Sensor and Simulation Notes

Note 157

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Capacitance Calculations for Satellites

Part I. Isolated Capacitances of Ellipsoidal Shapes with Comparisons to Some Other Simple Bodies

by

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Abstract

In this note, the free space electrostatic capacitance of conducting ellipsoids and spheroids is determined by solving Laplace's equation in ellipsoidal coordinates. The particular solutions are expressed in terms of incomplete elliptic integrals which are evaluated numerically to determine the capacitance. Degenerate cases of the general ellipsoid are also considered. In addition, an equivalent radius for each of the conducting bodies is defined and calculated. The results are presented in both graphical and tabular form.

ACKNOWLEDGEMENTS

I would like to thank Dr. Fred Tesche of Dikewood for suggesting the problem to me initially, and I especially would like to thank Dr. Carl Baum for his many ideas and helpful comments concerning the material included here. In addition, thanks go to Terry Brown, also of Dikewood, for his assistance in the numerical calculations.

I. INTRODUCTION

A current area of interest in EMP studies is that of attempting to understand system generated EMP. This effect occurs when a conducting obstacle in space (such as a satellite) is subjected to a high energy beam of gamma rays. These gamma rays cause electrons to be emitted from the conductor at one location, travel through space in the vicinity of the obstacle, and either be reabsorbed by the conductor, or escape to infinity. The result is that currents flow on and around the obstacle, providing a source of large electromagnetic fields.

A general discussion concerning the system generated EMP has been given previously [7]. A forthcoming note by Dr. C. E. Baum describes in detail this effect and proposes a new type of EMP simulator to test actual satellites for EMP hardness [11]. As mentioned in that note, one problem of interest is to compute the free space capacitance of various types of satellites as well as defining an equivalent radius for the body.

II. FORMULATION

The design of an actual satellite depends on many factors. In order to perform its desired functions, the satellite may need to be of a rather complicated geometry. It is difficult to define a general model with a geometry of enough complexity to exhibit the desired electrical characteristics of the actual satellite and of enough simplicity to allow an analytical or exact determination of these characteristics. As a first order approximation, one particular geometry, the ellipsoid, appears very promising for modeling arbitrarily shaped conducting bodies.

The problem of determining the free space electrostatic capacitance of perfectly conducting ellipsoids and spheroids has been considered by several authors [1, 2, 3]. It is also interesting that rather tight bounds for the capacitances of these bodies can be predicted by purely geometrical considerations. The numeral ranges of these bounds will be discussed in a later section of this note. A thorough discussion of this geometrical approach has been given by Polya and Szego [4]. The method presented here for determining the capacitance of an ellipsoid is essentially that given by Stratton [1].

Suppose that a perfectly conducting ellipsoid of semiaxes a, b, c, centered about the origin of a cartesian coordinate system carries a total charge Q. (See Figure 1.) One can state several conditions which the electrostatic potential, ϕ, produced by this charge must satisfy:

1. ϕ must satisfy Laplace's equation at all points not on the surface of the ellipsoid,
2. ϕ must be constant on the surface of the ellipsoid, and
3. ϕ must be regular at infinity.

The equation for the surface of an ellipsoid centered about the origin of a cartesian coordinate system may be written as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{1}$$

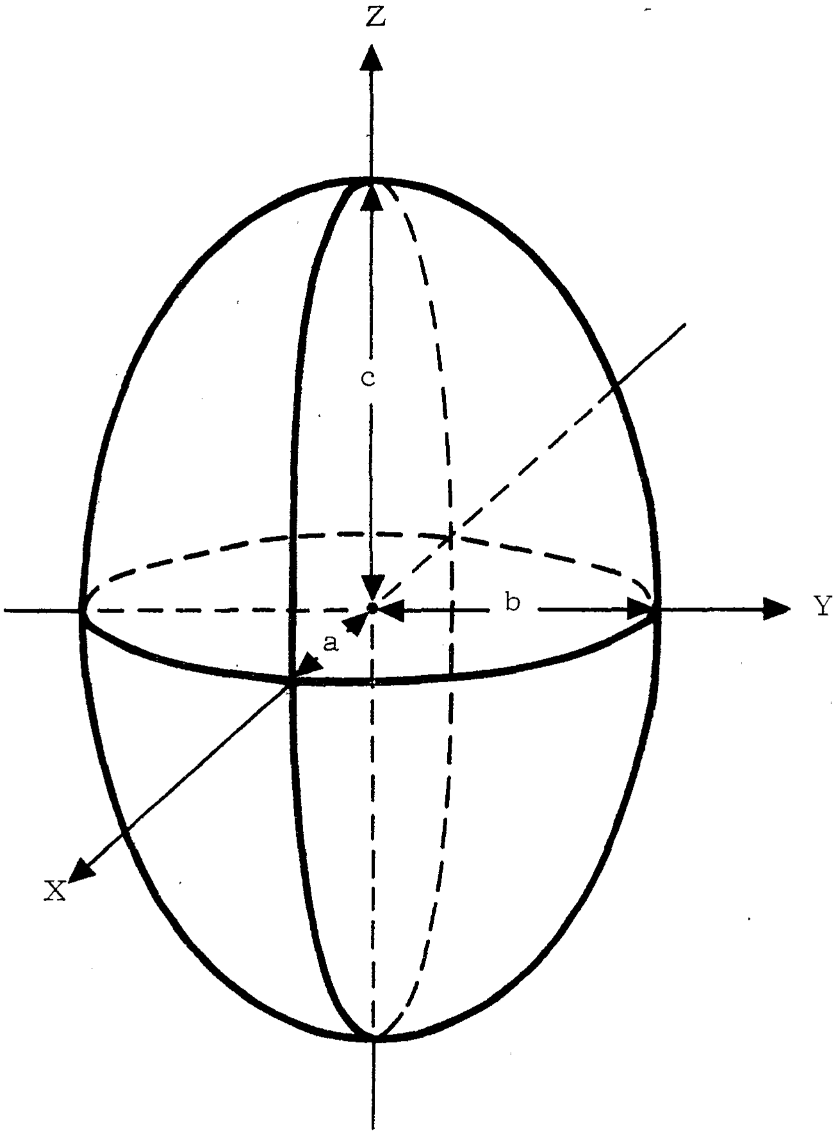

FIGURE 1. General ellipsoid $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,\ c > b > a\right)$.

For later convenience we will force the semiaxes to satisfy the inequality c>b>a. There is, however, a more general representation for the ellipsoidal surface which leads one to considerations in another coordinate system. The equations

$$\frac{x^2}{a^2+\xi} + \frac{y^2}{b^2+\xi} + \frac{z^2}{c^2+\xi} = 1 \ , \quad \xi > -c^2 \tag{2}$$

$$\frac{x^2}{a^2+\eta} + \frac{y^2}{b^2+\eta} + \frac{z^2}{c^2+\eta} = 1 \ , \quad -c^2 > \eta > -b^2 \tag{3}$$

$$\frac{x^2}{a^2+\zeta} + \frac{y^2}{b^2+\zeta} + \frac{z^2}{c^2+\zeta} = 1 \ , \quad -b^2 > \zeta > -a^2 \tag{4}$$

are the equations respectively of an ellipsoid, a hyperboloid of one sheet, and a hyperboloid of two sheets, all confocal with the ellipsoid given by equation (1). The three surfaces associated with the variables ξ, η, and ζ represent a new orthogonal curvilinear coordinate system usually referred to as the ellipsoidal coordinate system. Note that in equation (2) when $\xi = 0$, we have the surface of the conducting ellipsoid of semiaxes a, b, c centered about the origin.

