Analysis of Periodic Structures for Trapping Electrons in EMP Simulators

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Abstract

In the System Generated Electromagnetic Pulse (SGEMP) simulators a flux of high energy photons is introduced into a vacuum chamber. These photons strike the walls of the chamber, producing Compton electrons. As this effect is not present in the environment to be simulated, it is desirable to prevent these electrons from entering the working volume of the chamber.

In this paper a method of containing these electrons is considered. It is proposed that a grid of wires be placed parallel to the chamber wall and raised to a high negative potential, thus repelling the electrons and containing them between the chamber wall and the grid. The results of a computer study undertaken to evaluate this approach are presented.

The electrostatic potential is determined by the use of a line charge approximation for grids consisting of wire of small cross section. For wires of larger cross section a series solution is employed; the coefficients of the series are determined using a least squares fit. Finally, the effectiveness of the grids as electron traps is evaluated by calculating the trajectories of electrons having various kinetic energies, positions, and angles of departures.

This study was performed under subcontract to
The Dikewood Corporation
1009 Bradbury Drive, S.E.
University Research Park
Albuquerque, New Mexico 87106
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INTRODUCTION

One of the problems encountered in the study of EMP phenomena is the effect of a flux of gamma rays and X-rays impinging upon a satellite. To study this System Generated EMP it has been proposed to build a SGEMP simulator of the general configuration described by Baum\(^1\). The simulator is to consist of a metal chamber in which a very high vacuum is to be maintained, a source of high energy photons (gamma rays and X-rays), and a method of coupling the beam of photons into the chamber (Figure 1). The device to be studied is placed in the vacuum chamber and the photons allowed to impinge on it. It is desirable that the chamber simulate the environment of free space as closely as possible. This requires the damping of electromagnetic resonances of the chamber, reduction of the capacitance between the device under study and the chamber to acceptable levels, absorption and/or trapping of particles striking the chamber walls, etc.

In his note describing a SGEMP simulator, Baum discusses some of the problems and design considerations which arise in the construction of such a simulator. This paper considers a possible solution to one of these problems, the production of Compton electrons at the wall of the chamber\(^2\). These electrons are produced by two different causes. When the incident photon beam enters the chamber, not all the photons will strike the device under test; some will strike the wall of the chamber and possibly produce Compton electrons. Similarly, photons striking the device will produce Compton electrons at the surface of the device. Some of these electrons will return to the device under the influence of electromagnetic fields, but others may proceed to the wall of the chamber and there produce secondary Compton electrons. Since electrons produced
Figure 1. An SGEMP Simulator
Figure 2. A Wire Grid Near a Ground Plane
grating of parallel rods near a ground plane in the study of an EMP parallel plate simulator\(^{(5)}\). For other studies where the line charge approximation may not be used or the finite extent of the conducting plane must be considered, other methods, such as the electrolyte tank, have been used to find the potential distribution\(^{(6,7)}\). The above methods are confined to two-dimensional geometries. The electrolyte tank as well as resistor networks have been used to find the potential distribution of electrostatic lenses and deflector systems of three-dimensional geometry but only those which have axial symmetry\(^{(7)}\). For more general geometries, the expansion of the potential in terms of spatial harmonics may be employed as was done in this paper. Another possible approach would be the use of an integral equation formulation as has been done with thin wire structures; this technique would be of interest if the charge densities on the conductors were of principal concern\(^{(8)}\).

To determine the effectiveness of the grids in containing electrons, it is necessary to determine the fraction of electrons escaping to the working volume of the chamber as a function of their initial kinetic energy. To do this, the trajectories of the electrons under the influence of the potential distribution produced by the grids must be determined. The effect of an electrostatic field on the paths of electrons has been studied in vacuum tubes and in electron microscopes and cathode ray tubes. Experimental techniques such as the direct measurement of the electron flux density in the devices have been used. The effect of the potential distribution may also be simulated by means of a stretched rubber sheet\(^{(9)}\). In this method the rubber sheet is deformed in such a way that the effect of the resulting gravitational potential on balls rolled on the sheet simulates the effect of the electrostatic
potential on electrons. An analytical technique has been developed by
Hashimoto, et al., using conformal mapping and an axial ray theory to
find the trajectories of electrons in an electrostatic field\(^{(10)}\). The
disadvantage of many of these techniques is that they may only be used
for two-dimensional field configurations. Furthermore, many of the tech-
niques assume only small deflections of the electrons. The method em-
ployed in this paper of direct calculation of the electron trajectories
by numerical techniques does not have these drawbacks and has become
feasible with the use of high speed digital computers.

In this paper computer programs were developed which allowed the
work done by Tesche to be extended to three dimensions. Single-layer
and two-layer grids composed of wires of sufficiently small radii that
they could be approximated by lines of charge were studied in this manner.
Programs were then developed to study grids consisting of conductors of
larger radii. These last programs employ a series solution to Laplace's
equation and a least squared error fitting technique for the boundary
conditions. In all cases the potential distribution was found and po-
tential contour maps were drawn. The effectiveness of the grids was
found as a function of the energy and position of the electrons and of
the dimensions and potential of the grid by direct calculation of the tra-
jectories of electrons leaving the ground plane. Plots of the function of
electrons escaping as a function of energy and direction were made for a
uniform distribution of electrons being emitted from the ground plane.

Many of the calculations performed in this paper required the use
of a digital computer. The CDC 6600 located at Kirtland Air Force Base
at Albuquerque, New Mexico was utilized to do these calculations. Some
of the subroutines used in this paper, in particular BRUTE and DRAW4,
were developed at The Dikewood Corporation of Albuquerque, New Mexico.
1. **Simplifying Assumptions**

   In this section we will obtain the potential distribution due to a grid of wires placed parallel to the wall of the simulator chamber. The chamber wall will be assumed to be perfectly conducting and to be locally flat. The potential of the wall will be assigned a value of zero. The problem then becomes, under these circumstances, that of finding the potential distribution of a wire mesh above and parallel to a perfectly conducting ground plane. A further assumption will be made that the mesh consists of parallel cylindrical conductors and that the diameter of the wires is small compared to both the distance from the wires to the ground plane and the distance between parallel wires. This last assumption allows the conductors to be replaced by lines of charge as will be shown in Section I-4.

2. **Single Grid of Wires**

   The first geometry we will consider is that of a mesh of wires consisting of cylindrical conductors. The centerlines of all the conductors lie in a plane at a distance $b$ above and parallel to the ground plane. The conductors intersect at right angles and form a square grid. Assuming the conductors have a small diameter, we replace them with line charges located at the centerlines of the conductors as shown in Figure I-1.

   The configuration of Figure I-1 may be thought of as the superposition of two sets of parallel line charges, one set consisting of line charges extending in the $x$ direction and the other set consisting of...
Figure I-1. A Single Grid of Line Charges

line charges extending in the y direction. In this case, superposition may be used since the wires have been replaced by fixed line charges. It now remains to find the potential distribution due to these sets of line charges so that they may be combined to yield the solution for the grid.
Figure I-2. A Set of Parallel Line Charges

Figure I-2 shows an infinite set of line charges at a distance $b$ above a perfectly conducting ground plane. The line charges are parallel to each other and the ground plane. They extend to $\pm\infty$ in the $y$ direction with a distance $2a$ between adjacent line charges. F. M. Tesche\(^4\) has shown that for this geometry the potential distribution can be found in closed form by conformal mapping techniques and is

$$\Phi(x, z) = \frac{\rho}{4\pi \varepsilon_0} \ln \frac{\cosh \left( \frac{\pi(z+b)}{a} \right) - \cos \left( \frac{\pi x}{a} \right)}{\cosh \left( \frac{\pi(z-b)}{a} \right) - \cos \left( \frac{\pi x}{a} \right)}$$

(1)

where $\Phi(x, z)$ is the potential, $\rho$ is the line charge density, $\varepsilon_0$ is
the permittivity of free space, 2a is the spacing between line charges and b is the distance of the line charges from the ground plane.

From the above expression we see that in the limit as z becomes infinite the potential becomes

\[
\frac{\rho_o}{2\epsilon_o} \left( \frac{b}{a} \right).
\]

To obtain the potential distribution due to the grid of line charges shown in Figure I-1, we add the potential distributions for the two orthogonal sets of line charges. Let \( \phi_o(x,y) \) be the potential distribution function given by Equation (1) but with \( \rho_o \) equal to 1. The potential distribution for the geometry in Figure I-1 is then given by

\[
\phi(x,y,z) = \rho_1 \phi_o(x,z) + \rho_2 \phi_o(y,z)
\]

(2)

where \( \rho_1 \) and \( \rho_2 \) are the charge densities of the line charges extending in the y and x directions respectively. As z becomes infinite, the potential becomes

\[
\phi(x,y,z) \bigg|_{z=\infty} = \frac{\rho_1}{2\epsilon_o} \left( \frac{b}{a} \right) + \frac{\rho_2}{2\epsilon_o} \left( \frac{b}{a} \right)
\]

(3)

Confining our attention to the case where the charge densities of the two sets of line charges are identical, the potential distribution becomes

\[
\phi(x,y,z) = \rho [\phi_o(x,z) + \phi_o(y,z)]
\]

(4)

and the potential as z becomes infinite is

\[
\phi(x,y,z) \bigg|_{z=\infty} = \frac{\rho b}{2\epsilon_o a}
\]

(5)

where \( \rho \) is the line charge density of both sets of line charges.
3. *Double Grid of Wires*

From the potential contour plots and electron trajectory plots shown in Section IV for the single grid case analyzed above, it was noted that the electrons had a tendency to escape through the center of the grid apertures. It was thought that placing another grid of wires above and parallel to the first grid and shifted so that the crossing points of the wires of the second grid would lie directly above the center of the apertures of the first grid (Figure I-3) would reduce the number of electrons escaping.

Again we approximate the wires by line charges located on the centerlines of the wires. We only treat the case of grids with square apertures and with the distance between the first and second grid being the same as the distance between the first grid and the ground plane. The principle of superposition is again employed to find the total potential distribution.

For the purposes of this analysis of the double grid case, let the potential distribution function for the single grid shown in Figure I-1 with charge density equal to 1.0 be represented by $\phi_{SG}(x, y, z, a, b)$ where $2a$ is the spacing between parallel line charges and $b$ is the distance from the line charges to the ground plane. Here we have written the potential distribution as an explicit function of $a$ and $b$ as well as $x$, $y$, and $z$. The potential for the double grid configuration may then be written as

$$\phi_{DG}(x, y, z, a, b) = \rho_1 \phi_{SG}(x, y, z, a, b)$$

$$+ \rho_2 \phi_{SG}(x+a, y+a, z, a, 2b) \quad (6)$$

where $\phi_{DG}(x, y, z, a, b)$ is the potential distribution for the double
Figure I-3. A Double Grid of Line Charges
grid, \( \rho_1 \) is the line charge density on the lower grid, and \( \rho_2 \) is the line charge density on the upper grid. As \( z \) becomes infinite for this case the potential becomes

\[
\phi_D(x,y,z,a,b) \bigg|_{z \to \infty} = \rho_1 \frac{b}{\varepsilon_0} + \rho_2 \frac{2b}{\varepsilon_0} \tag{7}
\]

In this case, since we are considering two completely separate grids, we will not assume that the charge density on the two grids is the same.

4. Limits on Wire Size for Line Charge Approximation

In the single and double grid analysis, the wires were replaced by line charges. Intuitively, we know that for small distances from the line charge the potential distribution will be that of the logarithmic singularity of an isolated line charge and that a constant potential contour near the line charge would therefore be cylindrical. If this is indeed so, a cylindrical conductor placed so that its surface corresponds to such a contour and maintained at the same potential as the contour would produce the same field distribution outside of the conductor as the line charge. We will show that for a sufficiently small distance from the wire the contours are indeed cylindrical. We will also derive an approximate expression for the error encountered in using the line charge approximation.

To show that the contours near one of the line charges is cylindrical, consider Figure I-4. In this figure there is a set of parallel line charges spaced 2a apart and a distance b above the ground plane. The set of charges extends infinitely in the x direction and each line charge extends to \( \pm \infty \) in the y direction.

The potential of a point lying on a circle of radius \( \delta \) about the
Figure I-4. A Set of Parallel Line Charges With a Point on a Circle of Radius $\delta$

line charge on the $z$ axis is

$$\phi(x,z) = \frac{\rho_0}{4\pi\varepsilon_0} \ln \left[ \frac{\cosh((2b+\delta\sin(\theta))\pi/a) - \cos(\delta\cos(\theta)\pi/a)}{\cosh(\delta\sin(\theta))\pi/a - \cos(\delta\cos(\theta)\pi/a)} \right]$$

(8)

where $x$ and $z$ specify a point on the circle at an arbitrary angle $\theta$.

If $\delta\pi/a$ is small compared to 1.0, we may use the approximation

$$\cos(\delta\cos(\theta)\pi/a) \approx 1.0$$

(9)

If we further require that $\delta$ is small compared to $2b$, then

$$\cosh((2b+\delta\sin(\theta))\pi/a) = \cosh(2b\pi/a)$$

(10)

Using these approximations, the numerator of the term in brackets, which we will call $N$, becomes
\[ N = \cosh(2b\pi/a) - 1 \] (11)

To approximate the denominator of the term in brackets, the \( \cosh \) and \( \cos \) functions are expanded in a power series. The expansions for the functions are well known and are

\[
\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots
\] (12)

and

\[
\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots
\] (13)

Using these expansions, the denominator, which we will call \( D \), becomes

\[
D = \left( \frac{T_1^2 + T_2^2}{2!} \right) + \left( \frac{T_1^4 - T_2^4}{4!} \right) + \ldots
\] (14)

where

\[
T_1 = \delta \sin(\theta)\pi/a
\] (15)

and

\[
T_2 = \delta \cos(\theta)\pi/a.
\] (16)

Since we are assuming that \( \delta \pi/a \) is much less than 1.0, both \( T_1 \) and \( T_2 \) are much less than 1.0. Ignoring terms of order four and higher, the denominator becomes

\[
D = \frac{T_1^2 + T_2^2}{2!} = 1/2 \left[ \frac{\delta \pi}{a} \right]^2
\] (17)

The potential of Equation (8) is now

\[
\phi(x,z) = \frac{\rho_0}{4\pi \varepsilon_0} \ln \left[ \frac{\cosh(2b\pi/a) - 1}{1/2 \left[ \frac{\delta \pi}{a} \right]^2} \right]
\] (18)

Since this expression is not a function of \( \theta \), the potential is a constant on a circle of radius \( \delta \) if \( \delta \) is small compared to both \( \frac{a}{\pi} \) and \( 2b \).

