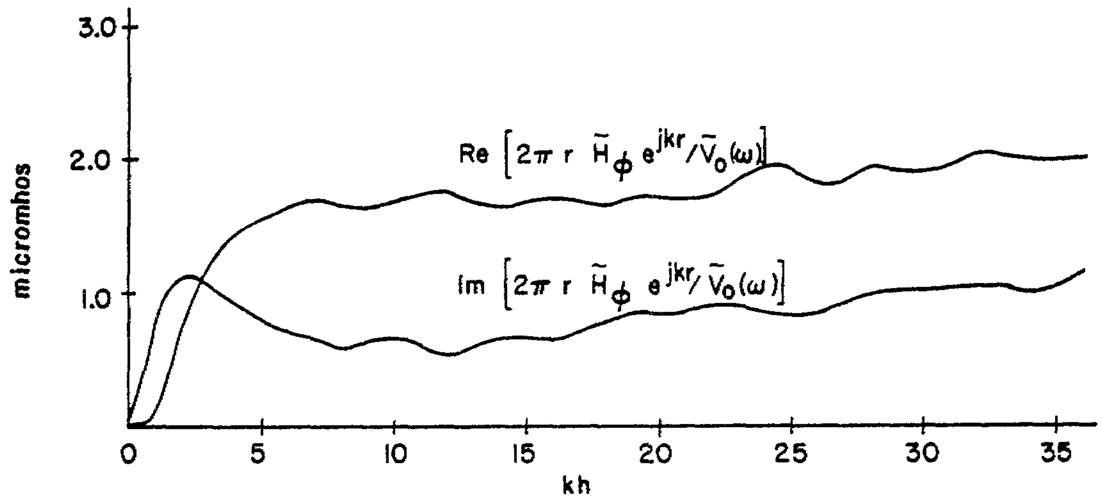

Figure 2a: Real and imaginary components of the magnetic field versus electric half-length of the impedance loaded dipole. Here r=h, z=o.

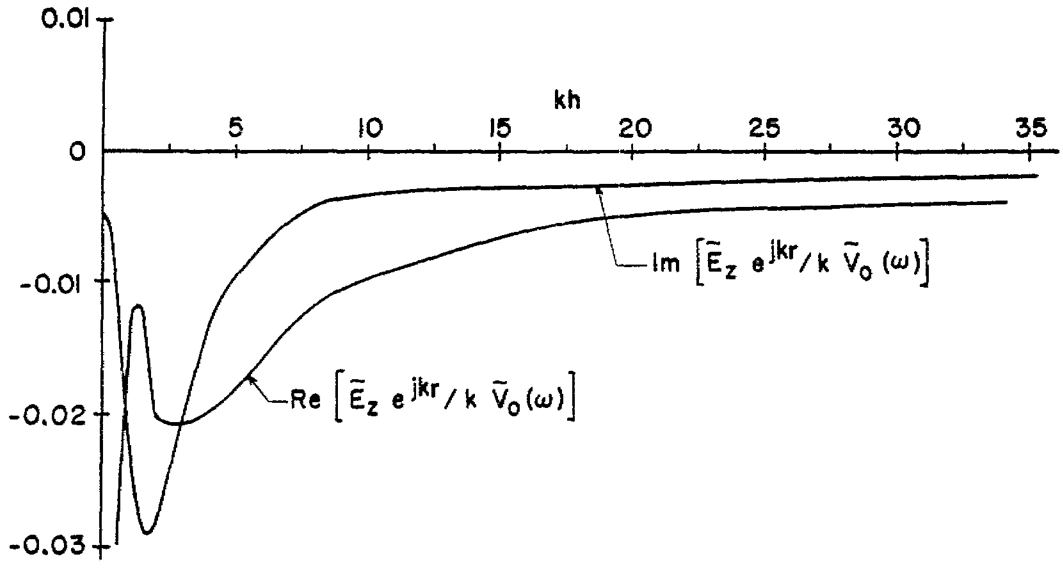

Figure 2b: Real and imaginary components of the electric field versus electrical half-length of the impedance loaded dipole. Here r = h, z = o.

the near zone predictions in the hope that the far zone approximation may be used to obtain at least qualitative predictions of the near zone fields.

The foregoing field components are used to obtain the time histories in Figures (3a) and (3b). Figure (3c) shows the time history of the far field approximation. Two interesting observations are that the "rise-times" for all three components are about the same which indicates qualitative validity of the far field approximation and the wave impedance (ratio of electric field to magnetic field) is near 300 ohms for the pulse duration. In obtaining these data, the parameters $V_o = 1$ and $h = 500$ ft. were used. Note that the time in the foregoing is actually the retarded time.