The pertinent field relations, in particular Laplace's equation, can be written in terms of the new coordinates as (see Moon & Spencer [5])

$$(\eta-\zeta)R_\xi \frac{\partial}{\partial\xi}\left(R_\xi \frac{\partial\phi}{\partial\xi}\right) + (\zeta-\xi)R_\eta \frac{\partial}{\partial\eta}\left(R_\eta \frac{\partial\phi}{\partial\eta}\right) + (\xi-\eta)R_\zeta \frac{\partial}{\partial\zeta}\left(R_\zeta \frac{\partial\phi}{\partial\zeta}\right) = 0 \tag{5}$$

where

$$R_\alpha = \sqrt{(\alpha+a^2)(\alpha+b^2)(\alpha+c^2)} \tag{6}$$

Since ϕ must be constant on the surface of the conducting ellipsoid, it must be independent of η and ζ. Hence, the potential, ϕ, must be a function depending only on ξ which satisfies equation (5) exterior to the conducting ellipsoid, represents the correct value of the potential on the surface of the conducting ellipsoid, and is regular at infinity.

Under these considerations, Laplace's equation reduces to

$$\frac{d}{d\xi}\left(R_\xi \frac{d\phi}{d\xi}\right) = 0 \tag{7}$$

where

$$R_\xi = \sqrt{\left(\xi + a^2\right)\left(\xi + b^2\right)\left(\xi + c^2\right)} \tag{8}$$

Integrating this equation, one obtains

$$\phi(\xi) = C_1 \int_\xi^\infty \frac{d\xi}{R_\xi} \tag{9}$$

The upper limit is chosen as indicated to produce the proper behavior of ϕ at infinity. When ξ becomes very large, R_ξ approaches $\xi^{3/2}$ and

$$\phi(\xi) \approx \frac{2C_1}{\sqrt{\xi}} \quad \text{as } \xi \to \infty \tag{10}$$

Note also that equation (2) can be written as

$$\frac{x^2}{1+\frac{a^2}{\xi}} + \frac{y^2}{1+\frac{b^2}{\xi}} + \frac{z^2}{1+\frac{c^2}{\xi}} = \xi \tag{11}$$

If $r^2 = x^2 + y^2 + z^2$ is the distance from the origin to any point on the ellipsoid ξ, then as ξ becomes very large $\xi \rightarrow r^2$. Hence

$$\phi(\xi) \simeq \frac{2C_1}{r} \quad \text{for} \quad \xi \rightarrow \infty \tag{12}$$

We know that whatever the charge distribution, the dominant term of the potential function for large distances from that charge distribution is the potential of a point charge at the origin equal to the total charge.

$$\phi(\xi) \simeq \frac{Q}{4\pi\epsilon r} \quad \text{for} \quad \xi \rightarrow \infty \tag{13}$$

Therefore, the solution for the complete potential is

$$\phi(\xi) = \frac{Q}{8\pi\epsilon}\int_{\xi}^{\infty} \frac{d\xi}{R_{\xi}} \tag{14}$$

Now, since the total charge on the $\xi = 0$ ellipsoid is Q, the electrostatic capacitance is found to be

$$C = \frac{8\pi\epsilon}{\int_{0}^{\infty} R_{\xi}^{-1}\, d\xi} \tag{15}$$

Due to the form of R_ξ, the capacitance can be written in terms of an incomplete elliptic integral of the first kind. (See Abromowitz and Stegun [10].)

$$C = \frac{8\pi\epsilon\lambda}{F(\phi|m)} \qquad (16)$$

where

$$F(\phi|m) = \int_0^{\phi} \left(1 - m \sin^2\theta\right)^{-\frac{1}{2}} d\theta \qquad (17)$$

and

$$\lambda = c\sqrt{1 - \left(\frac{a}{c}\right)^2} \qquad (18)$$

$$\phi = \text{arc cos}\left(\frac{a}{c}\right) \qquad (19)$$

$$m = \frac{1 - \left(\frac{b}{c}\right)^2}{1 - \left(\frac{a}{c}\right)^2}, \quad m < 1 \qquad (20)$$

III. REDUCTION TO DEGENERATE CASES

As mentioned before, the capacitance (or the equivalent radius) of the general ellipsoid can be calculated in terms of the incomplete elliptic integral of the first kind. However, this integral does reduce to more elementary forms for several particular geometries, namely the prolate and oblate spheroids, the sphere, the elliptic disk, and the circular disk.

CASE I. Prolate Spheroid (See Figure 2, c>b = a). Equation (15) reduces to

$$C = \frac{8\pi\epsilon}{\displaystyle\int_0^\infty \frac{d\xi}{\left(\xi + c^2\right)\sqrt{\xi + b^2}}} \quad , \tag{21}$$

and upon evaluation of this integral,

$$C = \frac{8\pi\epsilon c\sqrt{1 - \left(\frac{b}{c}\right)^2}}{\ell n\left(\dfrac{1 + \sqrt{1 - \left(\frac{b}{c}\right)^2}}{1 - \sqrt{1 - \left(\frac{b}{c}\right)^2}}\right)} \quad . \tag{22}$$

CASE II. Oblate Spheroid (See Figure 3, c = b>a). Integral is of the same form as (21), and

$$C = \frac{4\pi\epsilon c\sqrt{1 - \left(\frac{a}{c}\right)^2}}{\arctan\left(\sqrt{\left(\frac{c}{a}\right)^2 - 1}\right)} \tag{23}$$

Note: The inequality (c > b > a) results in the different forms for the evaluation of the integral in equation (21).

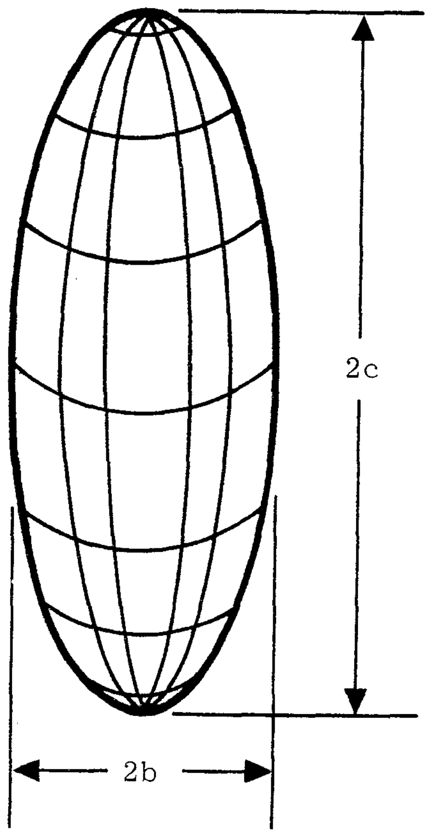

FIGURE 2. Prolate spheroid (c >b = a).

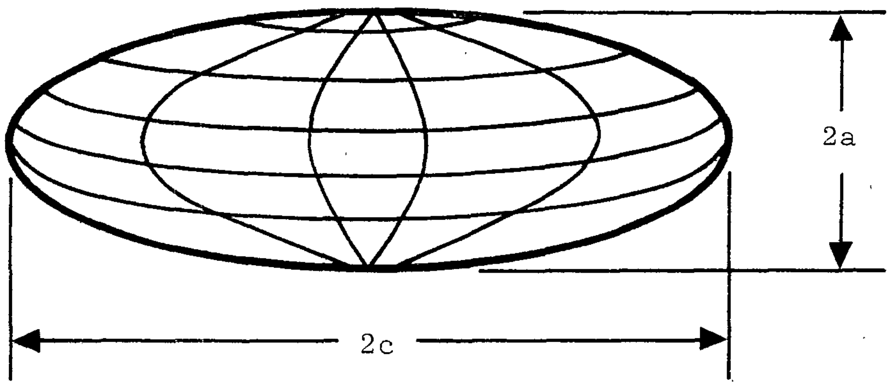

FIGURE 3. Oblate spheroid (c = b>a).