The above analysis has shown that the contours become cylindrical
as we approach the line charges. We will now derive an expression which
will provide an estimate of how severely the actual contours deviate from
being cylindrical for a given $\delta$. We will calculate the difference in
potential of two points, one at a distance $\delta$ and at an angle $\pi/2$ and the
other at a distance $\delta$ and at an angle $-\pi/2$. The potentials at these two
points are

$$
\phi(x, z) = \frac{\rho_o}{4\pi \varepsilon_o} \ln \left[ \frac{\cosh((2b+\delta)\pi/a) - 1}{\cosh(\delta\pi/a) - 1} \right]
$$

(19)

$$
\phi(x, z) = \frac{\rho_o}{4\pi \varepsilon_o} \ln \left[ \frac{\cosh((2b-\delta)\pi/a) - 1}{\cosh(-\delta\pi/a) - 1} \right]
$$

(20)

Subtracting these two expressions and using the fact that the $\cosh$ is an even function and the characteristics of logarithms, we find

$$
\phi(x, z) - \phi(x, z) = \frac{\rho_o}{4\pi \varepsilon_o} \ln \left[ \frac{\cosh((2b+\delta)\pi/a) - 1}{\cosh((2b-\delta)\pi/a) - 1} \right]
$$

(21)

We now write the $\cosh$ in terms of exponentials and factor the term $e^{\delta\pi/a}$ out of the numerator and the term $e^{-\delta\pi/a}$ out of the denominator.

Denoting the difference in potential as $\Delta \phi$, we write

$$
\Delta \phi = \frac{\rho_o}{4\pi \varepsilon_o} \ln \left[ \frac{e^{\delta\pi/a}(1/2) \left( e^{2b\pi/a} + e^{2(b+\delta)\pi/a} \right)}{e^{-\delta\pi/a}(1/2) \left( e^{2b\pi/a} + e^{2(b-\delta)\pi/a} \right)} \right]
$$

(22)

If we require that $\delta$ be small compared to both $b$ and $a/\pi$

$$
e^{-(b-\delta)2\pi/a} \approx e^{-2b\pi/a}
$$

(23)

and the difference in the potentials becomes
\[
\Delta \phi = \frac{\rho_o}{4\pi\varepsilon_o} \ln \left[ \frac{\delta \pi/a \left( \cosh(2b\pi/a) - 1.0 \right)}{e^{-\delta \pi/a} \left( \cosh(2b\pi/a) - 1.0 \right)} \right] = \frac{\rho_o}{2\pi\varepsilon_o} \left( \frac{\delta \pi}{a} \right) (24)
\]

Dividing this expression by the potential found in Equation (18) yields the fractional variation of potential between these two points. Denoting the potential of Equation (18) as \( \phi_o \), we find

\[
\frac{\Delta \phi}{\phi_o} = \frac{2 \delta \pi}{a} \ln \frac{\cosh(2b\pi/a) - 1}{\frac{1}{2} \left( \frac{\delta \pi}{a} \right)^2} \quad (25)
\]

This expression provides an estimate of the accuracy obtained by substituting line charges for the actual conductors for various radii of the conductors. It is informative to calculate this fractional variation of potential for a few sample cases. The results of these calculations are shown in Table I-1.

<table>
<thead>
<tr>
<th>( \delta/a )</th>
<th>( b/a )</th>
<th>( \Delta \phi/\phi_o )</th>
</tr>
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<tbody>
<tr>
<td>.003</td>
<td>1.0</td>
<td>.12%</td>
</tr>
<tr>
<td>.03</td>
<td>1.0</td>
<td>.48%</td>
</tr>
<tr>
<td>.03</td>
<td>1.0</td>
<td>1.7%</td>
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<tr>
<td>.1</td>
<td>1.0</td>
<td>7.1%</td>
</tr>
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<td>.003</td>
<td>.5</td>
<td>.15%</td>
</tr>
<tr>
<td>.01</td>
<td>.5</td>
<td>.68%</td>
</tr>
<tr>
<td>.03</td>
<td>.5</td>
<td>2.4%</td>
</tr>
<tr>
<td>.1</td>
<td>.5</td>
<td>12.7%</td>
</tr>
</tbody>
</table>

Table I-1 Fractional Variation of Potential

The analysis of this section was done using a set of parallel line charges. The results are applicable, however, to the grid of line
charges if the contributions to the potential of the other set or sets of line charges that make up the grid are small compared to that of the line charge representing the conductor. Since we are considering only small distances from the line charge, the potential due to this charge should be large compared to the other potential contributions if we avoid points near the intersections of the grid.
1. Need for Series Solution

In Section I-4 an error expression was derived and the error calculated for several cases. As can be seen from Table I-1, the error increases rapidly with increasing radii of the conductors. In one case with a radius of .1 the error is greater than 10 percent. The error calculations were not carried out for larger values of conductor radii since larger radii would not satisfy the conditions which must be fulfilled to use the error equation.

The failure of the line charge approximation to give sufficiently accurate results for the larger values of conductor radii led to the use of a series approach to the potential problem. Two configurations of conductors were considered, that of a set of conductors parallel to each other and to a ground plane and that of a grid formed by the intersection of two sets of parallel conductors which are parallel to a ground plane.

2. Parallel Cylindrical Conductor Case

The geometry for this case is shown in Figure II-1. It consists of a set of cylindrical conductors which are parallel to each other and also parallel to a perfectly conducting ground plane. The set of conductors extends to infinity in both the positive and negative x direction and each conductor extends to \( \infty \) in the y direction. The conductors have radius \( c \) and are maintained at a potential \( V_0 \).

To find the potential distribution for this configuration, Laplace's equation in two dimensions is solved in Cartesian coordinates by the
method of separation of variables.\(^\text{(12)}\) The solution is

\[
\phi(x,z) = [C_1 \cosh(Kz) + C_2 \sinh(Kz)] [C_3 \cos(Kx) + C_4 \sin(Kx)]
\]

(26)

where \(K, C_1, C_2, C_3,\) and \(C_4\) are constants which must be determined.

We must also consider the trivial solutions

\[
\phi(x,z) = C_5 + C_6x + C_7z + C_8xz
\]

(27)

where \(C_5, C_6, C_7,\) and \(C_8\) are also constants which must be determined.
We next divide the region above the ground plane into two regions and determine the proper form of the solution to be used in each region. Region I is such that $0 \leq z \leq b$ and region II is such that $z \geq b$. In region I the solution must be an even periodic function of $x$ of period $2a$. The potential must also satisfy the boundary condition on the ground plane and be equal to zero at $z = 0$. These requirements reduce the form of the solution to

$$\phi_I(x, z) = \sum_{n=1}^{\infty} A_n \sinh\left(\frac{n\pi}{a} z\right) \cos\left(\frac{n\pi}{a} x\right) + A_0 z \quad (28)$$

In region II the solution must also be an even periodic function of $x$ of period $2a$. It must further satisfy the condition that the potential at infinity must remain finite. These conditions reduce the form of the solution in region II to

$$\phi_{II}(x, z) = \sum_{n=1}^{\infty} B_n e^{-\frac{n\pi z}{a}} \cos\left(\frac{n\pi x}{a}\right) + B_0 \quad (29)$$

where $B_0$ and $B_n$ are constants and $\phi_{II}(x, z)$ is the potential in region II.

To find the coefficients $A_0$, $B_0$, $A_n$ and $B_n$, it is necessary to satisfy the boundary conditions on the surface of the cylinders and at the boundary of the two regions lying between the cylinders. These conditions are:

$$\begin{align*}
\phi_I(x, z) &= V_0 \quad \text{on the surface of} \\
\phi_{II}(x, z) &= V_0 \quad \text{the conductors} \\
\phi_I(x, z) &= \phi_{II}(x, z) \quad \text{at } z = b \\
\frac{d\phi_I(x, z)}{dz} &= \frac{d\phi_{II}(x, z)}{dz} \quad \text{the conductors}
\end{align*}$$

(30)
Since both $\phi_I(x,z)$ and $\phi_{II}(x,z)$ are even periodic functions of $x$ with periods of $2a$, satisfying the boundary conditions on that part of the boundary for which $0 \leq x \leq a$ insures that the boundary conditions will be satisfied at all points on the boundaries. For this reason only those portions of the configuration for which $0 \leq x \leq a$ will be considered.

Since it is impossible to calculate an infinite number of coefficients, the series in Equation (28) and Equation (29) were truncated. If the contributions to the total potential of the terms removed by this truncation is small, this procedure will yield a solution very close to the actual solution. However, the boundary conditions cannot be satisfied exactly with the truncated series. To obtain the best possible fit to the boundary conditions the method of least squared error was employed.

To apply the method of least squares to this problem, the square of the error in satisfying the boundary conditions as a function of the coefficients must be determined. We first consider the error in satisfying the boundary condition on the conductors. In region I this squared error at a point on the conductors is given by

$$\text{ERROR}^2 = [V_0 - \phi_I(x,z)]^2$$  \hspace{1cm} (31)

where $x$ and $z$ specify a point on the surface of the conductor in region I. In region II this error is given by

$$\text{ERROR}^2 = [V_0 - \phi_{II}(x,z)]^2$$  \hspace{1cm} (32)

where $x$ and $z$ specify a point on the conductors in region II.

We next consider the squared error contributions due to the requirements on the potential at the boundary between the two regions between the conductors. The error here arises from two sources, the difference in potential between region I and region II and the difference in the normal derivative between the two regions. The two errors are
calculated separately, squared, and then added to find the total squared error for points on this boundary. The first squared error is given by

\[ \text{ERROR}^2 = \left[ \phi_1(x,z) - \phi_{II}(x,z) \right]^2 \]  (33)

and the second by

\[ \text{ERROR}^2 = \left[ a \frac{d\phi_1(x,z)}{dz} - a \frac{d\phi_{II}(x,z)}{dz} \right]^2 \]  (34)

where the expressions are evaluated at points where \( z = b \) and \( c \leq x \leq a \).

Care must be taken in calculating the second squared error term above. When a function is represented by a series the derivative of the function cannot always be determined by a term-by-term differentiation of the series, as pointed out by Van Bladel.\(^{(13)}\) We know that this problem will occur, at least in the limiting case of very small radii conductors. To gain insight into how this problem occurs, we will look at the series solution of the set of parallel line charges problem in Figure I-2. Tesche\(^{(4)}\) has shown that this is

\[ \phi_1(x,z) = \frac{\rho_o}{\varepsilon_0 \left[ \frac{z}{2} + \sum_{n=1}^{\infty} \frac{1}{\gamma_n} \cos(\gamma_n x) e^{-\gamma_n b} \sinh(\gamma_n z) \right]} \]  (35)

in region I and

\[ \phi_{II}(x,z) = \frac{\rho_o}{\varepsilon_0 \left[ \frac{b}{2} + \sum_{n=1}^{\infty} \frac{1}{\gamma_n} \cos(\gamma_n x) e^{-\gamma_n b} \sinh(\gamma_n z) \right]} \]  (36)

in region II where region I and II are comprised of areas \( 0 \leq z \leq b \) and \( b \geq z \) respectively, and \( \gamma_n = n\pi/a \).

We will now differentiate these series term by term and evaluate the result at \( x = a \) and \( z = b \). In region I this results in the expression
\[
\frac{d\phi'_I(x,z)}{dz} \bigg|_{x=a, z=b} = \frac{\rho_o}{2\pi \epsilon_0} \left[ 1 + \sum_{n=1}^{\infty} \cos(\gamma_n a) \left( 1 + e^{-\gamma_n^2 b} \right) \right]
\]  

(37)

which does not converge. However, at a point a small distance away from this point at \( z = b - \delta \) and \( x = a \), the expression becomes

\[
\frac{d\phi'_I(x,z)}{dz} \bigg|_{x=a, z=b-\delta} = \frac{\rho_o}{2\pi \epsilon_0} \left[ 1 + \sum_{n=1}^{\infty} \cos(\gamma_n a) \left( e^{-\gamma_n^2 \delta} + e^{-\gamma_n(2b-\delta)} \right) \right]
\]  

(38)

which does converge. This indicates that the series representing the potential could be differentiated using the term by term procedure at all points except those on the boundary. To avoid the difficulty at the boundary, the derivative was calculated numerically at the boundary using the following technique. The derivative in region I at the boundary is given approximately by

\[
\frac{d\phi_I(x,z)}{dz} \bigg|_{z=b} = \frac{\phi_I(x,b) - \phi_I(x,b-\delta)}{\delta}
\]  

(39)

and the derivative with respect to \( z \) in region II is given approximately by

\[
\frac{d\phi_{II}(x,z)}{dz} \bigg|_{z=b} = \frac{\phi_{II}(x,b+\delta) - \phi_{II}(x,b)}{\delta}
\]  

(40)

Since \( \phi_I(x,z) \) and \( \phi_{II}(x,z) \) converge for all points in their respective regions, the above expressions must converge and will provide a good approximation to the derivative for sufficiently small \( \delta \). This method of calculating the derivative was used for finding the derivative at \( z = b \) of all series used.

Knowing the error at any point on the boundary, we now establish a set of points on the boundaries. The total squared error is found
by adding the squared error of each point. The resulting expression is differentiated with respect to each of the coefficients we wish to find and the result of each differentiation is set equal to zero. The resulting set of linear algebraic equations will have as their solution the values of the coefficients which will produce the minimum squared error. This was done for parallel conductor configuration and the equation was solved with the aid of a C.D.C. 6600 digital computer. The residual error was then determined by substitution of the coefficients into the equations for the potential and summing the squared error over the points (see Section II-5).

3. Crossed Cylindrical Conductor Case

The geometry for this case is shown in Figure II-2. It consists of two sets of parallel conducting cylinders like those of Figure II-1 but with one set orthogonal to the other. Both sets of cylinders are parallel to a perfectly conducting ground plane and the spacing between cylinders is the same for each set. This results in a grid with square apertures.

As in the preceding case, Laplace's equation was solved by the method of separation of variables. This time it is necessary to use Laplace's equation in three dimensions. The general solution in Cartesian coordinates is of the form

$$\phi(x,y,z) = [C_1 \cos(k_1 x) + C_2 \sin(k_1 x)] [C_3 \cos(k_2 y) + C_4 \sin(k_2 y)] [C_5 \cosh(\gamma z) + C_6 \sinh(\gamma z)]$$

where $C_1, C_2, C_3, C_4, C_5$ and $C_6$ are constants and $\gamma^2 = k_1^2 + k_2^2$. The trivial solutions which must also be considered are...
Figure II-2. A Set of Crossed Cylindrical Conductors

\[ \phi(x, y, z) = C_7 + C_8 x + C_9 y + C_{10} z + C_{11} xy + C_{12} xz + C_{13} yz + C_{14} xyz \] (42)

where \( C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, \) and \( C_{14} \) are constants. As in the case of the parallel conducting cylinders, the space above the ground plane is divided into two regions. Regions I and II are those spaces such that \( 0 \leq z \leq b \) and \( b \leq z \) respectively. In each of these regions the appropriate form of the solution is found.
In region I the solution must be an even periodic function of \(x\) with period \(2a\). The solution must also satisfy the boundary condition at the ground plane which states that the potential is zero at \(z = 0\). These restrictions reduce the form of the solution in region I to

\[
\phi_I(x,y,z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} A_{n,m} \cos(n\pi x/a) \cos(m\pi y/a) \sinh(\gamma_{n,m} z) + A_{0,0} z
\]  (43)

where the \(A_{n,m}\) terms are constants and \(\gamma_{n,m} = \frac{\pi}{a} \sqrt{n^2 + m^2}\).

