Some Limitations on Microwave Air-Conductivity Measurements

I. Introduction

Because of the importance of the air conductivity to the nuclear EMP problem it is often desired to measure this parameter in the nuclear EMP environment, simultaneously with measurements of the electromagnetic field components. One often suggested technique is the measurement of the amplitude and/or phase shift of a microwave signal which is transmitted from one antenna and received by another antenna in this conducting environment. However, this measurement can be influenced by several phemomena, including the electric field dependence of the electron mobility, changes in the antenna characteristics, and the magnetic field dependence of the electron mobility, causing a possible rotation of the microwave polarization. The last of these effects is of minor significance because, for sea level air, the EMP magnetic fields are not intense enough to raise the electron cyclotron (or Larmor) frequency to near the electron momentum transfer collision frequency in air. Hence, this last effect will not be considered in this note.

However, the electric field dependence of the electron mobility and changes in the microwave antenna characteristics can affect the validity of the air-conductivity measurement. Each of these effects will be considered with emphasis on the first because of its fundamental relationship to this type of air-conductivity measurement. We will be concerned principally with electron conductivity effects but most of the results can be used by extension for ionic conductivity effects.

II. Microwave Air Conductivity: Small Electric Fields

To better understand the adverse effects of the electric field dependence of the electron mobility on the microwave conductivity, first consider the simpler case in which the electric field is sufficiently small that the electron mobility is independent of the electric field strength. With this restriction we have, for a microwave with time dependence $e^{j\omega t}$, a complex electron mobility given by

$$\mu = \mu_r + j\mu_i \tag{1}$$

where

$$\mu_r = \frac{e}{m} \frac{\nu}{\omega^2+\nu^2} \tag{2}$$

and

$$\mu_i = \frac{-e}{m} \frac{\omega}{\omega^2+\nu^2} \tag{3}$$

where e/m is the electron charge to mass ratio, ω is the radian microwave frequency, ν is the electron momentum transfer collison frequency, and t is time. These mobility equations can be derived from simplified equations of motion for the conduction electrons. More accurate expressions involve integrals over the electron energy spectrum since ν is a function of the electron energy.[1] However, the assumption of a single collision frequency does not introduce a significant error.

If we now assume a collision frequency given by[2]

$$\nu = \frac{e}{m\mu_{DC}} \tag{4}$$

where μ_{DC} is the low frequency (and small electric field) value of the electron mobility, we then have

$$\mu_r = \frac{\mu_{DC}}{1 + (\frac{m}{e}\,\omega\,\mu_{DC})^2} \tag{5}$$

and

$$\mu_i = -\frac{(\frac{m}{e}\,\omega\mu_{DC})\mu_{DC}}{1 + (\frac{m}{e}\,\omega\mu_{DC})^2} \tag{6}$$

Using this last formulation we can see that in the limit of low frequencies ($\omega << \nu$)

$$\mu_r \approx \mu_{DC} \tag{7}$$

and

$$\mu_i \approx 0 \tag{8}$$

as we would expect. For high frequencies ($\omega >> \nu$) we have

$$\mu_r \approx 0 \tag{9}$$

and

$$\mu_i \approx -\frac{e}{m\omega} \tag{10}$$

which is another simple result.

Looking at the high frequency case just derived we can see that the microwave mobility is independent of μ_{DC}. Thus, a high frequency measurement does not really measure the conductivity of the gas for EMP purposes. Rather, it measures the electron number density, n, since the microwave conductivity is proportional to the product of the electron number density and the microwave mobility. Since such a measurement is independent of μ_{DC}, it is also unaffected by the dependence of the electron mobility on the electric field (to be considered later).

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1. Margenau, H., <u>Phys. Rev. 69</u>, 508 (1964)

2. All units are taken in the rationalized m.k.s. system unless otherwise specified.

Now, for dry STP air (273°K, 760 mm Hg) we have, using the definition of equation (4),

$$\mu_{DC} \approx 1.57 \text{ meter}^2/\text{volt-sec} \tag{11}$$

and thus

$$\nu \approx 1.1 \times 10^{11} \text{ sec}^{-1} \tag{12}$$

The microwave frequency, f, which can be considered the transition between the high and low frequency cases is (for dry STP air)

$$f = \frac{\nu}{2\pi} \approx 1.8 \times 10^{10} \text{ cps} = 18 \text{ gigacycles} \tag{13}$$

This parameter scales inversely with μ_{DC}. (Look ahead to figure 1 to see the variation of μ_{DC} (μ_e for $E = 0$) with density and water vapor content.) If the probing frequency is much higher than about 18 gigacycles then the electron density is measured; if it is much less, the air conductivity is measured. Thus far, we have considered two types of measurements: an air conductivity measurement or an electron density measurement. Ideally, we may wish to measure the air conductivity, but this measurement has some difficulties which we shall now discuss.