CASE III. Elliptic Disk (See Figure 4, $c > b > a = 0$). Equation (15) reduces to

$$C = \frac{8\pi\epsilon}{\int_0^\infty \frac{d\xi}{\sqrt{\xi\left(\xi+b^2\right)\left(\xi+c^2\right)}}} \tag{24}$$

The integral above is a complete elliptic integral of the first kind. Hence

$$C = \frac{4\pi\epsilon\, c}{K(m)} \tag{25}$$

where

$$K(m) = F\left(\frac{\pi}{2}\middle|m\right) = \int_0^{\pi/2} \left(1 - m \sin^2\theta\right)^{-1/2} d\theta \tag{26}$$

$$\text{and} \quad m = 1 - \left(\frac{b}{c}\right)^2 , \quad m < 1 \tag{27}$$

CASE IV. Circular Disk (See Figure 5, $c = b > a = 0$).

$$C = \frac{8\pi\epsilon}{\int_0^\infty \frac{d\xi}{\left(\xi + c^2\right)\sqrt{\xi}}} = 8\epsilon c \tag{28}$$

CASE V. Sphere (See Figure 6, $c = b = a$).

$$C = \frac{8\pi\epsilon}{\int_0^\infty \frac{d\xi}{\left(\xi + c^2\right)^{3/2}}} = 4\pi\epsilon c \tag{29}$$

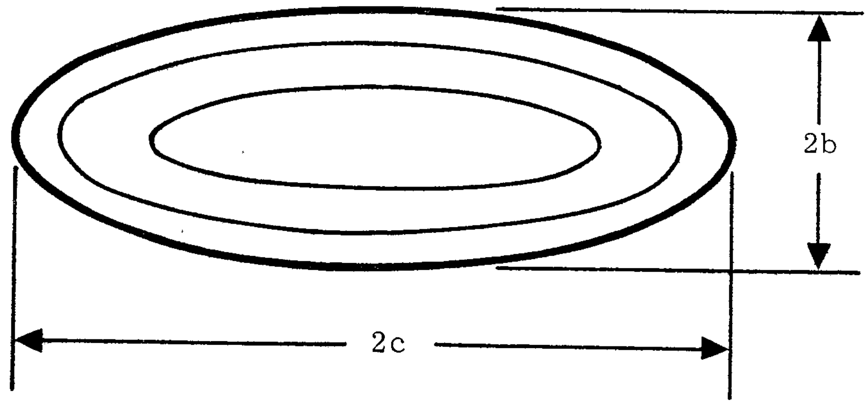

FIGURE 4. Elliptic disk (c>b>a = 0).

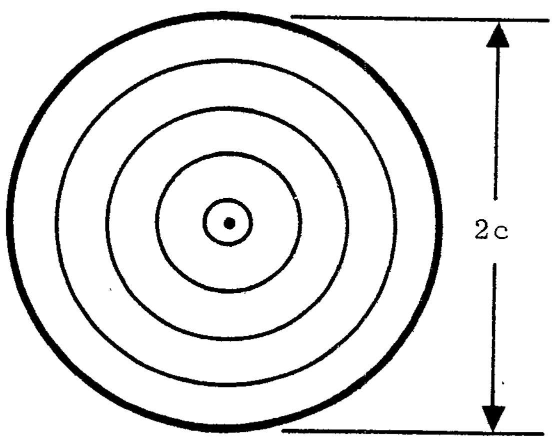

FIGURE 5. Circular disk (c = b>a = 0).

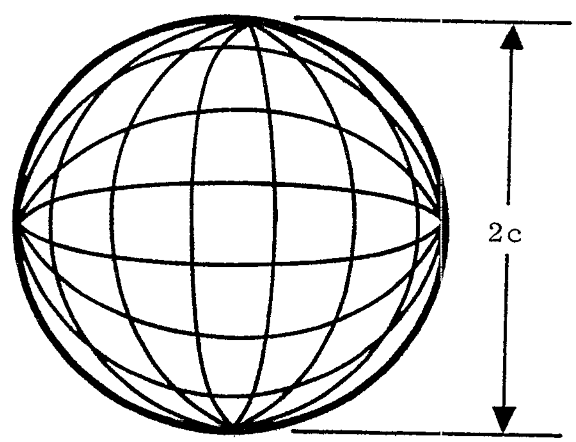

FIGURE 6. Sphere (c = b = a).

IV. EQUIVALENT RADII

A definition of an equivalent radius for conducting bodies of arbitrary shape has been suggested by Baum [7]. Suppose we know the capacitance of a particular conducting body as C, and we equate this capacitance to that of a sphere of radius r. Then solving for r, we define this r as the equivalent radius of the conducting body under consideration and denote it as r_{eq}. For example, consider the general ellipsoid. From equation (16) we have

$$C = \frac{8\pi\epsilon\lambda}{F(\phi\,|\,m)} \quad . \tag{30}$$

We know the capacitance of an isolated sphere of radius r is

$$C = 4\pi\epsilon r \quad . \tag{31}$$

Hence, the equivalent radius of the ellipsoid is

$$r_{eq} = \frac{2\lambda}{F(\phi|m)} \quad . \tag{32}$$

Finally we define the normalized equivalent radius as the equivalent radius divided by the largest semi-axis of the ellipsoid.

$$\frac{r_{eq}}{c} = \frac{2\lambda'}{F(\phi|m)} \tag{33}$$

$$\lambda' = \sqrt{1 - \left(\frac{a}{c}\right)^2} \tag{34}$$

$$\phi = \arc\cos\left(\frac{a}{c}\right) \tag{35}$$

$$m = \frac{1 - \left(\frac{b}{c}\right)^2}{1 - \left(\frac{a}{c}\right)^2} \quad , \; m < 1 \tag{36}$$

Note that once this equivalent radius is determined, then the capacitance of the conducting ellipsoid can be obtained immediately as

$$C = 4\pi\epsilon\, r_{eq} \quad . \tag{37}$$

V. NUMERICAL RESULTS

The numerical results are presented in terms of the normalized equivalent radius since the capacitance can be obtained from this directly.

Table 1 gives the normalized equivalent radius for the prolate spheroid for a variety of shapes ranging from the one extreme of the sphere to the other extreme of the infinitely thin needle. Figure 7 is the corresponding curve for this data.

Table 2 gives the normalized equivalent radius for the oblate spheroid for a variety of shapes ranging from the sphere to the infinitely thin circular disk. Figure 8 is the corresponding curve for this data.

Table 3 gives the normalized equivalent radius for the elliptic disk for a variety of shapes ranging from the circular disk to the infinitely thin line segment. Figure 9 is the corresponding curve for this data.