In region II the potential must also be an even periodic function of \(x\) with period \(2a\) and must satisfy the additional requirement that it remain finite as \(z\) goes to infinity. These requirements reduce the form of the solution in this region to

\[
\phi_{II}(x,y,z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B_{n,m} \cos(n\pi x/a) \cos(m\pi y/a) e^{-2\gamma_{n,m}} + B_{0,0}
\]  (44)

where the \(B_{n,m}\) terms are constants.

Since we have limited this discussion to grids having square apertures, additional simplifications result from the symmetry of the configuration. Consider the potential at two points in region I as shown in Figure II-3. The first point \(P\) is specified by the coordinates \(x = a\), \(y = \beta\), and \(z\). The second point \(P'\) is specified by the coordinates \(x' = \beta\), \(y' = a\) and \(z' = z\). These points are symmetrical about the line \(1-1'\). Since the grid is also symmetrical about the line, the potentials at these two points are the same. Looking at the equations for the potential at these two points, we see that the potential at point \(P\) is given by

\[
\phi_I(a,\beta,z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} A_{n,m} \cos(n\pi a/a) \cos(m\pi \beta/a) \sinh(\gamma_{n,m} z) + A_{0,0} z
\]  (45)
and the potential at point P' is given by

\[ \phi_1(\beta, \alpha, z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} A_{n,m} \cos(n\pi \beta/a) \cos(m\pi \alpha/a) \sinh(\gamma_{n,m} z) + A_{0,0} z \]

(46)

Subtracting these two expressions and interchanging the m and n indices in the summations of Equation (46) we

Figure II-3. Crossed Cylindrical Conductors - Top View

are then able to combine the two pair of summations into the expression
\[
0 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left\{ [A_{n,m} \cos(n\pi a/a) \cos(m\pi \beta/a) \sinh(\gamma_{n,m} z)] \\
- [A_{m,n} \cos(m\pi \beta/a) \cos(n\pi a/a) \sinh(\gamma_{m,n} z)] \right\}
\] (47)

Since \( \gamma_{n,m} = \gamma_{m,n} \) for all \( n \) and \( m \), this equation may be rewritten as

\[
0 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left\{ [A_{n,m} - A_{m,n}] \cos(n\pi a/a) \cos(m\pi \beta/a) \sinh(\gamma_{n,m} z) \right\}
\] (48)

For this to be true for all \( a, \beta \) and \( z \) in region I, \( A_{n,m} \) must equal \( A_{n,m} \) for all \( n \) and \( m \). Substituting this condition into Equation (48), the potential in region I now has the form

\[
\phi_1(x,y,z) = \sum_{n=0}^{\infty} \sum_{m=n}^{\infty} A_{n,m} \left[ \cos(n\pi x/a) \cos(m\pi y/a) \right. \\
+ \cos(m\pi x/a) \cos(n\pi y/a) \sinh(\gamma_{n,m} z) + A_{0,0} \left. \right] (49)
\]

A similar argument can be made for the potential solution in region II. If this is done, the solution in region II is of the form

\[
\phi_2(x,y,z) = \sum_{n=0}^{\infty} \sum_{m=n}^{\infty} B_{n,m} \left[ \cos(n\pi x/a) \cos(m\pi y/a) \right. \\
+ \cos(m\pi x/a) \cos(n\pi y/a) \right] e^{-2\gamma_{n,m}} + B_{0,0} (50)
\]

The simplification of the form of the potential solutions that was carried out above greatly reduces the number of coefficients which must be calculated. Also, since the solutions thus obtained are symmetrical about the line \( l-l' \), satisfying the boundary conditions on one side of this line insures that the boundary conditions will be satisfied on both sides of the line. If we further note that the functions are even periodic functions of \( x \) and \( y \), it can be seen that the boundary conditions need only be satisfied in the shaded area of Figure II-3 to insure that they are satisfied over all space.

The boundary conditions for this case, which are very similar to
those of the preceding case, are as follows:

\[
\begin{align*}
\phi_I(x,y,z) &= V_0 & \text{on the conductor} \\
\phi_{II}(x,y,z) &= V_0 \\
\phi_I(x,y,z) &= \phi_{II}(x,y,z) \\
\frac{d\phi_I(x,y,z)}{dz} &= \frac{d\phi_{II}(x,y,z)}{dz}
\end{align*}
\]

(51)

at \( z = b \) between conductors

For reasons previously mentioned, a truncated series of the solution was used to approximately satisfy the boundary conditions. A least squares fit was utilized to obtain the best possible approximation to the boundary conditions with the terms employed. The squared error was calculated only at points in the shaded area of Figure II-3. The residual mean squared error was also calculated for this case after the coefficients were determined (see Section II-5).

4. Application of the Line Charge Solutions to the Large Radii Cylindrical Conductor Cases

It was shown in Section I that for small radii cylindrical conductors a solution for the potential could be found by replacing the conductors with line charges and using conformal mapping techniques. For large radii conductors we found that the accuracy of this method decreases rapidly with increasing radii. However, it is still desirable to use this line charge model to give a first approximation which is then improved by the use of the series solution. To do this, we write the potential in the form

\[
\phi_T = \phi_{LC} + \phi_S
\]

(52)

where \( \phi_T \) is the total potential, \( \phi_{LC} \) is the potential from the line charge solution, and \( \phi_S \) is the series solution.
The line charge solution term has a different form for each of the two configurations considered in this section. For the parallel cylinder case, this is merely the solution for the set of parallel line charges as found by Tesche\(^4\) and given in Equation (1). By adjusting the value of the charge density term of this solution, it may be made to match the potential of the conductor at one point on the conductor. We chose to have this point be the one at \(z = b\) and \(x = c\).

For the crossed conductor case, the line charge solution used is that of the single grid, i.e., Equation (4). Similarly for this case the charge density term may be adjusted so that the solution has the exact value of the potential of the conductor at \(x = a, y = c,\) and \(z = b\).

Using the line charge solution to obtain a first approximation to the potential affects only the squared error term due to the matching of the potential at the conductor. The squared error terms due to the matching conditions at \(z = b\) between the conductors are not affected. The squared error term at the conductor boundary may be easily modified to take account of the line charge solution as will be shown.

5. **Least Squared Error Fitting**

In this section we have stated that the boundary conditions will be approximately satisfied using a least squared error fitting procedure. The error terms for the various configurations and sections of the boundaries have been determined. These terms involve functions which are represented by series with unknown coefficients. We will now show how the coefficients of these series may be obtained to insure the best fit to the boundary conditions in the least squares sense.\(^{14}\) For the cases under consideration it is convenient to consider errors at a point \(P_i\) which can be represented by
where $A_n$ and $B_n$ are the coefficients to be determined and $C(P_i)$, $\varepsilon^1_n(P_i)$ and $\varepsilon^2_n(P_i)$ may be functions of position. The squared error is

$$[\text{ERROR}]^2 = \left[C(P_i) + \sum_{n=0}^{N} A_n \varepsilon^1_n(P_i) + \sum_{n=0}^{N} B_n \varepsilon^2_n(P_i)\right]^2 \quad (54)$$

Summing the error over a number of points, we have a total squared error of

$$[\text{TOTAL ERROR}]^2 = \sum_i \left[C(P_i) + \sum_{n=0}^{N} A_n \varepsilon^1_n(P_i) + \sum_{n=0}^{N} B_n \varepsilon^2_n(P_i)\right]^2 \quad (55)$$

where \(\sum\) is used to indicate the sum over the various points.

To minimize the total squared error, we differentiate this expression with respect to the coefficients and set the resulting expressions equal to zero. Differentiating with respect to $A_m$ yields

$$0 = \sum_i 2C(P_i) + \sum_{n=0}^{N} A_n \varepsilon^1_n(P_i) + \sum_{n=0}^{N} B_n \varepsilon^2_n(P_i) \varepsilon^1_m(P_i) \quad (56)$$

$$m = 0, 1, 2...N$$

Differentiating with respect to $B_m$ yields

$$0 = \sum_i 2C(P_i) + \sum_{n=0}^{N} A_n \varepsilon^1_n(P_i) + \sum_{n=0}^{N} B_n \varepsilon^2_n(P_i) \varepsilon^2_m(P_i) \quad (57)$$

$$m = 0, 1, 2...N$$

With simple algebraic manipulation, Equation (56) may be rewritten as

$$\sum_{n=0}^{N} A_n \varepsilon^1_n(P_i) \varepsilon^1_m(P_i) + \sum_{n=0}^{N} B_n \varepsilon^2_n(P_i) \varepsilon^1_m(P_i) = - \sum_i C(P_i) \varepsilon^1_m(P_i) \quad (58)$$

$$m = 0, 1, 2...N$$

Similarly, Equation (57) becomes
Since \( m \) may take on values from zero to \( N \), we see that we now have

\[ 2N + 2 \text{ equations in } 2N + 2 \text{ unknowns, } A_0 \text{ through } A_N \text{ and } B_0 \text{ through } B_N. \]

If all the parameters of these equations are known except for the coefficients, the coefficients may be found by solving this set of linear algebraic equations.

\[
\sum_{n=0}^{N} A_n \sum_{i}^{1} \varepsilon_n^1(P_i) \varepsilon_m^2(P_i) + \sum_{n=0}^{N} B_n \sum_{i}^{2} \varepsilon_n^2(P_i) \varepsilon_m^2(P_i) = - \sum_{i}^{C} \varepsilon_m^2(P_i)
\]

(59)

\( m = 0, 1, 2 \ldots N \)

Figure II-4. A Set of Parallel Cylindrical Conductors
The previous analysis may now be applied to the parallel conducting cylinders case. In Figure II-4 we see that there are three line segments where it is necessary to match the boundary conditions in the space such that $0 \leq x \leq a$. The segment $l_1$ is that part of the cylinder to the right of the $z$ axis and for which $z < b$. The segment $l_2$ is that part of the cylinder to the right of the $z$ axis for which $z > b$. Line segment $l_3$ is the boundary at $z = b$ outside of the cylinders.

For a point lying on the line segment $l_1$, the error is of the form

$$\text{ERROR} = V_0 - \phi_1 (x, z) = V_0 - \sum_{n=0}^{N} A_n \eta_n (x, z)$$

where

$$\eta_n = \cos(n\pi x/a) \sinh(n\pi z/a) \text{ for } n > 0$$

$$\eta_0 = z$$

For points lying on the line segment $l_2$, the error is of the form

$$\text{ERROR} = V_0 - \phi_2 (y, z) = V_0 - \sum_{n=0}^{N} B_n \xi_n (x, z)$$

where

$$\xi_n = \cos(n\pi x/a) e^{-n\pi z/a}$$

For points lying on the line segment $l_3$, the error resulting from discontinuity in the potential is of the form

$$\text{ERROR} = \left[ \phi_1 (x, z) - \phi_2 (x, z) \right] = \sum_{n=0}^{N} A_n \eta_n (x, z) - \sum_{n=0}^{N} B_n \xi_n (x, z)$$

and the error resulting from discontinuity of the normal derivative of the potential is one of the form
\[
\text{ERROR} = a \left[ \frac{d\phi_I(x,z)}{dz} - \frac{d\phi_{II}(x,z)}{dz} \right] = \sum_{n=0}^{N} \left[ A_n \eta_n(x,z) - B_n \xi_n(x,z) \right] 
\]

where \( \eta_n(x,z) \) and \( \xi_n(x,z) \) are as defined previously and

\[
\eta_n(x,z) = \text{the derivative of } \eta_n(x,z) \text{ with respect to } z \text{ found numerically as described in Section II-2 multiplied by } a, \text{ and}
\]

\[
\xi_n(x,z) = \text{the derivative of } \xi_n(x,z) \text{ with respect to } z \text{ found numerically as described in Section II-2, multiplied by } a. \quad (66)
\]

Comparing these expressions with Equation (53), we see that on segment 1

\[
\begin{align*}
C(P_i) &= V_0 \\
\varepsilon_1^n(P_i) &= -\eta_n(x,z) \\
\varepsilon_2^n(P_i) &= 0
\end{align*}
\]

and on segment 2

\[
\begin{align*}
C(P_i) &= V_0 \\
\varepsilon_1^n(P_i) &= 0 \\
\varepsilon_2^n(P_i) &= -\xi_n(x,z)
\end{align*}
\]

On segment 3 the error due to the discontinuity of the potential results in

\[
\begin{align*}
C(P_i) &= 0 \\
\varepsilon_1^n(P_i) &= \eta_n(x,z) \\
\varepsilon_2^n(P_i) &= -\xi_n(x,z)
\end{align*}
\]

and the error due to the discontinuity of the normal derivative is
\[ c(P_1) = 0 \]
\[ \varepsilon_{n}^{1}(P_1) = \eta_{n}(x,z) \]  
\[ \varepsilon_{n}^{2}(P_1) = -\xi_{n}(x,z) \]  

Substituting these relationships into Equation (58) and summing over points on the three line segments, we obtain the following:

\[ \sum_{n=0}^{N} A_n \sum_{l=1}^{3} \eta_{n}(x,z) \eta_{m}(x,z) + \sum_{n=0}^{N} A_n \sum_{l=3}^{13} \eta_{n}(x,z) \eta_{m}(x,z) \]
\[ + \sum_{n=0}^{N} B_n \sum_{l=3}^{13} -\xi_{n}(x,z) \eta_{m}(x,z) + \sum_{n=0}^{N} A_n \sum_{l=3}^{13} \eta_{n}(x,z) \eta_{m}(x,z) \]
\[ + \sum_{n=0}^{N} B_n \sum_{l=3}^{13} -\xi_{n}(x,z) \eta_{m}(x,z) = \sum_{l=1}^{11} V_{0}\eta_{m}(x,z) \]
\[ m = 0, 1, 2 \ldots N \]

Substituting into Equation (59), we obtain the following:

\[ \sum_{n=0}^{N} B_n \sum_{l=2}^{12} \xi_{n}(x,z) \xi_{m}(x,z) + \sum_{n=0}^{N} A_n \sum_{l=3}^{13} -\eta_{n}(x,z) \xi_{m}(x,z) \]
\[ + \sum_{n=0}^{N} B_n \sum_{l=3}^{13} \xi_{n}(x,z) \xi_{m}(x,z) + \sum_{n=0}^{N} A_n \sum_{l=3}^{13} -\eta_{n}(x,z) \xi_{m}(x,z) \]
\[ + \sum_{n=0}^{N} B_n \sum_{l=3}^{13} -\xi_{n}(x,z) \xi_{m}(x,z) = \sum_{l=2}^{12} V_{0}\xi_{m}(x,z) \]
\[ m = 0, 1, 2 \ldots N \]

