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On the Behavior of Thin-Wire Scatterers and Antennas Arbitrarily Located Within a Parallel Plate Region, I (The Formulation)

by

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Abstract

In this note, an integro-differential equation for the current on a thin wire arbitrarily positioned between two parallel plates is formulated. Both the scattering and active antenna problems are considered, and the numerical techniques for solving the equation is presented. Typical results of the input current in the frequency domain are given, but the time domain results and an adequate parameter study are not included. These will be presented in a future note after the computations are completed.

Acknowledgement

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I. Introduction

Through the use of recently developed numerical techniques, many problems that are difficult to treat classically may be analyzed. The problem of determining the behavior of a radiating antenna or a scattering element within a parallel plate region is one example. By formulating an integral equation for the current distribution on the antenna or scattering element, the method of moments may then be employed to obtain the solution. The effect of the parallel plates is taken into account by using the appropriate Green's function for the region.

Aside from the academic interest in this problem, the results of such an analysis are helpful in the design and evaluation of EMP (electromagnetic pulse) simulators. The parallel plates are a useful device for achieving relatively large electromagnetic field strengths with conventional excitation techniques, but some interaction between the test obstacle and the plates is to be expected. The results of this analysis will aid in evaluating this interaction and provide guidelines for future simulator development.

Previous investigators have considered some special cases of this problem. Rao[1] has treated the case of a driven antenna extending perpendicularly across the parallel plate region in such a way that it is shorted to the plate at one end, and driven by a delta gap against the other plate. His results, which include plots of the input admittance of the antenna as a function of the plate separation, show a discontinuity as the separation passes through the value $kh = \pi$, but the actual effect of the presence of the plates is somewhat masked by the resonate behavior of the antenna itself which occurs at roughly the same frequency.

Taylor[2] has considered a symmetrically located thin receiving antenna (scattering element) which again is perpendicular to the two plates. Current at the center of the antenna is plotted as the plate separation is varied, but no noticeable discontinuity or singularity in the current is observed as the plate separation passes through the resonate values of $kd = \pi$, 2π, 3π, etc.

Recently, Scheer and Neureuther[3] have considered an infinite array of parallel antennas which are inclined at some angle with respect to the axis where the antenna centers are located. While this is not exactly the same as the present problem under consideration (see Figure 1), the special case of the antennas being co-linear is the same as that of an antenna being perpendicular to the two parallel plates.

In a recent note by Lee and Latham[7] the scattering from a cylinder in a parallel plate region was studied in both the frequency and time domain. As in the previous cases, the obstacle was perpendicular to the plates and no resonant effects were noted as the plate separation varied.

The analysis and results presented in this note are for an antenna or scattering body at an arbitrary inclination angle within the parallel plate region. For this problem, some interesting interaction effects between the plates and the thin wire are observed, especially for the wire being parallel to the plates so that the coupling is greatest. As the wire rotates and becomes nearly perpendicular to the plates, these effects become much weaker.

In this note, the main emphasis is in studying the radiating and scattering properties of a thin cylinder within the parallel plates. Special attention is paid to the formulation of the appropriate integral equation for the unknown current on the cylinder, and also to the numerical technique for solving the equation. Some typical results in the frequency domain are presented, but neither the time response nor an adequate parameter study is included. These will be presented in a future note as soon as the computations have been completed.

II. Formulation and Solution

To develop an integral equation for the antenna current, consider the geometry of the problem as shown in Figure 1a. The wire antenna of total length L and radius a is located with its center a distance h above one plane. The antenna is inclined at some angle θ with the $\hat{z}$ axis and the second plate is at a distance d from the first. A small voltage source of V volts is assumed to be the driving source for the antenna problem, and for the scattering problem, a TEM wave is assumed for the incident field as in Figure 2.

Through the use of image theory, the parallel planes may be replaced by two infinite sets of images. One set of images remains parallel to the original antenna, while the second set is at an angle 2θ with respect to the antenna. The phasing of the sources of the parallel images is the same as the primary source, but that of the non-parallel image is 180 degrees out of phase with the primary source.

Notice that since the image currents flow in more than one direction, it is necessary to have more than one component of the vector potential $\overline{A}$ to describe the fields in this problem. Thus, the relatively straightforward formulation for the Hallén type integral equations used in Refs. 1, 2, and 3 is not sufficient here. For the present case, a Hallén type integral equation similar to that described by Mei[4] for curved wires must be employed if it is desired to treat the problem in this manner.

On the other hand, an integro-differential equation of the Pocklington form can be developed, much in the same manner as done by Harrington[5] for the arbitrary thin-wire antenna. This latter approach has been taken for the present problem.

Consider for the moment, the electric field at an observation point, $\overline{r}_o$, produced by one of the images, say the n^{th} image. This is expressible in terms of the vector and scalar potentials arising from the n^{th} image as

$$\overline{E}_n(\overline{r}_o) = - j\omega\mu\overline{A}_n(\overline{r}_o) - \nabla_o\phi_n(\overline{r}_o) \tag{1}$$

where

$$\overline{A}_n(\overline{r}_o) = \int_{n^{th}\ \text{image}} I_n(\xi_n')\hat{\xi}_n'(\overline{r}_s)G(\overline{r}_o,\overline{r}_s)d\xi_n' \tag{2}$$

and

$$\phi_n(\overline{r}_o)=\frac{1}{\varepsilon}\int_{n^{th}\text{ image}} \rho_n(\xi_n')G(\overline{r}_o,\overline{r}_s)d\xi_n'. \tag{3}$$

Here $\hat{\xi}_n'(\overline{r}_s)$ is the unit vector tangent to the n^{th} image at the point given by $\overline{r}_s$. $G(\overline{r}_o,\overline{r}_s)$ is the free space Green's function denoted by

$$G(\overline{r}_o,\overline{r}_s) = \frac{e^{-jk|\overline{r}_o-\overline{r}_s|}}{4\pi|\overline{r}_o-\overline{r}_s|}. \tag{4}$$

Figure 1b shows the relevant quantities in these expressions.

