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ANALYSIS OF THE MOEBIUS LOOP MAGNETIC FIELD SENSOR

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ABSTRACT

An analysis of the Moebius loop which provides the relationship between magnetic field and output voltage is presented. This analysis accounts for all of the loop electrical phenomena involved over the broad range of frequencies encountered in measurement of fast pulses; specifically, from dc to frequencies corresponding to a loop electrical diameter of from about 6.4 degrees to 12.8 degrees. In the usual application, the loop is small, so that this corresponds to high (typically much greater than 100 MHz) frequencies. It is found that the relationship given is valid up to the stated 6.4 degrees limit for any loop coax and connecting balanced line impedance combination, but that the validity of extensions above this limit depend on the relationship between these impedances. Loop response as a function of frequency is deduced, from which useful approximations are derived, one of which is that below a certain frequency, which is defined here, loop output voltage is proportional to the derivative of the field. Finally, a detailed numerical example is given.

INTRODUCTION

The Moebius loop magnetic field sensor is a circular loop consisting of two solid-shield coaxial ''arms'' which are split at the top to form a gap which is very small compared to loop dimensions, as shown in Figure 1. The center conductor of each coaxial arm is connected to the shield of the opposite arm, as shown at Points A and B. The loop is otherwise closed, driving a balanced, shielded line as indicated. The loop is usually used in the presence of ionizing radiation, generally where small physical size is also desirable or required.

A similar loop, but with the center conductors connected to each other at the gap, called a split-shield loop, has been discussed extensively by Libby[1] and others. By comparison with this loop, Baum[2] showed that the subject Moebius connection provides (1) twice the loop output voltage, and (2) when used in an ionizing radiation environment, greatly reduced common-mode signal due to such radiation; also, the differential mode signal due to ionizing radiation can be

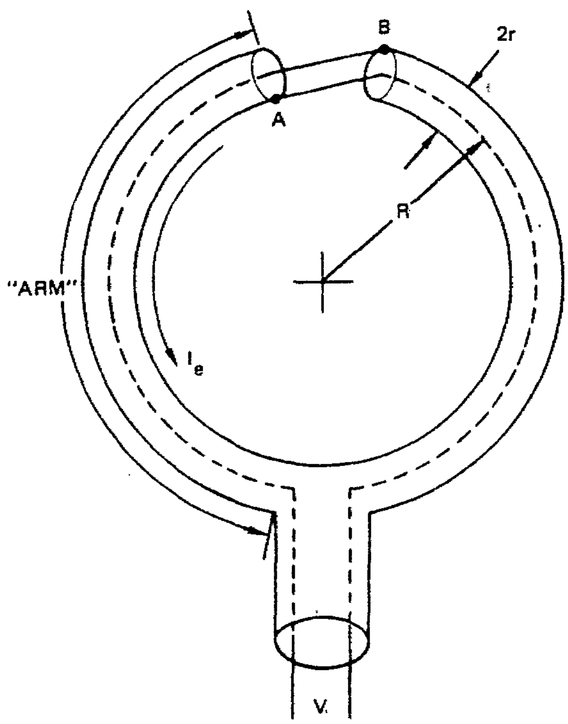


Figure 1. Moebius Loop

made essentially equal. Baum then demonstrated that this is at the expense of increasing the loop time-constant by a factor of four. As will be shown later, this does not compromise operation over a very important frequency range. The Moebius loop is thus a very effective magnetic field sensing device which has good noise rejection properties in the presence of ionizing radiation.

The purpose of this paper is to present the derivation of the relationship between the magnetic or B field component of the incident field, and the resultant loop output voltage V. This analysis will be valid for frequencies such that the electrical length, θ, of one of the ''arms'' of Figure 1 is between 10 and 20 degrees at the highest frequency of interest (the precise value depends upon details which will be discussed) and will extend, on the low end, to dc. Thus the response of the loop over the range of frequencies corresponding to fast pulse excitation, the usual application, will be quantified. This relationship, accounting for all of the loop electrical phenomena, agrees with both the previously noted increase in time constant by a factor of four and the doubled sensitivity. More often than not during cursory analyses, the correct doubled sensitivity is predicted but the increased time constant is not accounted for. The importance of accounting for increased time constant increases with the relative high frequency content of the transient measured.

In addition to enhancing understanding of the operation of the loop, thus assuring correct application, this analysis is also desirable since, in order to perform an accurate calibration of such a loop, one must obtain simultaneous time-correlated precise measurements of a continuous wave (CW) radiating system and loop output voltage and/or current, then relate the two using Maxwell's equations, repeating the process at a great number of frequencies. Since analysis is therefore required anyway, it would be preferable to use calibration data at a much smaller number of frequency points, or a pulse input, to refine and/or verify an analytical loop model.