Rearranging and gathering terms for the \( A_n \) and \( B_n \) coefficients, Equation (71) becomes
\[
\sum_{n=0}^{N} \left[ A_n \left\{ \sum_{l=1}^{3} \eta_n(x,z) \eta_m(x,z) + \sum_{l=3}^{1} \eta_n(x,z) \eta_m(x,z) \right\} + B_n \left\{ \sum_{l=3}^{1} \left[ -\xi_n(x,z) \eta_m(x,z) - \xi_n(x,z) \eta_m(x,z) \right] \right\} - \sum_{l=2}^{1} \xi_n(x,z) \xi_m(x,z) \right\} = \sum_{l=2}^{1} v_0\eta_m(x,z)
\]
\[m = 0, 1, 2, \ldots N\] (73)

Rearranging the terms in Equation (72) yields
\[
\sum_{n=0}^{N} \left[ A_n \left\{ \sum_{l=1}^{3} -\eta_n(x,z) \xi_m(x,z) - \eta_n(x,z) \xi_m(x,z) \right\} + B_n \left\{ \sum_{l=2}^{1} \xi_n(x,z) \xi_m(x,z) + \sum_{l=3}^{1} \xi_n(x,z) \xi_m(x,z) \right\} + \xi_n(x,z) \xi_m(x,z) \right\} = \sum_{l=2}^{1} v_0\xi_m(x,z)
\]
\[m = 0, 1, 2, \ldots N\] (74)

If the line charge solutions are used to provide a first approximation to the potential as mentioned in Section II-4 the above equations are modified slightly. It may be seen that the effect of the line charge solution will be taken into account merely by replacing the term \(v_0\) with \(v_0 - \phi_{LC}(x,z)\) where \(\phi_{LC}(x,z)\) is the line charge solution. This results in the right hand side of Equation (73) becoming
\[
\sum_{l=1}^{1} \left[ v_0 - \phi_{LC}(x,z) \right] \eta_m(x,z)
\]
(75)

and the right hand side of Equation (74) becoming
\[
\sum_{l=2}^{1} \left[ v_0 - \phi_{LC}(x,z) \right] \xi_m(x,z)
\]
(76)
We will now apply the least squares error fitting scheme to the grid of crossed cylindrical conductors. Referring to Figure II-5, which shows the crossed cylindrical conductor configuration, we see that there are three surfaces over which the boundary conditions must be satisfied in the space where $0 \leq x \leq a$ and $0 \leq y \leq a$. The surface $S_1$ is that part of the conductor such that $z < b$, and is to one side of a diagonal of the aperture. The surface $S_2$ is specified the same way as $S_1$ but such that $z > b$. The surface $S_3$ is the area at $z = b$ to one side of a diagonal of the aperture and not on the conductor. For points lying on the surface $S_1$ the error is of the form

$$\text{ERROR} = V_0 - \phi_I(x,y,z) = V_0 - \sum_{n=0}^{N} \sum_{m=0}^{N} A_{n,m} n_{n,m}(x,y,z)$$

where

$$n_{n,m}(x,y,z) = \left[ \cos(n\pi x/a) \cos(m\pi y/a) + \cos(m\pi x/a) \cos(n\pi y/a) \right] \sinh(\gamma_{n,m} z)$$

and

$$\gamma_{n,m} = \frac{\pi}{a} \sqrt{n^2 + m^2}.$$ (79)

For points lying on the surface $S_2$ the error is of the form

$$\text{ERROR} = V_0 - \phi_{II}(x,y,z) = V_0 - \sum_{n=0}^{N} \sum_{m=n}^{N} B_{n,m} \xi_{n,m}(x,y,z)$$ (79)

where

$$\xi_{n,m} = [\cos(n\pi x/a) \cos(m\pi y/a) + \cos(m\pi x/a) \cos(n\pi y/a)] e^{-z\gamma_{n,m}}$$ (80)

For points on the surface $S_3$ the error due to the discontinuity of the potential is
\[
\text{ERROR} = [\phi_I(x,y,z) - \phi_{II}(x,y,z)] = \sum_{n=0}^{N} \sum_{m=0}^{N} [A_{n,m} n_{n,m}(x,y,z) - B_{n,m} n_{n,m}(x,y,z)]
\]

(81)

Figure II-5. A Set of Crossed Cylindrical Conductors

and the error due to the discontinuity of the normal derivative of the potential is
ERROR = \[ a \left[ \frac{d\phi_I(x,y,z)}{dz} - \frac{d\phi_{II}(x,y,z)}{dz} \right] = \sum_{n=0}^{N} \sum_{m=n}^{N} \left[ A_{n,m} \eta_{n,m}(x,y,z) \right. \]

\[ - B_{n,m} \xi_{n,m}(x,y,z) \] (82)

where $\eta_{n,m}(x,y,z)$ and $\xi_{n,m}(x,y,z)$ are as defined previously and

$\eta_{n,m}^{\prime}(x,y,z) = \text{the derivative of } \eta_{n,m}(x,y,z) \text{ with respect to } z$

*calculated numerically as shown in Section II-2 multiplied by } a,

$\xi_{n,m}^{\prime}(x,y,z) = \text{the derivative of } \xi_{n,m}(x,y,z) \text{ with respect to } z$

*calculated numerically as shown in Section II-2 multiplied by } a.

Since the two summations in the equations are finite, they may be regarded as merely a more convenient method of writing a single sum. Thus, the identification of terms to be substituted into Equations (58) and (59) may be carried out much as was done in the parallel conducting cylinders case. The error terms for this case are seen to be very similar to those of the parallel conducting cylinders case. Therefore we may write by inspection the following equations:

\[ \sum_{n=0}^{N} \sum_{m=n}^{N} A_{n,m} \left[ \sum_{S_1} \eta_{n,m}(x,y,z) \eta_{k,p}(x,y,z) + \sum_{S_3} \eta_{n,m}(x,y,z) \eta_{k,p}(x,y,z) \right. \]

\[ + \eta_{n,m}^{\prime}(x,y,z) \eta_{k,p}^{\prime}(x,y,z) \] + $B_{n,m} \left[ \sum_{S_3} \right] - \xi_{n,m}(x,y,z) \eta_{k,p}(x,y,z)$

\[ \left. - \xi_{n,m}^{\prime}(x,y,z) \eta_{k,p}^{\prime}(x,y,z) \right] = \sum_{S_1} \nabla V_{0} \eta_{k,p}(x,y,z) \]
\[ \sum_{n=0}^{N} \sum_{m=n}^{N} \left[ A_{n,m} \sum_{S_3} -\eta_{n,m}(x,y,z) \xi_{l,p}(x,y,z) - \eta_{n,m}(x,y,z) \xi_{l,p}(x,y,z) \right] \\
+ B_{n,m} \left[ \sum_{S_2} \xi_{n,m}(x,y,z) \xi_{l,p}(x,y,z) + \sum_{S_3} \xi_{n,m}(x,y,z) \xi_{l,p}(x,y,z) \right] \\
+ \xi_{n,m}(x,y,z) \xi_{l,p}(x,y,z) \right] = \sum_{S_2} V_0 \xi_{l,p}(x,y,z) \\
\xi = 0, 1, 2, \ldots N \\
P = 0, 1, 2, \ldots N \\
P \geq \xi \\
(85) \\
(86) \\
(87)

As in the parallel conducting cylinders case, the effect of using the line charge solution to obtain a first approximation to the potential is merely to modify the right hand term in Equations (84) and (85). The right hand term of Equation (84) becomes
\[ \sum_{S_1} \left( V_0 - \phi_{LC}(x,y,z) \right) \eta_{l,p}(x,y,z) \]
and the right hand term of Equation (85) becomes
\[ \sum_{S_2} \left( V_0 - \phi_{LC}(x,y,z) \right) \xi_{l,p}(x,y,z) \]
where \( \phi_{LC}(x,y,z) \) is the appropriate line charge solution.

Thus we see that the equations for the two configurations are very similar. The complexity of the grid of crossed cylindrical conductors case is much greater, however, due to the double summations,
the more complicated regions over which the error must be calculated, and the larger number of points which must be used for a reasonable least squares fit. In both cases we have found a set of linear simultaneous algebraic equations which may be solved on a digital computer using any standard method such as Gaussian elimination. It should be noted that the number of error terms used must be larger than the total number of coefficients, and preferably should be much larger than the number of coefficients.
SECTION III

ELECTRON TRAJECTORIES

1. Need for Electron Trajectories

In the preceding sections the potential distribution for a number of configurations has been calculated. This information does not tell us the effectiveness of the various grids in preventing electrons which leave the ground plane with an initial velocity from reaching the interior of the chamber. To evaluate this fully, the actual trajectories of the electrons leaving the ground plane must be determined. The procedure used to find the trajectories of the electrons is discussed in this section. However, some limits for the behavior of the electrons can be deduced from the potential distribution. These are discussed in the third part of this section.

2. Calculation of Electron Trajectories

In this part we are concerned with the actual path of the electrons as they leave the ground plane. We assume a high vacuum and small electron flux density so that the electrons do not interact with other particles or among themselves. If the initial velocity of an electron is \( \vec{V} \) and the initial position of the electron is \( \vec{x} \), we may calculate a new position and velocity at a small increment of time later by assuming that the increment of time is sufficiently small so that the electric field over the distance of travel of the electron is essentially constant. To do this, we must first find the equation of motion of the electron.

The electric field at a point is given by the negative gradient of the potential, and the force exerted on the electron by the electric
field is therefore given by

\[ F = -e \nabla \phi \]  

(88)

where \( \phi \) is the potential and \( e \) is the charge of the electron. From Newton's first law of motion, the acceleration produced by this force is

\[ \ddot{a} = -\frac{e \nabla \phi}{m} \]  

(89)

where \( m \) is the mass of electron. Integrating twice with respect to time and supplying the proper constants of integration, and assuming a small increment of time \( \Delta t \), the new position of the electron may be well approximated by

\[ \bar{x}' = -\frac{e}{2m} \nabla \phi(\Delta t)^2 + \bar{v}(\Delta t) + \bar{x} \]  

(90)

Similarly, the velocity of the electron after an amount of time \( \Delta t \) is given by

\[ \bar{v}' = -\frac{e}{2m} \nabla \phi(\Delta t) + \bar{v} \]  

(91)

These equations in their normalized form were used to obtain the trajectories of the electrons. This was done by starting with the electron at the ground plane and from the initial velocity and position calculating the new velocity and position at a time \( \Delta t \) later. We then repeat the procedure until the entire trajectory is determined. The equations in their present form are not convenient, however, for this purpose. We desire to write them in a form normalized with respect to the potential as \( z \to \infty \) and in terms of the initial energy.

If the initial kinetic energy of the electron is \( \tau \), then the magnitude of the initial velocity is

\[ |v| = \sqrt{\frac{2\tau}{m}} \]  

(92)
where \( m \) is the mass of the electron. The minimum kinetic energy that an electron may have and still escape is the potential energy of an electron at \( z = \infty \). This corresponds to a minimum escape velocity magnitude of

\[
|V_e| = \sqrt{\frac{2e\phi_\infty}{m}} \quad (93)
\]

where \( \phi_\infty \) is the potential as \( z \to \infty \).

We now rewrite equation (90) in terms of \( V\phi/\phi_\infty \) and \( |V|/|V_e| \).

\[
\ddot{x}' = -\frac{e\phi_\infty}{2m} \frac{V\phi}{\phi_\infty} (\Delta t)^2 + \frac{\ddot{V}}{|V_e|} |V_e| (\Delta t) + \ddot{x} \quad (94)
\]

But

\[
\frac{e\phi_\infty}{m} = \frac{1}{2} |V_e|^2 \quad (95)
\]

Therefore, Equation (94) becomes

\[
\ddot{x}' = -\frac{1}{2} \left( \frac{|V_e|}{\sqrt{2}} \right)^2 \frac{V\phi}{\phi_\infty} + \frac{\ddot{V}}{|V_e|} |V_e| (\Delta t) + \ddot{x} \quad (96)
\]

Letting

\[
T = \frac{|V_e|}{\sqrt{2}} \quad (97)
\]

Equation (96) becomes

\[
\ddot{x}' = -\frac{1}{2} \frac{V\phi}{\phi_\infty} \Delta T + \sqrt{2} \frac{\ddot{V}}{|V_e|} \Delta T + \ddot{x} \quad (98)
\]

Writing Equation (91) in terms of \( V\phi/\phi_\infty \) and \( V/|V_e| \) and dividing by \( V_e \), we have

\[
\frac{\ddot{V}}{|V_e|} = \frac{e\phi_\infty}{m|V_e|} \left( \frac{\Delta \phi}{\phi_\infty} \right) \Delta t + \frac{\ddot{V}}{|V_e|} \quad (99)
\]

Since

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$$\frac{e\phi}{m} = \frac{1}{2} \left| \frac{\Delta \phi}{\Delta t} \right|^2$$ (100)

Equation (99) becomes

$$\frac{V'}{V_e} = \frac{1}{2} \left( \frac{\Delta \phi}{\Delta t} \right) \left| \frac{V_e}{V_e} \right| \Delta t + \frac{V}{V_e}$$ (101)

Substituting $\Delta t = \frac{V_e}{\Delta t}^2$, this equation is now

$$\frac{V'}{V_e} = \frac{1}{\sqrt{2}} \frac{\Delta \phi}{\Delta t} + \frac{V}{V_e}$$

We now have the equations in the normalized form that was utilized in calculating the actual electron trajectories. The trajectories were calculated for various positions on the ground plane and for various energies and angles of departure. The fraction of electrons escaping for each set of parameters was recorded in order to evaluate the effectiveness of the grids.

3. Some Approximations for Determining Whether an Electron Will Escape

Although the trajectory of an electron must be followed in most cases to see if an electron will escape, in some cases it is possible to determine this fact from the potential distribution data. The first case to be considered is that of a lower limit on the kinetic energy which must be possessed by the electron in order to escape. As stated in the preceding part, this energy corresponds to the potential energy that an electron would have at $z$ equals infinity and is given by $-e\phi_\infty$.

It was found that the following approximation has significance for higher energy electrons in the line charge cases. If we approximate the field distribution of the grid by assuming the potential increases linearly in the $z$ direction and is constant in the $x$ and $y$ direction, the only effect of the electric field is to exert a force on the electron
in the negative z direction. Since this is true, the electron will escape if and only if its initial velocity in the z direction is greater than or equal to the escape velocity. Therefore, the electron will escape if the following equation is satisfied:

$$\sqrt{\frac{2\pi}{m}} \cos \theta \geq |v_e|$$  \hspace{1cm} (103)

where $\theta$ is the angle between the z axis and the velocity vector of the electron.
SECTION IV

COMPUTER SOLUTIONS

1. Preliminaries

In the preceding sections, methods have been described for calculating the potential distribution for the various conductor configurations considered in this paper. A scheme has also been presented to find the trajectories of electrons under the influence of an electrostatic potential. Utilizing this information and a digital computer, we evaluate the effectiveness of the various grids in containing Compton electron produced on the ground plane.