Of special interest is the component of the scattered electric field which is tangent to the primary source. This direction is denoted by $\hat{\xi}$. Taking the $\hat{\xi}$ component of Eq. (1), substituting in Eqs. (2) and (3) and making use of the continuity equation for the total current

$$\frac{dI_n}{d\xi_n'} = -\frac{1}{j\omega}\rho(\overline{r}_s), \tag{5}$$

the following relation for the scattered tangential electric field results

$$-j\omega\varepsilon E_n^{sca}(\overline{r}_o) = k^2\int_{n^{th}\text{ image}} I_n(\xi_n')G(\overline{r}_o,\overline{r}_s)\hat{\xi}_n'(\overline{r}_s)\cdot\hat{\xi}(\overline{r}_o)d\xi_n'$$

$$+\int_{n^{th}\text{ image}} \frac{dI_n}{d\xi_n'}\frac{\partial G}{\partial\xi}(\overline{r}_o,\overline{r}_s)d\xi_n' \tag{6}$$

Integrating the last integral by parts and using the fact that $I_n = 0$ at the end points of the n^{th} image, results in the following relation

$$-j\omega\varepsilon E_n^{sca}(\overline{r}_o) = \int_{n^{th}\text{ image}} I_n(\xi_n')\left[-\frac{\partial}{\partial\xi_n'}\frac{\partial}{\partial\xi}+k^2\hat{\xi}_n'\cdot\hat{\xi}\right]G(\overline{r}_o,\overline{r}_s)d\xi_n' \tag{7}$$

The scattered tangential electric field at the point ξ on the antenna then consists of a sum over all of the n terms given in Eq. (7). Denote by the symbol $\mathcal{L}$ the operator within the brackets of Eq. (7). Then, the scattered field produced by the images as well as by the primary current itself may be written as

$$-j\omega\varepsilon E^{sca}(\xi) = \sum_{n=-\infty}^{\infty} \int_{n^{th}\ \text{image}} I_n(\xi_n')\mathcal{L}G(\bar{r}_o,\bar{r}_s)d\xi_n' \tag{8}$$

where it is assumed that when n = 0, the source and observation points are both on the physical antenna.

At this point it is convenient to separate Eq. (8) into two parts, one of which arises from images parallel to the antenna, and the others which are non-parallel. If the n^{th} image is parallel to the physical antenna, then

$$\hat{\xi}_n' \cdot \hat{\xi} = 1$$

and (9)

$$-\frac{\partial}{\partial\xi_n'} G = \frac{\partial}{\partial\xi} G .$$

Hence, for these images the operator $\mathcal{L}$ is modified and it is possible to define it as a new operator $\mathcal{L}'$, where

$$\mathcal{L}' = \frac{\partial^2}{\partial\xi^2} + k^2 \quad . \tag{10}$$

For the non parallel images, it is noted that the quantity $\hat{\xi}_n' \cdot \hat{\xi}$ always has a constant value of cos 2θ, since the antenna is not a curved wire. Hence, the operator $\mathcal{L}$ takes the form

$$\mathcal{L} = \left(-\frac{\partial}{\partial\xi_n'}\frac{\partial}{\partial\xi} + k^2 \cos 2\theta\right) \tag{11}$$

With these two operators, it is possible to write an integral equation for the current by imposing the boundary condition that $E^{inc} + E^{sca} = E^{tot} = 0$ on the surface of the conducting wire. Hence, we have from Eq. (8)

$$j\omega\varepsilon E^{inc}(\xi) = \sum_{\substack{n \text{ for} \\ \text{parallel} \\ \text{images}}} \int_{n^{th}\ \text{image}} I_n\mathcal{L}'Gd\xi_n' + \sum_{\substack{n \text{ for} \\ \text{non-parallel} \\ \text{images}}} \int_{n^{th}\ \text{image}} I_n\mathcal{L}Gd\xi_n' \tag{12}$$

where the first sum accounts for all images parallel to the primary source (including the self current) and the second sum is for all of the other, non-parallel images.

From image theory, the values of the image currents I_n, are expressible in terms of the still unknown current on the antenna. Specifically, for the images parallel to the antenna, $I_n = I_o$. However, for the non-parallel images, it is found that $I_n = -I_o$. Rearranging the orders of summation, integration and operation by $\mathcal{L}$ or $\mathcal{L}'$ yields the following integral equation for the antenna current.

$$j\omega\varepsilon E^{inc}(\xi) = \int_o^L I_o(\xi')\left[-\mathcal{L}\sum_{n=-\infty}^{\infty}\frac{e^{-jkR_n^{(1)}}}{4\pi R_n^{(1)}} + \mathcal{L}'\sum_{n=-\infty}^{\infty}\frac{e^{-jkR_n^{(2)}}}{4\pi R_n^{(2)}}\right]d\xi' \qquad (13)$$

where

$$R_n^{(1)} = \left[\Big((L-\xi-\xi')\sin\theta - 2(h+nd)\Big)^2 + \Big((\xi-\xi')\cos\theta\Big)^2\right]^{\frac{1}{2}}$$

$$R_n^{(2)} = \left[\Big((\xi-\xi')\sin\theta - 2nd\Big)^2 + \Big((\xi-\xi')\cos\theta\Big)^2\right]^{\frac{1}{2}} \qquad n \neq 0 \qquad (14)$$

$$R_o^{(2)} = \left[(\xi-\xi')^2 + a^2\right]^{\frac{1}{2}} \qquad n = 0 \quad (a = \text{antenna wire radius}).$$

In determining the solution $I(\xi')$ in Eq. (13), the method of moments as described by Harrington[5] is used to reduce the integro-differential equation to a system of linear algebraic equations. In order to evaluate the matrix elements required for the solution, the two doubly infinite sums must be evaluated efficiently, so as to reduce the computational time and effort. As readily seen from the sums, the n^{th} term behaves asymptotically as

$$n^{th} \text{ term} \sim \frac{e^{-j2nkd}}{n}. \qquad (15)$$

As a result, the sums will converge uniformly in the limit as n approaches infinity, provided that $kd \neq \pi, 2\pi, 3\pi$, etc. For these values of plate separation, the terms behave as $1/n$ which indicates that the sums in Eq. (13) are divergent.