Figures 10, 11, 12, and 13 show plots of the normalized equivalent radius of a general ellipsoid. Note that the degenerate cases appear as boundary lines or points on this general plot.

b/c	r_{eq}/c	b/c	r_{eq}/c	b/c	r_{eq}/c
.01	.188733	.34	.539942	.67	.776757
.02	.217110	.35	.547540	.68	.783649
.03	.238016	.36	.555098	.69	.790530
.04	.255443	.37	.562616	.70	.797401
.05	.270791	.38	.570099	.71	.804262
.06	.284739	.39	.577546	.72	.811114
.07	.297670	.40	.584960	.73	.817957
.08	.309827	.41	.592343	.74	.824790
.09	.321369	.42	.599696	.75	.831615
.10	.332413	.43	.607021	.76	.838432
.11	.343045	.44	.614318	.77	.845239
.12	.353327	.45	.621590	.78	.852040
.13	.363310	.46	.628836	.79	.858832
.14	.373034	.47	.636059	.80	.865617
.15	.382531	.48	.643259	.81	.872395
.16	.391827	.49	.650438	.82	.879165
.17	.400946	.50	.657596	.83	.885929
.18	.409905	.51	.664733	.84	.892686
.19	.418720	.52	.671852	.85	.899436
.20	.427405	.53	.678951	.86	.906180
.21	.435971	.54	.686034	.87	.912917
.22	.444430	.55	.693099	.88	.919649
.23	.452788	.56	.700148	.89	.926374
.24	.461056	.57	.707181	.90	.933094
.25	.469239	.58	.714199	.91	.939808
.26	.477345	.59	.721203	.92	.946516
.27	.485377	.60	.728191	.93	.953220
.28	.493343	.61	.735167	.94	.959917
.29	.501245	.62	.742129	.95	.966610
.30	.509088	.63	.749079	.96	.973298
.31	.516876	.64	.756016	.97	.979981
.32	.524612	.65	.762941	.98	.986660
.33	.532300	.66	.769855	.99	.993333

TABLE 1. Normalized equivalent radius of the prolate spheroid.

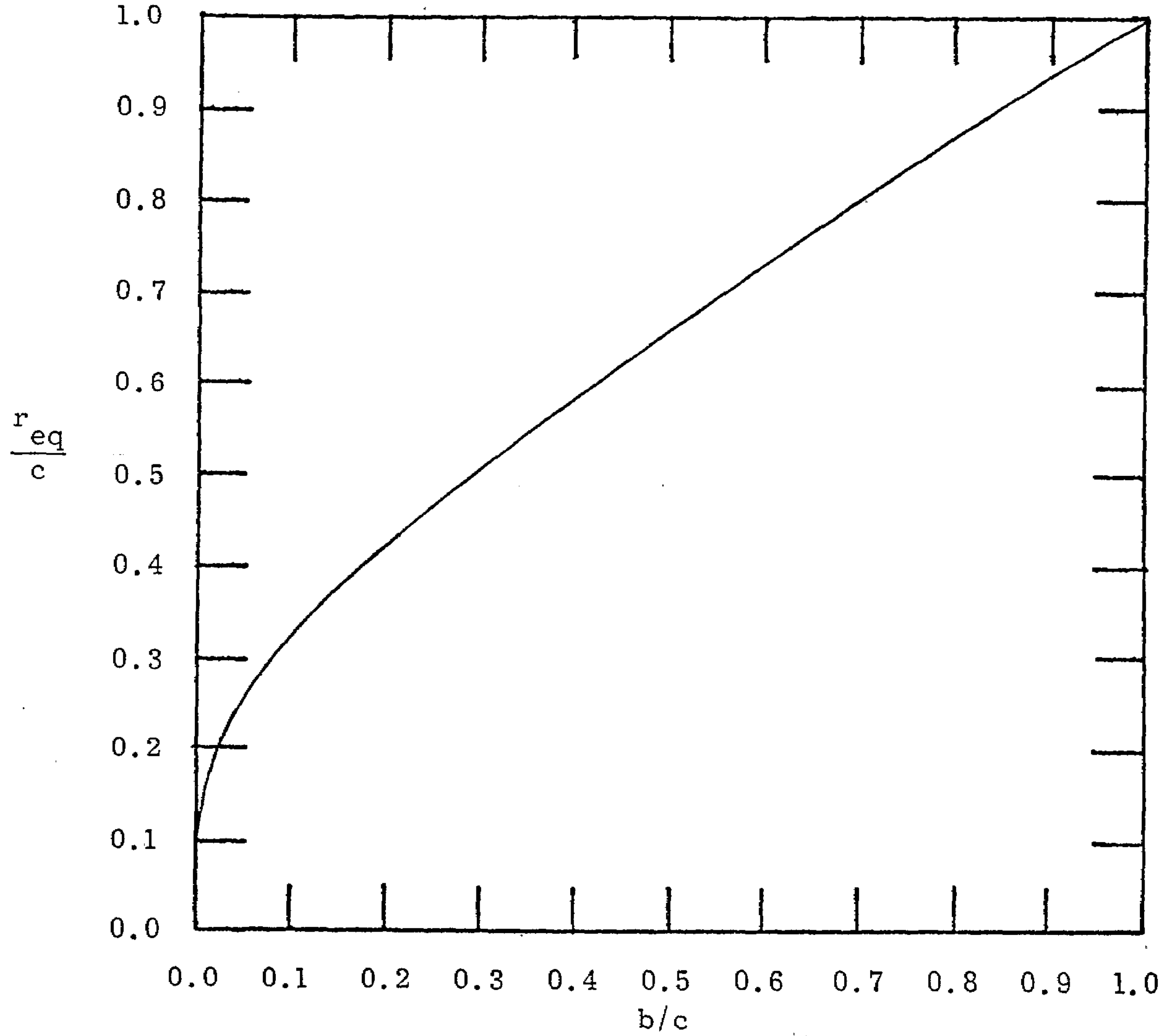

FIGURE 7. Normalized equivalent radius of the prolate spheroid ($c > b = a$).

$\frac{a}{c}$	$\frac{r_{eq}}{c}$	$\frac{a}{c}$	$\frac{r_{eq}}{c}$	$\frac{a}{c}$	$\frac{r_{eq}}{c}$
.01	.640667	.34	.768397	.67	.887369
.02	.644702	.35	.772116	.68	.890866
.03	.648725	.36	.775826	.69	.894358
.04	.652737	.37	.779528	.70	.897843
.05	.656737	.38	.783224	.71	.901323
.06	.660726	.39	.786911	.72	.904798
.07	.664704	.40	.790591	.73	.908267
.08	.668671	.41	.794263	.74	.911731
.09	.672627	.42	.797928	.75	.915188
.10	.676573	.43	.801586	.76	.918641
.11	.680508	.44	.805237	.77	.922090
.12	.684433	.45	.808881	.78	.925531
.13	.688347	.46	.812517	.79	.928968
.14	.692252	.47	.816146	.80	.932401
.15	.696146	.48	.819769	.81	.935826
.16	.700031	.49	.823385	.82	.939247
.17	.703906	.50	.826994	.83	.942665
.18	.707771	.51	.830597	.84	.946076
.19	.711626	.52	.834192	.85	.949483
.20	.715473	.53	.837781	.86	.952884
.21	.719310	.54	.841364	.87	.956281
.22	.723138	.55	.844939	.88	.959672
.23	.726957	.56	.848509	.89	.963059
.24	.730767	.57	.852072	.90	.966441
.25	.734568	.58	.855629	.91	.969818
.26	.738361	.59	.859180	.92	.973192
.27	.742145	.60	.862725	.93	.976562
.28	.745920	.61	.866264	.94	.979921
.29	.749687	.62	.869796	.95	.983281
.30	.753445	.63	.873322	.96	.986638
.31	.757195	.64	.876843	.97	.989987
.32	.760938	.65	.880357	.98	.993341
.33	.764671	.66	.883866	.99	.996713

TABLE 2. Normalized equivalent radius of the oblate spheroid.