Potential contour plots as well as information on the trajectories of the electrons are shown for each configuration. For cases involving a series solution for the potential, comparisons are made between the error obtained using a line charge approximation and the error obtained using the series and line charge solution together. Due to limitations of time and resources, an exhaustive examination of all cases was not attempted. The programs which have been written allow such a study to be made and suggestions in the future work section would hopefully reduce the time needed to do this.

Before proceeding with the results of the computer study, we present the drawing below to clarify the labeling used in this section. The ground plane corresponds at all times to the x-y plane. The angles \( \theta \) and \( \beta \) are used in the following parts of this section in specifying the direction of the electrons leaving the ground plane. The values of the angle \( \theta \) are given in degrees and those of the angle \( \beta \) are given in radians.
2. Single Grid of Small Radii Conductors

We first consider the case of Section I in which the conductors have a diameter which is sufficiently small so that they may be approximated by line charges. We present potential contour maps for four spacings of conductors. Since the geometry is three dimensional, the potential is plotted in several sections normal to the grid. Five planes were used, the first plane being the x-z plane. The other four planes were parallel to this plane and are located at \( y = a/4, y = a/2, y = 3a/4, \) and \( y = a. \) Since the potential is periodic, the plot of the potential in a plane located at \( y = 5a/4 \) is the same as that in the plane at \( y = 3a/4, \) etc. The potentials in these plots have been normalized so that the potential at \( z = \infty \) is 1.0.
Figure IV-2a. Normalized Potential Contour Plot for $b/a = 1.0$ and $z = 0$
Figure IV-2b. Normalized Potential Contour Plot for \( \frac{b}{a} = 1.0 \) and \( z = \frac{a}{4} \)
Figure IV-2c. Normalized Potential Contour Plot for $b/a = 1.0$ and $z = a/2$
Figure IV-2d. Normalized Potential Contour Plot for 
b/a = 1.0 and z = 3a/4
Figure IV-2e. Normalized Potential Contour Plot for b/a = 1.0 and z = a
Figure IV-3a. Normalized Potential Contour Plot for 
b/a = .5 and z = 0
Figure IV-3b. Normalized Potential Contour Plot for $b/a = .5$ and $z = a/4$
Figure IV-3c. Normalized Potential Contour Plot for 
h/a = .5 and z = a/2
Figure IV-3d. Normalized Potential Contour Plot for \( b/a = 0.5 \) and \( z = 3a/4 \)
Figure IV-3e. Normalized Potential Contour Plot for $b/a = .5$ and $z = a$.
The preceding plots were informative as to the potential distribution, but it is not readily apparent whether an electron leaving the ground plane will escape. In order to determine this, plots of the electron trajectories were made using the scheme described in Section III. The grid and the trajectories are shown in isometric projection.

All of the electrons in one plot have the same energy and angles of departure. The initial positions of the electrons are evenly distributed on the ground plane beneath one aperture of the grid. Due to the periodicity of the potential, no other initial positions of the electrons and only those values of \( \beta \) between 0 and \( \pi/4 \) radians need be considered. The electron trajectories were calculated until the electron either returned to the ground plane or the \( z \) component of its position was equal to 2, at which time it was deemed to have escaped. For clarity, only one aperture of the grid is shown in the plots.

In the interest of brevity, a complete set of trajectories for normalized energies of 1.1, 1.3, and 2.0 with value of \( \theta \) of 0°, 10°, 20° and 30° with values of \( \beta \) of 0, \( \pi/8 \), and \( \pi/4 \) radians are given only for the spacing \( b/a = 1.0 \). For the spacing of \( b/a = .5 \), only the trajectories for a normalized energy of 1.1 are presented for comparison.
Figure IV-4a. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.1, $B = 0$ radian, and $\theta = 0^\circ$
Figure IV-4b. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.1, $B = 0$ radian, and $\theta = 10^\circ$
Figure IV-4c. Electron Trajectories for \( b/a = 1.0 \), Normalized Energy of 1.1, \( B = 0 \) radian, and \( \theta = 20^\circ \).
Figure IV-4d. Electron Trajectories for $b/a = 1.0$, 
Normalized Energy of 1.1, $\beta = 0$ radian, and $\theta = 30^\circ$
Figure IV-4e. Electron Trajectories for b/a = 1.0, Normalized Energy of 1.1, B = π/8 radians, and θ = 10°
Figure IV-4f. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.1, $B = \frac{\pi}{8}$ radians, and $\theta = 20^\circ$
Figure IV-4g. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.1, $B = \pi/8$ radians, and $\theta = 30^\circ$
Figure IV-4h. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.1, $B = \pi/4$ radian, and $\theta = 10^\circ$
Figure IV-4i. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.1, $\beta = \pi/4$ radians, and $\theta = 20^\circ$
Figure IV-4j. Electron Trajectories for $b/a = 1.0$, Normalized Energy of $1.1$, $B = \pi/4$ radians, and $\theta = 30^\circ$
Figure IV-5a. Electron Trajectories for b/a = 1.0, Normalized Energy of 1.3, B = 0 radian, and θ = 0°
Figure IV-5b. Electron Trajectories for b/a = 1.0, Normalized Energy of 1.3, B = 0 radian, and θ 10°
Figure IV-5c. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.3, $B = 0$ radian, and $\theta = 20^\circ$
Figure IV-5d. Electron Trajectories for \( b/a = 1.0 \), Normalized Energy of 1.3, \( B = 0 \) radian, and \( \theta = 30^\circ \)
Figure IV-5a. Electron Trajectories for $b/a = 1.0$, normalized energy of 1.3, $B = \pi/8$ radians, and $\theta = 10^\circ$. 
Figure IV-5f. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.3, $B = \pi/8$ radians, and $\theta = 20^\circ$
Figure IV-5g. Electron Trajectories for \( \frac{b}{a} = 1.3 \),
Normalized Energy of 1.3, \( B = \pi/8 \) radians, and \( \theta = 30^\circ \)
Figure IV-5h. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.3, $B = \pi/4$, and $\theta = 10^\circ$
Figure IV-51. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 1.3, $B = \pi/4$, and $\theta = 20^\circ$
Figure IV-5j. Electron Trajectories for b/a = 1.0, Normalized Energy of 1.3, $\beta = \pi/4$, and $\theta = 30^\circ$
Figure IV-6a. Electron Trajectories for b/a = 1.0,
Normalized Energy of 2.0, B = 0 radian, and θ = 10°
Figure IV-6b. Electron Trajectories for b/a = 1.0,
Normalized Energy of 2.0, B = 0 radian, and θ = 10°
Figure IV-6c. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 2.0, $B = 0$ radian, and $\theta = 20^\circ$
Figure IV-6d. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 2.0, $B = 0$ radian, and $\theta = 30^\circ$
Figure IV-6c. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 2.0, $B = \pi/8$ radians, and $\theta = 10^\circ$
Figure IV-6f. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 2.0, $B = \pi/8$ radians, and $\theta = 20^\circ$
Figure IV-6g. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 2.0, $B = \pi/8$ radians, and $\theta = 30^\circ$.
Figure IV-6h. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 2.0, $B = \pi/4$ radians, and $\theta = 10^\circ$
Figure IV-6i. Electron Trajectories for b/a = 1.0, Normalized Energy of 2.0, B = π/4 radians, and θ = 20°
Figure IV-6j. Electron Trajectories for $b/a = 1.0$, Normalized Energy of 2.0, $B = \pi/4$ radians, and $\theta = 30^\circ$
Figure IV-7a. Electron Trajectories for b/a = .5, Normalized Energy of 1.1, \( B = 0 \) radian, and \( \theta = 0^\circ \)
Figure IV-7b. Electron Trajectories for $b/a = .5$, Normalized Energy of 1.1, $B = 0$ radian, and $\theta = 10^{\circ}$
Figure IV-7c. Electron Trajectories for b/a = .5, Normalized Energy of 1.1, B = 0 radian, and $\theta = 20^\circ$
Figure IV-7d. Electron Trajectories for $b/a = .5$, Normalized Energy of 1.1, $B = 0$ radian, and $\theta = 30^\circ$
Figure IV-7e. Electron Trajectories for \( b/a = 0.5 \), Normalized Energy of 1.1, \( B = \pi/8 \) radians, and \( \theta = 10^\circ \)
Figure IV-7f. Electron Trajectories for $b/a = .5$, Normalized Energy of 1.1, $B = \pi/8$ radians, and $\theta = 20^\circ$
Figure IV-7g. Electron Trajectories for $b/a = 0.5$, Normalized Energy of 1.1, $B = \pi/8$ radians, and $\theta = 30^\circ$
Figure IV-7h. Electron Trajectories for $b/a = .5$, Normalized Energy of $1.1$, $B = \pi/4$ radians, and $\theta = 10^\circ$
Figure IV-71. Electron Trajectories for $b/a = .5$, Normalized Energy of 1.1, $\beta = \pi/4$ radians, and $\theta = 20^\circ$
Figure IV-7j. Electron Trajectories for $b/a = 0.5$, Normalized Energy of 1.1, $\phi = \pi/4$ radians, and $\theta = 30^\circ$. 
The preceding plots provided information as to the trajectories followed by the electrons. It was noticed that particularly for the 0 = 0 case the electrons tended to be forced to the center of the apertures. This led to the consideration of the double grid configuration. Otherwise, the results were much as anticipated and little new information was obtained from the detailed information about the trajectories.

To obtain a quantitative measure of the effectiveness of the grids, the trajectory program was run using 100 initial positions for the electrons and recording the fraction escaping and the fraction contained. Actual electron trajectory plots were not produced in these runs. The results of the runs are summarized in graphical form on the following pages.
Figure IV-8a. Fraction of Electrons Escaping ($F_e$) for $b/a = 1.0$

Figure IV-8b. Fraction of Electrons Escaping ($F_e$) for $b/a = 1.0$
Figure IV-8c. Fraction of Electrons Escaping ($F_e$) for $b/a = 1.0$

Figure IV-9a. Fraction of Electrons Escaping for $b/a = 0.5$
Figure IV-9b. Fraction of Electrons Escaping for \( b/a = .5 \)

Figure IV-9c. Fraction of Electrons Escaping for \( b/a = .5 \)
3. **Double Grid of Small Radii Conductors**

The double grid of small radii conductors was considered in an effort to reduce the number of electrons that escape through the center of the aperture of the single grid. The potential for this configuration is given by Equation (6). In this equation there are different charge densities for each of the two grids. For the purposes of analysis in this paper, the charge density on the upper grid was set to a value of 1/2 of that of the lower grid. This causes the contribution to the potential at $z = \infty$ to be the same for each grid. This somewhat arbitrary choice was made since the relationship between the ratio of the charge densities and the voltage on the wire approximated by the line charges is not a simple one. For example, let the grids be specified by the dimensions $a = 1$ and $b = 1$. For the same charge density on the two grids we will now look at the ratio of the voltages on wires of varying radii which would be approximated by the line charges. If $\Delta$ is the ratio of the voltage required on the wire of the lower grid to that of the upper grid, we obtain the following table by the use of Equation (18).

<table>
<thead>
<tr>
<th>radius of conductor</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.581</td>
</tr>
<tr>
<td>.01</td>
<td>.633</td>
</tr>
<tr>
<td>.001</td>
<td>.741</td>
</tr>
</tbody>
</table>

Table IV-1. Ratio of Voltages Required on Conductors to be Approximated by Equal Charge Densities.

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If the possibility exists of different radii wires being employed for
the upper grid than are used for the lower grid, the problem becomes
even greater.

Using the two to one charge density ratio mentioned above, contour
plots were made for the double grid configuration as was done for the
single grid. It is interesting to note that the potential contours
for this configuration tend to be parallel to the ground plane indicating
a potential distribution that tends to increase linearly in the z direc-
tion. The effect of this on the electron trajectories will be discussed
later.
Figure IV-10a. Normalized Potential Contour Plots for b/a = 1.0 and z = 0
Figure IV-10b. Normalized Potential Contour Plots for $b/a = 1.0$ and $z = a/4$
Figure IV-10c. Normalized Potential Contour Plots for $b/a = 1.0$ and $z = a/2$
Figure IV-10d, Normalized Potential Contour Plots for $b/a = 1.0$ and $z = 3a/4$
Figure IV-10e, Normalized Potential Contour Plots for b/a = 1.0 and z = a
Electron trajectory plots were made for this case as for the single grid case. In general, the actual trajectory plots will not be presented for this and the following cases, but one final plot follows. This plot is for electrons of a normalized energy of 1.1 and with the electrons rising vertically out of the ground plane. It illustrates the tendency of the second grid, particularly for the $\theta = 0$ case, to "straighten out" the trajectories of the electrons. In the single grid case, some of the electrons with an energy of 1.1 and $\theta = 0$ were returned to the ground plane primarily due to a change in direction. The straightening effect of the second grid makes this behavior much more unlikely.
Figure IV-11. Electron Trajectories for b/a = 1.0, Normalized Energy of 1.1, B = 0 radian, and \( \theta = 0^\circ \)
The following are plots of the fraction of electrons escaping as
a function of energy and direction. As in the single grid case, 100
initial positions were used for the electrons at each energy and direc-
tion. In considering the results for the double grid case, one should
keep in mind that the potential has been normalized to be 1.0 at \( z = \infty \).
It is not, therefore, equivalent to the single grid case which produces
a potential of 1.0 at \( z = \infty \) with the addition of another grid which
also produces a potential of 1.0 at \( z = \infty \). Even so, the double grid
configuration does not seem to have sufficiently better characteristics
to warrant a detailed study. Therefore, only the results of one spacing
of the grids is presented.
Figure IV-12a. Fraction of Electrons Escaping ($F_e$) for $b/a = 1.0$

Figure IV-12b. Fraction of Electrons Escaping ($F_e$) for $b/a = 1.0$
Figure IV-12c. Fraction of Electrons Escaping ($F_e$) for $b/a = 1.0$
4. **Parallel Cylindrical Conductors Case**

The potential distribution for a series of cylindrical conductors set parallel to a ground plane was found using the approach of Section II-2 and the least squares fitting of Section II-5. The coefficients for the series for the potential are given in Appendix A. The line charge solution was used as described in Section II-4 to give a first approximation to the potentials used in the analysis of this configuration. For the purposes of comparison the mean squared error was calculated in the same manner for three different solutions. The solutions used for the comparison are the line charge solution only, the series solution only, and the series and line charge solution together. The results of the error calculation for these three different solutions are shown below.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Mean squared error (volts)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b/a c/a</td>
<td>Line charge approximation</td>
</tr>
<tr>
<td>1.0 .25</td>
<td>$4.69 \times 10^{-3}$</td>
</tr>
<tr>
<td>1.0 .1</td>
<td>$4.54 \times 10^{-4}$</td>
</tr>
<tr>
<td>.5 .25</td>
<td>$2.00 \times 10^{-2}$</td>
</tr>
<tr>
<td>.5 .1</td>
<td>$1.41 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table IV-2. Mean Squared Error for Various Solutions

All the series calculations were made using fifteen terms of the series in each region and with potential of the conductor set to 1.0.
The first thing we note from the table is that the line charge approximation becomes better as the radii of the conductor approximated becomes smaller and as the spacing between the conductor and the ground plane becomes larger. The series solution also becomes better as the distance from the conductors to the ground plane increases, but the error increases with decreasing radii. In both cases where the ratio c/a is .1, the error of the line charge solution alone is comparable to that of the series solution alone. It would appear that this ratio is the approximate value at which the use of the series solution results in significant improvement, at least for the number of terms of the series calculated here. As Table IV-2 shows the combination of the series and line charge solution results in appreciable improvement over the use of either one separately.