The B versus V relationship will first be derived for the case where the electrical length, θ, of each ''arm'' is less than 10 degrees, but where the frequency is high enough so the depth of penetration of the field into the coaxial shield is much less than the shield thickness. The B versus V relationship is derived by relating ''source'' voltage, V_{AB}, which the changing magnetic field develops across the gap, and the current I_e shown in Figure 1, to ''load'' voltage and current. The ''load'' is the impedance which the balanced line presents to the source voltage, V_{AB}, by the connection of the conductors at points A and B. Next, the conditions under which this relationship may be extended to frequencies for which the electrical length of one arm is between θ = 10 degrees and θ = 20 degrees will be presented. Finally, the response of the loop as a function of frequency, including the low frequency case, i.e., for frequencies where the magnetic field completely penetrates the (nonferrous) loop shield is analyzed, from which useful approximations are deduced. The result of this analysis is then applied to a particular practical problem, providing a numerical example.

RELATIONSHIP BETWEEN MAGNETIC FIELD AND OUTPUT VOLTAGE (B VERSUS V) FOR θ < 10 DEGREES

The emf developed in the loop by the field is given by the Faraday law of electromagnetic induction:

$$\mathcal{E} = \oint_{\text{Loop Boundary}} \vec{E} \cdot d\vec{\ell} = - \iint_{\text{Loop Area}} \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} \, dA, \tag{1}$$

where the line integral is taken from A to B. For the case where the electrical length of one arm is 10 degrees, the electrical diameter of the loop is only $\phi = 2/\pi$ (10) = 6.4 degrees; evidently for this case the spatial variation of the rate of change of flux, i.e., the variation at any instant t_1 of:

$$\left. \frac{\partial \vec{B}}{\partial t} \right|_{t=t_1} \cdot \hat{n}$$

(where t_1 is arbitrary) is minimal over the loop so that Equation (1) becomes

$$\mathcal{E} = - A \frac{dB}{dt} \tag{2}$$

where A is the area of the loop perpendicular to the B field. It will be assumed here that B is perpendicular to a plane through the loop, so that A = loop area.

For θ <10 degrees, I_e is substantially constant in amplitude and phase, so the free space inductance of the loop shield is:

$$L = \mu_o R \left[\log_e \left(\frac{8R}{r}\right) - 2\right] \tag{3}$$

where R and r are as shown in Figure 1 (r is the radius from the center conductor to the outside of the loop coax shield). For these conditions, the radiation resistance is approximately:

$$R = 31{,}000 \frac{A^2}{\lambda^4} - \text{ohms}$$

The ratio of the magnitude of the inductive reactance to the radiation resistance is therefore:

$$\frac{\omega L}{31{,}000 \frac{A^2}{\lambda^4}} = \frac{\frac{2\pi c}{\lambda} \mu_o R \left[\log_e \left(\frac{8R}{r}\right) - 2\right]}{31{,}000 \frac{\pi^2 R^4}{\lambda^4}}$$

for R/r >5, this becomes, inserting values for μ_o and c:

$$\left|\frac{\text{inductive reactance}}{\text{radiation resistance}}\right| > \frac{2\pi\ (3 \times 10^8)\ 4\pi\ (10^{-7})\ 1.7}{31{,}000\ \pi^2} \left(\frac{\lambda}{R}\right)^3 = 0.0132 \left(\frac{\lambda}{R}\right)^3$$

Even for θ as high as 20 degrees, R is only 6.4 degrees, so $\lambda/R \geq 56$ and the inductive reactance dominates by more than 2,000 to 1. Therefore, by Lenz's law the emf of Equation (2) is reduced by an opposing emf $\mathcal{E}'$ of magnitude

$$\mathcal{E}' = L \frac{dI_e}{dt} \tag{4}$$

The voltage V_{AB} across the gap is therefore:

$$V_{AB} = - A \frac{dB}{dt} + L \frac{dI_e}{dt} \tag{5}$$

Because the gap is so short, the potential along the center conductors across the gap is essentially constant, so V_{AB} at the top of the gap may be represented as in Figure 2A. The polarity shown results from the negative sign in Equation (5).

The inside of each arm comprises a transmission line of length $\theta < 10$ degrees, so that the voltage between center conductor and the inside of the shield is essentially constant from the gap to the loop output, as shown in Figure 2A. We may consider this phenomenon independently of phenomena on the outside of the shield, because currents on the inside of the shield are essentially localized to the inside surface of the shield, for the same reason that shield external currents are essentially all on the outside at frequencies under consideration here. The net result is the voltage 2 V_{AB} across the load line of impedance Z. This line is assumed to be terminated in its characteristic impedance and may therefore be represented as the lumped impedance Z in Figure 2A.