III. Microwave Air Conductivity: Large Electric Fields

In the nuclear EMP environment, the large electric fields raise the electron temperature and make the electron mobility (as "seen" by the EMP electric field) and conductivity functions of this large electric field. The electron mobility, μ_e, shown in figure 1, is a function of the molecular density, N, of the air under consideration, the molecular fraction of water vapor, f_{H_2O}, present, and the magnitude of the electric field.[3] However, the effective mobility and thus conductivity is not necessarily the same as that "seen" by the large EMP electric field. If we assume that the radian microwave frequency is much less than the electron momentum transfer collision frequency, there is no imaginary component to the effective microwave conductivity. The effective microwave conductivity can then be calculated by using a perturbation analysis (for sufficiently small microwave electric fields) on the basic conductivity relationships. However, if our results require that the electron temperature change because of the microwave electric field, as they will require for the microwave field parallel to the EMP electric field, then for validity of our results the probing frequency will need to be always much less than the electron energy exchange collision frequency (which is in turn much less than the electron momentum transfer collision frequency).[4]

Thus, as illustrated in figure 2, we may assume electric fields and current densities associated with both the EMP and the microwave signal,

3. These curves are constructed from the measurements of Dr. Phelps of Westinghouse using the techniques outlined in EMP Theoretical Notes VI and XII by this author.