On the following pages are presented potential contour plots for the four cases presented in Table IV-2. To reiterate, 15 terms were used in each solution region and the potential of the conductor was normalized to 1.0. The solution used to obtain these plots is the combination of the line charge and series solution.
Figure IV-13. Normalized Potential Contour Plot for $b/a = 1.0$ and $c/a = .25$
Figure IV-14. Normalized Potential Contour Plot for $b/a = 1.0$ and $c/a = 0.1$
Figure IV-15. Normalized Potential Contour Plot for \( \frac{b}{a} = .5 \) and \( \frac{c}{a} = .25 \)
Figure IV-16. Normalized Potential Contour Plot for b/a = .5 and c/a = .1
Electron trajectories were found for this case in the same manner as was done previously except that only the $\theta = 0$ angle needed to be considered. The data obtained from these trajectories are shown in graphical form on the following pages. Since we are now using conductors of finite radii, the possibility of the electrons striking the conductors must be considered. Since we do not know the type of collision to expect between the electrons and the conductors, the percentage of the electrons striking the conductor is merely listed. If the collision is elastic, the program may be easily modified to continue the trajectories after the collision. If the electrons are absorbed by the conductors, the fraction of electrons striking the conductors should be added to the fraction of electrons contained. If secondary Compton electrons are produced from these electrons, and data is available concerning the initial velocity distribution of the secondary electrons, the program may be modified to account for this phenomenon.

Another consideration that arises due to the finite size of the conductors is that Compton electrons could be produced on the conductors themselves by the incoming radiation.\(^{(15)}\) Closely related to this is the fact that the conductor would shield a portion of the ground plane from the incoming radiation.\(^{(15)}\) Both of these phenomena are dependent on the direction of the incoming radiation. These effects were ignored in the present analysis, but could be accounted for if more detailed information concerning them were available.

It is useful to calculate the ratio $V_0/V_\infty$ for the various spacings of conductors used for this configuration where $V_0$ is the potential of the conductor and $V_\infty$ is the potential at $z = \infty$. If the energy of the electron, normalized to the potential energy of an electron at $z = \infty$, is
less than this value of $V_0/V_\infty$, it is impossible for the electron to strike the conductor.

<table>
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<th>$V_0/V_\infty$</th>
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<td>1.135</td>
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<td>.1</td>
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<td>.25</td>
<td>1.28</td>
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<tr>
<td>.5</td>
<td>.1</td>
<td>1.74</td>
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Table IV-3. $V_0/V_\infty$ for Various Spacings of Conductors

The data for the plots presented on the following pages were obtained using 30 initial positions for the electrons on the ground plane. Due to the increase in computer time needed to calculate each trajectory, the number of trajectories had to be reduced from the 100 used in the line charge case. For similar reasons, normalized energies of 1.1, 1.3, and 2.0 were used for the $a/b = 1.0$ and $c/a = .25$ spacing only and the normalized energy of 1.1 was used for the other spacings of $b/a = 1.0$ and $c/a = .1$, $b/a = .5$ and $c/a = .25$, and $b/a = .5$ and $c/a = .1$. 
Figure IV-17. Fraction of Electrons Escaping ($F_e$) for $b/a = 1.0$ and $c/a = 0.25$.

Figure IV-18. Fraction of Electrons Striking Conductor ($F_c$) for $b/a = 1.0$ and $c/a = 0.25$. 
Figure IV-19. Fraction of Electrons Escaping ($F_e$) for a Normalized Energy of 1.1
5.0 Crossed Cylindrical Conductors Case

The potential distribution for the grid of crossed cylindrical conductors was found using the approach of Section II-3 and the least squares fitting of Section II-5. The actual coefficients for the series for the cases studied are given in Appendix A. For the purpose of comparison, the error was calculated using the line charge portion of the solution along and for the line charge and series solution combined. Due to the longer time necessary for calculation, the solution obtained by using only the series form was not calculated. Table IV-4 shows the error obtained for these solutions.

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<th>Dimensions</th>
<th>Mean Squared Error (volts)$^2$</th>
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<td>Line charge Approximation</td>
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<tr>
<td>b/a c/a</td>
<td></td>
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<tr>
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<td>$3.93 \times 10^{-2}$</td>
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<tr>
<td>1.0 .1</td>
<td>$7.60 \times 10^{-3}$</td>
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<tr>
<td>.5 .25</td>
<td>.172</td>
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<td>.5 .1</td>
<td>.243</td>
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Table IV-4. Mean Squared Error for Two Solutions

On the following pages, potential contour plots are shown for the spacings shown in Table IV-4. The potentials are those found using the line charge and series solutions together. The potential of the conductors was normalized to 1.0 and 15 terms of the series solution were used in each region.
Figure IV-20a. Normalized Potential Contour Plot for $b/a = 1.0$, $c/a = 0.25$ and $z = 0$
Figure IV-20b. Normalized Potential Contour Plot for \( b/a = 1.0, c/a = .25 \) and \( z = a/4 \)
Figure IV-20c. Normalized Potential Contour Plot for $b/a = 1.0$, $c/a = .25$ and $z = a/2$
Figure IV-20d. Normalized Potential Contour Plot for $b/a = 1.0$, $c/a = 0.25$ and $z = 3a/4$
Figure IV-20e. Normalized Potential Contour Plot for b/a = 1.0, c/a = .25 and z = a
Figure IV-21a. Normalized Potential Contour Plot for $b/a = .5$, $c/a = .25$ and $z = 0$
Figure IV-21b. Normalized Potential Contour Plot for b/a = .5, c/a = .25 and z = a/4
Figure IV-21c. Normalized Potential Contour Plot for 
b/a = .5, c/a = .25 and z = a/2
Figure IV-21d, Normalized Potential Contour Plot for $b/a = .5$, $c/a = .25$ and $z = 3a/4$
Figure IV-21e. Normalized Potential Contour Plot for $b/a = .5$, $c/a = .25$ and $z = a$
Figure IV-22a. Normalized Potential Contour Plot for $b/a = 1.0$, $c/a = .1$ and $z = 0$
Figure IV-22b. Normalized Potential Contour Plot for b/a = 1.0, c/a = .1 and z = a/4
Figure IV-22c. Normalized Potential Contour Plot for \( b/a = 1.0, \ c/a = 0.1 \) and \( z = a/2 \)
Figure IV-22d, Normalized Potential Contour Plot for 
b/a = 1.0, c/a = .1 and z = 3a/4
Figure IV-22e. Normalized Potential Contour Plot for \( b/a = 1.0, c/a = 0.1 \) and \( z = a \)
Figure IV-23a. Normalized Potential Contour Plot for $b/a = .5$, $c/a = .1$ and $z = 0$
Figure IV-23b. Normalized Potential Contour Plot for $b/a = .5$, $c/a = .1$ and $z = a/4$
Figure IV-23c. Normalized Potential Contour Plot for $b/a = .5$, $c/a = .1$ and $z = a/2$
Figure IV-23d. Normalized Potential Contour Plot for b/a = .5, c/a = .1 and z = 3a/4
Figure IV-23e. Normalized Potential Contour Plot for $b/a = .5$, $c/a = .1$ and $z = a$
Electron trajectory calculations were performed for this case as was done for the other cases except that 25 initial positions for the electrons were used. The problems encountered due to the electrons striking the conductor are the same for this case as they are for the large radii parallel conductor case and the percentage of the electrons striking the conductors are shown. Energies of 1.1, 1.3 and 2.0 and values of $\beta$ of 0 and $\pi/4$ radians were used for one spacing only but the 1.1 energy and $\beta$ value of 0 radians was used for all spacings to provide comparison as was done in the parallel conducting cylinder case. The table below of $V_0/V_\infty$ for the dimensions considered for this case is provided to determine if it is possible for an electron of a given energy to strike the conductors. $V_0$ is the potential of the conductors and $V_\infty$ is the potential of $z \to \infty$.

<table>
<thead>
<tr>
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<th>$c/a$</th>
<th>$V_0/V_\infty$</th>
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<td>1.039</td>
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<td>1.436</td>
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</table>

Table IV-5. $V_0/V_\infty$ for Various Dimensions

6. **High Energy Approximation**

It was mentioned in Section III-3 that the approximation of Equation (103) was found to be good for high energy electrons for the line charge cases. To illustrate this, the graph of Figure IV-27 was produced.
Figure IV-24a. Fraction of Electrons Escaping ($F_e$) for $b/a = 1.0$ and $c/a = .25$

Figure IV-24b. Fraction of Electrons Escaping ($F_e$) for $b/a = 1.0$ and $c/a = .25$
Figure IV-25a. Fraction of Electrons Striking the Conductors ($F_c$) for $b/a = 1.0$ and $c/a = .5$

Figure IV-25b. Fraction of Electrons Striking the Conductors ($F_c$) for $b/a = 1.0$ and $c/a = .5$
Figure IV-26a. Fraction of Electrons Escaping ($F_e$) for a Normalized Energy of 1.1

Figure IV-26b. Fraction of Electrons Striking Conductor ($F_c$) for a Normalized Energy of 1.1
Figure IV-27. High Energy Approximation Comparison.

The dotted line represents the plot of the percentage of electrons that would escape according to Equation (103) for an energy of 2.0. The other lines are the percentage of electrons that escape from the electron trajectory data for the various line charge cases where the energy is 2.0. It can be seen that this approximation is quite good for a number of the cases and an investigation of this sort for higher energies shows that the agreement becomes better as the energy increases. It is not possible to compare this approximation to the large radii conductor cases due to the electrons that strike the conductors.
SECTION V

CONCLUSIONS AND FUTURE WORK

1. Conclusions

It has been found that the incoming radiation in an EMP simulator will produce Compton electrons on the wall of the simulator. In this paper techniques have been developed to determine the effectiveness of using a grid of wires parallel to the wall of the chamber in reducing the number of these electrons escaping to the working volume of the chamber. Several configurations have been studied and various radii of the conductors have been considered.

It is not the intention of this paper to present an exhaustive parametric study of the problem. The computer programs to do this have been written and a more complete study could be undertaken if sufficient computer time were available. In Section V-2 methods are suggested which, it is believed, would substantially reduce the time required for these calculations.

For those cases where the conductors could be approximated by line charges, the solution for the potential distribution was found in closed form. The single grid case was studied in some detail and data presented which characterizes the grids. A brief study was made of the double grid configuration but the results did not justify a more detailed investigation at this time. For each case, the normalized energies between 1.0 and about 2.0 or 3.0 were found to be the only ones for which actual calculations of the electron trajectories were required. For normalized energies greater than 2.0 or 3.0 the approximation of Equation (103) was found to be sufficiently accurate.

For cases of grids of conductors with large radii the potential
was found using the closed form line charge solution as a first approximation with a series solution added to improve the accuracy. A least squared error technique was used to match the boundary conditions. Electron trajectory calculations were performed for each of the cases.

In all cases, reducing the distance from the ground plane to the conductors resulted in a decrease in the number of electrons escaping for a given energy. As the conductors approach the ground plane their potential must increase in order to maintain the same potential at infinity. Thus, the electrons approaching close to the conductor encounter a higher potential gradient and are more likely to be repelled the closer the conductors are to the ground plane.

Significant differences are noted between the line charge cases and the corresponding large radii cases, especially when the comparison is made to the crossed conductor cases. The rather large radii of the conductor for which the most data was obtained makes these differences particularly apparent. More data was produced for this case to demonstrate the effects and problems which are encountered when using large radii conductors.

It is believed that the data presented in this paper provides insight into the features of the various configurations and shows the comparative effectiveness of the grids. As other design considerations arise, one may wish to more thoroughly investigate a particular case. The programs in Appendix B provide this capability and may be modified to include such effects as secondary Compton electrons produced on the grids, etc.

2. **Future Work**

It would be advisable to pursue a more detailed study of the problem
than was possible in this paper. To do this we suggest the following method for reducing the time necessary for the electron trajectory calculations. The programs as they are presently constituted recalculate the gradient of the potential for each iteration from the expression for the potential. It would appear to be faster to produce a table from which the gradient of the potential could be found by a "look up" procedure and a few algebraic manipulations. Since the series solutions are of the form of the product of functions each of which is a function of only one coordinate variable, this may be easily accomplished. A table could be made of the value of these functions in the x, y, and z directions and the potential at any point found by "looking up" the appropriate values and multiplying to find the potential. The gradient could be found in a similar manner. A similar table would reduce the time needed to calculate the line charge portion of the gradient. The symmetry of the problem helps to reduce the amount of storage necessary for this procedure.

The symmetry of the problem also could be used to reduce the number of electron trajectories it is necessary to calculate. For the $\beta = 0$ case, only those initial positions on the ground plane for which $0 < y < a$ need to be considered and for the $\beta = \pi/4$ case only those positions lying on and to one side of the line $x = y$ need be considered.

As mentioned previously, a further investigation into the effects of the electrons striking the wires and the effects of radiation incident on the wires should be undertaken.
REFERENCES


APPENDIX A

SERIES COEFFICIENTS

1. General

In this appendix are presented the series coefficients found in solving the potential distribution problem of the parallel and crossed cylindrical conductors cases. The forms of the equations used were modified by constant multipliers of the coefficients for ease of comparison. The form of the solution and the grid of points used for the fitting of the solutions are shown for each geometry. The coefficients are then tabulated. Unless otherwise specified, all solutions for which data is presented used 15 terms of the solution in each region.