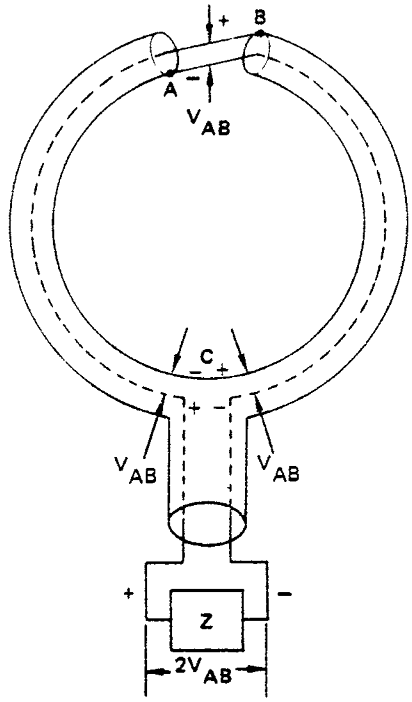
Since the loop output voltage 2 V_{AB} is composed of exactly equal and opposite potential variations above and below the common reference Point C, it follows that the circuit of Figure 2B is exactly equivalent to that of Figure 2A.

Consider the current path BCDA of Figure 3. The impedance presented to this current is Z/2. Now consider the current path BECA. The impedance presented to this current is also Z/2. These currents are consequently equal, therefore,

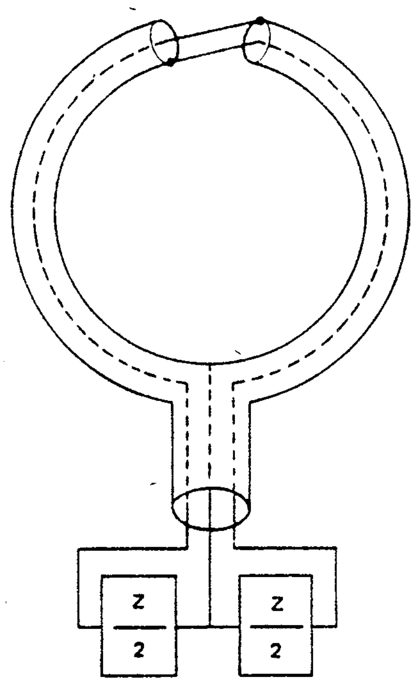
$$I_e = 2I_i \tag{6}$$

Thus, one may draw the equivalent circuit of Figure 4A from which the simpler circuit of Figure 4B directly follows. From Figure 4B:

$$V_{AB} = - V_{BA} = - 2I_i \left(\frac{Z}{4}\right) = - I_i \frac{Z}{2} \tag{7}$$



(A) SOURCE AND LOAD VOLTAGES



(B) EQUIVALENT CIRCUIT FOR LOAD

Figure 2. Equivalent Circuit

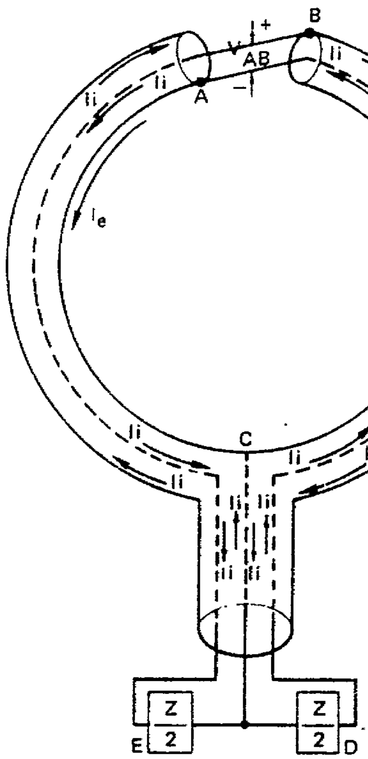


Figure 3. Relationship Between I_e and I_i

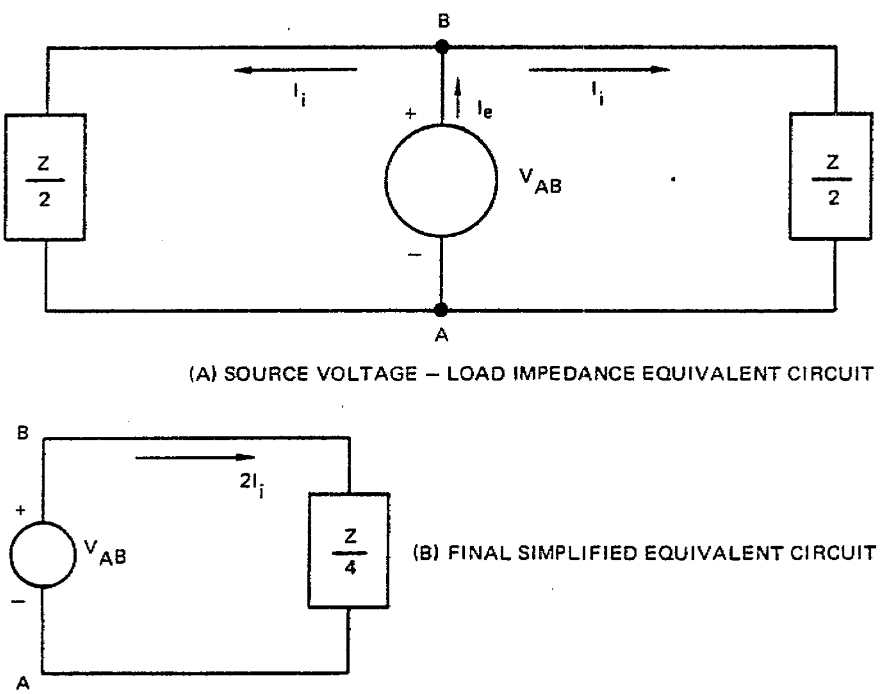


Figure 4. Source Voltage – Load Impedance Equivalent Circuit

The relationship between B and V follows from Equations (5), (6) and (7). From these equations, obtain:

$$A \frac{dB}{dt} = I_i \frac{Z}{2} + 2L \frac{dI_i}{dt} \tag{8}$$

Assuming $I_i = 0$ at $t = 0$, integration of Equation (8) yields:

$$B = \frac{1}{A}\left(\frac{Z}{2} \int_0^t I_i \, dt + 2LI_i\right) \tag{9}$$

In terms of the loop output voltage $V = I_i Z$, Equation (9) becomes:

$$B(t) = \frac{1}{2A} \int_0^t V(t) \, dt + \left(\frac{2L}{AZ}\right) V(t) \tag{10}$$

Equation (10) is the basic relationship sought.

B VERSUS V RELATIONSHIP FOR $\theta > 10$ DEGREES

The validity of the analysis in the section above is based upon the validity of Equations (2), (4) and (7). The validity of each of these equations depends upon small θ. It is easily shown that Equation (2) is valid with good approximation for electrical ''arm'' lengths of $\theta = 15$ degrees to $\theta = 20$ degrees. The analysis however, is somewhat involved, so it is included as the Appendix to this paper. Consider now the other assumptions relating to θ. Libby[1] has shown that, for θ much greater than 10 degrees, the external shield impedance apparently remains essentially an inductive reactance with inductance very nearly exactly that of Equation (3). One need not be concerned about the slightly nonuniform distribution of I_e; since Equation (2) is valid and the source impedance is unaltered, I_e is unchanged at Points A and B, and ''L'' may be viewed as a lumped inductance between. The only remaining troublesome factor is the validity of Equation (7). For $\theta > 10$ degrees, Figure 4A must be replaced by the more involved representation of Figure 5A, where the lines of characteristic impedance Z_o are terminated in the impedances Z/2. The validity of Equation (7) therefore depends upon the validity of the assumed load impedance. Since Z_i is the impedance looking either way from the source,

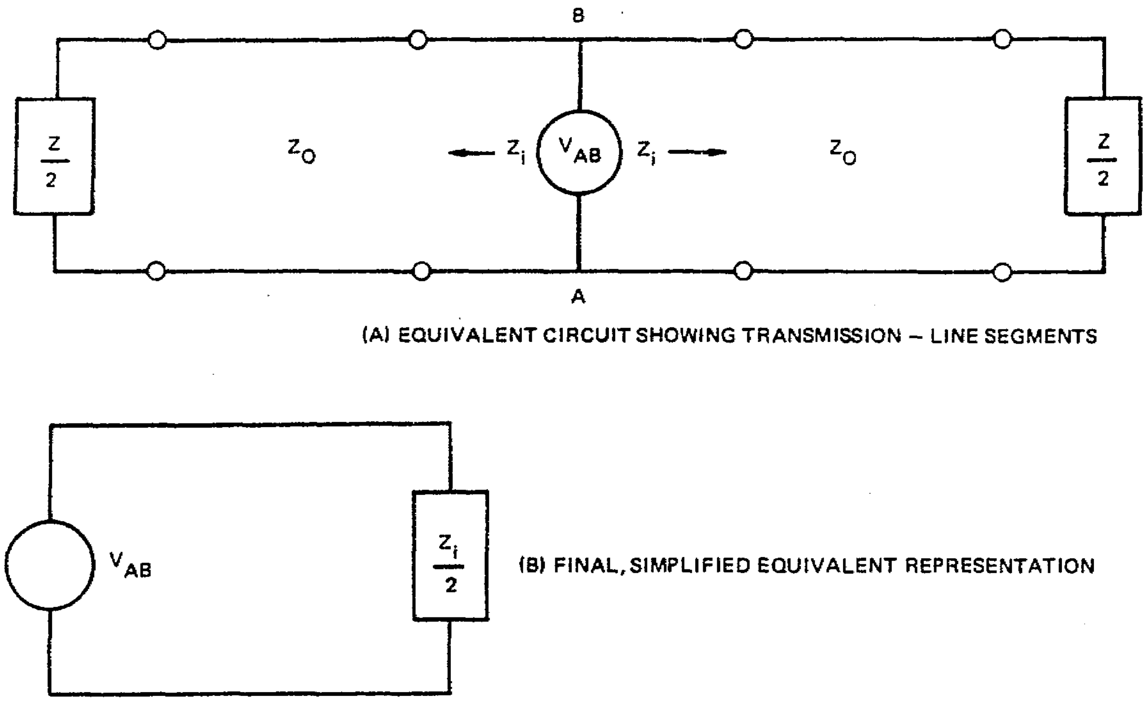


Figure 5. Equivalent Representation

one obtains the equivalent circuit of Figure 5B. Neglecting the attenuation constant:

$$\frac{Z_i}{2} = \frac{Z}{4}\left[\frac{1 + \hat{j}\ \left(Z_o/\frac{Z}{2}\right)\ \tan\theta}{1 + \hat{j}\ \left(\frac{Z}{2}/Z_o\right)\ \tan\theta}\right]$$

$$= \frac{Z}{4}\left[\frac{1 + \tan^2\theta + \hat{j}\ \left(Z_o/\frac{Z}{2} - \frac{Z}{2}/Z_o\right)\ \tan\theta}{1 + \left(\frac{Z/2}{Z_o}\ \tan\theta\right)^2}\right] \qquad (11)$$