For these plate separations, the validity of interchanging the $\sum$, $\int$ and $\mathcal{L}$ operations in Eq. (12) is open to question. For example, consider the antenna being perpendicular to the two image planes. If the operations are performed in the order of Eq. (13), namely $\int \mathcal{L} \sum$, a divergent sum results. However, if the operations are done in the order $\int \sum \mathcal{L}$, it is noted that the terms in the sum fall off as $\exp(-j2nkd)/n^2$, since each term corresponds to the axial electric field along the axis of a radiating antenna. This sum will converge even if $kd = \pi$, 2π, etc. If the antenna is tilted slightly, then the electric field contribution due to one of the images will have a $1/n$ term, since the observation point is no longer along the axis of the image. This resulting sum, will not converge either. Since all of the sums are convergent for $kd \neq \pi$, 2π, etc., it is best to exclude the resonant values of plate separations so that the integral equation in Eq. (13) remains valid.

Even though the sums in Eq. (13) do converge for non-resonant values of kd, they do so rather slowly. The technique of using the Poisson summation formula is employed by Rao[1], and is one method for summing the series rapidly. This is useful, however, for source and observation points relatively widely separated.[6] For the present problem, the summation of the series was effected by subtracting term by term the series e^{-j2kdn}/n from the original sum, and then adding the closed form sum

$$\sum_{n=1}^{\infty} \frac{e^{-j2kdn}}{n} = -\ln(1 - e^{-j2kd}) \tag{16}$$

to the final result. This technique yielded an accuracy within 10^{-4} in less than 100 terms of the series for all combinations of source and observation points.

III. Numerical Results

The integro-differential equation for the antenna current has been solved using conventional numerical techniques described in the literature.[3,4,5] The magnitude of the current distribution on a symmetrically positioned half-wave dipole antenna is shown in Fig. 3 for various inclination angles of the antenna. The plate separation, $d = 1.49\ \lambda$ is very nearly that of one of the resonant separations. It is observed that for this length of the antenna, the functional form of the antenna current is relatively insensitive to the inclination angle. The same insensitivity to the plate separation was also noted.

The magnitude of the current at the input of the antenna does vary markedly with the plate separation and antenna location. This is seen in Figs. 4 and 5, where plots of the input admittance, $Y = G+jB$, are given as a function of the plate separation d for various antenna inclinations. In Fig. 4a the input conductance of an antenna of length $L = \lambda/2$ which is located midway between the parallel plates is shown as a function of plate separation. Figure 4b shows the input susceptance for the same antenna. Figures 5a and 5b show the input conductance and susceptance for the antenna located at $h = d/4$ within the parallel plates. For each of these curves, the value of G or B which is obtained as d approaches infinity (free space case) is indicated by an arrow.

As may be seen from these curves, the solution for the antenna current remains continuous for certain antenna locations at some specific values of kd, where the sums are infinite but for other values of the resonant kd, the current behaves in a discontinuous manner. For the symmetrically located quarter wave dipole, there are no discontinuities in the solution at $kd = 2\pi$, 4π, etc. and for the dipole with center at $h = d/4$, the current is continuous for $kd = 4\pi$, 8π, etc. This behavior of the current can be readily understood in terms of the modal representation of the fields within the parallel plates. As the plate separation increases, modes which were evanescent in nature begin to propagate and carry away energy. This gives rise to the discontinuities in the antenna current. For certain antenna positions, some modes do not couple to the antenna, due to the geometry. Hence, there are no discontinuities in the current as the particular modes in question begin to propagate.

In obtaining the time response of the antenna or scattering body, it is necessary to obtain the frequency response of the structure and then perform a Fourier inversion. The results in Figures 6 and 7 show the frequency response for the driven antenna. In these curves, only the $\theta = 0^{\circ}$ and $\theta = 90^{\circ}$ cases have been plotted. The responses for other inclination angles lie somewhere within the limits provided by these curves. Figures 6a and 6b give the input conductance and susceptance for an antenna of length L = .3d and $\Omega = 2\ln(L/a) = 10$ which is located at h = d/2, shown as a function of d in wavelengths. Figures 7a and 7b present the same data as in Figures 6, but with the antenna at h = d/4.

The scattering problem is treated in Figures 8a and 8b. The incident field is assumed to be TEM with respect to the $\hat{z}$ axis as shown in Figure 2. For this particular case the scattering body is assumed to lie in the x, z plane so that the angle $\phi = 0^{\circ}$. The frequency response of the current at the center of the scattering element is presented for two different antenna inclinations. In these cases, the scattering element is symmetrically located in the center of the plates, at h = d/2. Notice that for $\theta = 0^{\circ}$ there is no coupling between the scatterer and the incident field. Hence, the wire current is zero. When $\theta = 90^{\circ}$, this coupling is maximum, but the discontinuities in the current are non-existant for reasons previously explained. At $\theta = 45^{\circ}$, these discontinuitities are present, but they are not as great as in the driven case. As a result, the time response of the scattering body will not be affected as much by this resonant plate behavior as will that of the driven antenna.