4. See EMP Theoretical Notes VI and XII for a discussion of these two types of collision frequencies.

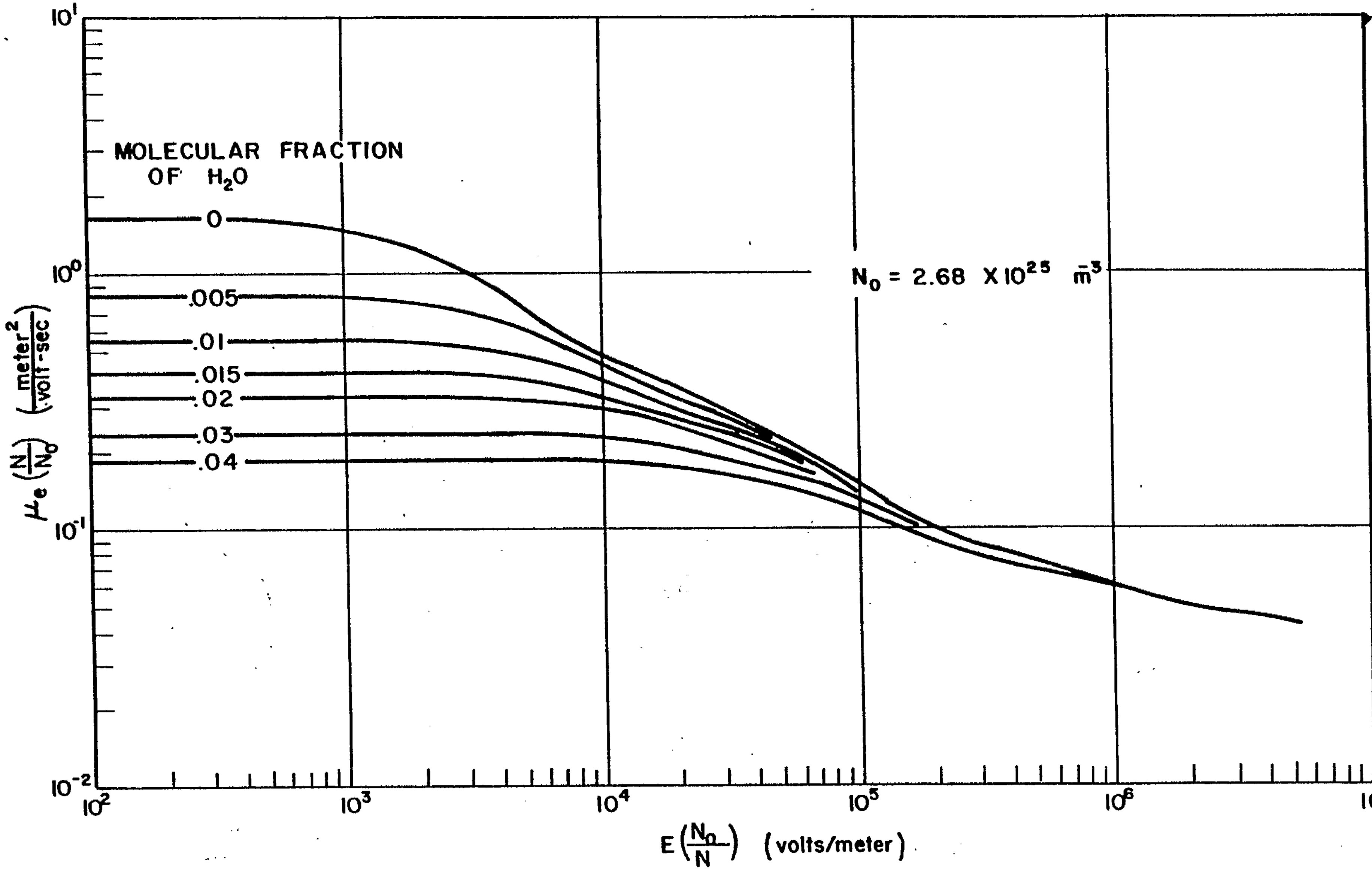

Fig. 1. ELECTRON MOBILITY IN AIR

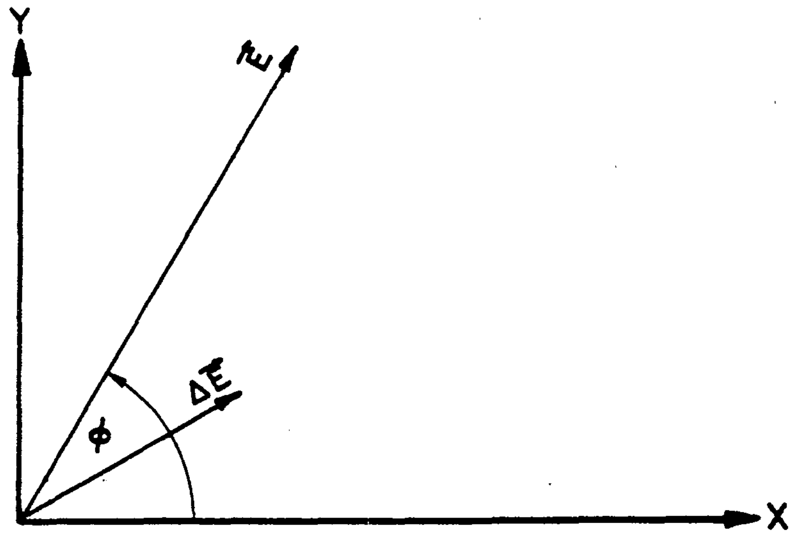

Fig. 2. COORDINATE SYSTEM FOR (σ_Δ) CALCULATION

given by

	EMP	microwave signal
electric field	$\vec{E}$	$\Delta\vec{E}$
magnitude	E	ΔE
components	E_x, E_y	ΔE_x, ΔE_y
conduction current density	$\vec{J}$	$\Delta\vec{J}$
magnitude	J	ΔJ
components	J_x, J_y	ΔJ_x, ΔJ_y

The basic equation to be considered is simply

$$\vec{J} = \sigma (E) \vec{E} \tag{14}$$

or

$$J_x = \sigma (E) E_x \tag{15}$$

and

$$J_y = \sigma (E) E_y \tag{16}$$

where

$$\sigma (E) = en \mu_e(E) \tag{17}$$

where e is the magnitude of the electron charge, n is the electron number density, and $\mu_e(E)$ (or just μ_e) is the electron mobility, a function of the magnitude of the total electric field present. Because of the assumed small size of the microwave electric field the EMP fields and currents are essentially equal to the total fields and currents so that equations (14) through (17) apply directly both to the EMP fields and currents and to the total fields and currents. Note that the total conduction current density is parallel to the total electric field. This last property will not necessarily apply to the microwave conduction current density and electric field, but the effective microwave conductivity can be calculated by using a perturbation analysis on equations (14) through (17).

