Sensor and Simulation Notes

Note 150

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A Method of Calculating Impedance and Field Distribution of a Multi-Wire Parallel Plate Transmission Line Above a Perfectly Conducting Ground

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Abstract

Using image theory and superposition, the potential distribution for a number of thin, parallel wires located above a ground plane so as to form a transmission line is calculated. The wires are arranged so as to approximate a two plate transmission line above a perfectly conducting ground and TEM mode propagation is assumed. Charges on each wire are calculated so that all wires in each "plate" are at the same potential and the resulting potential function is used to calculate the field distribution and impedance of the line. The potential function for each wire is assumed to be that of a line charge so that one requires all dimensions to be large compared to a wire radius for this approximation to hold. An estimate of the effects of finite ground conductivity is also included.

Acknowledgement

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I. Introduction

The effect of the nearby, finitely conducting earth on a horizontally polarized transmission line is an important question which is far from being well understood theoretically. The specific effects of the finite conductivity of the earth are particularly difficult to treat analytically or model experimentally. Therefore, in this note only the effect of a perfectly conducting ground (earth) on the impedance and field distribution of a transmission line will be calculated, with the assumption of TEM propagation. A two-wire transmission line will be considered first and the results extended to a multi-wire line where the wires are arranged so as to approximate a two-plate transmission line. An estimate of the effects of finite ground conductivity will also be made, but one must remember that the original assumption of TEM mode propagation is not valid for a finitely conducting ground. Throughout this note the assumption that the wires are very thin is made; ie, it is assumed that the wire diameter is much smaller than other characteristic lengths. This assumption is necessary in order to be able to use the superposition of the potentials of line charges to approximate the fields due to the wires. The resulting potential function for various arrangements of wires will be given and the electric field distribution and transmission line impedance calculated from this potential function.

II. The Two-Wire Line in Free Space

First, consider a two-wire transmission line in free space. This problem has already been considered in a previous note.[1] (See Figure 1A.) From this note, the potential function for such a line is given by

$$\phi = u + iv = \ell n\left[\frac{z + a}{z - a}\right] \tag{1}$$

where

$$z = x + iy \tag{2}$$

Thus

$$u = \frac{1}{2}\ell n\left[\frac{\left(\frac{x}{a} + 1\right)^2 + \left(\frac{y}{a}\right)^2}{\left(\frac{x}{a} - 1\right)^2 + \left(\frac{y}{a}\right)^2}\right] \tag{3}$$

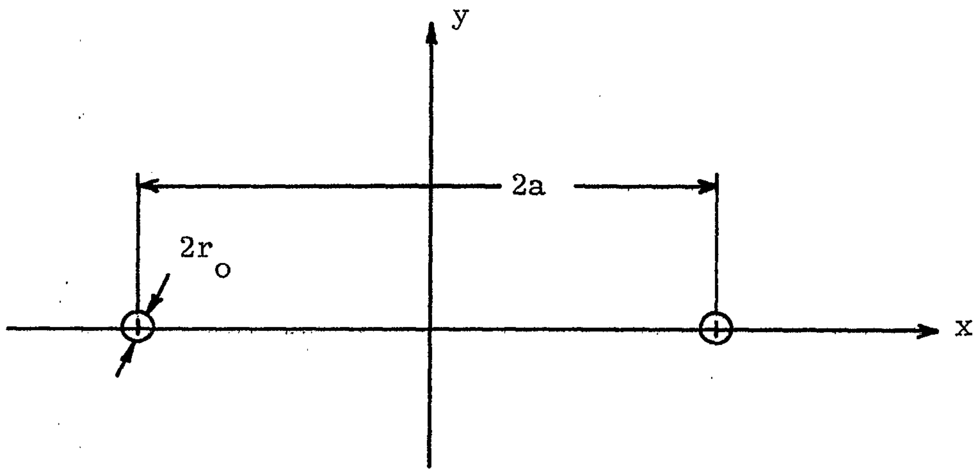


Figure 1A. Two-Wire Line in Free Space.

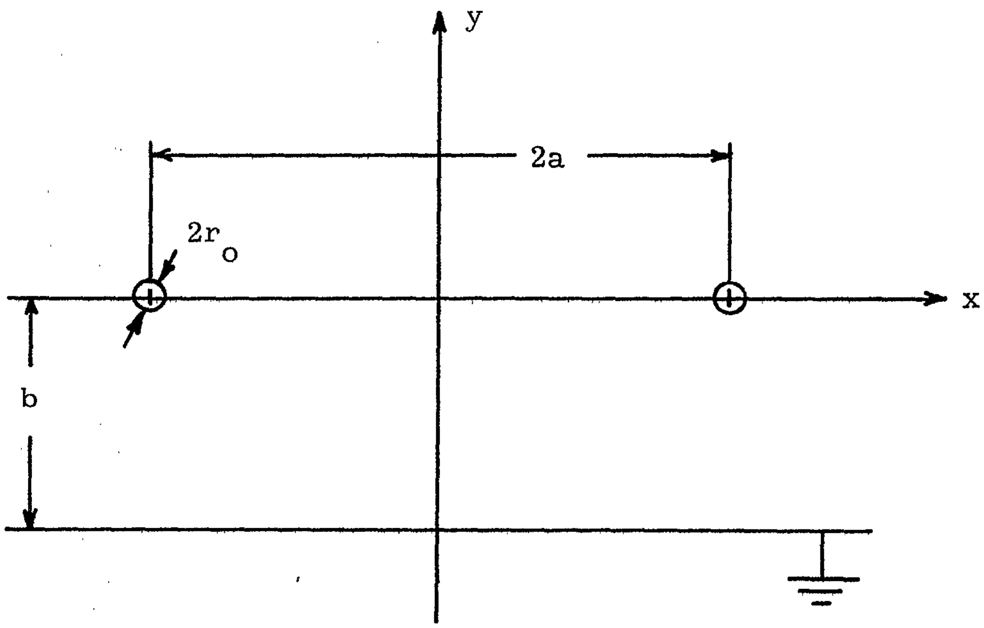


Figure 1B. Two-Wire Line Above a Perfectly Conducting Ground.