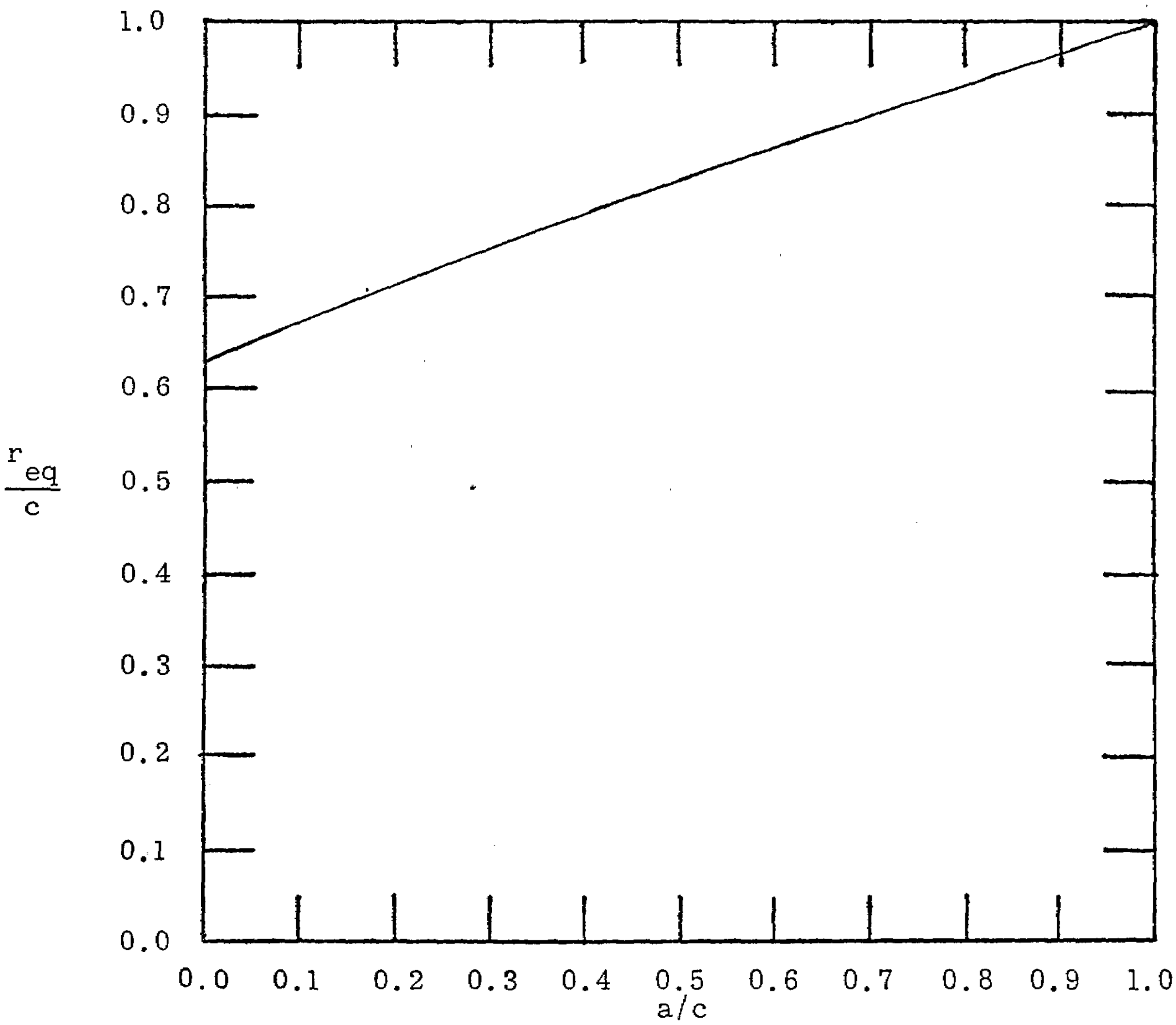

FIGURE 8. Normalized equivalent radius of the oblate spheroid (c = b> a).

$\frac{b}{c}$	$\frac{r_{eq}}{c}$	$\frac{b}{c}$	$\frac{r_{eq}}{c}$	$\frac{b}{c}$	$\frac{r_{eq}}{c}$
.01	.166901	.34	.398392	.67	.526324
.02	.188724	.35	.402736	.68	.529854
.03	.204343	.36	.407038	.69	.533370
.04	.217079	.37	.411300	.70	.536872
.05	.228095	.38	.415523	.71	.540360
.06	.237948	.39	.419710	.72	.543835
.07	.246957	.40	.423861	.73	.547297
.08	.255317	.41	.427979	.74	.550746
.09	.263163	.42	.432064	.75	.554182
.10	.270589	.43	.436117	.76	.557606
.11	.277665	.44	.440141	.77	.561017
.12	.284442	.45	.444136	.78	.564417
.13	.290963	.46	.448103	.79	.567806
.14	.297260	.47	.452043	.80	.571183
.15	.303360	.48	.455957	.81	.574548
.16	.309284	.49	.459846	.82	.577903
.17	.315052	.50	.463711	.83	.581247
.18	.320677	.51	.467552	.84	.584580
.19	.326173	.52	.471371	.85	.587903
.20	.331553	.53	.475167	.86	.591216
.21	.336824	.54	.478942	.87	.594519
.22	.341996	.55	.482697	.88	.597812
.23	.347077	.56	.486431	.89	.601096
.24	.352073	.57	.490145	.90	.604370
.25	.356989	.58	.493841	.91	.607634
.26	.361832	.59	.497518	.92	.610890
.27	.366605	.60	.501177	.93	.614136
.28	.371313	.61	.504819	.94	.617373
.29	.375961	.62	.508443	.95	.620602
.30	.380550	.63	.512051	.96	.623822
.31	.385086	.64	.515642	.97	.627034
.32	.389569	.65	.519218	.98	.630237
.33	.394004	.66	.522778	.99	.633433

TABLE 3. Normalized equivalent radius of the elliptic disk.