Thus, given $\vec{E}$ and $\sigma(E)$, we can calculate that $\Delta\vec{J}$ which will result from a given $\Delta\vec{E}$ by considering the first order terms in a Taylor expansion for $\Delta\vec{J}$, i.e.,

$$\Delta J_x = \frac{\partial J_x}{\partial E_x} \Delta E_x + \frac{\partial J_x}{\partial E_y} \Delta E_y \tag{18}$$

and

$$\Delta J_y = \frac{\partial J_y}{\partial E_x} \Delta E_x + \frac{\partial J_y}{\partial E_y} \Delta E_y \tag{19}$$

This can be rewritten in matrix form as

$$(\Delta J_x, \Delta J_y) = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \cdot \begin{pmatrix} \Delta E_x \\ \Delta E_y \end{pmatrix} \tag{20}$$

or simply as

$$\Delta\vec{J} = (\sigma_\Delta) \cdot \Delta\vec{E} \tag{21}$$

where

$$\sigma_{xx} = \frac{\partial J_x}{\partial E_x} \tag{22}$$

$$\sigma_{xy} = \frac{\partial J_x}{\partial E_y} \tag{23}$$

$$\sigma_{yx} = \frac{\partial J_y}{\partial E_x} \tag{24}$$

$$\sigma_{yy} = \frac{\partial J_y}{\partial E_y} \tag{25}$$

Equation (21) is a more general form of the conductivity relationship between conduction current density and electric field, and should be adequate for sufficiently small ΔE so that the first terms of the expansion for $\Delta\vec{J}$ accurately describe $\Delta\vec{J}$. This, of course, presumes that at least one of the partial derivatives used is non-zero.

We can proceed now to calculate the components of (σ_Δ) using equations (14) through (17) together with the relation

$$E = (E_x{}^2 + E_y{}^2)^{1/2} \tag{26}$$

Thus for σ_{xx}

$$\sigma_{xx} = \frac{\partial(\sigma(E)E_x)}{\partial E_x}$$

$$= \sigma(E) + E_x \frac{\partial\sigma(E)}{\partial E} \times \frac{1}{2} \; (E_x{}^2 + E_y{}^2)^{-1/2} \times 2E_x$$

$$= \sigma(E) + \frac{E_x{}^2}{E} \frac{\partial\sigma\,(E)}{\partial E}$$

$$= \sigma(E) \left[1 + \frac{E_x{}^2}{\sigma(E)E} \; \frac{\partial\sigma(E)}{\partial E} \right] \tag{27}$$

or using equation (17)

$$\sigma_{xx} = \sigma(E) \left[1 + \frac{E_x{}^2}{\mu_e E} \; \frac{\partial\mu_e}{\partial E} \right] \tag{28}$$

for σ_{xy}

$$\sigma_{xy} = \frac{\partial(\sigma(E)E_x)}{\partial E_y}$$

$$= E_x \frac{\partial\sigma(E)}{\partial E} \times \frac{1}{2} (E_x{}^2+E_y{}^2)^{-1/2} x2E_y$$

$$= \frac{E_xE_y}{E} \frac{\partial\sigma(E)}{\partial E}$$

$$= \sigma(E) \frac{E_xE_y}{\sigma(E)E} \frac{\partial\sigma(E)}{\partial E} \tag{29}$$

or

$$\sigma_{xy} = \sigma(E) \frac{E_xE_y}{\mu_e E} \frac{\partial\mu_e}{\partial E} \tag{30}$$

for σ_{yx}

$$\sigma_{yx} = \frac{\partial(\sigma(E)E_y)}{\partial E_x}$$

$$= E_y \frac{\partial\sigma(E)}{\partial E} \times \frac{1}{2} (E_x{}^2+E_y{}^2)^{-1/2} x2E_x$$

$$= \frac{E_yE_x}{E} \frac{\partial\sigma(E)}{\partial E}$$

$$= \sigma(E) \frac{E_yE_x}{\sigma(E)E} \frac{\partial\sigma(E)}{\partial E} \tag{31}$$

or

$$\sigma_{yx} = \sigma(E) \frac{E_yE_x}{\mu_e E} \frac{\partial\mu_e}{\partial E} \tag{32}$$

and we can see that

$$\sigma_{yx} = \sigma_{xy} \tag{33}$$

or that the matrix is symmetric, and finally for σ_{yy}

$$\sigma_{yy} = \frac{\partial(\sigma(E)E_y)}{\partial E_y}$$

$$= \sigma(E)+E_y\frac{\partial\sigma(E)}{\partial E} \times \frac{1}{2} (E_x{}^2+E_y{}^2)^{-1/2} x2E_y$$

$$= \sigma(E)+\frac{E_y{}^2}{E} \frac{\partial\sigma(E)}{\partial E}$$

$$= \sigma(E) \left[1 + \frac{E_y{}^2}{\sigma(E)E} \frac{\partial\sigma(E)}{\partial E}\right] \tag{34}$$

or

$$\sigma_{yy} = \sigma(E)\left[1 + \frac{E_y^{\,2}}{\mu_e E}\frac{\partial\mu_e}{\partial E}\right] \tag{35}$$