IV. Conclusion

An integral equation for the current on a thin-wire antenna or scatterer arbitrarily located within a parallel plate waveguide has been formulated and then solved numerically. For certain resonant plate separations, the Green's function for the problem is found to be non-summable and the interchange of the processes of integration, summation and differentiation must be done carefully to ensure a physically correct solution.

In the numerical treatment of the problem, this resonance problem has been avoided by calculating the response of the antenna or scatterer for plate separations slightly different from the resonant values, but never for these values exactly. The numerical results show that the currents on the wire are, at times, discontinuous as the plate separation passes through one of the resonant values. Sometimes, however, the wire current is not affected by the plate resonance. This effect may be attributed to the coupling of the wire structure to the propagating and evanescent modes within the parallel plate waveguide.

Various curves have been presented showing the behavior of the input current (input admittance) of a driven half wave dipole antenna at different locations within the plates. In addition, the frequency response of the driven antenna and a scattering element are presented. In these cases, the discontinuities in the current are noted. The effect of these discontinuities on the time behavior of the wire is still undetermined. A more comprehensive study of the scattering problem in both the frequency and time domain will be the subject of a forthcoming note.

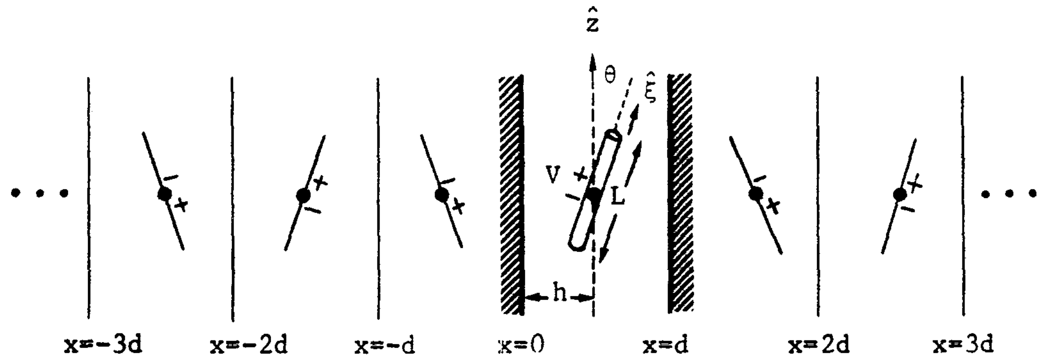

Figure 1a. Geometry of the antenna in a parallel plate region. $L=.5\lambda$, $\Omega=2\ln(L/a)=10$, a=wire radius.

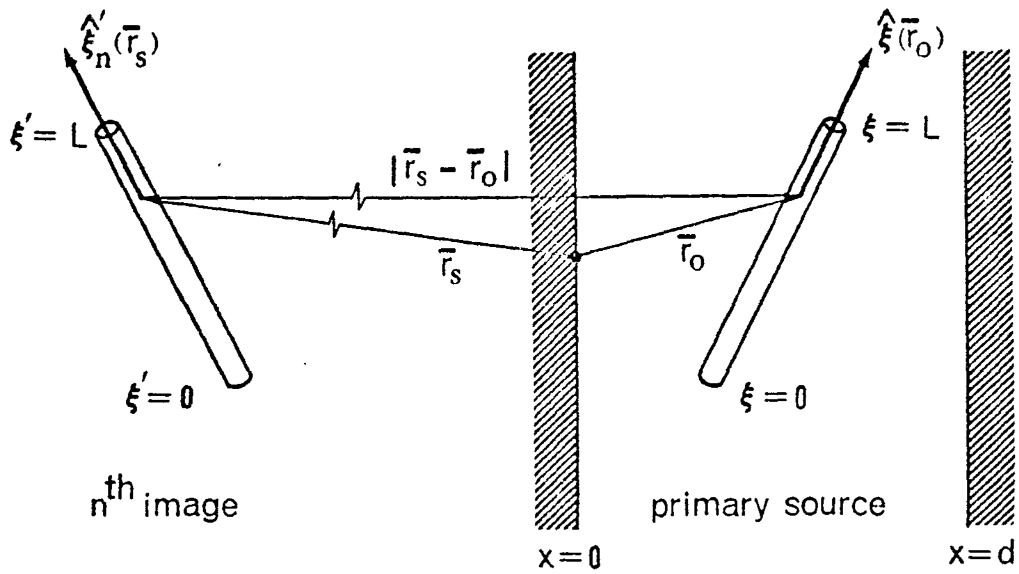

Figure 1b. Relevant quantities in defining the Green's function for the parallel plate.

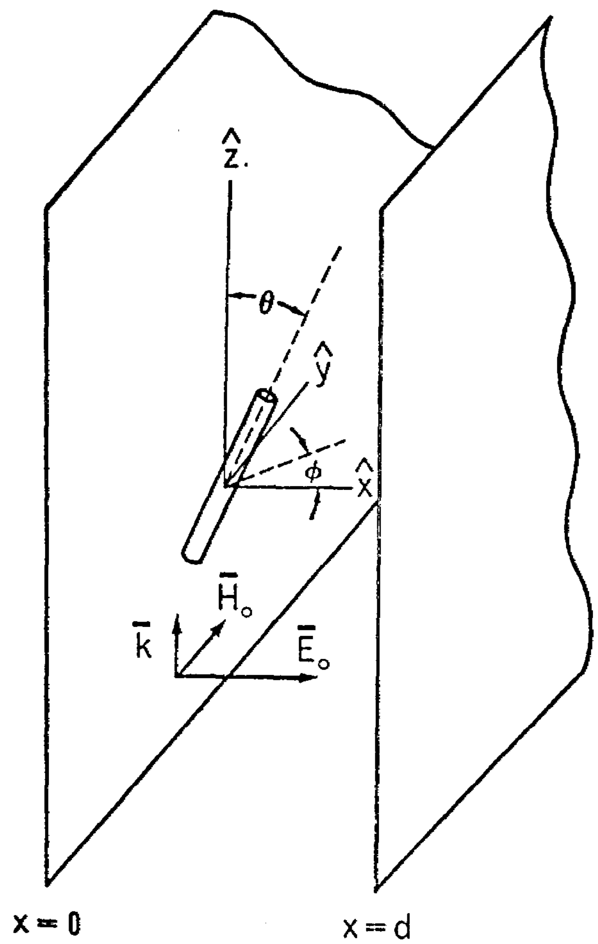

Figure 2. Orientation of the wire scattering element with a TEM wave incident.