$$v = \arctan\left[\frac{-2y/a}{\frac{x^2}{a^2} + \frac{y^2}{a^2} - 1}\right] \qquad (4)$$

and

u = constant is an equipotential surface

v = constant is a stream line

The electric field is calculated by taking the derivative of the potential function and it is shown in Reference 1 that

$$E_{x_{rel}} = \frac{1}{2}\left[\frac{1 + x/a}{\frac{y^2}{a^2} + \left(1 + \frac{x}{a}\right)^2} + \frac{1 - x/a}{\frac{y^2}{a^2} + \left(1 - \frac{x}{a}\right)^2}\right] \qquad (5)$$

$$E_{y_{rel}} = \frac{1}{2}\left[\frac{y/a}{\frac{y^2}{a^2} + \left(1 + \frac{x}{a}\right)^2} - \frac{y/a}{\frac{y^2}{a^2} + \left(1 - \frac{x}{a}\right)^2}\right] \qquad (6)$$

where Ex_{rel} and Ey_{rel} are normalized so that Ex_{rel} is equal to 1 at the origin (x = 0, y = 0). The actual field in volts per meter is given by multiplying the relative fields by

$$E_x\Big|_{\substack{x=o\\y=o}} = \frac{\Delta V}{2a} f_E \qquad (7)$$

where

ΔV is the voltage across the line

2a is the wire spacing

and

$$f_E \simeq \frac{2}{\ln\left(\frac{2a}{r_o}\right)} \qquad (8)$$

where r_o is the wire radius and it is assumed that

$$\frac{a}{r_o} << 1 \tag{9}$$

It is also shown in Reference 1 that the geometrical factor for such a line is approximately just

$$f_g \simeq \frac{1}{\pi} \ell n \left(\frac{2a}{r_o}\right) \tag{10}$$

again assuming (9) is correct. Then the pulse impedance Z is given by

$$Z = \sqrt{\frac{\mu_o}{\varepsilon_o}} f_g \simeq 120 \pi f_g \tag{11}$$

where the inductance per unit length of line is

$$L' = \mu_o f_g \tag{12}$$

and the capacitance per unit length is

$$C' = \frac{\varepsilon_o}{f_g} \tag{13}$$

III. The Two-Wire Line Above a Perfectly Conducting Ground

Now consider the same two-wire line discussed in section II with the addition of a perfectly conducting ground along the line $y = -b$ (see Figure 1B). This gives two image wires such that one can approximate the potential function by

$$\phi = u + iv = \ell n\left[\frac{z + a}{z - a}\right] + \ell n\left[\frac{z - a + 2ib}{z + a + 2ib}\right] \tag{14}$$

Such superposition is only strictly correct in the limit as $r_o \rightarrow 0$; ie, for line charges. But as long as r_o is much smaller than other characteristic lengths, equation 14 should be a good approximation for the potential. Thus

$$u = \frac{1}{2} \ln\left[\frac{\left(\frac{x}{a}+1\right)^2 + \frac{y^2}{a^2}}{\left(\frac{x}{a}-1\right)^2 + \frac{y^2}{a^2}}\right] + \frac{1}{2} \ln\left[\frac{\left(\frac{x}{a}-1\right)^2 + \left(\frac{y}{a}+\frac{2b}{a}\right)^2}{\left(\frac{x}{a}+1\right)^2 + \left(\frac{y}{a}+\frac{2b}{a}\right)^2}\right] \tag{15}$$

$$v = \arctan\left[\frac{-2y/a}{\frac{x^2}{a^2}+\frac{y^2}{a^2}-1}\right] + \arctan\left[\frac{-2\left(\frac{y}{a}+\frac{2b}{a}\right)}{\frac{x^2}{a^2}+\left(\frac{y}{a}+\frac{2b}{a}\right)^2-1}\right] \tag{16}$$

Now define an effective wire separation a_{Eff} such that

$$\frac{1}{a_{Eff}} = \frac{1}{u_o} \left.\frac{\partial u}{\partial x}\right|_{\substack{x=o \\ y=o}} \tag{17}$$

where u_o is the potential at one of the wires. To find u_o, let $z = \xi - a$. Then

$$\phi = \ln\left[\frac{\xi}{\xi - 2a}\right] + \ln\left[\frac{\xi - 2a + 2ib}{\xi + 2ib}\right] \tag{18}$$

Now, assume ξ is very small, ie

$$|\xi| = r_o << 1 \tag{19}$$

Then

$$u \simeq \ln\left[\frac{r_o}{2a}\right] + \frac{1}{2} \ln\left[1 + \frac{a^2}{b^2}\right] \tag{20}$$

giving

$$|u_o| = \ln\left[\frac{2a}{r_o}\right] - \frac{1}{2} \ln\left[1 + \frac{a^2}{b^2}\right] \tag{21}$$

From equation 15

$$\left.\frac{\partial u}{\partial x}\right|_{\substack{x=o\\y=o}} = \frac{2}{a}\left[1 - \frac{1}{\frac{4b^2}{a^2} + 1}\right] = \frac{8b^2}{a(4b^2 + a^2)} \tag{22}$$

Now,

$$\left.E_x\right|_{\substack{x=o\\y=o}} = \frac{\Delta V}{2a_{Eff}} = \frac{\Delta V}{2a} f_E \tag{23}$$

where

$$f_E = \frac{a}{a_{Eff}} \tag{24}$$

and ΔV is the voltage across the line. From equations 17, 21, and 22, the efficiency factor, f_E is

$$f_E = \frac{8b^2}{4b^2 + a^2}\left[\ell n\left(\frac{2a}{r_o}\right) - \frac{1}{2}\,\ell n\left(1 + \frac{a^2}{b^2}\right)\right]^{-1} \tag{25}$$

Note that this equation has the limiting form given in equation 10 as b becomes large (ground plane goes to infinity).

Now, define relative electric fields by

$$E_{x_{rel}} = \frac{\frac{\partial u}{\partial x}}{\left.E_x\right|_{\substack{x=o\\y=o}}} \tag{26}$$

$$E_{y_{rel}} = \frac{\frac{\partial u}{\partial y}}{\left.E_x\right|_{\substack{x=o\\y=o}}} \tag{27}$$

Then

$$E_{x_{rel}} = \frac{a(4b^2 + a^2)}{8b^2} \left\{ \frac{a + x}{y^2 + (a + x)^2} + \frac{a - x}{y^2 + (a - x)^2} - \frac{a + x}{(y + 2b)^2 + (a + x)^2} - \frac{a - x}{(y + 2b)^2 + (a - x)^2} \right\} \tag{28}$$

$$E_{y_{rel}} = \frac{a(4b^2 + a^2)}{8b^2} \left\{ \frac{y}{y^2 + (a + x)^2} - \frac{y}{y^2 + (a - x)^2} - \frac{y + 2b}{y^2 + (a + x)^2} + \frac{y + 2b}{y^2 + (a - x)^2} \right\} \tag{29}$$

It can easily be shown that these expressions just reduce to equations 5 and 6 for the fields in free space as the conducting plane moves far away, ie as $b \to \infty$.

