NOTE 13

ELECTRIC FIELD AND CURRENT DENSITY MEASUREMENTS IN MEDIA OF CONSTANT CONDUCTIVITY

by

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Sensor and Simulation Notes XIII

Electric Field and Current Density Measurements in Media of Constant Conductivity

I. Introduction

The problem of measuring electric fields in air, in the presence of rapidly varying radiation and conductivity, leads one to use a sensor with a load impedance much larger than the sensor impedance so that the electric field or potential is being sampled but not loaded. However, measurement of electric field below the ground plane where the characteristics of the medium are vastly different changes the requirements for an electric field sensor. If such a medium, e.g., soil, salt water, etc., can be considered to have a conductivity (and dielectric constant) which is not significantly changed by the ionizing radiation or the electric field present, then the restriction of an infinite load impedance can be removed.

The general technique which this note will discuss involves removing the conductance from a certain portion of this medium and transferring it to the load impedance, eliminating the need for isolating electronics. In effect, this electric field sensor will be matched to the conducting medium in which it is located, the sensor having the same bulk parameters as the equivalent volume of the medium. Finally, this sensor will be generalized to the extent that the sensor parameters are different from those of the medium but the frequency response is still flat.

II. Equivalent Impedance Electric Field or Current Density Probe

To develop this particular kind of electric field probe consider the medium with electric field, E, and current density, J, given in figure 1. The medium is described by permittivity, ϵ_1, permeability, μ_1, and conductivity, σ_1. The electric field and current density related by

$$J = \sigma_1 E \tag{1}$$

will be taken as the components in a given direction for which a measurement is desired. This will be considered independently from any other vector components of these quantities which may be present.

If a restriction is placed that all distances of concern to the sensor shall be less than any distances over which this electric field is changing, then over a restricted volume of the medium equipotential planes can be constructed perpendicular to the electric field. Quantitatively, if the electric field is considered in its radian frequency components, ω, as being of the form $e^{j(\omega t - kz)}$, where the z-axis is taken as the direction of propagation and the propagation constant, k, is

$$k = (\omega^2 \mu_1 \epsilon_1 - j\omega\mu_1\sigma_1)^{1/2} \tag{2}$$

then the distance, (Δl), over which the phase changes by one radian, is just the wave number, i.e., the reciprocal of the real part of the propagation constant. Thus,

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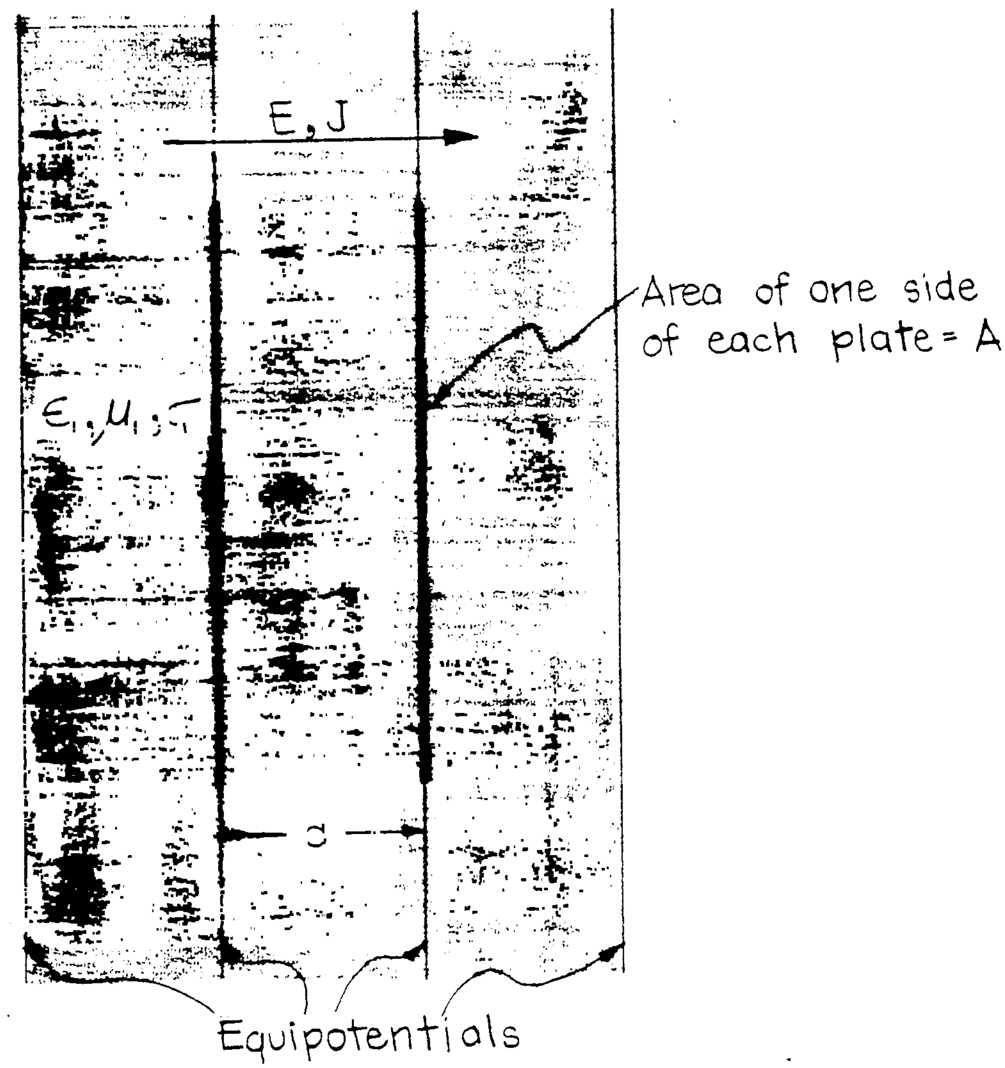