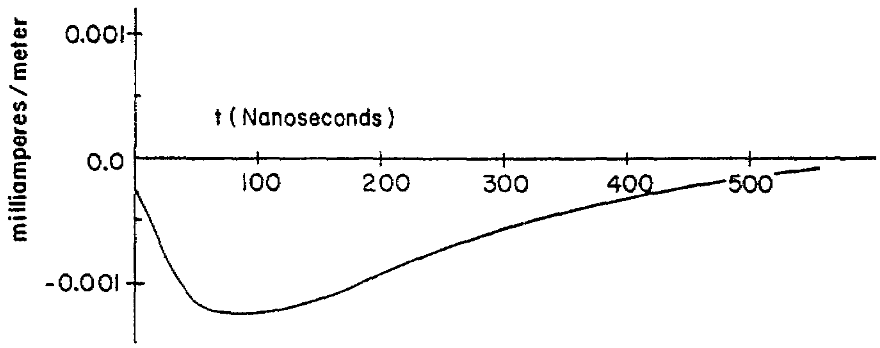

Figure 3a: Time history of the magnetic field $H_{\phi}(r, z, t)$ for $T_2 = 2 \times 10^7 \text{ sec}^{-1}$, $r = h$, $z = o$.

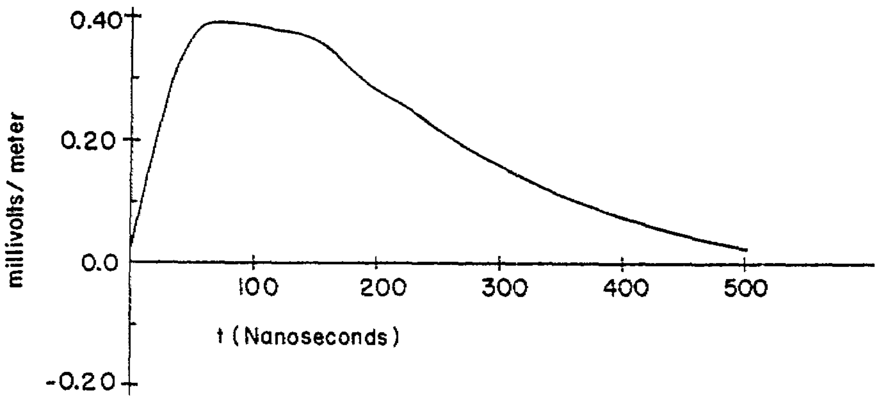

Figure 3b: Time history of the electric field $E_z(r, z, t)$ for $T_2 = 2 \times 10^7 \text{ sec}^{-1}$, $r = h$, $z = o$.

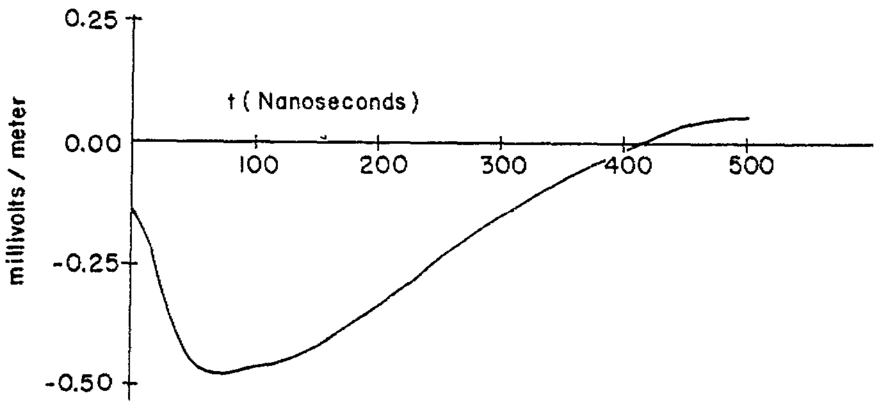

Figure 3c: Time history of the far field approximation to the electric field $E_{\theta}(r, z, t)$ for $T_2 = 2 \times 10^7 \text{ sec}^{-1}$, $r = h$, $z = o$.

LIST OF REFERENCES

1. C. D. Taylor, "Cylindrical Transmitting Antenna: Tapered Resistivity and Multiple Impedance Loadings," IEEE Trans. Ant. Prop., Vol. AP-16, No. 2, pp. 176-79, March 1968.

2. S. A. Schelkunoff, <u>Advanced Antenna Theory</u>, New York: Wiley, 1952, p. 131.

3. C. W. Harrison, Jr., C. D. Taylor, E. A. Aronson, and M. L. Houston, "An Accurate Representation of the Complete Electromagnetic Field in the Vicinity of a Base-Driven Cylindrical Monopole," Sandia Corporation Report SC-R-68-1965, January 1968.