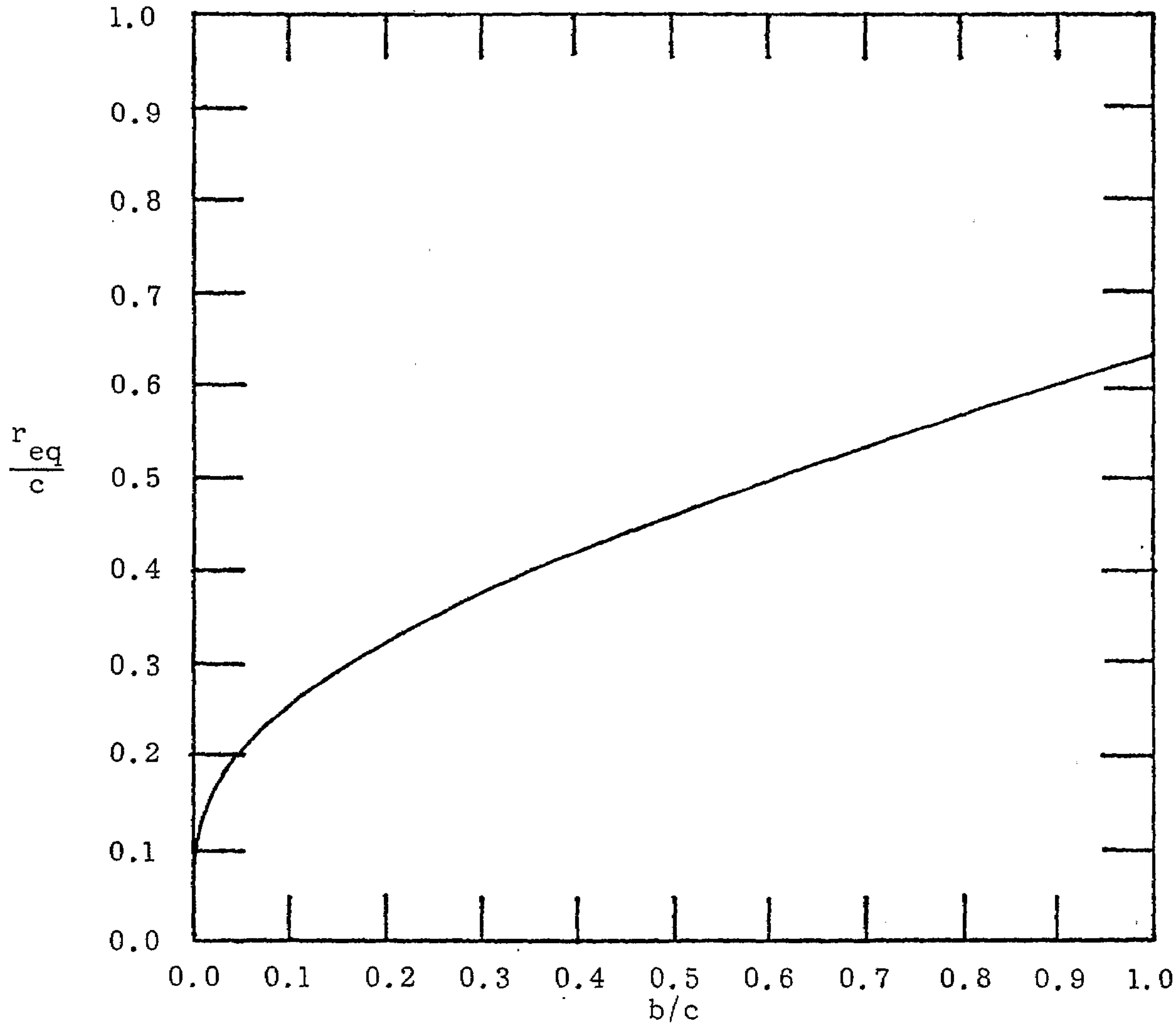

FIGURE 9. Normalized equivalent radius of the elliptic disk ($c > b > a = 0$).

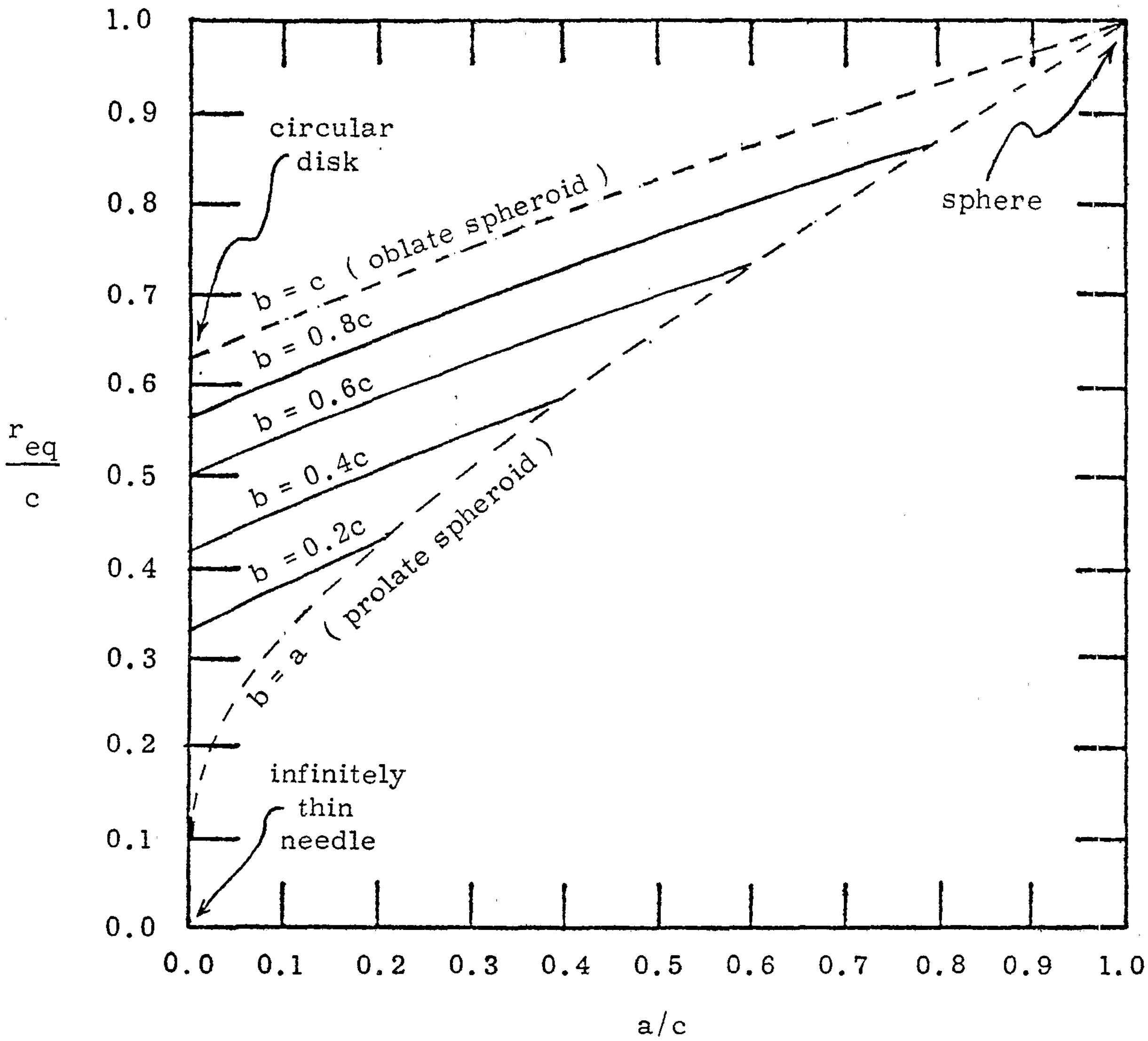

FIGURE 10. Normalized equivalent radius of the general ellipsoid (c>b>a).