Fig. 1 Electric Field and Current in Medium

$$\Delta l = \left(Re\left[(\omega^2\mu_1\epsilon_1 - j\omega\mu_1\sigma_1)^{1/2}\right]\right)^{-1} \qquad (3)$$

or

$$\Delta l = \frac{1}{\omega\sqrt{\mu_1\epsilon_1}}\ \frac{1}{Re\left[(1 - j\frac{\sigma_1}{\omega\epsilon_1})^{1/2}\right]} \qquad (4)$$

For $\frac{\sigma_1}{\omega\epsilon_1} >> 1.0$ this reduces to

$$\Delta l \simeq \sqrt{\frac{2}{\omega\mu_1\sigma_1}} \qquad (5)$$

which is the familiar skin depth equation. Given a certain Δl characterized by the sensor dimensions then from equation (4) a frequency can be calculated which will be the upper frequency limit for which the following approximations will hold.

For frequencies below this limit equipotential planes can be considered as in figure 1. Along any equipotential surface a conductor can be placed (assuming good electrical contact to the medium) without disturbing either the electric field or the current density. For the development of a sensor consider two identical conducting plates, each of area, A, (one side), and placed on equipotential planes of spacing d, such that any line of field or current which passes through one plate passes through the other plate. This defines a cylindrical volume of cross sectional area, A, and height, d. Measurement of the potential difference between these two plates gives both the electric field and the current density. If no current is drawn from these plates the potential difference, V, will be just Ed.

Now that this cylindrical volume is defined one can try to replace it with lumped electrical elements which have the same electrical characteristics in the medium. In this case as far as the electric field and current density are "concerned" the medium is not disturbed and thus the fields and equipotentials are not disturbed either. Figure 2 illustrates how this is done. Consider first the conduction current, I, through this volume

$$I = JA = \sigma_1 EA \qquad (6)$$

where A, as previously defined, is the cross sectional area of this cylindrical volume, but since

$$V = Ed \qquad (7)$$

where d is the cylindrical height, equal to the plate spacing, the current through this volume can be carried by a conductance, G_m, given by