These results can be summarized in the form

$$(\sigma_\Delta) = \sigma(E)\begin{pmatrix} 1 + \frac{E_x^{\,2}}{\mu_e E}\frac{\partial\mu_e}{\partial E} & \frac{E_x E_y}{\mu_e E}\frac{\partial\mu_e}{\partial E} \\ \frac{E_x E_y}{\mu_e E}\frac{\partial\mu_e}{\partial E} & 1 + \frac{E_y^{\,2}}{\mu_e E}\frac{\partial\mu_e}{\partial E} \end{pmatrix} \tag{36}$$

Referring to figure 2 we can put (σ_Δ) in a convenient form by use of the angle, ϕ, the angle between $\vec{E}$ and the x axis. Thus

$$(\sigma_\Delta) = \sigma(E)\begin{pmatrix} 1 + \cos^2\phi\frac{E}{\mu_e}\frac{\partial\mu_e}{\partial E} & \cos\phi\sin\phi\frac{E}{\mu_e}\frac{\partial\mu_e}{\partial E} \\ \cos\phi\sin\phi\frac{E}{\mu_e}\frac{\partial\mu_e}{\partial E} & 1 + \sin^2\phi\frac{E}{\mu_e}\frac{\partial\mu_e}{\partial E} \end{pmatrix} \tag{37}$$

In this form we can see a common factor in all the matrix components, i.e., $\frac{E}{\mu_e}\frac{\partial\mu_e}{\partial E}$, which is plotted in figure 3 versus the density-normalized electric field. The curves in this figure are derived from the curves of figure 1 by taking the slope (on a log-log plot) of the curves in figure 1:

$$\frac{E}{\mu_e}\frac{\partial\mu_e}{\partial E} = \frac{\partial(\ln(\mu_e))}{\partial(\ln(E))} \tag{38}$$

As such, the curves of figure 3 are less accurate than those of figure 1, but the purpose of figure 3 is to show the difficulty involved in relating a low frequency microwave conductivity measurement to the air conductivity. For convenience, figure 4 is included to give the saturation molecular fraction of water vapor, f_{sat}, as a function of the temperature and the molecular density of the air under consideration.

The low-frequency microwave conductivity matrix can be reduced to its simplest form by defining one of the coordinate axes of figure 2, say the x axis, to be parallel to $\vec{E}$, i.e., set $\phi = 0$. Then

$$(\sigma_\Delta) = \sigma(E)\begin{pmatrix} 1 + \frac{E}{\mu_e}\frac{\partial\mu_e}{\partial E} & 0 \\ 0 & 1 \end{pmatrix} \tag{39}$$