2. Parallel Cylindrical Conductor Case

The form of the solutions used is

\[ \phi_I(x,z) = \sum_{n=1}^{N} A_n \frac{\sinh(n\pi z/a)}{\sinh(n\pi b/a)} \cos(n\pi x/a) + A_0 \frac{z}{b} + (\text{pic}) \phi_{LC}(x,z) \]  

(A.1)

in region I and

\[ \phi_{II}(x,z) = \sum_{n=0}^{N} B_n e^{-\frac{n\pi}{a} (z-b)} \cos(n\pi x/a) + (\text{pic}) \phi_{LC}(x,z) \]  

(A.2)

in region II, where \( \phi_{LC}(x,z) \) is the line charge solution with leading constants omitted and is given by

\[ \phi_{LC}(x,z) = \ln \frac{\cosh(\pi(z+b)/a) - \cos(\pi z/a)}{\cosh(\pi(z-b)/a) - \cos(\pi x/a)} \]  

(A.3)

The term pic is a constant adjusted to insure that the potential given by the line charge portion of the solution alone is 1.0 on the surface of the conductor at \( x = c \). For this geometry the points used for the
matching of the boundary conditions were located on both the upper and lower surfaces of the conductor at \( x < c(n-1)/20, \ n = 1, 2, \ldots 20 \) and at \( z < b \) between the conductors at \( x = c + (a-c)(n-1)/20, \ n = 1, 2, \ldots 20 \). The coefficients obtained for this case are shown in the following tables.
Table A-1. Series Coefficients for b/a = 1 and c/a = .25.

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pic = .1468

Mean Squared Error = $1.064 \times 10^{-5}$
Table A-2. Series Coefficients for $b/a = 1$ and $c/a = .1$.

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$pic = .1168$

Mean Squared Error = $7.480 \times 10^{-5}$
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\(\xi C = 0.2767\)

Mean Squared Error = \(1.558 \times 10^{-5}\)

Table A-3. Series Coefficients for \(b/a = 0.5\) and \(c/a = 0.25\).
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<td>$-1.7519 \times 10^{-2}$</td>
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<tr>
<td>5</td>
<td>$1.8596 \times 10^{-2}$</td>
<td>$-1.6503 \times 10^{-2}$</td>
</tr>
<tr>
<td>6</td>
<td>$1.7653 \times 10^{-2}$</td>
<td>$-1.5977 \times 10^{-2}$</td>
</tr>
<tr>
<td>7</td>
<td>$1.5713 \times 10^{-2}$</td>
<td>$-1.4369 \times 10^{-2}$</td>
</tr>
<tr>
<td>8</td>
<td>$1.4484 \times 10^{-2}$</td>
<td>$-1.3372 \times 10^{-2}$</td>
</tr>
<tr>
<td>9</td>
<td>$1.2317 \times 10^{-2}$</td>
<td>$-1.1451 \times 10^{-2}$</td>
</tr>
<tr>
<td>10</td>
<td>$1.0933 \times 10^{-2}$</td>
<td>$-1.0225 \times 10^{-2}$</td>
</tr>
<tr>
<td>11</td>
<td>$8.7026 \times 10^{-3}$</td>
<td>$-8.1817 \times 10^{-3}$</td>
</tr>
<tr>
<td>12</td>
<td>$7.3616 \times 10^{-3}$</td>
<td>$-6.9463 \times 10^{-3}$</td>
</tr>
<tr>
<td>13</td>
<td>$5.2748 \times 10^{-3}$</td>
<td>$-4.9956 \times 10^{-3}$</td>
</tr>
<tr>
<td>14</td>
<td>$4.1563 \times 10^{-3}$</td>
<td>$-3.9510 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table A-4. Series Coefficients for $b/a = .5$ and $c/a = .1$. 

Mean Squared Error = $2.465 \times 10^{-5}$
3. **Crossed Cylindrical Conductors Case**

The forms of the solution used are

\[ \phi_I(x, y, z) = \sum_{n=0}^{N} \sum_{m=0}^{N} \{A_{n,m} \left[ \cos(n \pi x/a) \cos(m \pi y/a) + \cos(m \pi x/a) \right] \cos(n \pi y/a) \sinh(\gamma_{n,m} z)/2 \sinh(\gamma_{n,m} b) \} + A_{\infty} z/b \]

\[ + (\text{pic}) \phi_{LC}(x, y, z) \]  \hspace{1cm} (A.4)

in region I and

\[ \phi_{II}(x, y, z) = \sum_{n=0}^{N} \sum_{m=0}^{N} \{B_{n,m} (\frac{1}{2}) \left[ \cos(n \pi x/a) \cos(m \pi y/a) \right] \cos(m \pi x/a) \cos(n \pi y/a) \} e^{-\gamma_{n,m} (z-b)} + (\text{pic}) \phi_{LC}(x, y, z) \]  \hspace{1cm} (A.5)

in region II where \( \gamma_{n,m} = \sqrt{n^2 + m^2 \left( \frac{\pi}{a} \right)^2} \) and \( \phi_{LC}(x, y, z) \) is the line charge solution for a single crossed grid with the leading coefficients omitted and is given by

\[ \phi_{LC}(x, y, z) = \ln \frac{\cosh(\pi (z+b)/a) - \cos(\pi x/a)}{\cosh(\pi (z-b)/a) - \cos(\pi x/a)} + \ln \frac{\cosh(\pi (z+b)) - \cos(\pi y/a)}{\cosh(\pi (z-b)) - \cos(\pi y/a)} \]  \hspace{1cm} (A.6)

The term \( \text{pic} \) is a constant whose value is adjusted to insure that the potential given by the line charge portion of the solution is 1.0 at \( x = a \) and \( y = c \) on the conductors. For this geometry the points used for the matching of the boundary conditions were located at \( x = c (n-1)/8 \) and \( x = c + (a-c) n/9 \) for \( n = 1, 2, \ldots 9 \) and at \( y = c (m-1)/8 \) and \( y = c + (a-c) m/9 \) for \( m = 1, 2, \ldots 9 \). If the \( x \) and \( y \) coordinates of a point corresponded to those of the conductors, points used were on the upper and lower surfaces of the conductor. If the \( x \) and \( y \) coordinates of a point corresponded to the aperture between conductors,
the point for the matching of the boundary conditions was located at 
\( \text{z} = b \). Only those points for which \( x \geq y \) were used. The coefficients 
obtained for this case are shown in the following tables.
<table>
<thead>
<tr>
<th>$n,m$</th>
<th>$A_{n,m}$</th>
<th>$B_{n,m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>$4.0074 \times 10^{-2}$</td>
<td>$.1104$</td>
</tr>
<tr>
<td>01</td>
<td>$4.0260 \times 10^{-2}$</td>
<td>$.1997$</td>
</tr>
<tr>
<td>02</td>
<td>$.1052$</td>
<td>$.1260$</td>
</tr>
<tr>
<td>03</td>
<td>$.1064$</td>
<td>$.1101$</td>
</tr>
<tr>
<td>04</td>
<td>$6.272 \times 10^{-2}$</td>
<td>$-7.1596 \times 10^{-2}$</td>
</tr>
<tr>
<td>11</td>
<td>$-/.1722$</td>
<td>$-7.8816 \times 10^{-2}$</td>
</tr>
<tr>
<td>12</td>
<td>$-/.1461$</td>
<td>$-4.8780 \times 10^{-2}$</td>
</tr>
<tr>
<td>13</td>
<td>$-4.5895 \times 10^{-2}$</td>
<td>$-5.2738 \times 10^{-2}$</td>
</tr>
<tr>
<td>14</td>
<td>$-2.0106 \times 10^{-2}$</td>
<td>$-3.9059 \times 10^{-2}$</td>
</tr>
<tr>
<td>22</td>
<td>$-2.7186 \times 10^{-2}$</td>
<td>$-1.1611 \times 10^{-2}$</td>
</tr>
<tr>
<td>23</td>
<td>$5.7667 \times 10^{-3}$</td>
<td>$-4.4008 \times 10^{-2}$</td>
</tr>
<tr>
<td>24</td>
<td>$2.5202 \times 10^{-3}$</td>
<td>$-3.2261 \times 10^{-2}$</td>
</tr>
<tr>
<td>33</td>
<td>$1.5504 \times 10^{-2}$</td>
<td>$-2.5466 \times 10^{-2}$</td>
</tr>
<tr>
<td>34</td>
<td>$1.7077 \times 10^{-2}$</td>
<td>$-3.4254 \times 10^{-2}$</td>
</tr>
<tr>
<td>44</td>
<td>$5.1119 \times 10^{-3}$</td>
<td>$-1.1367 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

$\text{pic} = 8.5354 \times 10^{-2}$

$\text{Mean Squared Error} = 6.275 \times 10^{-4}$

Table A-5. Series Coefficients for $b/a = 1$ and $c/a = .25$. 
<table>
<thead>
<tr>
<th>n,m</th>
<th>$A_{n,m}$</th>
<th>$B_{n,m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,0</td>
<td>-7.4821 x 10^{-2}</td>
<td>- .1020</td>
</tr>
<tr>
<td>0,1</td>
<td>-3.913 x 10^{-2}</td>
<td>-6.8826 x 10^{-2}</td>
</tr>
<tr>
<td>0,2</td>
<td>3.6006 x 10^{-3}</td>
<td>-2.3307 x 10^{-2}</td>
</tr>
<tr>
<td>0,3</td>
<td>8.7687 x 10^{-3}</td>
<td>-1.8746 x 10^{-2}</td>
</tr>
<tr>
<td>0,4</td>
<td>-5.9525 x 10^{-4}</td>
<td>-1.4771 x 10^{-2}</td>
</tr>
<tr>
<td>1,1</td>
<td>-4.7650 x 10^{-2}</td>
<td>-4.2002 x 10^{-2}</td>
</tr>
<tr>
<td>1,2</td>
<td>-3.3415 x 10^{-2}</td>
<td>-2.5861 x 10^{-2}</td>
</tr>
<tr>
<td>1,3</td>
<td>-1.7365 x 10^{-2}</td>
<td>-1.4637 x 10^{-2}</td>
</tr>
<tr>
<td>1,4</td>
<td>-1.2492 x 10^{-2}</td>
<td>-1.1782 x 10^{-2}</td>
</tr>
<tr>
<td>2,2</td>
<td>-7.2520 x 10^{-3}</td>
<td>-3.8021 x 10^{-3}</td>
</tr>
<tr>
<td>2,3</td>
<td>-6.0480 x 10^{-3}</td>
<td>-5.8940 x 10^{-3}</td>
</tr>
<tr>
<td>2,4</td>
<td>-5.0636 x 10^{-3}</td>
<td>-6.0690 x 10^{-3}</td>
</tr>
<tr>
<td>3,3</td>
<td>-1.6965 x 10^{-3}</td>
<td>-9.3250 x 10^{-4}</td>
</tr>
<tr>
<td>3,4</td>
<td>-2.6606 x 10^{-3}</td>
<td>-2.6163 x 10^{-3}</td>
</tr>
<tr>
<td>4,4</td>
<td>-1.3321 x 10^{-3}</td>
<td>-3.0276 x 10^{-4}</td>
</tr>
</tbody>
</table>

pic = 7.6011 x 10^{-2}

Mean Squared Error = 2.542 x 10^{-3}

Table A-6. Series Coefficients for b/a = 1 and c/a = .1.
<table>
<thead>
<tr>
<th>( n,m )</th>
<th>( A_{n,m} )</th>
<th>( B_{n,m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,0</td>
<td>.1154</td>
<td>-.2314</td>
</tr>
<tr>
<td>0,1</td>
<td>.1755</td>
<td>-.4097</td>
</tr>
<tr>
<td>0,2</td>
<td>.2782</td>
<td>-.2726</td>
</tr>
<tr>
<td>0,3</td>
<td>.2662</td>
<td>-.2450</td>
</tr>
<tr>
<td>0,4</td>
<td>.1599</td>
<td>-.1639</td>
</tr>
<tr>
<td>1,1</td>
<td>-.3252</td>
<td>-.1526</td>
</tr>
<tr>
<td>1,2</td>
<td>-.2639</td>
<td>-9.2444 x 10^{-2}</td>
</tr>
<tr>
<td>1,3</td>
<td>-5.4500 x 10^{-2}</td>
<td>-.1117</td>
</tr>
<tr>
<td>1,4</td>
<td>-9.6815 x 10^{-3}</td>
<td>-9.0271 x 10^{-2}</td>
</tr>
<tr>
<td>2,2</td>
<td>-4.2613 x 10^{-2}</td>
<td>-2.2307 x 10^{-2}</td>
</tr>
<tr>
<td>2,3</td>
<td>4.2115 x 10^{-2}</td>
<td>-9.6293 x 10^{-2}</td>
</tr>
<tr>
<td>2,4</td>
<td>2.8580 x 10^{-2}</td>
<td>-7.4525 x 10^{-2}</td>
</tr>
<tr>
<td>3,3</td>
<td>4.6487 x 10^{-2}</td>
<td>4.2115 x 10^{-2}</td>
</tr>
<tr>
<td>3,4</td>
<td>5.6085 x 10^{-2}</td>
<td>-7.9811 x 10^{-2}</td>
</tr>
<tr>
<td>4,4</td>
<td>1.8237 x 10^{-2}</td>
<td>-2.7425 x 10^{-2}</td>
</tr>
</tbody>
</table>

\( \pi = .1833 \)

\( \text{Mean Squared Error} = 4.089 \times 10^{-3} \)

Table A-7. Series Coefficients for \( b/a = .5 \) and \( c/a = .25 \).
<table>
<thead>
<tr>
<th>n, m</th>
<th>( A_{n,m} )</th>
<th>( B_{n,m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,0</td>
<td>-0.1051 ( \times 10^{-2} )</td>
<td>-0.1741</td>
</tr>
<tr>
<td>0,1</td>
<td>-6.5391 ( \times 10^{-2} )</td>
<td>-0.1336</td>
</tr>
<tr>
<td>0,2</td>
<td>-1.9930 ( \times 10^{-3} )</td>
<td>-3.9764 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>0,3</td>
<td>1.1666 ( \times 10^{-2} )</td>
<td>-3.3880 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>0,4</td>
<td>7.4902 ( \times 10^{-3} )</td>
<td>-2.4173 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>1,1</td>
<td>-7.7180 ( \times 10^{-2} )</td>
<td>-7.1494 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>1,2</td>
<td>-5.5072 ( \times 10^{-2} )</td>
<td>-4.1261 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>1,3</td>
<td>-2.7487 ( \times 10^{-2} )</td>
<td>-2.3332 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>1,4</td>
<td>-1.9461 ( \times 10^{-2} )</td>
<td>-1.7427 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>2,2</td>
<td>-1.2776 ( \times 10^{-2} )</td>
<td>-4.1782 ( \times 10^{-3} )</td>
</tr>
<tr>
<td>2,3</td>
<td>-1.0835 ( \times 10^{-2} )</td>
<td>-7.6550 ( \times 10^{-3} )</td>
</tr>
<tr>
<td>2,4</td>
<td>-7.3335 ( \times 10^{-3} )</td>
<td>-9.4613 ( \times 10^{-3} )</td>
</tr>
<tr>
<td>3,3</td>
<td>-4.7821 ( \times 10^{-3} )</td>
<td>-1.2948 ( \times 10^{-3} )</td>
</tr>
<tr>
<td>3,4</td>
<td>-5.6570 ( \times 10^{-3} )</td>
<td>-1.5460 ( \times 10^{-3} )</td>
</tr>
<tr>
<td>4,4</td>
<td>3.9989 ( \times 10^{-3} )</td>
<td>2.2703 ( \times 10^{-3} )</td>
</tr>
</tbody>
</table>