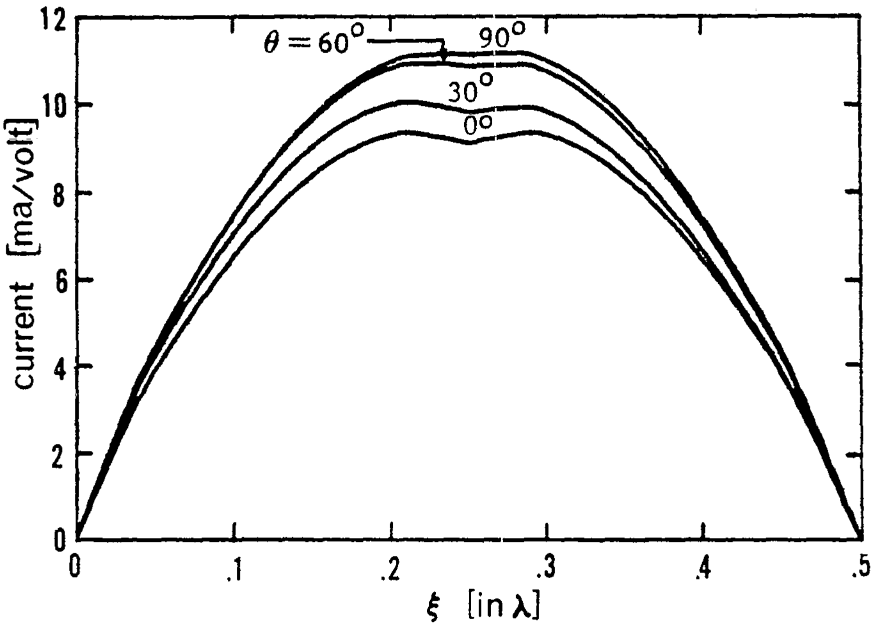

Figure 3. Magnitude of the antenna current distribution for various inclination angles. $L = .5\lambda$, $d = 1.49\lambda$, $h = d/2$ and $\Omega = 10$.