$$G_m = \frac{I}{V} = \sigma_1\frac{A}{d} \qquad (8)$$

Next, consider the displacement current, I_D, through this volume

$$I_D = \epsilon_1 \frac{\partial E}{\partial t} A, \qquad (9)$$

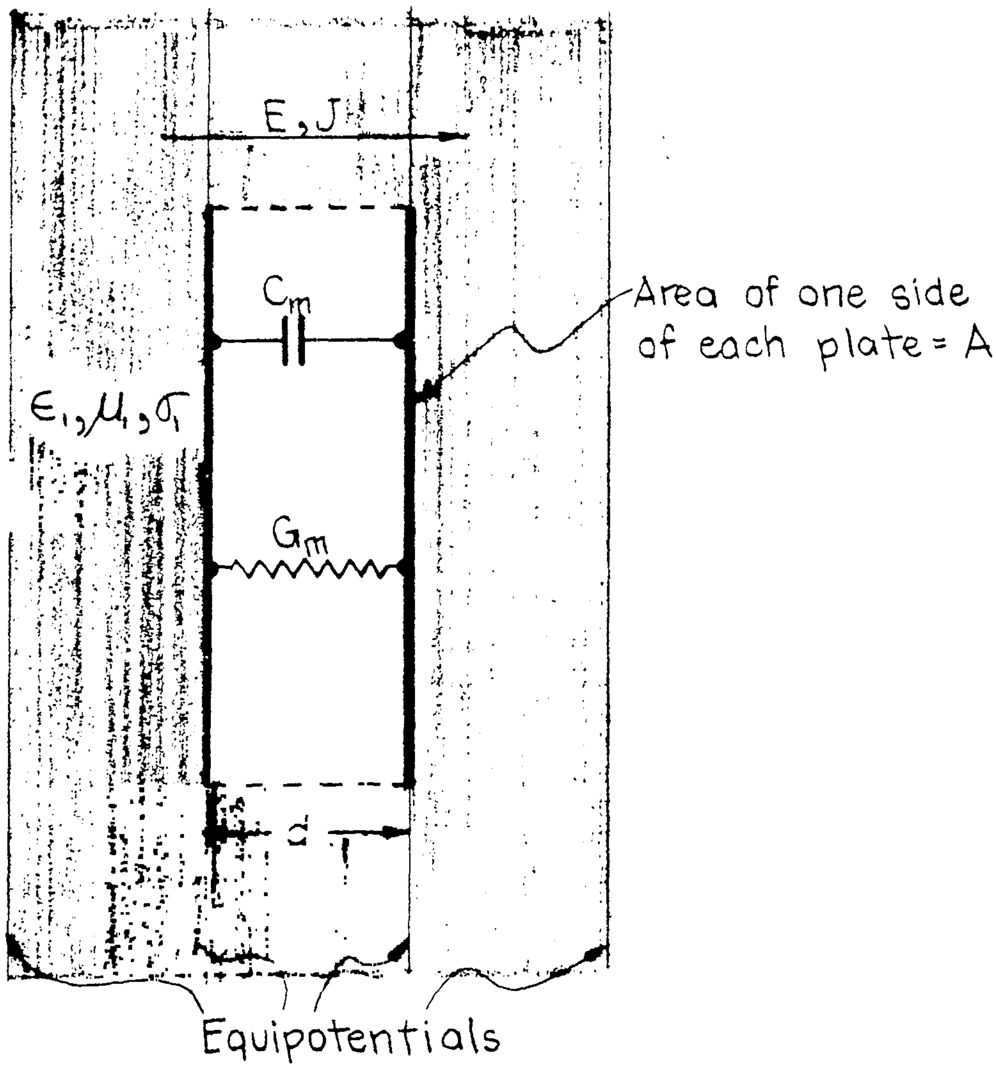

Fig. 2 Replacement of Volume in Medium by Lumped Constants

but since

$$\frac{\partial V}{\partial t} = d\frac{\partial E}{\partial t} \tag{10}$$

the displacement current can be carried by a capacitance, C_m, given by

$$C_m = I_D/(\frac{\partial V}{\partial t}) = \varepsilon_1 \frac{A}{d} \tag{11}$$

This volume of medium can then be replaced by two conducting plates at the ends of the cylinder to properly terminate the field lines and connected by two lumped circuit elements, a conductance and a capacitance, associated with the conductivity and permittivity of the medium. Below the upper frequency limit implied in equation (4) it is not necessary to include any lumped inductance.