Now consider the effect of the ground plane on the geometrical factor

$$f_g \equiv \frac{\Delta u}{\Delta v} = \frac{2|u_o|}{2\pi} = \frac{|u_o|}{\pi} \tag{30}$$

Thus

$$f_g = \frac{1}{\pi}\left[\ell n\left[\frac{2a}{r_o}\right] - \frac{1}{2}\ \ell n\left[1 + \frac{a^2}{b^2}\right]\right] \tag{31}$$

and the ground plane has the effect of lowering the value of f_g, as expected (see equation 10).

IV. A Multi-Wire Transmission Line Above a Perfectly Conducting Ground

Now let us consider a transmission line above a ground which consists of a number of parallel wires lying along two planes (see Figure 2). The wires in the plane $x = -a$ are at the potential $+V/2$ and those in the plane $x = +a$ are at a potential $-V/2$. Each plate of the transmission line consists of N wires equally spaced a distance d apart, and the bottom wire is a distance b above a perfectly conducting ground.

One might first consider treating this problem in a method similar to that used in the previous sections, ie, merely

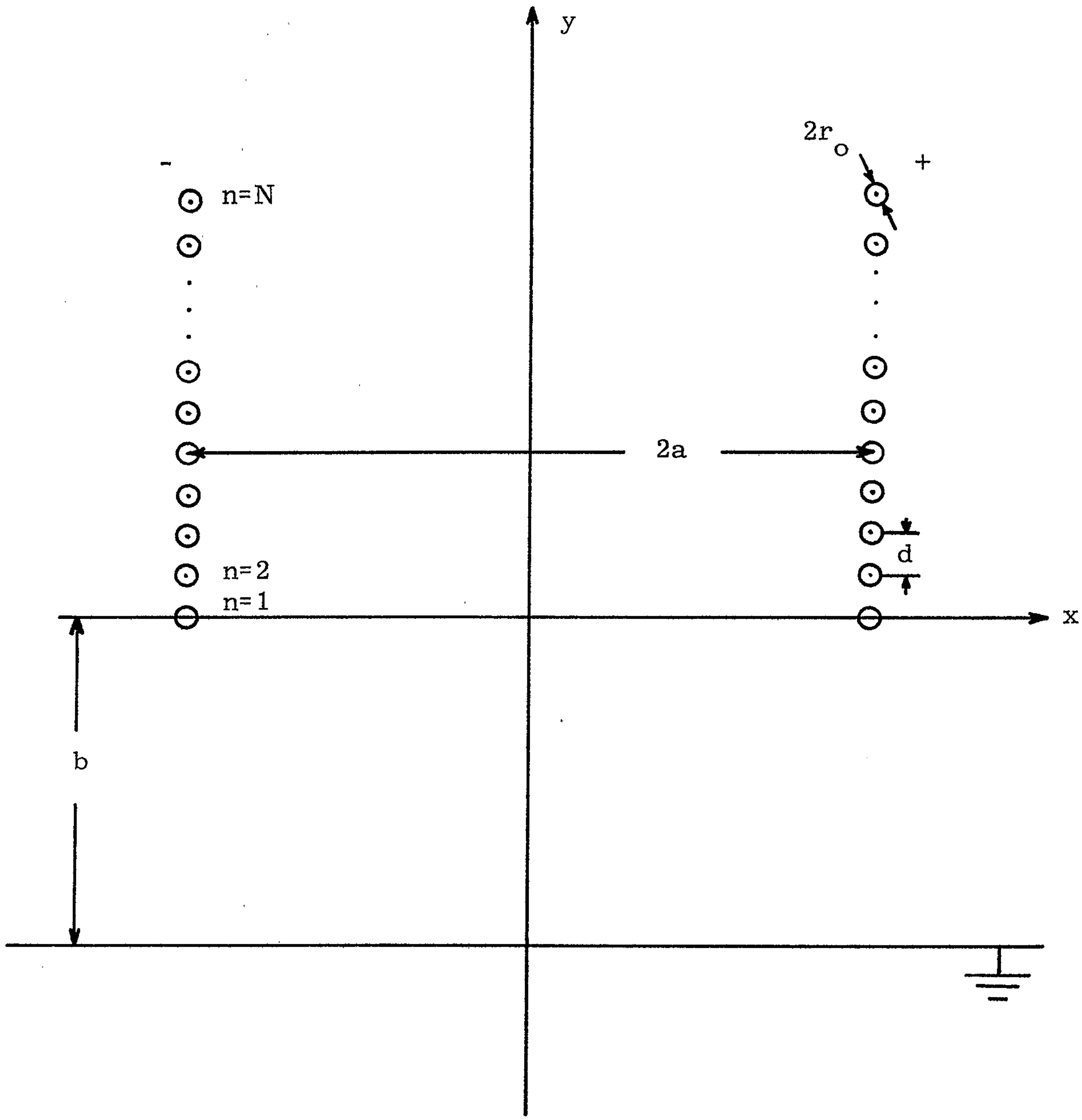


Figure 2. N-Wire Line Above a Perfectly Conducting Ground.

superimposing the potentials of a number of line charges to get an expression for the total potential. One runs into an added complication for multiple wires though. We require the magnitude of the potential at the surface of each of the wires to be the same. If one assumes equal charges on the wires, this will not be true. Thus we must solve for the charge on each wire such that the potentials are the same.

To solve this problem we will find Maxwell's potential coefficients for the multi-conductor system being considered, invert this matrix to find the capacitance coefficients, and thus find the charge required on each wire of the line.[2,3]

A more complete discussion of this technique can be found in references 2 and 3 and thus will just be outlined here. Basically, if we have a set of N conductors, each with a given charge, superposition tells us that the potential on the n^{th} conductor can be written as

$$u_n = p_{n1}q_1 + p_{n2}q_2 + \cdots + p_{nN}q_N \tag{32}$$

where we have a set of N linear equations since $1 \leq n \leq N$. (q_i is the charge on the i^{th} conductor.)