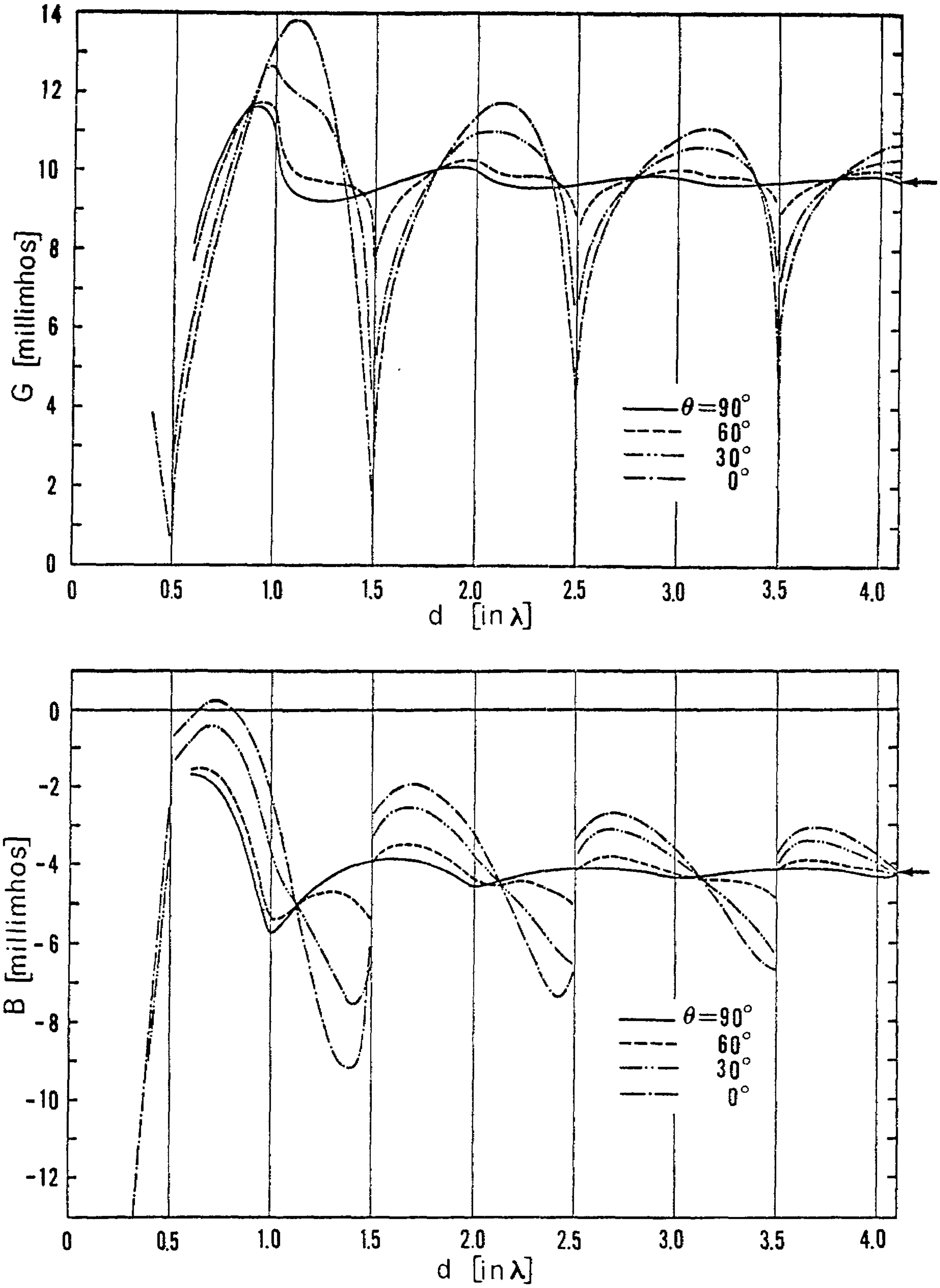

Figure 4. Plots of the input conductance G, and susceptance B, of the antenna as a function of the plate separation. The arrow indicates the computed free space values. $L = \lambda/2$, $h = d/2$, $\Omega = 10$.

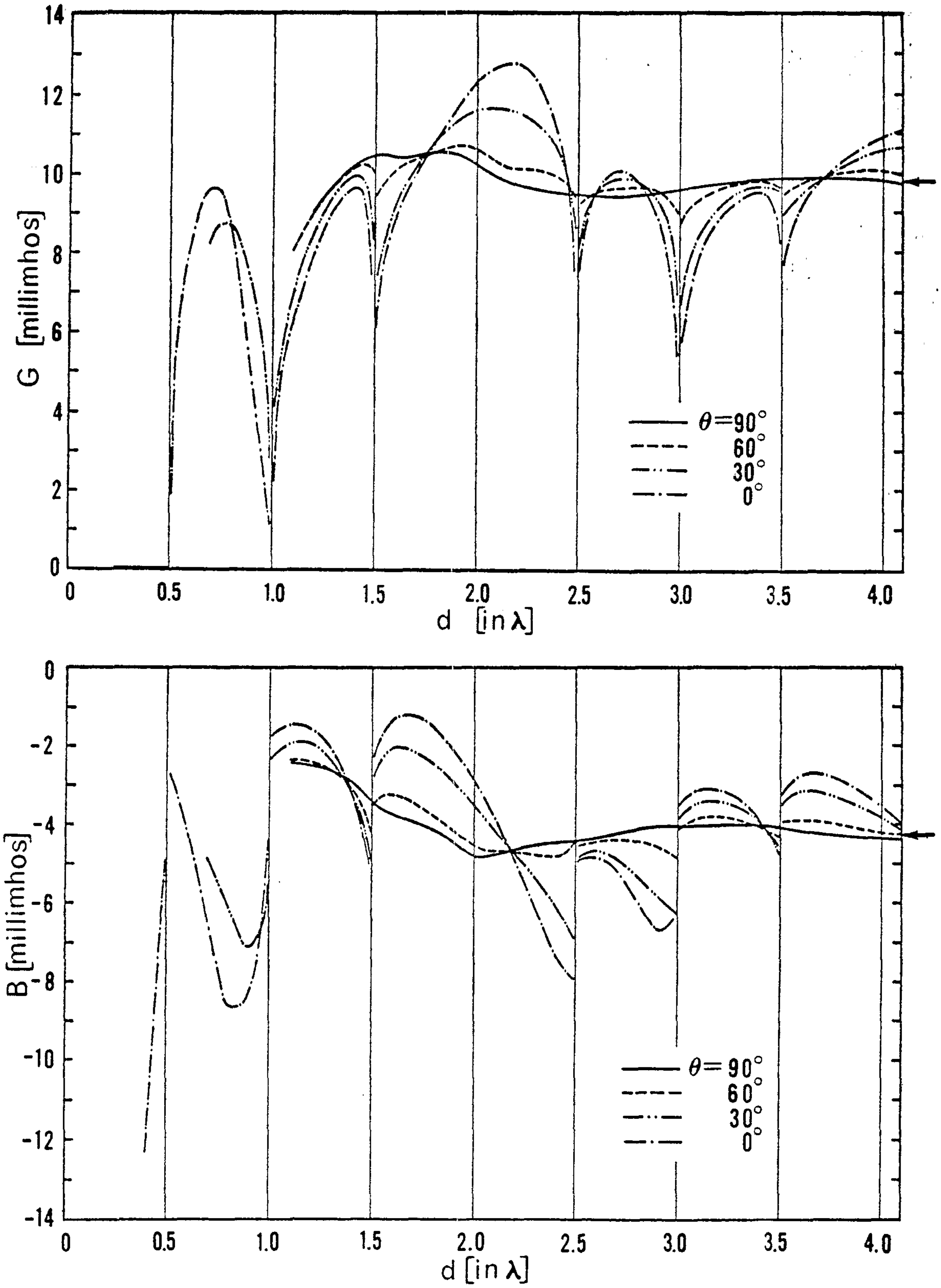

Figure 5. Plots of the input conductance G, and susceptance B, of the antenna as a function of the plate separation. The arrow indicates the free space values. $L = \lambda/2$, $h = d/4$, $\Omega = 10$.