Suppose now that this lumped conductance is greater than or equal to the reciprocal of the differential impedance, Z, of a particular twinax cable. Then the twinax cable can be directly connected to the two conducting plates and an appropriate conductance subtracted from G_m. This is illustrated in figure 3 for the case

$$G_m = \frac{1}{Z} \tag{12}$$

in which the sensor dimensions have been chosen to match the conductance of the medium to the cable impedance (Z). The capacitance of the medium has also been matched to that of the sensor by using a non-conducting dielectric (permittivity $\varepsilon_2 \leq \varepsilon_1$) and by extending inward the conducting walls such that (neglecting corrections near the edges)

$$\frac{w}{d} = \frac{\varepsilon_2}{\varepsilon_1} \tag{13}$$

However, there are many other methods of matching the capacitance. Ideally, the twinax cable also lies along an equipotential plane midway between the two plates and/or uses an isolation technique such as supression of the net currents on the cable inductively (to be described in another SSN).

To illustrate what such a sensor might look like assume that the medium is characteristic of the Nevada Test Site where

$$\begin{aligned} \varepsilon_1 &\simeq 16\ \varepsilon_0 \\ \sigma_1 &\simeq .02 \text{ mho/meter} \end{aligned} \tag{14}$$

Using the sensor design of figure 3 assume that

$$Z = 100\ \Omega$$

$$\varepsilon_2 = 2.26\ \varepsilon_0 \text{ (polyethylene)} \tag{15}$$

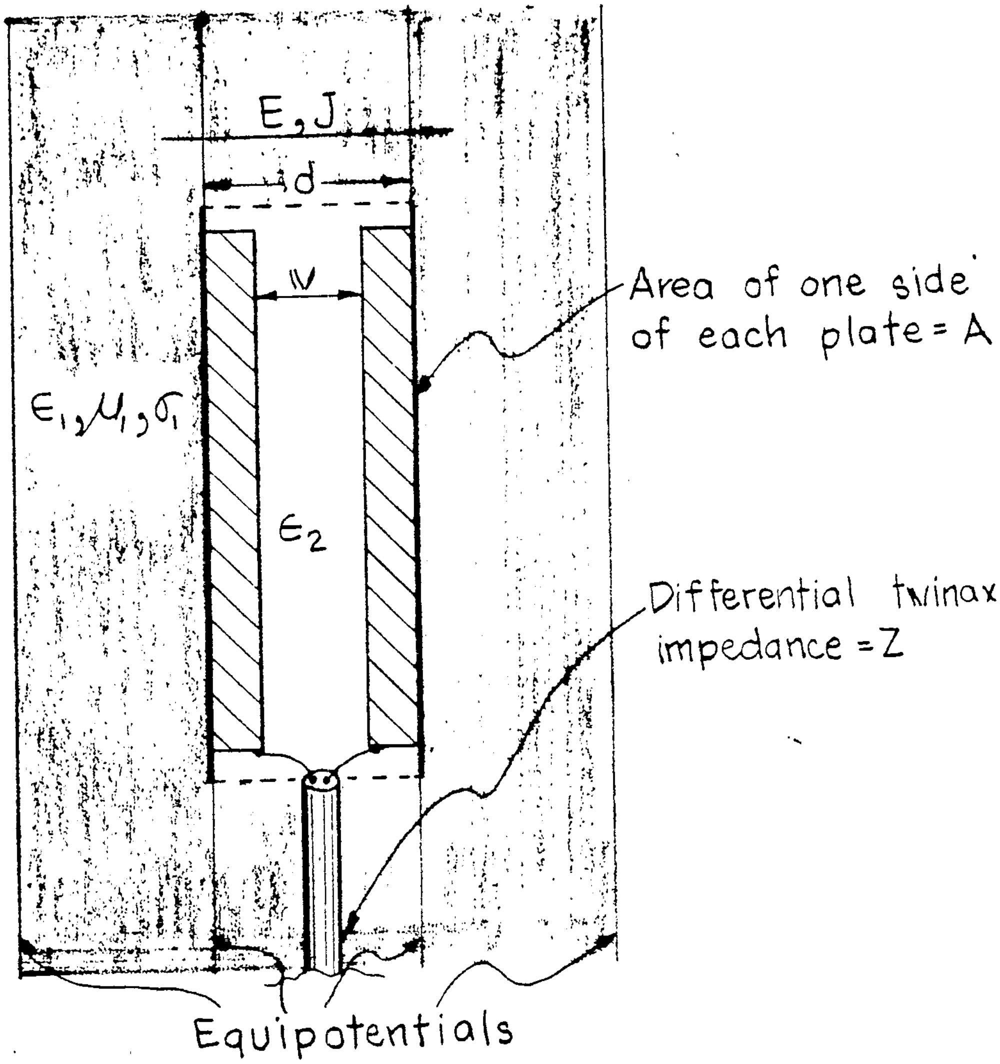

Fig. 3 Electric Field or Current Sensor Matched to Medium.

Then combining equations (8) and (12)

$$\frac{A}{d} = \frac{1}{Z\sigma_1} = 0.5 \text{ meter} \tag{16}$$

If d is taken as

$$d = .1 \text{ meter} = 10 \text{ cm} \tag{17}$$

and the conducting plates are assumed to be circular disks of radius, r, then

$$A = \pi r^2 = .05 \text{ m}^2 = 500 \text{ cm}^2 \tag{18}$$

and

$$r = .126 \text{ m} = 12.6 \text{ cm} \tag{19}$$

Finally, from equation (13)

$$\frac{w}{d} = .141 \tag{20}$$

and

$$w = .0141 \text{ meter} = 1.41 \text{ cm} \tag{21}$$

This set of dimensions can easily be obtained in a practical sensor design.

A quite different case to consider is a medium consisting of sea water in which

$$\epsilon_1 \approx 80\, \epsilon_o$$