Evidently, from Equation (11) we have the condition that if $Z_o \approx Z/2$, $Z_i/2 \approx Z/4$, i.e., if the balanced line impedance is about double the loop coax impedance, the B versus V relationship of Equation (10) is valid with good approximation to between (from the analysis in the Appendix) θ = 15 degrees and θ = 20 degrees. If Z_o is much different than Z/2, and θ > 10 degrees, one must evaluate the effect of the ''mismatch'' using Equation (11).

FREQUENCY DOMAIN PROPERTIES: USEFUL APPROXIMATIONS

The loop may be viewed, over the frequency range of interest here, as a linear system with input B(t) and output V(t). Designating the transfer function of the loop by H(jω), the frequency-domain relationship is:

$$V(j\omega) = B(j\omega)\,H(j\omega), \text{ or } H(j\omega) = \frac{V(j\omega)}{B(j\omega)} \qquad (12)$$

Where V(jω) and B(jω) are the Fourier transforms of V(t) and B(t), respectively. From the ω space equivalent of Equation (10), H(jω) is readily found to be:

$$H(j\omega) = \frac{1}{\frac{1}{j(2\omega A)} + \frac{2L}{AZ}} \qquad (13)$$

Simplifying and expressing in polar form:

$$H(j\omega) = \left(\frac{ZA}{2L}\right)\frac{1}{\sqrt{1 + \frac{1}{\left[\frac{f}{Z/(8\pi L)}\right]^2}}}\; e^{j\,\phi} \qquad (14)$$

where $\phi = \arctan\left[\frac{1}{f\Big/\left(\frac{Z}{8\pi L}\right)}\right]$

for $f \gg \frac{Z}{8\pi L}$, $H(j\omega) \approx \frac{ZA}{2L}$, i.e., output voltage is proportional to the B field.

For $f \ll \frac{Z}{8\pi L}$, $\phi \approx 90$ degrees, so from this and the magnitude expression,

$$H(j\omega) \approx j\,\frac{ZA}{2L}\left[\frac{f}{Z/(8\pi L)}\right] = j\;2A\omega \qquad (15)$$

or the output voltage is proportional to the derivative of the field.

The above relationships are quantified in the plot of Figure 6, showing loop transfer function amplitude (solid line) and phase versus frequency. Frequency is expressed in terms of a ratio to the quantity Z/(8πL), as indicated. From