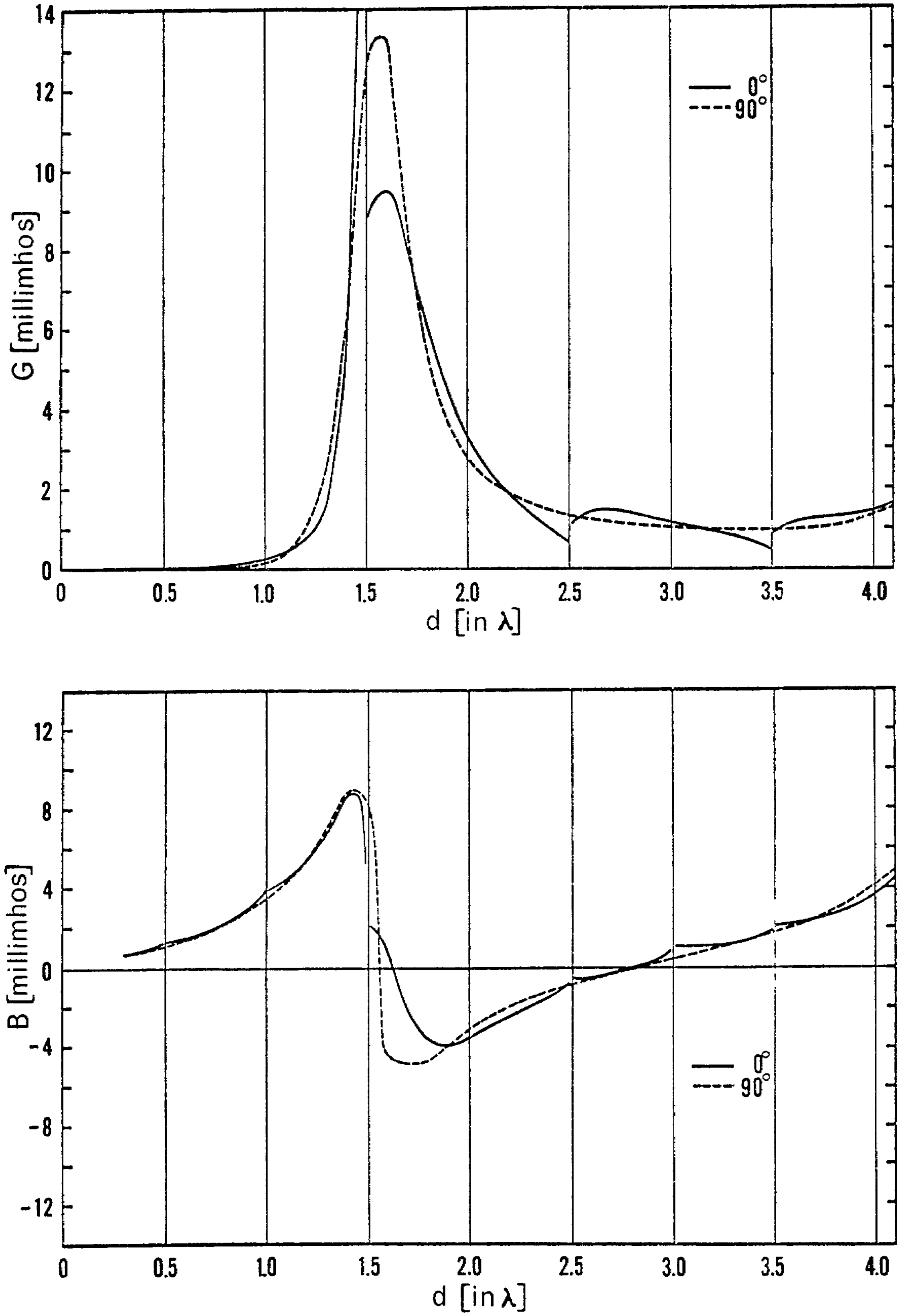

Figure 6. Frequency response of the input admittance, G and B, of the antenna. In this case, L = .3d, h = d/2, and Ω = 10.