$$\sigma_1 \approx 5.3 \text{ mhos/meter} \quad (25°C) \tag{22}$$

Here the conductivity is so high that if equation (12) is used to calculate the required conductance of the sensor then A/d will be extremely small. In fact, σ_1 is so large that if A/d is taken to be

$$\frac{A}{d} = 0.5 \text{ meter} \tag{23}$$

as in the previous example, then from equation (8)

$$G_m = 2.65 \text{ mho} \tag{24}$$

while

$$\frac{1}{Z} = .01 \text{ mho} \tag{25}$$

Therefore, returning to figure 1, if the cable is attached directly to the two plates, the change in the conductance between these plates is only of the order of 0.5%, a negligible perturbation. In such a case then this much simplier procedure is quite adequate. It is therefore possible to match an electric field sensor to a conducting medium which does not have a time or field dependent conductivity by replacing a cylindrical volume of the medium by equivalent lumped parameters. Such a sensor can directly drive a terminated cable by including the cable impedance in these lumped parameters. Thus, no active electronic devices are needed at the sensor.

III. Sensor Calibration

In this scheme of sensor design, the sensor is matched to the medium. Therefore, variation in the conductivity and permittivity of the medium in which a measurement is intended to be made must be considered in sensor design. One way to get around this problem is illustrated in figure 4. In this construction the sensor parameters are matched to the medium by varying the sensor conductance and capacitance through the adjustment of a trimmer conductance, G_t, and a trimmer capacitance, C_t, such that

$$G_m = \sigma_1 \frac{A}{d} = \frac{1}{Z} + G_t \tag{26}$$

and

$$C_m = \varepsilon_1 \frac{A}{d} = \varepsilon_2 \frac{A}{w} + C_t \tag{27}$$

The sensor would thus be constructed so that $\frac{1}{Z}$ was less than the minimum expected value of G_m, and so that $\varepsilon_2 A/w$ was less than the minimum expected value of C_m. Provision would need to be made for adjusting these trimmers while the sensor was in the medium, ideally in the intended measurement location. If the medium were soil and the sensor was not too far from the surface this might be done mechanically by non-conducting rods turning in non-conducting tubes leading from the sensor to the surface.