The p_{ij}'s are called Maxwell's potential coefficients and form a N × N matrix. These potential coefficients are a function of the conductor geometry only and have the general properties

$$p_{ij} = p_{ji} \tag{33}$$

$$p_{ij} \geq 0 \text{ for all } i,j \tag{34}$$

By inverting the p_{ij} matrix, or equivalently, solving the set of linear equations in equation 32, one obtains another N × N matrix, namely K_{pq}, where a given K_{pq} is called a capacitance coefficient. One can write

$$Q_n = K_{n1}U_1 + K_{n2}U_2 + \cdots + K_{nN}U_N \tag{35}$$

where $1 \leq n \leq N$ and Q_n is the charge on the n^{th} conductor. The capacitance coefficient is given by the expression

$$K_{rs} = \frac{\text{cofactor of } p_{rs} \text{ matrix}}{\text{determinant of } p_{rs} \text{ matrix}} \tag{36}$$

$$K_{rs} = K_{sr}$$

$$K_{rs} \geq 0 \text{ for } r = s \qquad (37)$$

$$\leq 0 \text{ for } r \neq s$$

Once we know the capacitance coefficients, K_{rs}, one can use equation 35 to calculate the charge required on each wire such that the potential on each wire is the same, ie let

$$U_1 = U_2 = \cdots = U_N = U_o \qquad (38)$$

Then the charge on the n^{th} wire is

$$Q_n = U_o \sum_{m=1}^{N} K_{nm} \qquad (39)$$

Once the charge on each wire is known, one can easily go back to superpositioning the potentials and fields of a number of line charges to get TEM mode expressions for a multiple wire line.

Now, consider a transmission line consisting of N-pairs of thin wires above a perfectly conducting ground. To calculate p_{rs}, assume some charge q_n on each of the wires to obtain the normalized complex potential function

$$\phi(x,y) = \sum_{n=1}^{N} \{q_n \ell n [\frac{z + a - i(n - 1)d}{z - a - i(n - 1)d}] + q_n \ell n [\frac{z - a - i(n - 1)d + 2i[b + (n - 1)d]}{z + a - i(n - 1)d + 2i[b + (n - 1)d]}]\} \qquad (40)$$

We want an expression for the potential, u_m, on the surface of the m^{th} wire ($m = 1$ for the wire closest to the ground plane).

Let

$$z = \xi - a + i(m - 1)d \qquad (41)$$

Then

$$\phi(x,y) = \sum_{n=1}^{N} \left\{ \ln\left[\frac{\xi - i(n-m)d}{\xi - 2a - i(n-m)d}\right] + \ln\left[\frac{\xi - 2a + 2i[b + (\frac{n+m-2}{2})d]}{\xi + 2i[b + (\frac{n+m-2}{2})d]}\right] \right\} \tag{42}$$

Now, assume

$$|\xi| = r_o << a,b,d \tag{43}$$

where r_o is the wire radius. Then one can write

$$|u_m| \simeq q_m \ln\left[\frac{2a}{r_o}\right] + \frac{1}{2}\sum_{\substack{n=1\\ n\neq m}}^{N} \left\{ q_n \ln\left[\frac{[(n-m)^2 d^2 + 4a^2]^2}{(n-m)^4 d^4 + 4(n-m)^2 a^2 d^2}\right] \right\}$$

$$- \frac{1}{2}\sum_{n=1}^{N} \left\{ q_n \ln\left[1 + \frac{a^2}{[b + (\frac{n+m-2}{2})d]^2}\right] \right\} \tag{44}$$

Now we have an expression for the potential at each of the wires given a constant charge at each wire. From this we can determine Maxwell's potential coefficients since

$$u_m = \sum_{n=1}^{N} q_n p_{mn} \tag{45}$$

From the above equations

$$p_{mn} = \frac{1}{2} \ln\left[\frac{[(n-m)^2 d^2 + 4a^2]^2}{(n-m)^4 d^4 + 4(n-m)^2 a^2 d^2}\right] - \frac{1}{2} \ln\left[1 + \frac{a^2}{[b + (\frac{n+m-2}{2})d]^2}\right] \tag{46}$$

for $m \neq n$ and

$$p_{nn} = \ell n\left[\frac{2a}{r_o}\right] - \frac{1}{2}\,\ell n\left[1 + \frac{a^2}{[b + (\frac{n-1}{2})d]^2}\right] \qquad (47)$$

for m = n.

Thus, knowing p_{mn} one can use equation 36 or some other numerical inversion technique to obtain K_{mn}. Once K_{mn} is known, equation 39 can be used to find the required charge on each wire for a given potential. Using equation 39, the complex potential function for all the wires at a constant voltage is given by

$$\phi_{cv}(x,y) = \sum_{n=1}^{N}\left\{Q_n \ell n\left[\frac{z + a - i(n-1)d}{z - a - i(n-1)d}\right]\right.$$

$$\left. + Q_n \ell n\left[\frac{z - a - i(n-1)d + 2i[b + (n-1)d]}{z + a - i(n-1)d + 2i[b + (n-1)d]}\right]\right\} \qquad (48)$$

where cv indicates that all the wires in each plate are at the same constant voltage.

Now, define an effective plate separation a_{Eff} such that

$$\frac{1}{a_{Eff}} = \frac{1}{u_o}\left.\frac{\partial u_{cv}}{\partial x}\right|_{\substack{x=o \\ y=\frac{(N-1)}{2}d}} \qquad (49)$$

From equation 48, it can be shown that

$$\left.\frac{\partial u_{cv}}{\partial x}\right|_{\substack{x=o \\ y=\frac{N-1}{2}d}} = \sum_{n=1}^{N}\left\{Q_n\left[\frac{2/a}{1 + [\frac{(N-1)}{2}\frac{d}{a} - (n-1)\frac{d}{a}]^2}\right]\right.$$

$$\left. - Q_n\left[\frac{2/a}{1 + [\frac{(N-1)d}{2a} + (n-1)\frac{d}{a} + \frac{2b}{a}]^2}\right]\right\} \qquad (50)$$

and since

$$Q_n = \sum_{m=1}^{N} K_{nm} u_o \tag{51}$$

$$\frac{1}{a_{Eff}} = \sum_{n=1}^{N} \left(\sum_{m=1}^{N} K_{nm} \right) \left\{ \frac{2/a}{1 + [\frac{(N-1)d}{2a} - (n-1)\frac{d}{a}]^2} \right.$$

$$\left. - \frac{2/a}{1 + [\frac{(N-1)d}{2} + (n-1)d + 2b]^2} \right\} \tag{52}$$

Now define an electric field efficiency factor f_E where

$$f_E = \frac{a}{a_{Eff}} \tag{53}$$

Then

$$E_x \Big|_{\substack{x=o \\ y=\frac{(N-1)}{2}d}} = \frac{\Delta V}{2a_{Eff}} = \frac{\Delta V}{2a} f_E \tag{54}$$

where ΔV is the difference in potential across the two sides of the line.