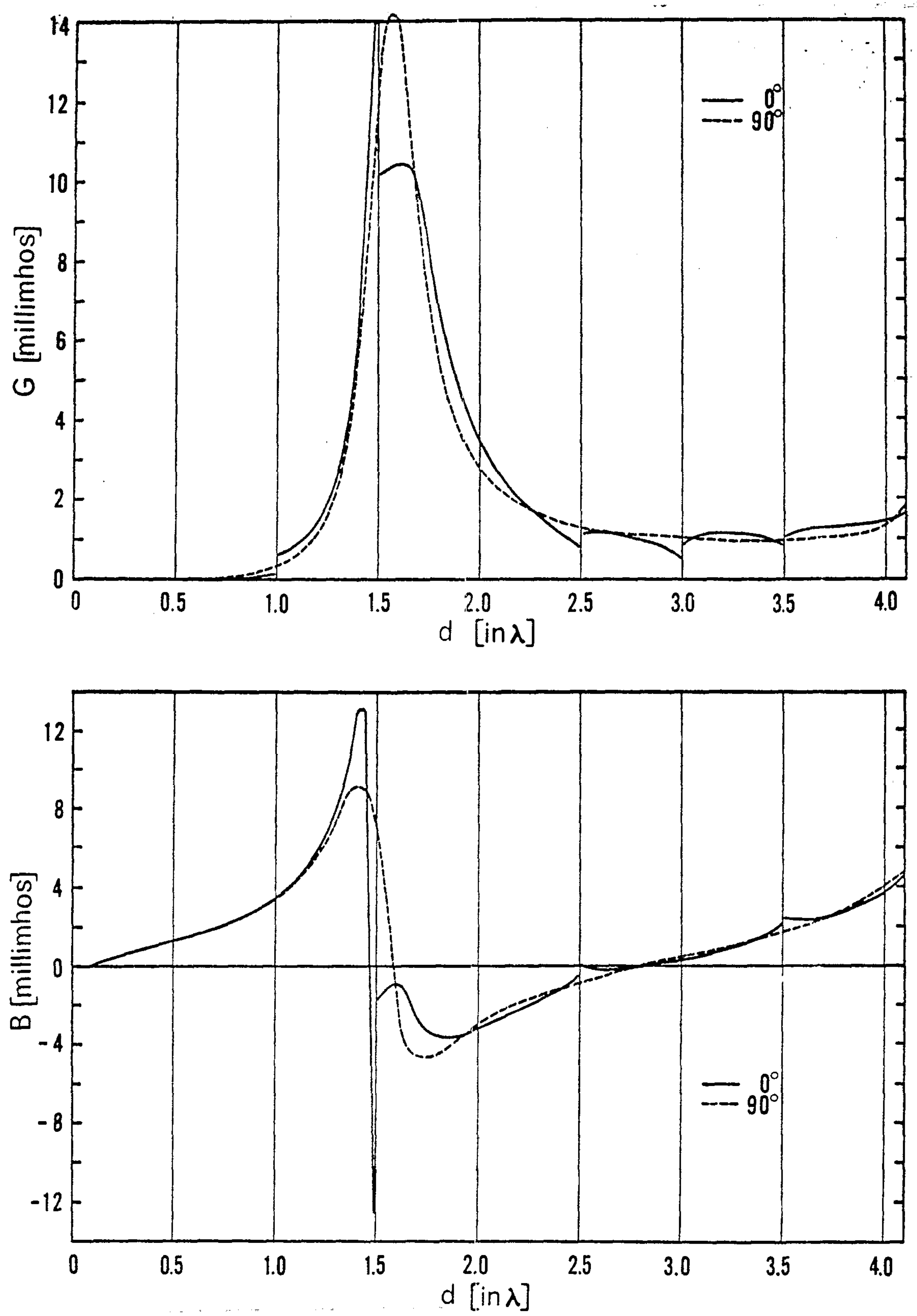

Figure 7. Frequency response of the input admittance, G and B, of the antenna. L = .3d, h = d/4 and Ω = 10.

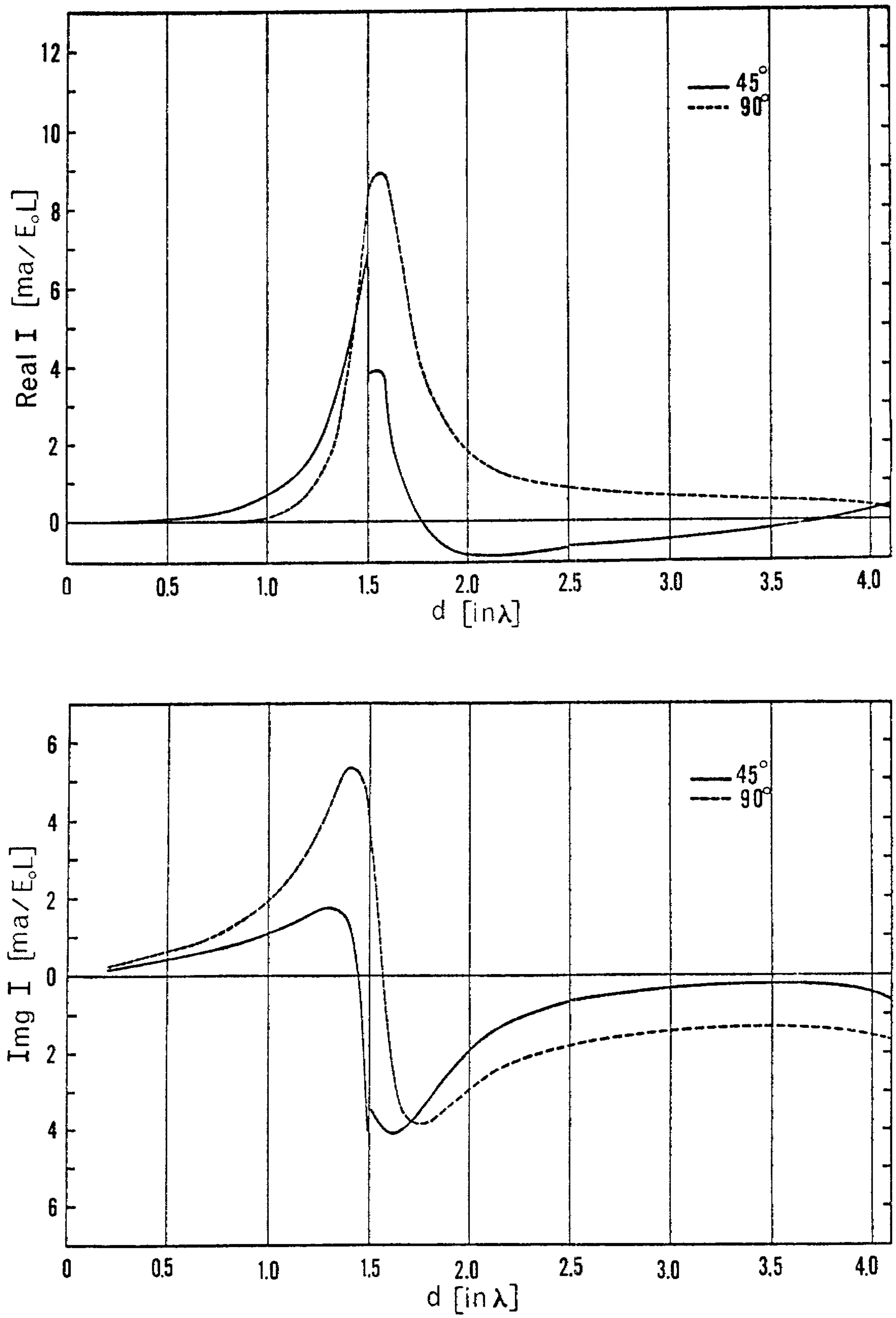

Figure 8. Plots of the real and imaginary parts of the current at the midpoint of a scattering element with a TEM wave of magnitude E_o incident. $L = .3d$, $h = d/2$, $\Omega = 10$, $\phi = 0^o$.

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