Another, more elegant, calibration scheme (also in figure 4) may consist of an extra pair of planar electrodes, significantly larger than and parallel to the sensor electrodes so that these additional electrodes are also on equipotentials. Then by applying a voltage, V', to these outer electrodes (separated by d') one can read a voltage V across the twinax leads at the recorder location. If the sensor is matched to the medium then for all frequencies of interest

$$V = \frac{d}{d'} V' \tag{28}$$

By adjusting G_t at low frequencies and C_t at high frequencies the requirement of equation (28) should be met.

After performing this calibration procedure V' could then be removed. Since the calibration electrodes are also on equipotentials (for the field component of interest) they will not disturb this field and can be left in place. However, for a medium such as soil there may be problems arising from the electrical contact between the soil and both the sensor electrodes and the calibration electrodes. If the contact resistance between the soil and the sensor electrodes is significant compared to

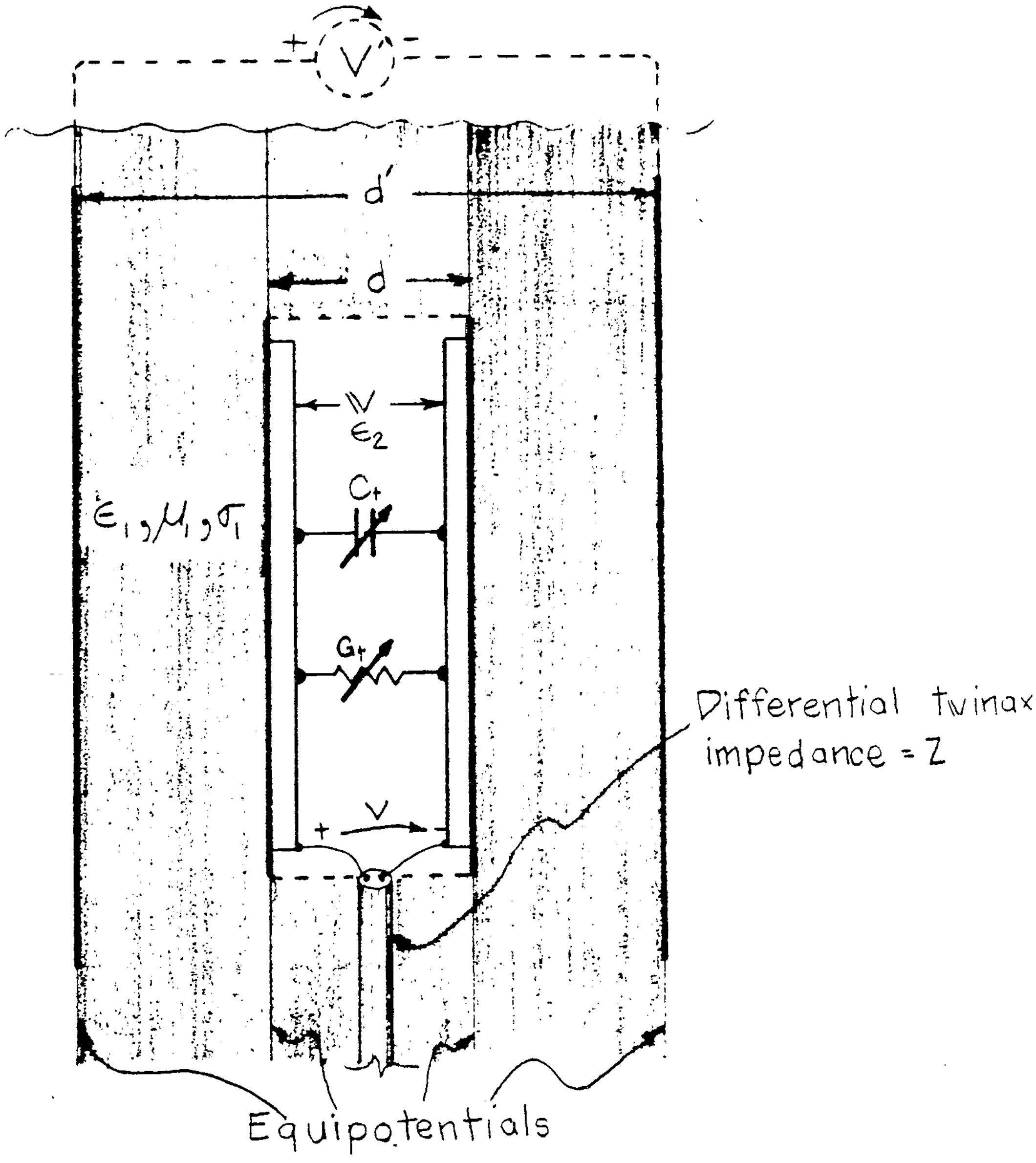

Fig. 4 Calibration System for Matching Sensor to Medium.