Now let us define a relative electric field normalized to the electric field strength in the x-direction at the center of the wire plates. Then

$$E_{x_{rel}}(x,y) = \frac{a}{f_E} \frac{\partial u}{\partial x}$$

$$= \frac{a}{f_E} \sum_{n=1}^{N} \left\{ \left(\sum_{m=1}^{N} K_{nm} \right) \left[\frac{x+a}{(x+a)^2 + [y-(n-1)d]^2} \right. \right.$$

$$\left. - \frac{x-a}{(x-a)^2 + [y+(n-1)d]^2} \right]$$

$$+ \left(\sum_{m=1}^{N} K_{nm}\right)\left[\frac{x-a}{(x-a)^2 + [y+(n-1)d+2b]^2} - \frac{x+a}{(x+a)^2 + [y+(n-1)d+2b]^2}\right]\Bigg\} \quad (55)$$

$$E_{y_{rel}}(x,y) = \frac{a}{f_E}\frac{\partial u}{\partial y}$$

$$= \frac{a}{f_E}\sum_{n=1}^{N}\Bigg\{\left(\sum_{m=1}^{N} K_{nm}\right)\left[\frac{y-(n-1)d}{(x+a)^2 + [y-(n-1)d]^2} - \frac{y-(n-1)d}{(x-a)^2 + [y-(n-1)d]^2}\right]$$

$$- \left(\sum_{m=1}^{N} K_{nm}\right)\left[\frac{y+(n-1)d+2b}{(x+a)^2 + [y+(n-1)d+2b]^2} - \frac{y+(n-1)d+2b}{(x-a)^2 + [y+(n-1)d+2b]^2}\right]\Bigg\} \quad (56)$$

Thus we have defined the relative fields such that

$$E_{x_{rel}}\left(x = 0,\ y = \frac{(N-1)}{2}d\right) = 1 \quad (57)$$

Now let us calculate the geometrical factor for a multiple wire transmission line above a perfectly conducting ground. The potential on the bottom wire is given by

$$u_1 = \left(\sum_{m=1}^{N} K_{1m}\right)\ln\left[\frac{2a}{r_o}\right] + \frac{1}{2}\sum_{n=2}^{N}\left\{\ln\left[\frac{[(n-1)^2d^2 + 4a^2]^2}{(n-1)^4d^4 + 4(n-1)^2a^2d^2}\right]\left(\sum_{m=1}^{N} K_{nm}\right)\right\}$$

$$- \frac{1}{2}\sum_{n=1}^{N}\left\{\ln\left[1 + \frac{a^2}{[b + (\frac{n-1}{2})d]^2}\right]\left(\sum_{m=1}^{N} K_{nm}\right)\right\} \quad (58)$$

Then, the geometrical factor, f_g, is given by

$$f_g = \frac{\Delta u}{\Delta v} = \frac{u_1}{\pi \sum_{n=1}^{N} \left(\sum_{m=1}^{N} K_{nm} \right)} \tag{59}$$

All the expressions in this section are for a multiple wire transmission line a distance b above a perfectly conducting ground. To consider such a line in free space, one can just take the limit as b becomes very large. It is easily seen that this is equivalent to dropping certain terms in the expressions for the potentials and fields. As a check on this formulation one can take the limit of large b and large N and compare the results to existing calculations for the field distribution and impedance of a finite width, parallel plate transmission line.[4] For N = 51 and $b/a = 10^6$ good agreement with the results of reference 4 were obtained (e.g., the result for f_g with 51 wires was about 4% higher than for the flat plate treated in reference 4).

One should also remember that all of these expressions are based on the superposition of the potentials and fields of line charges. Since real wires have some finite radius r_o, all of the expressions developed here are only approximations which are valid only as long as the wire radius is small when compared to all other characteristic lengths; ie, we require

$$\frac{r_o}{a} << 1 \tag{60}$$

$$\frac{r_o}{b} << 1 \tag{61}$$

and

$$\frac{r_o}{d} << 1 \tag{62}$$

V. An Estimate of the Effects of a Finitely Conducting Ground

All of the calculations thus far have assumed an infinitely conducting ground in order to make the problem tractable. However, one can at least estimate the effects of a finitely conducting ground in the limit of low frequency. One should note that the transmission line equations are really only valid for wavelengths large compared to the cross-sectional dimensions of

Since the presence of a ground decreases the geometrical factor of a transmission line,

$$Z_{FS} < Z_{FCG} < Z_{PCG} \tag{67}$$

Note that with a finitely conducting ground we can no longer assume TEM mode propagation; ie.