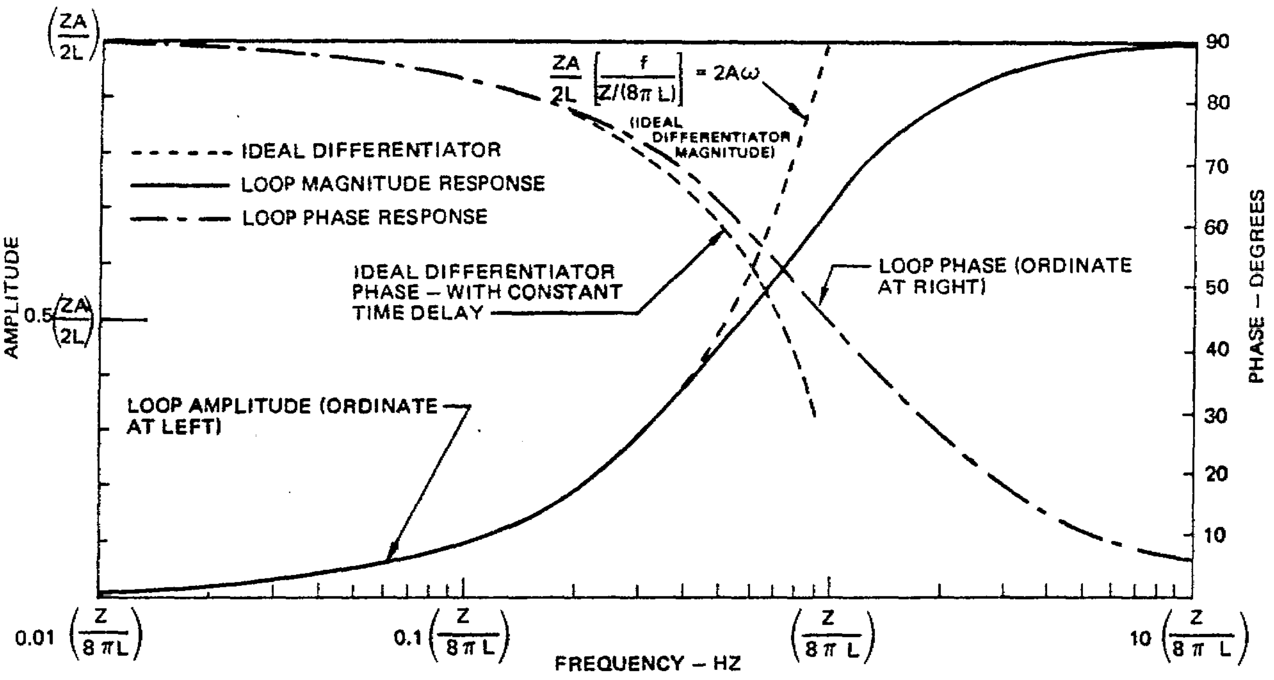


Figure 6. Transfer Function $H(j2\pi f) = \frac{V(j2\pi f)}{B(j2\pi f)}$

the above discussion, evidently $0 \leq |H(j\omega)| \leq ZA/2L$, so the magnitude is expressed in terms of fractions of ZA/2L. To provide comparison with the ideal differentiator defined by Equation (15), the magnitude of this equation is also shown, in dashed lines.

Now for relatively low frequency, the phase $\phi \approx \pi/2$. Using the fact that for small angles $\tan x \approx x$, straight-forward trigonometric manipulation leads to the approximate relationship:

$$\phi = \arctan\left[\frac{1}{f/(\frac{Z}{8\pi L})}\right] \approx \frac{\pi}{2} - \frac{f}{(\frac{Z}{8\pi L})} \qquad (16)$$

From elementay linear system theory, linear variation of phase with frequency corresponds to a constant time shift. The magnitude of this delay, τ, may be determined from the elementary result that $2\pi f\tau = f/(\frac{Z}{8\pi L})$, so $\tau = 4\frac{L}{Z}$.

The approximately linear phase relationship of equation (16) is shown by the dashed-line phase curve of Figure 6, where it is evident that the approximation is very good to about 0.5 $(\frac{Z}{8\pi L})$ to 0.6 $(\frac{Z}{8\pi L})$. Therefore, from both amplitude and phase consideration it follows that the operation of the loop may be very adequately characterized as a differentiator with constant time delay $4 \frac{L}{Z}$ for frequencies below about 0.6 $(\frac{Z}{8\pi L})$.

The entire analysis up to this point has assumed that frequency is high enough so that field penetration is such that current flow is essentially all on the surface (outside and inside) of the loop shield. Let us now proceed to examine the validity of this analysis at frequencies low enough so that the magnetic field penetrates the nonferrous loop shield. At such frequencies, referring to Figure 2a, one merely has a loop consisting of two turns. This is readily seen by tracing a dc current path looking toward the loop from the load. Thus, beginning at the (+) side of Z in Figure 2a, this path goes to Point B, around the loop to Point A, and back to the (-) side of Z. At these frequencies the inductive reactance is negligible so that

$$V = 2A \frac{dB}{dt} \tag{17}$$

for V = 0 at t = 0,

$$B = \frac{1}{2A} \int_{o}^{t} V dt \tag{18}$$

Equation (18) is exactly the same as Equation (10) if the contribution of the term (2L/AZ)V is negligible, which it is for low frequencies.

Evidently the amplitude-phase curves of Figure 6, extended indefinitely on the low end, are valid for two cases: (1) frequency high enough so that field penetration is negligible and (2) frequency low enough so that there is complete magnetic field penetration. To get an idea of the frequency range of uncertainty between these two analyses, and therefore assess the significance of this uncertainty, one must know the shield thickness and material. This will be examined in the practical example presented in the next section. Also, although

Using this value, with Z = 78 ohms and A the loop area, Equation (10) becomes:

$$B(t) = 4.4\ (10^{2}) \int_{0}^{t} V(t)\ dt + 1.34\ (10^{-6})\ V(t) \text{ - Teslas} \qquad (19)$$

For one arm of length π (0.75) 0.0254 = 0.0595 - meter to be less than 10 degrees at the highest frequency, this frequency must correspond to the wavelength λ = 36 (0.0595) = 2.15 - meters, which gives an upper frequency limit of validity of Equation (14) of

$$f = \frac{3\ (10^{8})}{2.15} = 140 \text{ MHz (10 degrees criterion, propagation velocity = c)}$$

As noted in the previous section, one may, under propitious conditions, extend this to the frequency corresponding to an electrical arm length of from 15 to 20 degrees. Considering the mismatch, let us see if the extension to 15 degrees is valid. For this slightly higher frequency, we are entering the range where the use of the more accurate transmission line representation of Figure 5a becomes necessary. From Equation (11), the departure from the nominal value of Z/4 is, for the present impedances and θ = 15 degrees:

$$\frac{Z}{4}\left[\frac{1 + \tan^{2} 15^{\circ} + \hat{j}\left(\frac{50}{39} - \frac{39}{50}\right)\tan 15^{\circ}}{1 + \left[\left(\frac{39}{50}\right)\tan 15^{\circ}\right]^{2}}\right] = \frac{Z}{4}\,[1.03 + \hat{j}\ (0.124)]$$

which is a negligibly small amplitude and phase error for signal components well above 140 MHz. Thus Equation (10) holds with good approximation to at least 1.5 (140) = 210 MHz.