$1/G_m$ then the sensor will not be matched to the medium and the fields will be distorted. Similar problems result from contact resistance associated with the calibration electrodes. In the case of the sensor electrodes this problem may be alleviated by perforating the electrodes and placing a blotter on the side of the electrodes opposite the soil. A small tank of a chosen salt solution can keep the blotter and thus the interface moistened with a highly conducting solution. Similar techniques may be used for the calibration electrodes but this may be more difficult. To avoid unnecessary scattering of other field components it may be desirable to eliminate these calibration electrodes, remove them after calibration, or segment them so that they can be broken into smaller electrical units which can be disconnected from one another at will.

By designing a sensor with provisions for such calibration procedures as outlined in this section, greater flexibility will be achieved. Then these features can be used as desired.

IV. Generalization of Sensor

Thus far one specific case has been considered on the basis of replacing a volume of a dissipative medium by a sensor with the same lumped parameters as the medium. However, it is possible to construct an electric field probe of this same general design in which the lumped elements do not match those of the medium but in which the sensor response is still frequency independent. Again in figure 1 consider the two sensor electrodes in the medium of interest. With no loads attached to these plates the open circuit voltage, V, is just Ed as before. Setting the external electric field to zero one can measure the admittance between the two plates giving a conductance, G_m, as before, plus a fringing conductance, G_f, from current along paths not directly between the plates, and a capacitance, C_m, as before, plus a fringing capacitance, C_f, from electric field lines along paths not directly between the plates. Then by Thevenin's Theorem this configuration has an equivalent circuit of a voltage source, V, in series with this admittance just described. This is exactly the left portion (before the first dotted line) of the equivalent circuit shown in figure 5.

Now one can proceed to "add" various loads to the sensor. First "add" a negative conductance, $-G_m$, and a negative capacitance, $-C_m$. This is essentially equivalent to removing the lumped elements shown in figure 2 and is illustrated by the negative elements between the two dotted lines in figure 5. Finally, add the lumped sensor elements from figure 4 defined as

$$G_s = \frac{1}{Z} + G_t \tag{29}$$

and

$$C_s = \frac{\varepsilon_2 A}{w} + C_t \tag{30}$$

where the symbols mean the same as those in equations (11),(12), and (13). G_s and C_s are illustrated by the positive elements after the last dotted line (on the right) of figure 5, and the equivalent circuit of this sensor is complete.