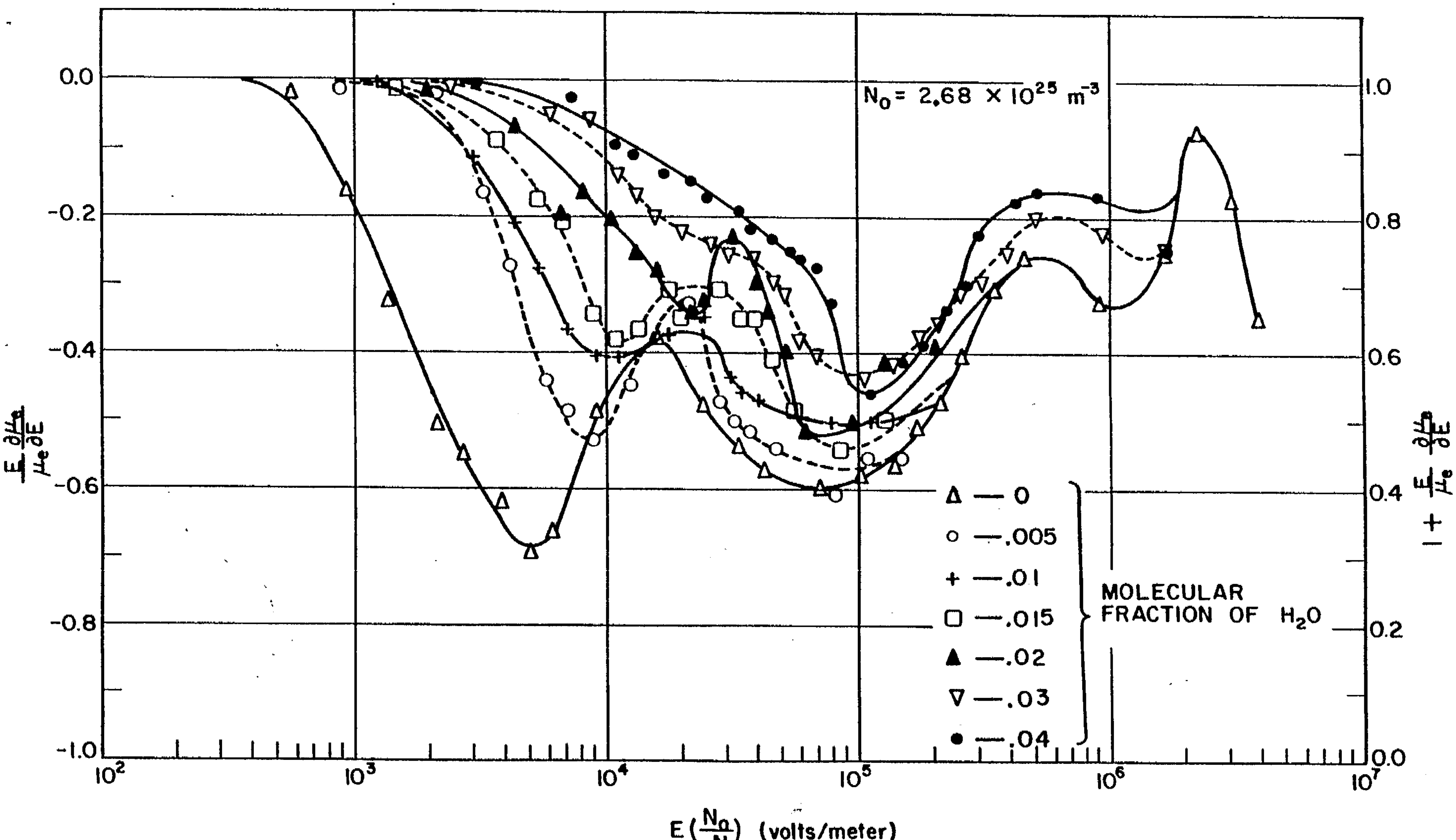

Fig. 3. EFFECTIVE LOW FREQUENCY MICROWAVE MOBILITY CORRECTIONS

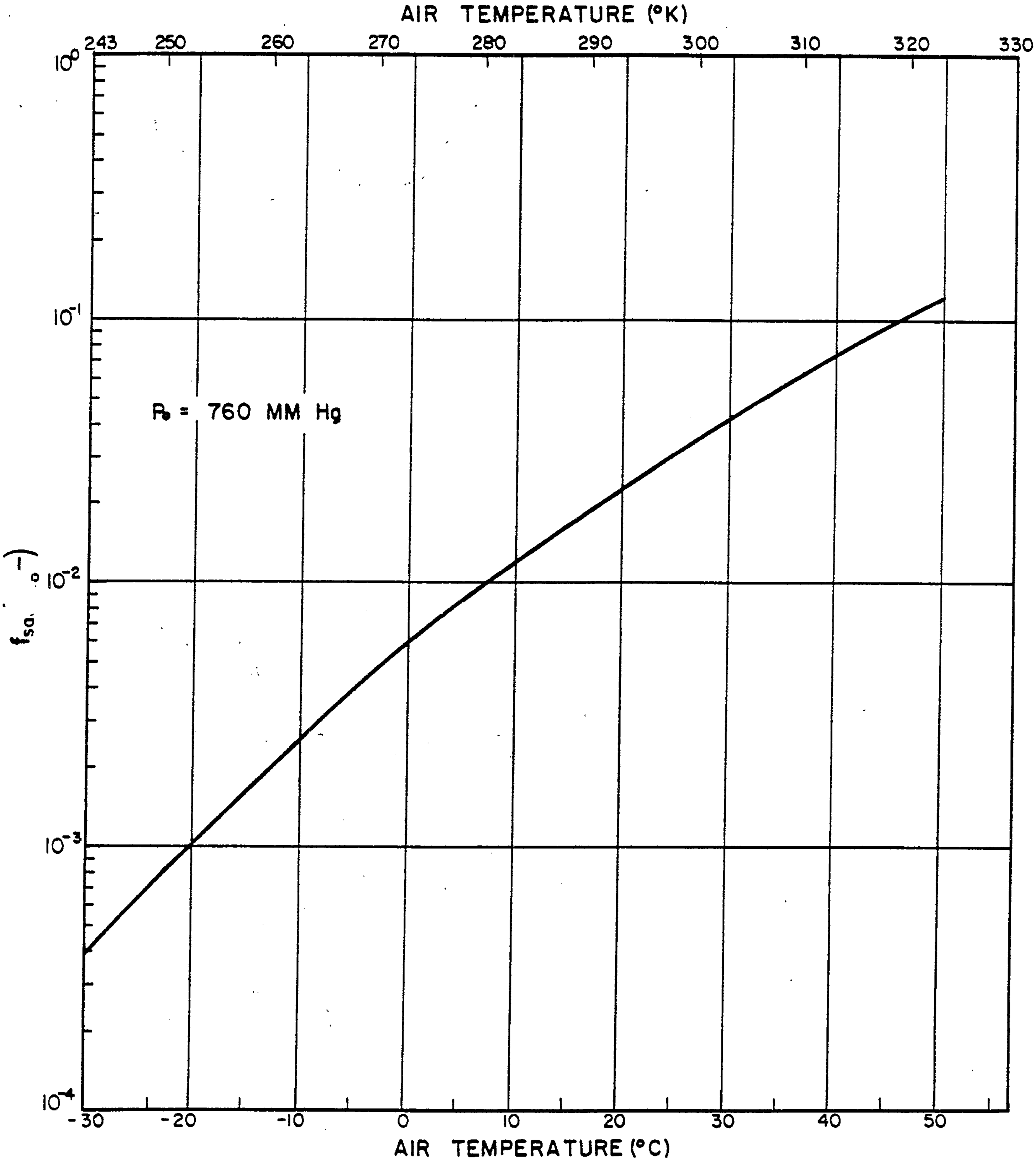

Fig. 4. SATURATION MOLECULAR FRACTION OF WATER VAPOR IN AIR

and the matrix is diagonalized. With this last result we can define $\sigma_{//}$ as the conductivity for the component of the microwave electric field parallel to the EMP electric field and $\sigma_{\perp}$ as the conductivity for the component of the microwave electric field perpendicular to the EMP electric field. Thus

$$(\sigma_{\Delta}) = \begin{pmatrix} \sigma_{//} & 0 \\ 0 & \sigma_{\perp} \end{pmatrix} \tag{40}$$

where

$$\sigma_{//} = \sigma(E) \left[1 + \frac{E}{\mu_e} \frac{\partial \mu_e}{\partial E} \right] \tag{41}$$

and

$$\sigma_{\perp} = \sigma(E) \tag{42}$$

These last results can be written in different form as

$$\sigma_{//} = en \frac{\partial(\mu_e E)}{\partial E} \tag{43}$$

and

$$\sigma_{\perp} = en\mu_e \tag{44}$$

showing that, for the microwave electric field parallel to the EMP electric field, the appropriate mobility is the derivative of the electron drift velocity with respect to the electric field; while for the microwave electric field perpendicular to the EMP electric field, the appropriate mobility is the ratio of the electron drift velocity to the electric field. These latter two quantities are both functions of the EMP electric field. The quantity μ_e is plotted in figure 1 while the correction factor to μ_e, $1 + \frac{E}{\mu_e} \frac{\partial \mu_e}{\partial E}$, for microwave electric fields parallel to the EMP electric fields is plotted in figure 3 (using the right vertical scale).

Some qualitative information can be gained from the results of equations (41) and (42). Apparently, if a low frequency and low intensity microwave electric field is polarized such that it is always perpendicular to the EMP electric field (at the measurement location), then this microwave field can measure $\sigma(E)$, the air conductivity pertinent to the EMP electric field. However, the measurement geometry must be chosen to ensure that the two electric fields are perpendicular. Otherwise, there will be two different conductivities and thus two different propagation constants affecting the microwave signal. This latter effect tends to rotate the microwave polarization if the microwave is being transmitted as a free wave.