Let the output signal be passed through a 78-ohm to 50-ohm impedance matching transformer for input to a 50-ohm coax line connected to an oscilloscope. The oscilloscope voltage for this case is thus $\sqrt{50/78}$ times the loop output voltage. If the loop output voltage V(t) in Equation (14) is replaced by the oscilloscope voltage, the right-hand side of this equation must be multiplied by $\sqrt{78/50}$ yielding:

$$B(t) = 5.5\ (10^2) \int_0^t V(t)\ dt + 1.68\ (10^{-6})\ V(t) \text{ - Teslas} \qquad (20)$$

Now consider the frequency behavior of this loop. For estimation purposes, the initial analysis of this paper will be considered approximately valid for a shield thickness of one skin-depth. For 0.012 inch thick copper, this implies a frequency of greater than about 47 kHz. Since $Z/(8\pi L) \approx 53.7\ (10^6)$, the frequency 47 kHz corresponds to $47/53.7\ (10^{-3})\ (Z/8\pi L)$ on the curve of Figure 6, which is completely off the curve to the left. Evidently the loop response is relatively quite low here. Obviously for the considerably lower frequency case of complete magnetic field penetration, the response must be lower still. There is nothing to indicate any discontinuity in the response, and considering the very small proportion of the probable spectrum of the signal being measured involved, one is inclined to conclude that little if any error is made in assuming that Equation (10) is valid at frequencies low enough so that the skin depth is greater than the shield thickness.

From the discussion of the previous section, the following approximations may be made (all at the oscilloscope input):

A. $B\ (t - 3 \times 10^{-9}) \approx 5.5\ (10^2) \int_0^t V(t)\ dt$ - Teslas (3 nanosecond delay)

for $f < 0.6\ (53.7)\ 10^6$, or $f < 32.1$ MHz

B. For $V(t) \approx K \sin \omega t$,

$B(t) \approx K\ (1.68)\ 10^{-6} \sin (\omega t + \phi)$ provided $f > 104.7$ MHz

CONCLUSIONS

The relationship between Moebius loop output voltage and magnetic field was derived. This relationship was shown to be valid between dc and frequencies for which a loop arm is between 10 and 20 electrical degrees in length. The condition under which the upper frequency can be extended above 10 degrees was shown to be the degree to which the impedance of one of the loop arms matches an impedance corresponding to half the balanced connecting line impedance. For frequencies corresponding to less than 10 degrees, the basic relationship derived is valid for any loop coax and balanced line impedance, i.e., the amount of ''mismatch'' is unimportant for this case.

Some approximations were deduced, probably the most significant of which (for pulse work) was the definition of the frequency below which the loop acts very nearly as an ideal differentiator.

APPENDIX

Consider a plane sinusoidal electromagnetic wave traveling in the direction of the x-axis, in the coordinate system of Figure A1, with $\vec{B}$ (t,x) perpendicular to the x-y plane. In the ensuing analysis, the time is held constant at $t = t_1$ so as to concentrate on spatial behavior. Now since $\partial \vec{B}(t_1, x)/\partial t \cdot \hat{n}$ is, by definition, invariant with y (and z), it follows that, for the loop,*

$$\frac{\partial \vec{B}(t_1, x)}{\partial t} \cdot \hat{n} \, dA = \frac{\partial B(t_1, x)}{\partial t} \left(2\sqrt{R^2 - (x-x_1)^2}\right) dx \qquad \text{A-1}$$

as illustrated in Figure A-1.