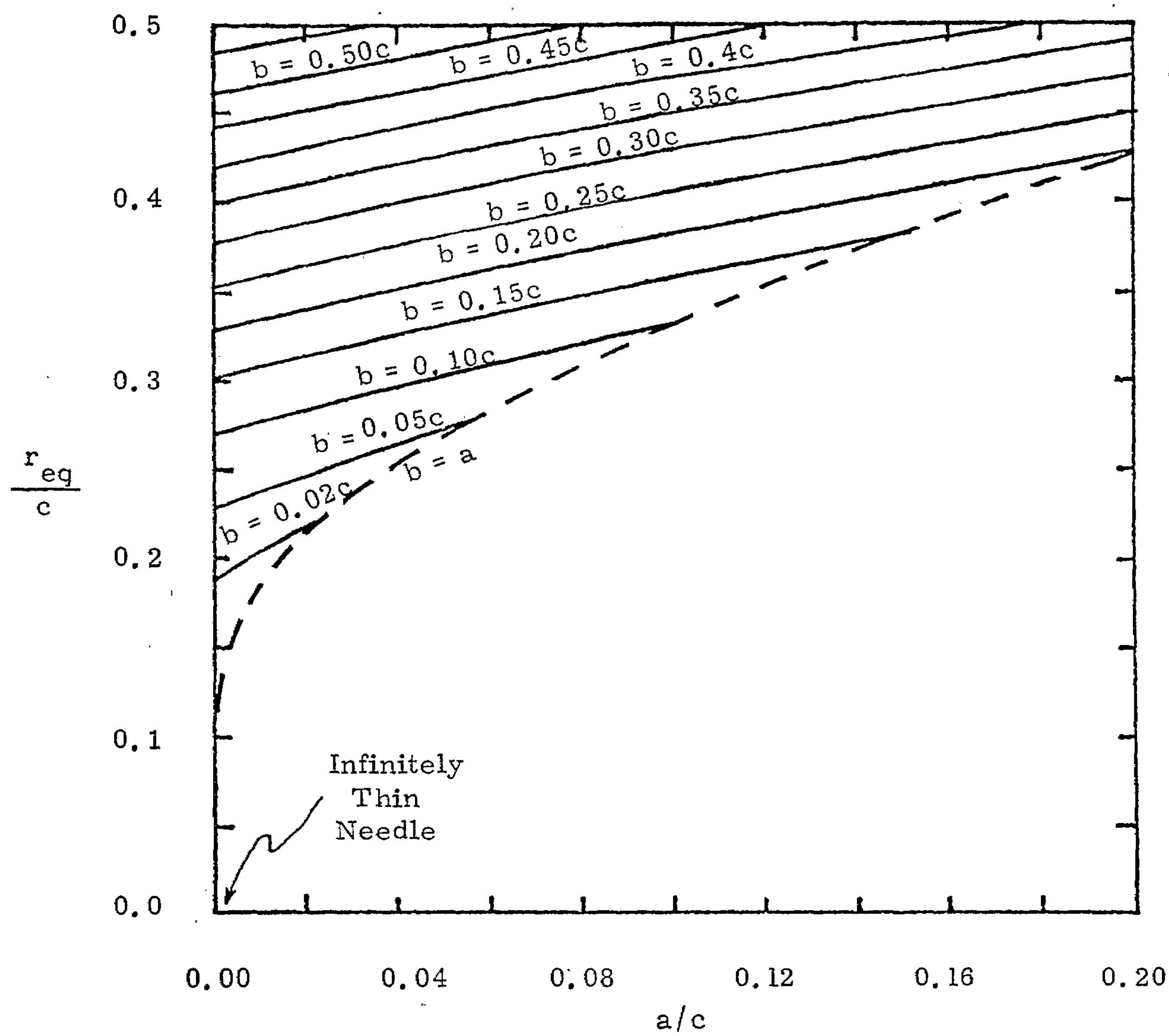

FIGURE 11. Normalized equivalent radius of the general ellipsoid (c>b>a, expanded view).

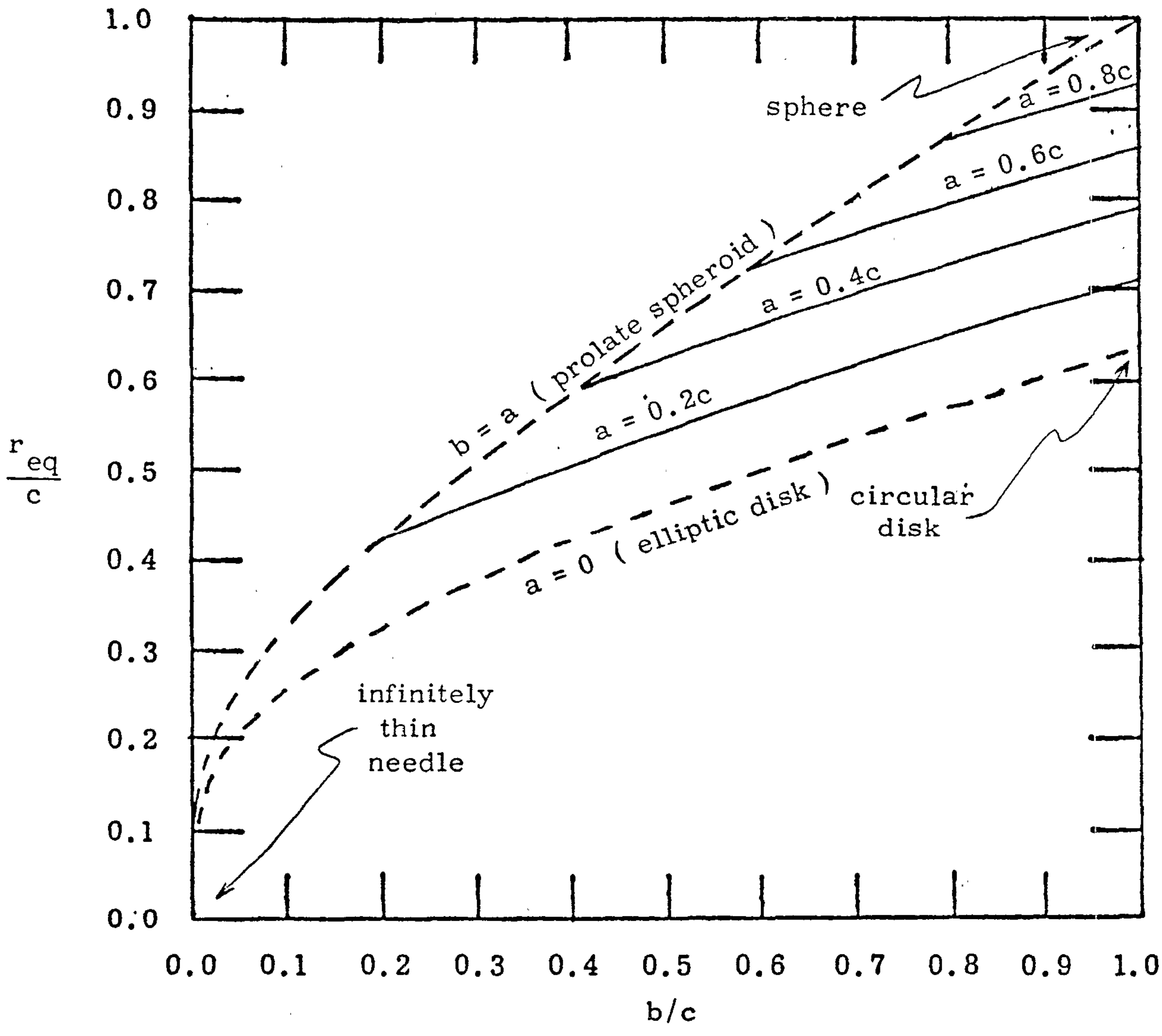

FIGURE 12. Normalized equivalent radius of the general ellipsoid (c>b>a).