$$\frac{E}{H} \neq \sqrt{\frac{\mu_o}{\varepsilon_o}} = 377\ \Omega \tag{68}$$

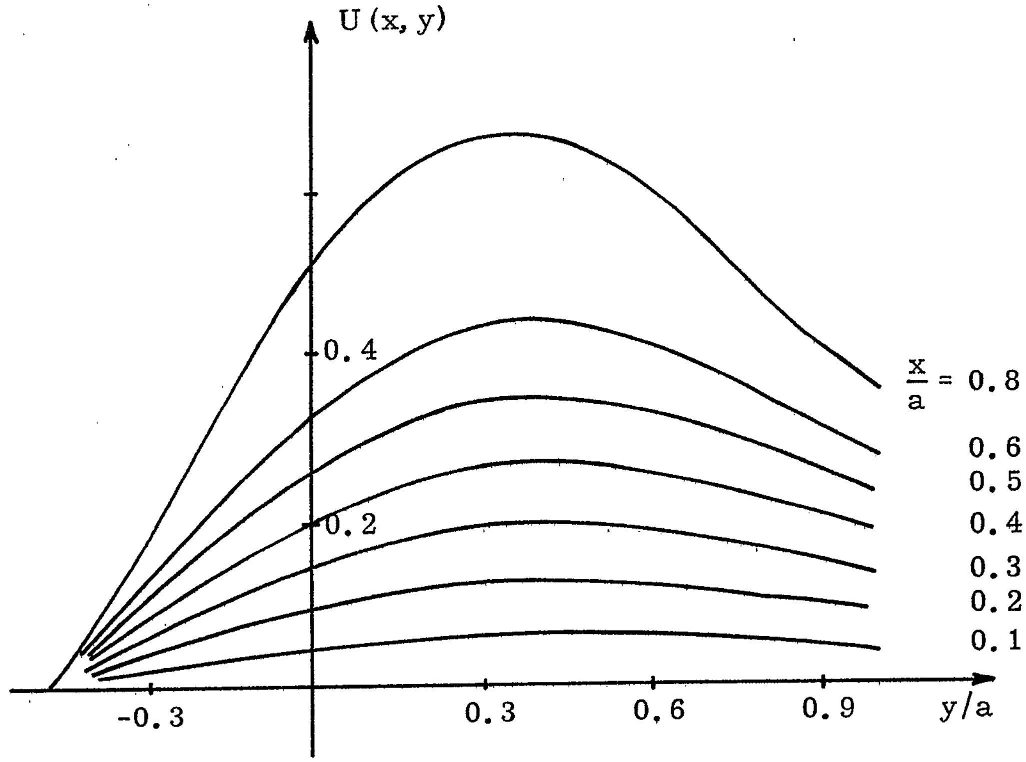
Thus the above equations are only estimates with limited validity.

VI. Conclusions and Summary

The expressions developed in Section IV of this note for the geometrical factor, efficiency factor, and electric field components of a multi-wire transmission line have been programmed for computer evaluation. A complete parametric study has not been carried out as of yet, but some preliminary results are shown in Figures 3 to 6. The number of wires making up each plate of the transmission line was chosen to be 51 somewhat arbitrarily, but it was found that data with N = 51 and large b/a agreed fairly well with Reference 4 and with experimental results using teledeltos plots. Thus one is led to believe that 51 wires is a pretty good approximation of a solid plate. On the other hand, numerical inversion of the $N \times N$ p_{rs} matrix is relatively simple, but increasing N much beyond 51 would require excessive amounts of computer time.

One should note that the technique used here to approximate a solid conducting plate by a number of thin, parallel wires all at the same potential can easily be generalized to other transmission line geometries. For instance, one might wish to study various curved plates in order to obtain better field uniformity. One would simply choose each wire's location on the surface of interest and develop an expression for the potential similar to that of equation 40. By requiring the wires to be at the same potential, one can approximate various equipotential surfaces formed by conducting sheets.

The use of a number of thin wires to approximate a solid sheet is more than just a theoretical technique, however. Many EMP simulators use wire mesh and/or a number of parallel wires instead of solid metal sheets. Both cost and mechanical considerations require the use of such "sparse" structures as approximations of solid ground planes and conducting plates.



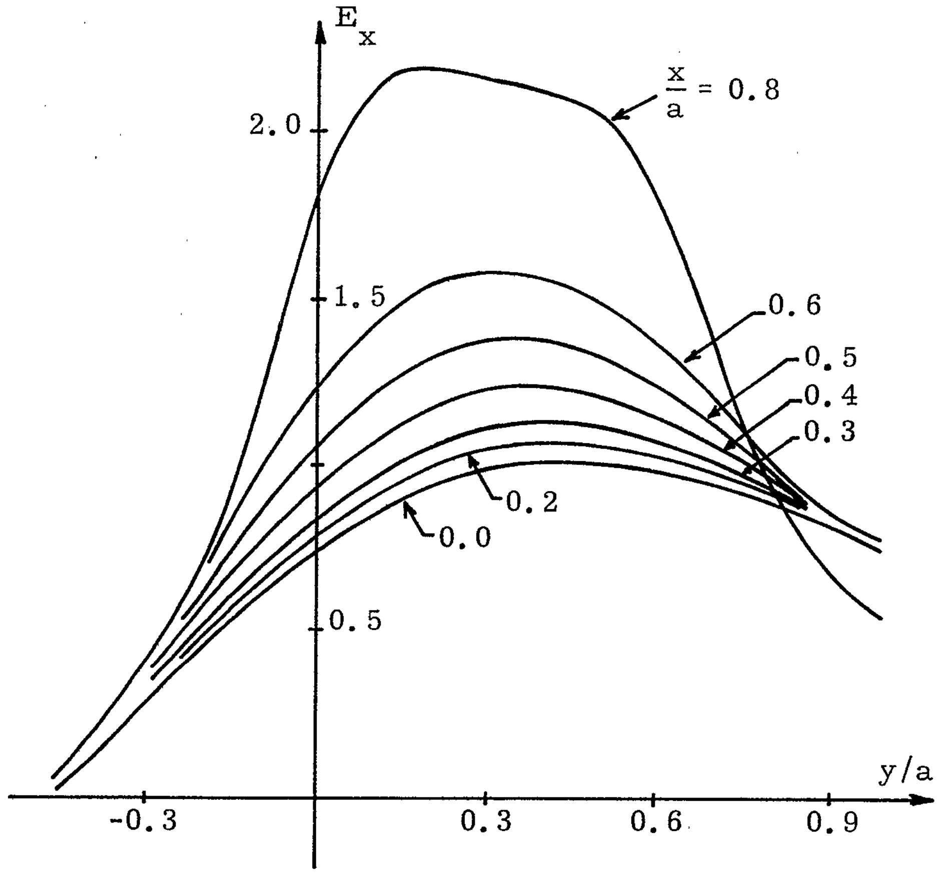
$N = 51$

$\frac{r_o}{a} = 2.65 \times 10^{-5}$

$\frac{d}{a} = 1.28 \times 10^{-2}$

$\frac{w}{a} = (N-1)d/a = .64$

Figure 3. Normalized Potential as a Function of y/a with x/a as a Parameter.