We have arrived at what seems to be an unusual result, i.e., the possibility of the measurement of a non-linear quantity, $\sigma(\vec{E})$, by means of the effect of this parameter on a much smaller quantity, $\Delta\vec{E}$, than the principal qualtity, $\vec{E}$, which is present. This technique might be applicable to other non-linear phenomena of similar mathematical form. Of course, even though we have referred to a microwave in this discussion, the analysis applies to other forms of electromagnetic measuring techniques in which a perturbing electric field is used. The above analysis for the electron conductivity in air is not necessary for the ion conductivity since the ion mass is much larger than the electron mass. This keeps the ion temperature thermal for electric fields of interest, and the ion mobility is independent of electric field strength.

IV. Microwave Antenna Limitations

In addition to the basic problem of the behavior of the quantity to be measured (the air conductivity) there are problems associated with the measurement system. In particular, if we are concerned with using the modulation of a transmitted microwave signal as a measurement system, we are faced with special problems associated with the necessary antennas for the transmission and reception of the microwave signal. These problems arise from the fact that the antenna characteristics are, in general, dependent on the electromagnetic parameters of the medium in which the antenna is placed. For example, if the directivity of the transmitting antenna (a measure of the solid angle spreading of the transmitted wave) changes significantly, the electric field strength at the receiving antenna is correspondingly changed. Such an effect would appear as a false attenuation of the microwave signal and thus an error in an attenuation measurement. Also if the effective aperture of the receiving antenna (a measure of the efficiency of collection of the signal) changes significantly, there will be another apparent attenuation in the measurement. Both of these effects must be considered (and, ideally, avoided) for a valid measurement.

We might try to avoid these antenna problems by choosing the geometry such that the antennas can be shielded to some extent from the nuclear radiation environment to significantly reduce the air conductivity in the vicinity of the antennas. However, the use of this technique introduces other problems involving the transition of the transmitted microwave signal from the "non-conducting" region (near the transmitting antenna) to a conducting region (the measurement region) and back to a "non-conducting" region (near the receiving antenna). At the interfaces between these regions the transmitted signal can be both reflected and refracted (i.e., have its direction altered on passing through the boundary). To avoid reflections we can make the thickness of the boundary region large compared to a signal wave length or we can measure the signal reflected back into the transmitting antenna as a check. To avoid refractions we can make the boundaries perpendicular to the direction of propagation of the probing signal.

These antenna limitations apply for both high and low frequency measurements (as defined in Section II). The seriousness of these limitations will be affected to some extent by the probing frequency. Depending on this latter parameter we may wish to measure attenuation or phase shift, both of which will be affected (although differently) by the antenna limitations, but when measuring phase shift, we must also consider the attenuation because, in general, the attenuation and phase shift information are contained together. For a phase shift measurement the attenuation can be removed from the measurement by the use of at least two independent measurements of the phase and amplitude information contained in the transmitted signal. Finally, we may increase the path length of the probing signal through the conducting region to the extent that any errors due to antenna limitations are small compared to the effects of the intended measurement on the probing signal. Thus it would seem that we can avoid the antenna limitations on our measurement by a good choice of the various parameters at our disposal.

V. Summary

We have considered two general types of microwave measurements concerning the air conductivity: an electron density measurement if the probing frequency is much higher than the electron momentum transfer collision frequency, and an air conductivity measurement if the probing frequency is much lower than the electron momentum transfer collision frequency. If we wish to measure the air conductivity during the time when the ions are the important contribution, then the probing frequency will also need to be much lower than the ion collision frequency, or much less than about 2 gigacycles.

However, if we wish to measure the air conductivity by the low frequency technique then we should consider the anisotropy introduced into the air conductivity (as measured by the microwave) by the intense EMP electric field. Specifically, the polarization of the microwave electric field should be chosen to be perpendicular to the EMP electric field. Otherwise, the air conductivity as measured by the microwave will have correction factors such as illustrated in figure 3.

Besides the basic problems associated with the physics of the air conductivity there are environmental problems associated with the microwave transmission and reception system. The electrical characteristics of the antenna systems may be a function of the environment and introduce errors into the measurement. If we minimize these adverse effects and increase the sensitivity of the microwave to the air conductivity or electron density (such as by increasing the length of the microwave transmission path) we may reduce these errors to acceptable levels.

Finally, we should note that the general technique outlined in this note is not the only method for measuring the air conductivity. For example, we could measure both the EMP electric field and current density and from these measurements infer the air conductivity.

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