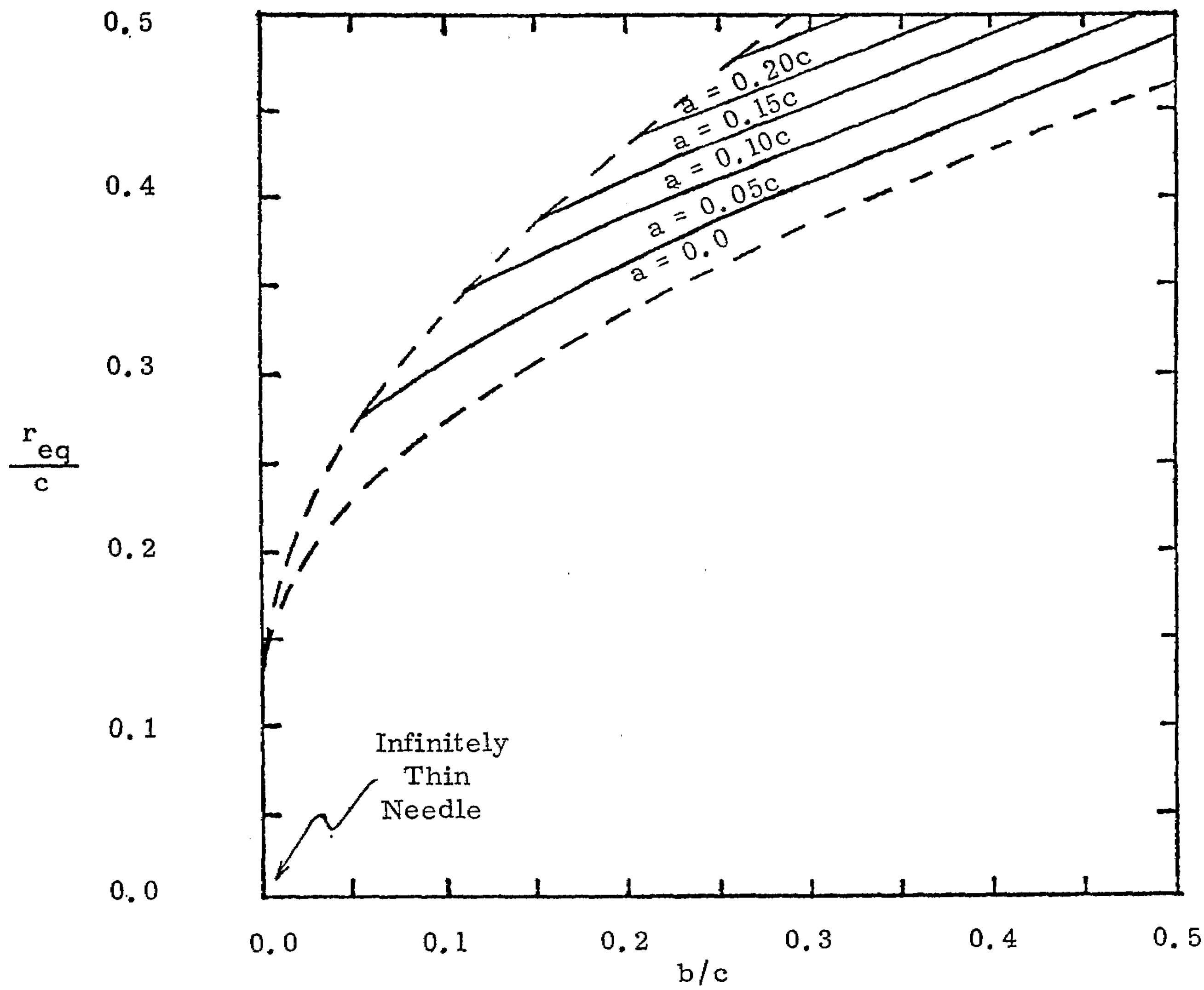

FIGURE 13. Normalized equivalent radius of the general ellipsoid (c>b>a, expanded view).

VI. COMPARISON WITH OTHER DATA

Suppose we wish to determine the capacitance or equivalent radius of a conducting body which is not an ellipsoid or one of its degenerate cases. It has been suggested by Baum [7] that the results obtained for the ellipsoid might be used to approximate the capacitance of the body under consideration. If we circumscribe the given body with an ellipsoid, the capacitance of this ellipsoid can serve as an upper bound on the capacitance of the body itself. Similarly, if we inscribe the given body with an ellipsoid, we can obtain a lower bound on the capacitance of the body. A simple average of these two bounds might serve as a first order approximation to the true capacitance of the body under consideration.

As an example, suppose we wish to determine the capacitance of a right circular cylinder of length 2c and diameter 2a. We inscribe an ellipsoid with semiaxes (c, a, a) and circumscribe an ellipsoid with semiaxes ($\sqrt{2}c$, $\sqrt{2}a$, $\sqrt{2}a$). Note in this case, the ellipsoids are actually spheroids due to the inherent symmetry of the cylinder. This produces bounds for the capacitance. If $c = 2a$, then

$$8.27 < \frac{C}{c\epsilon} < 11.7 \quad . \tag{38}$$

An approximate value of the capacitance is the average of these bounds. Hence,

$$\frac{C}{c\epsilon} \simeq 9.97 \quad . \tag{39}$$

The capacitance of a right circular cylinder for $c = 2a$ has been calculated numerically by Smythe [8] to be

$$\frac{C}{c\epsilon} = 9.88 \quad . \tag{40}$$

Polya and Szego [4] discuss several types of bounds and approximations based on the geometrical parameters of conducting solids such as the volume, surface area, and mean curvature. One interesting approximation proposed is related to the surface area.

$$C \simeq 4\pi\epsilon \left(\frac{S}{4\pi}\right)^{1/2} \tag{41}$$

where S is the total surface area of the solid. For example, the cylinder described previously has a surface area of

$$S = \frac{5}{2}\pi c^2, \tag{42}$$

and consequently the capacitance is approximately

$$\frac{C}{\epsilon c} \simeq 9.93 \tag{43}$$

Table 4 presents data obtained in the same way as that above for various conducting bodies. The two dimensional bodies are approximated with elliptic disks and the three dimensional ones with ellipsoids or spheroids. Whenever possible, capacitances calculated by other means have also been indicated for comparison. The square plate is of side 2c, the cube is of side 2c, the right circular cylinder is of length 2c and diameter 2a, and the rectangular parallelopiped is of length 2c, width 2b, and height 2a. (Note: $c \geq b \geq a$ for all bodies.)

body	bounds of $\frac{C}{\epsilon c}$ obtained using similar ellipsoids	average of bounds	approximation of $\frac{C}{\epsilon c}$ from surface area	calculated value of $\frac{C}{\epsilon c}$	investigator
square plate	$8.00 < \frac{C}{\epsilon c} < 11.3$	9.66	9.02	8.92	Harrington [9]
cube	$12.6 < \frac{C}{\epsilon c} < 21.7$	17.2	17.4	16.5	Reitan and Higgins [6]
cylinder (c=a)	$12.6 < \frac{C}{\epsilon c} < 17.8$	15.2	15.4	15.0	Smythe [8]
cylinder (c=2a)	$8.27 < \frac{C}{\epsilon c} < 11.7$	9.97	9.93	9.88	Smythe [8]
cylinder (c=4a)	$5.88 < \frac{C}{\epsilon c} < 8.32$	7.10	6.66	6.97	Smythe [8]
cylinder (c=8a)	$4.47 < \frac{C}{\epsilon c} < 6.32$	5.40	4.58	5.22	Smythe [8]
parallelopiped (a=0.8c b=c)	$11.7 < \frac{C}{\epsilon c} < 20.3$	15.4	16.2	-	-
parallelopiped (a=0.6c b=0.8c)	$10.0 < \frac{C}{\epsilon c} \leq 17.3$	13.7	13.7	-	-
parallelopiped (a=0.4c b=0.6c	$8.26 < \frac{C}{\epsilon c} < 14.3$	11.3	11.2	-	-
parallelopiped (a=0.2c b=0.4c)	$6.38 < \frac{C}{\epsilon c} < 11.1$	8.72	8.27	-	-

TABLE 4. Capacitance approximations for simple bodies.

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