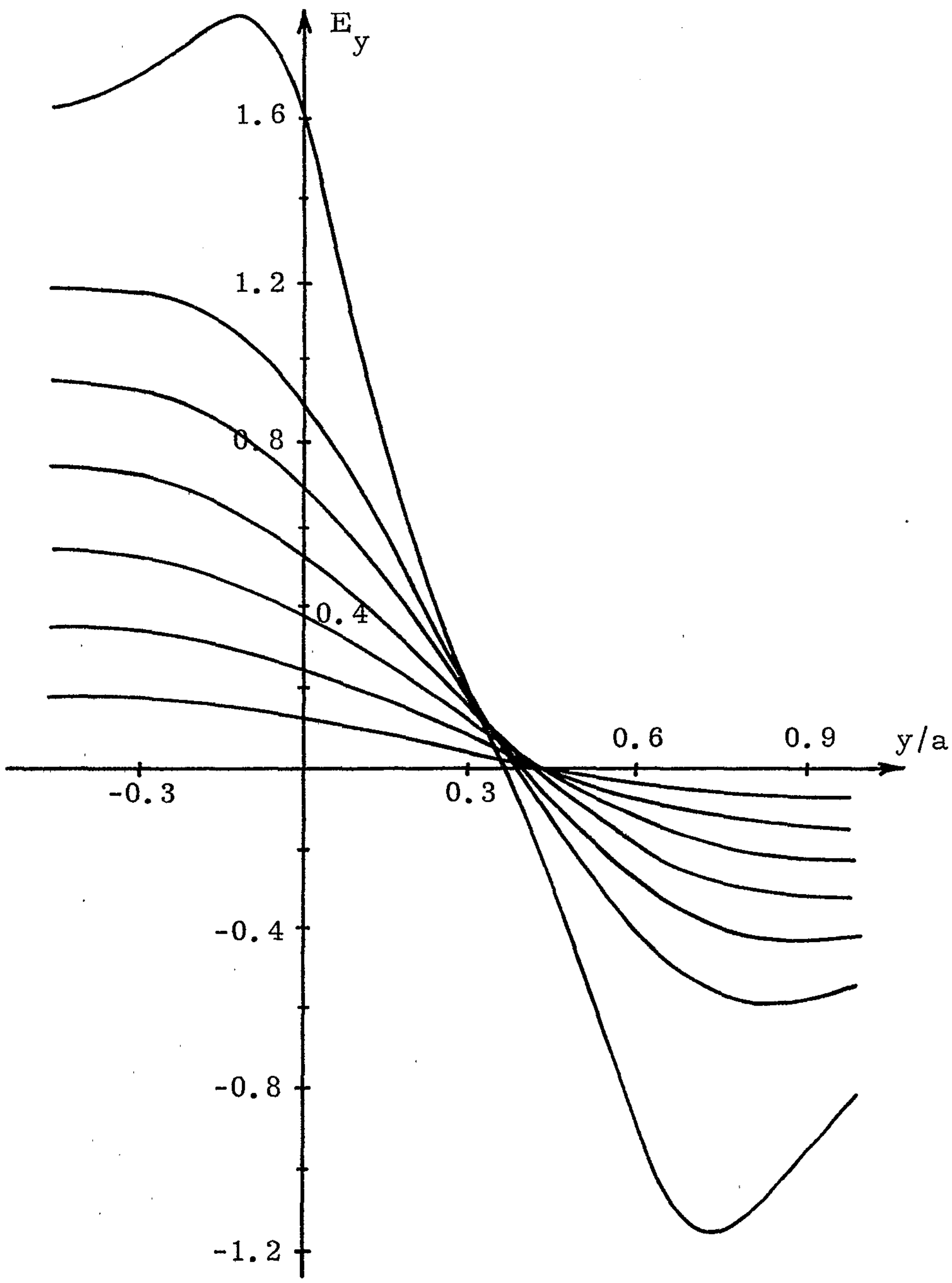
$N = 51$

$\frac{r_o}{a} = 2.65 \times 10^{-5}$

$\frac{d}{a} = 1.28 \times 10^{-2}$

$\frac{w}{a} = (N-1)d/a = .64$

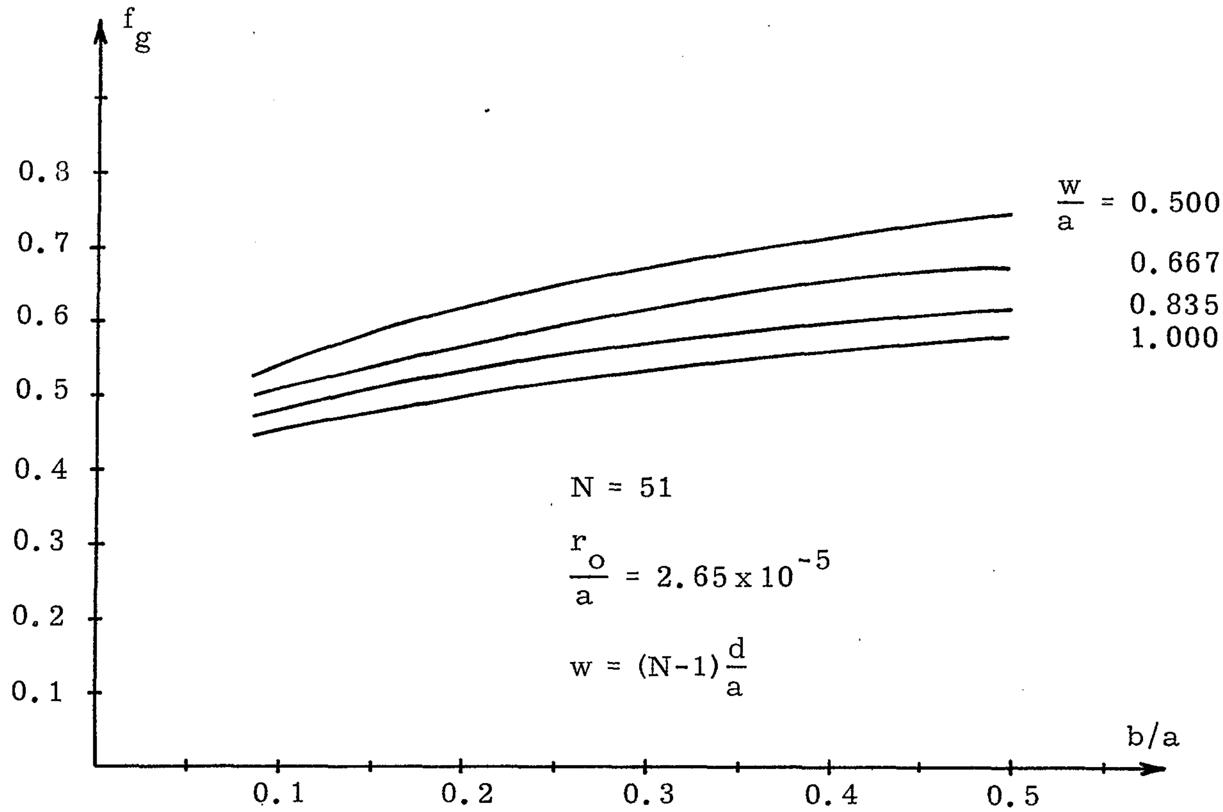
Figure 4. Relative Electric Field in the x-Direction as a Function of y/a with x/a as a Parameter.