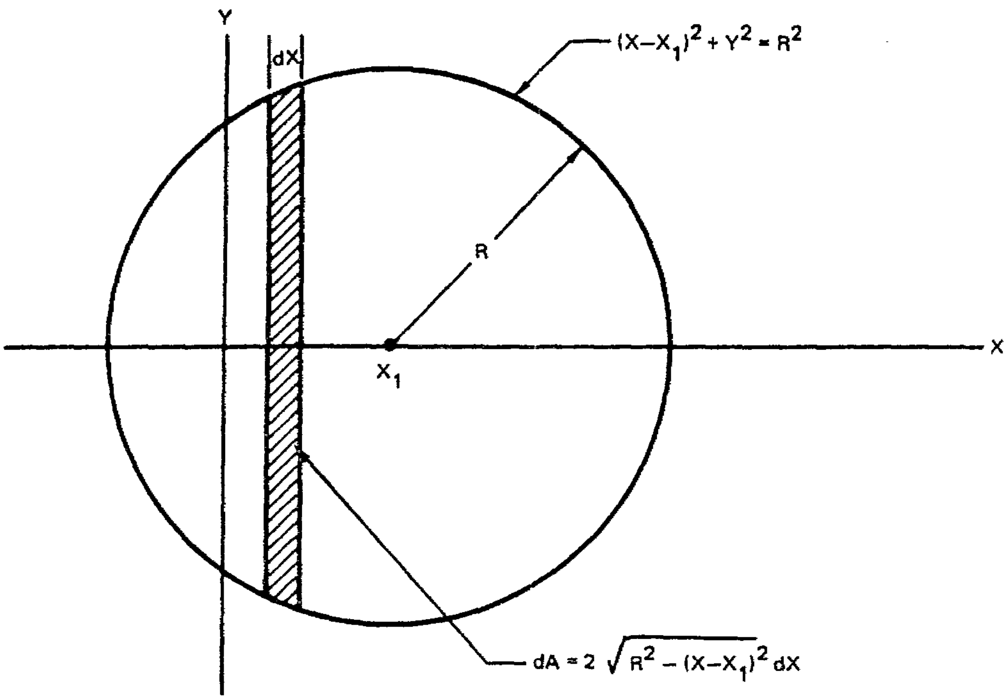


Figure A.1. Equation for the Loop Boundary, Illustrating a Differential Area Element

*The analysis of this appendix is limited to frequencies corresponding to a loop electrical diameter of about 10°, with a loop thickness, as noted in the discussion on radiation resistance, of less than one fifth the diameter. For these conditions, scattering effects can be neglected.

Now the assumption which leads to Equation (2) is that $\partial B(t_1, x)/\partial t$ is essentially constant over the loop area. Evidently the greatest variation in this quantity over the loop, for the present (sinusoidal) case, is for x near zero where t_1 is chosen so that $\partial B(t_1, 0)/\partial t = 0$. This is because, for this condition, $\partial B(t_1, x)/\partial t$ is of the form Ksin $\omega x/v$, where v is the propagation velocity. For this case, for values near zero,

$$\frac{\partial \vec{B}(t_1, x)}{\partial t} \cdot \hat{n} \approx \left(\frac{K\omega}{v}\right) x$$

Equation (A2) is valid within ± 10 percent for $\omega x/v$ up to ± 45.25 degrees. First one asks if the assumption that

$$\iint_{\substack{\text{Loop}\\ \text{Area}}} \frac{\partial \vec{B}(t_1, x)}{\partial t} \cdot \hat{n}\, dA \approx \pi R^2 \frac{\partial B(t_1, x_1)}{\partial t} \qquad \text{A-3}$$

is valid over this range. That is, may the rate of change of flux density at loop center be taken as constant over the loop area over this range because of averaging?

Using the approximation of Equation (A.2) this assumption gives:

$$\frac{\partial B(t_1, x_1)}{\partial t} A = \pi R^2 \left(\frac{K\omega}{v}\right) x_1$$

where x_1 is the value of x at the loop center. From Equations (A-1) and (A-2)

$$\iint_{\substack{\text{Loop}\\ \text{Area}}} \frac{\partial \vec{B}(t_1, x)}{\partial t} \cdot \hat{n}\, dA = \int_{x_1-R}^{x_1+R} 2\left(\frac{K\omega}{v}\right) x \left(\sqrt{R^2 - (x-x_1)^2}\right) dx \qquad \text{A-4}$$

Using the substitution $\phi = x-x_1$, Equation (A-4) becomes:

$$2\left(\frac{K\omega}{v}\right) \int_{-R}^{R} (\phi + x_1) \sqrt{R^2-\phi^2} \, d\phi = \left(\frac{K\omega}{v}\right) x_1 \pi R^2 \qquad \text{A-5}$$

which demonstrates the validity of Equation (A.3) for angles $\omega x/v < \pm 45.25$ degrees. For larger angles, the fact that $\partial B(t_1, x)/\partial t$ varies sinusoidally with x must be accounted for. Because the loop effectively weights field contributions near the center of the loop so much more than those near the edges, we assume that a variation in $\partial B(t_1, x)/\partial t$ (due to varying x) of ± 10 percent from the value at loop center is acceptable. For loop center at $\omega x/v = \pm 45.25$ degrees this gives 45.25 degrees ± 5.55 degrees. This is the ''worst case'' variation (beyond the quasi-linear region) since such variation decreases for increasing angle. Thus, the loop diameter is about 11.1 degrees so that a loop ''arm'' is

$$\frac{\pi}{2}(11.1) \approx 17.5 \text{ degrees}$$

Considering the nature of the approximations made it may be concluded that a maximum loop arm length of from 15 to 20 degrees may be used without undue concern over the validity of Equation (2).

REFERENCES

1. L. L. Libby. Special Aspects of Balanced Shielded Loops. Proceedings of the IRE, September 1946.

2. C. E. Baum. Characteristics of the Moebius Strip Loop. Electromagnetic Pulse Sensor and Simulator Notes, Note 7, December 1964.