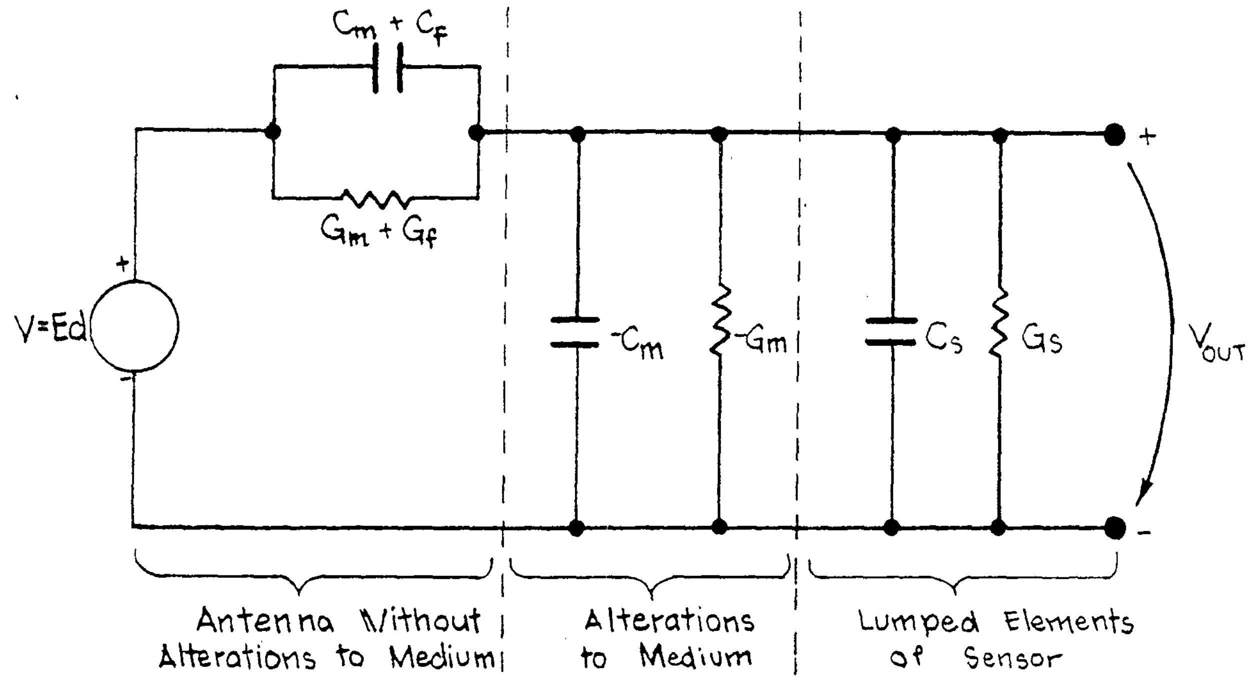

Fig. 5 Equivalent Circuit for Parallel Plate Sensor

Now, if one equates

$$G_s = G_m \tag{31}$$

and

$$C_s = C_m \tag{32}$$

then the admittance in parallel with V_{out} is identically zero and therefore

$$V_{out} = V = Ed \tag{33}$$

However, the procedure of equations (31) and (32) is precisely what has been developed in Section II in matching the sensor to the medium. In this development the same results were obtained as in equation (33), thus showing the consistency of the equivalent circuit of figure 5 in this case.

Now consider the elements of the equivalent circuit in a more general case where the restrictions of equations (31) and (32) are not assumed. Then in the limit of zero frequency any capacitance will be unimportant and one can define a voltage transfer ratio, t_{v_0}, as

$$t_{v_0} = \frac{V_{out}}{V} = \frac{G_m + G_f}{(G_m + G_f) + (G_s - G_m)} \tag{34}$$

or

$$t_{v_0} = \frac{G_m + G_f}{G_s + G_f} \tag{35}$$

For high frequencies such that conductances are unimportant compared to the capacitances then one can define another voltage transfer ratio, t_{v_1}, as

$$t_{v_1} = \frac{V_{out}}{V_{in}} = \frac{C_m + C_f}{(C_m + C_f) + (C_s - C_m)} \tag{36}$$

or

$$t_{v_1} = \frac{C_m + C_f}{C_s + C_f} \tag{37}$$

For flat frequency response it is required that

$$t_{v_0} = t_{v_1} \tag{38}$$

This is equivalent to equating the time constants of the sensor admittance (the elements to the left of the left dotted line) and the sensor load (all the rest). Thus, under the restriction of equation (38) it is possible to trim both C_s and G_s so that the voltage transfer ratio is greater or less than 1.0, as is desired within the limits imposed by the possible variation of these two parameters.

It is also possible to distort the geometry of the sensor plates and obtain similar results using the steps outlined in this section, although the computations will be more difficult. In either case in which the sensor is not matched to the medium, the calibration as in Section III becomes more difficult because the calibration plates have to be farther apart and larger to avoid the fringing fields from the sensor.

V. Summary

It is possible to construct a sensor for measuring a component of the electric field (or current density) in a dissipative medium without the need for active electronics. This sensor can be made to match both the conductivity and the permittivity of such a medium, assuming that these parameters are independent of frequency in their range of importance and that they are not changed significantly by the expected field strength or ionizing radiation. However, if other reasons dictate, this sensor need not match the parameters of the medium but may still have a response independent of frequency.

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