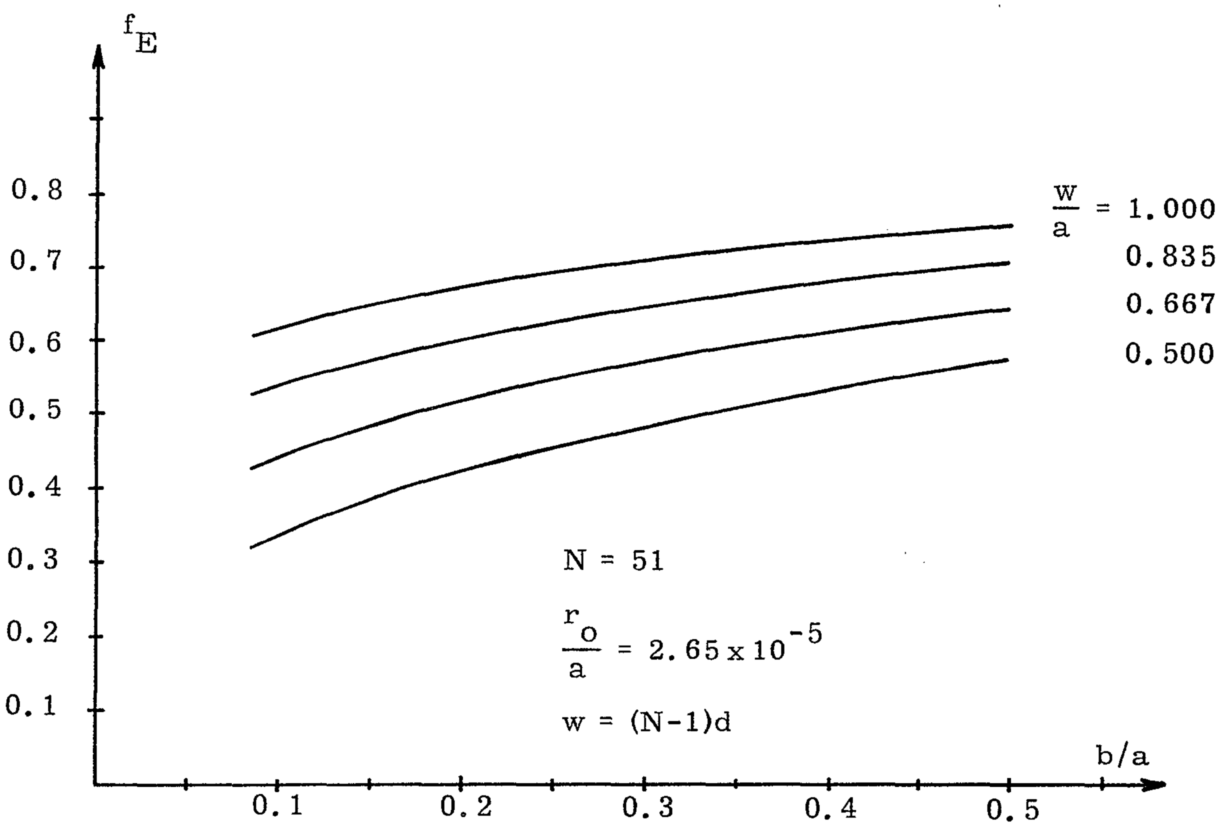
$N = 51$ $\qquad \frac{d}{a} = 1.28 \times 10^{-2}$

$\frac{r_o}{a} = 2.65 \times 10^{-5}$ $\qquad \frac{w}{a} = (N-1)\frac{d}{a} = .64$

Figure 5. Relative Electric Field In The y-Direction as a Function of y/a with x/a as a Parameter.



A. Geometrical Factor, f_g, Versus Height Above Ground



B. Efficiency Factor, f_E, Versus Height Above Ground

Figure 6. Effect of Ground Plane on f_g and f_E of a Horizontally Polarized Transmission Line.

Thus, the assumption made in this note that each plate of the transmission line is made up of a number of parallel wires is probably closer to the real case for a large horizontally polarized simulator than the assumption of a solid conducting plate would be.

The effect of replacing a solid conducting sheet by a grid of wires is considered in several other Sensor and Simulation Notes. Reference 4 treats the case of thin wires, as is assumed here. Reference 5 discusses relatively "thick" wires. Reference 6 goes on to calculate the difference in inductance between a grid of parallel wires and a planar conducting sheet and it is found that there is no change in inductance for certain ratios of wire size to wire spacing. (See Reference 6 for details.) If one applies the results of these references with the technique presented here, at least a fairly good qualitative understanding of the differences between a solid plate and a grid of wires can be obtained.

In summary then, this note has presented a method of calculating the TEM field distribution of a multi-wire parallel plate transmission line above a perfectly conducting ground. The method depends on the fact that the TEM field can be calculated from a potential function which satisfies Laplace's equation (rather than the more complicated wave equation). The potentials of a number of line charges are superpositioned to approximate thin wires, and the charge on each wire is chosen so that all the wires making up one "plate" of the transmission line are at the same potential. Maxwell's potential coefficients and a simple matrix inversion are used to solve for the proper charges. Once the potential function is known, relatively standard techniques are used to find the electric field at various points and the impedance of the transmission line.

References

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3. James Clerk Maxwell, *A Treatise on Electricity and Magnetism*, Clarendon Press, 1904.

4. Carl E. Baum, Sensor and Simulation Note 21, Impedances and Field Distributions for Parallel Plate Transmission Line Simulators, June 1966.

5. Lennart Marin, Sensor and Simulation Note 118, Effect of Replacing One Conducting Plate of a Parallel-Plate Transmission Line by a Grid of Rods, October 1970.

6. Carl E. Baum, Sensor and Simulation Note 127, Further Considerations for Multiturn Cylindrical Loops, April 1971.