\( \text{pic} = 0.1385 \)

Mean Squared Error = 9.183 \( \times 10^{-3} \)

Table A-8. Series Coefficients for \( b/a = 0.5 \) and \( c/a = 0.1 \).
To illustrate the effect of using more terms in each solution region, a run was made using 21 terms in each region for the \( b/a = 1 \) and \( c/a = 0.25 \) crossed cylindrical conductor case. The coefficients obtained are shown in Table A-9. It can be seen that although the mean squared error was reduced substantially, the \( B_{0,0} \) term changed by only \( \pm 0.6\% \). Since this term, together with the line charge solution, specifies the potential as \( z \) becomes infinite, little improvement in the potential solution is to be expected by the use of the additional terms.
<table>
<thead>
<tr>
<th>n,m</th>
<th>A&lt;sub&gt;n,m&lt;/sub&gt;</th>
<th>B&lt;sub&gt;n,m&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>4.7666 x 10⁻²</td>
<td>- .1120</td>
</tr>
<tr>
<td>01</td>
<td>3.6365 x 10⁻²</td>
<td>- .2098</td>
</tr>
<tr>
<td>02</td>
<td>.1210</td>
<td>- .1206</td>
</tr>
<tr>
<td>03</td>
<td>.1222</td>
<td>- .1021</td>
</tr>
<tr>
<td>04</td>
<td>9.1829 x 10⁻²</td>
<td>-8.6371 x 10⁻²</td>
</tr>
<tr>
<td>05</td>
<td>6.5801 x 10⁻²</td>
<td>-5.7384 x 10⁻²</td>
</tr>
<tr>
<td>11</td>
<td>- .1948</td>
<td>-8.6447 x 10⁻²</td>
</tr>
<tr>
<td>12</td>
<td>- .1645</td>
<td>-2.5389 x 10⁻²</td>
</tr>
<tr>
<td>13</td>
<td>-5.5210 x 10⁻²</td>
<td>-2.0104 x 10⁻²</td>
</tr>
<tr>
<td>14</td>
<td>-1.4014 x 10⁻²</td>
<td>-2.8432 x 10⁻²</td>
</tr>
<tr>
<td>15</td>
<td>2.9949 x 10⁻²</td>
<td>-4.1628 x 10⁻²</td>
</tr>
<tr>
<td>22</td>
<td>3.0044 x 10⁻²</td>
<td>1.0158 x 10⁻²</td>
</tr>
<tr>
<td>23</td>
<td>2.3377 x 10⁻³</td>
<td>-9.9481 x 10⁻⁴</td>
</tr>
<tr>
<td>24</td>
<td>2.0358 x 10⁻²</td>
<td>-2.7769 x 10⁻²</td>
</tr>
<tr>
<td>25</td>
<td>3.8575 x 10⁻²</td>
<td>-3.2589 x 10⁻²</td>
</tr>
<tr>
<td>33</td>
<td>1.8698 x 10⁻²</td>
<td>-8.8491 x 10⁻³</td>
</tr>
<tr>
<td>34</td>
<td>3.6226 x 10⁻²</td>
<td>-3.0328 x 10⁻²</td>
</tr>
<tr>
<td>35</td>
<td>3.9910 x 10⁻²</td>
<td>-2.9802 x 10⁻²</td>
</tr>
<tr>
<td>44</td>
<td>1.6167 x 10⁻²</td>
<td>-1.5809 x 10⁻²</td>
</tr>
<tr>
<td>45</td>
<td>3.2051 x 10⁻²</td>
<td>-2.6455 x 10⁻²</td>
</tr>
<tr>
<td>55</td>
<td>1.2009 x 10⁻²</td>
<td>-9.4318 x 10⁻³</td>
</tr>
</tbody>
</table>

pic = 8.5354 x 10⁻²

Mean Squared Error = 3.6146 x 10⁻⁴

Table A-9. Series Coefficients for b/a = 1 and c/a = .25.
APPENDIX H

COMPUTER PROGRAMS

In this appendix are presented the computer programs used to obtain the data for this paper. These programs appear in pairs. The EMPEF programs calculated the series coefficients for the conducting cylinders cases. Program EMPEF #1 calculates the coefficients for the parallel conducting cylinders case and program EMPEF #2 calculates the coefficients for the crossed conducting cylinders case. Program BRPHI #1 was used to obtain the potential contour plots for all the line charge approximation cases and program BRPHI #2 was used to produce the potential contour plots for the series solution cases. Program ETRA #1 was used to calculate the electron trajectories for the line charge approximation cases and program ETRA #2 calculated the electron trajectories for the series solution cases. All the programs were run on the CDC 6600 computer located at Kirtland Air Force Base at Albuquerque, New Mexico.
**EMPEF #1**

```plaintext
PROGRAM EMPEF(INPUT,OUTPUT)
  DIMENSION IX (100),TRI (10000),AA (100,100),BB (100),XX (100)
  EQUIVALENCE (AA,TRI)
  1000 FORMAT (3F7.3,3I4)
  1001 FORMAT (1H,10G11.4)
  1002 FORMAT (15H MATRIX AA N=+113+)
  1003 FORMAT (15H MATRIX BB )
  1004 FORMAT (15H MATRIX X )
  10 CONTINUE
    READ 1000,AA,BL,LL,KK,NN
    PRINT 1000,AA,BL,LL,KK,NN
    IF (AA.EQ.0.0) GO TO 400
    READ 1000,VO,ETRM
C REMOVE THE CARDS BETWEEN THIS AND THE NEXT COMMENT CARD TO CHANGE PROG.
    AC=0.0
    IF (ETRM.EQ.1.0) GO TO 500
    GO TO 30
  30 CONTINUE
    IF (ETRM.EQ.1.0) GO TO 365
    DO 250 J=1,NN
    DO 200 N=1,NN
    DO 100 L=1,LL
    X=C0(1.00*L-1.0)/1.00*LL
    Y=R-SORT(C0*2.*X+2)
    YP=R-SORT(C0*2.*X+2)
    AA(J+NN,N+NN)=PI1(N*X,YN,AA+R+C)*PI1(J*X,YN+AA+R+C)*AA(J+NN+N+NN)
    .AA(J+NN+NN)=PI2(N*X,YP+AB+C)*PI2(J*X,YP+AB+C)*AA(J+NN+N+NN)
  100 CONTINUE
    DO 200 K=1,KK
```

183
X = C \times (A - C) \times (1.0 \times X - 1.0) / (1.0 \times KK)

P1J = P1I(J, X, B, A, B, C)
P2J = P1I(J, X, B, A, B, C)
P3J = P3J2

D1J = D1I(J, X, B, A, B, C)
D2J = D2I(J, X, B, A, B, C)
D3J = D3J2

AA(J, N) = AA(J, N) \times P1I(N, X, B, A, R, C) \times D1I(N, X, R, A, B, C) \times D1J

AA(J, N, N) = AA(J, N, N) \times P1I(N, X, B, A, R, C) \times D1I(N, X, R, A, B, C) \times D1J

CONTINUE

DO 750 L = 1, LL
    X = C \times (A - C) \times (1.0 \times L - 1.0) / (1.0 \times LL)
    Y = P(SORT(C1 = X + 2))
    Z = R(SORT(C2 = X + 2))
    BR(J, N) = BR(J, N) \times P1I(J, X, Y, N, A, B, C) \times (V0 - PIC(X, Y, A, B, C, AC))
    RH(J, N) = RH(J, N) \times P1I(J, X, Y, P, A, Y, C) \times (V0 - PIC(X, Y, P, A, B, C, AC))

CONTINUE

C ********** THIS IS THE NEXT COMMENT CARD **********

NSO = NSO
NSO = NSO

X = 1
J = 0

DO 325 J = 1, NS0
    AA(J) = AA(J, K)
    IF(J, F0, N) K = K + 1
    IF(J, F0, N) J = 0

CONTINUE

PRINT 1002 + N
    IF(N, I, 30) PRINT 1001, (AA(I), I = 1, NS0)

PRINT 1003
PRINT 1001, (BR(I), I = 1, N)
CALL MATSOLV( N, J, X, I, TRI, AA, 0, BR, XX)
PRINT 1004
PRINT 1001, (XX(I), I = 1, N)

DO 350 I = 1, 10000

AA(I) = 0.0
DO 360 I = 1, 10000
BR(I) = 0.0

CONTINUE

C ALSO REMOVE THE FOLLOWING CARDS TO CHANGE PROGRA

CONTINUE

ERR1 = 0.0
ERR2 = 0.0
ERRS = 0.0
DO 375 L = 1, LL
X = C°(1.0°L-1.0)/1.0°LL
YN = R°-SORT(C°-X°2)
YP = R°-SORT(C°-X°2)
ERR1 = E0 - PIC(X, YN, H+A+C)
ERR2 = E0 - PIC(Y, YP, A+B+C)
DO 370 N = 1, NN
ERR1 = ERR1 - P11(N, X, YN, A+B+C) * XX(N)
ERR2 = ERR2 - P12(N, X, YP, A+B+C) * XX(N+NN)
370 CONTINUE
ERR3 = ERR1 * ERR1 * ERR2 * ERR2
375 CONTINUE
ERR3 = ERR3 + ERR3
ERR2 = 0.0
DO 385 K = 1, KK
X = C°(1.0°K-1.0°)/1.0°KK
DO 380 N = 1, NN
ERR1 = ERR1 + P11(N, X, Y, A, B, C) - XX(N+N)
ERR2 = ERR2 + P12(N, X, Y, A, B, C) - XX(N+NN)
380 CONTINUE
ERR3 = ERR3 + ERR1 + ERR2 + ERR2
385 CONTINUE
ERR3 = ERR3 / (2.0°(LL+KK))
PRINT 1005, ERRS
1005 FORMAT(LEN = 8, SQUARED ERROR = 8.11, 4)
500 CONTINUE
IF(AN .EQ. 0.0) GO TO 20
STOP REMOVING CARDS AT THIS POINT
GO TO 10
400 CONTINUE
STOP
END
FUNCTION DP11(N, X, Y, A, B, C)
DP11 = (PI1(N, X, Y, A, B, C) - PI1(N, X, Y, A, B, C))/B°.10
RETURN
END
FUNCTION DP12(N, X, Y, A, B, C)
DP12 = (PI2(N, X, Y, A, B, C) - PI2(N, X, Y, A, B, C))/B°.10
RETURN
END
FUNCTION PI1(N, X, Y, A, B, C)
PI1 = 3.14159
AN = FLOAT(N)
FACT = (AN-1.0)°PI/A
IF(AN .EQ. 1.0) GO TO 100
PI1 = SINH(FACT°Y)°COS(FACT°X)/SINH(FACT°B)
GO TO 200
100 PI=31.4159
200 CONTINUE
RETURN
END
FUNCTION PI2(X,Y,A,B,C)
PI=3.14159
AN=FLOAT(N)
FACT=(AN-1.0)*PI/A
PI2=EXP(FACT*B)*COS(FACT*X)
RETURN
END
FUNCTION SINH(X)
SINH=(EXP(X)-EXP(-X))/2.0
RETURN
END
FUNCTION COSH(X)
COSH=(EXP(X)+EXP(-X))/2.0
RETURN
END
FUNCTION PC(X,Y,A,B,C)
PI=3.14159
T1=COSH(P1*(Y+A)/A)-COS(P1*X/A)
T2=COSH(P1*(Y-B)/A)-COS(P1*X/A)
PIC=ACOS(T1/T2)
RETURN
END
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MATH000053
LAST
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186
EMPEF #2

PROGRAM EMPEF (INPUT, OUTPUT)
DIMENSION IX (100), TR(10000), AA(100, 100), BB(100), XX(100)
EQUIVALENCE (AA, TR)
1000 FORMAT (3F7.3, 41G)
1001 FORMAT (2H, 10G11.4)
1002 FORMAT (1SH, MATRIX AA N=, I3)
1003 FORMAT (1SH, MATRIX BB )
1004 FORMAT (1SH, MATRIX X )
1 CONTINUE
READ 1000, A, B, C, N, N2, KK, NN
PRINT 1000, A, B, C, N, N2, KK, NN
IF (A, EQ, 0.0) GO TO 400
READ 1000, VO
C REMOVE THE CARDS BETWEEN THIS AND THE NEXT COMMENT CARD TO CHANGE PROG.
AC=0.0
LN=NN
IF (NN, GT, 0) AC=1.0/(PIC(C, B, A, B, C, 1.0) + PIC(A, B, A, R, C, 1.0))
NN=KK
DO 20 J=1, KK
20 VN=NN+KK-J
IF (VN, LE, 5) GO TO 370
ISTEP=0.0
IJ1=0
IJ2=1
DO 300 J=1, NN
IJ1=IJ1+1
IJ2=IJ2+1
IF (~) IJ1+ISTEP
IF (IJ2, GT, KK) IJ1=1
IF (IJ2, GT, KY) ISTEP=ISTEP+1
IF (IJ2, GT, KK) IJ2=IJ1+ISTEP
NSTEP=0.0
IN1=0
IN2=1

DO 300 N=1,NN
IN1=IN1+1
IN2=IN1+NSTEP

IF(IN2.GT.KK) IN1=1
IF(IN2.GT.KK) NSTEP=NSTEP+1
IF(IN2.GT.KK) IN2=IN1+NSTEP

LL=NL+1

DO 300 L=1,LL

Y=FLOAT(L1-1)*C/FLOAT(N1-1)

IF(Y.GT.C) Y=C+FLOAT(L1-N1)*(A-C)/FLOAT(N2)

DO 300 L2=1,LL

X=FLOAT(L2-1)*C/FLOAT(N1-1)

IF(Y.GT.C) X=C+FLOAT(L2-N1)*(A-C)/FLOAT(N2)

ZP=R+SORT(C*2-Y*2)

GO TO 300

100 ZP=R

ZN=R

200 CONTINUE

PJ1=PI1(IN1,1,N2,Y,Y,ZN,A,B,C)
PJ2=PI2(IN1,1,N2,Y,Y,ZP,A+B+C)

AA(J,N)=PI1(IN1,IN2,Y,Y,ZN,A+B+C)*PJ1+AA(J,N)
AA(J,N*NN+NN)=PI2(IN1,IN2,Y,Y,ZP,A+B+C)*PJ2+AA(J,N*NN+NN)

IF(Y.LT,C) GO TO 250

DPJ1=DPJ1(IN1,IN2,Y,ZN,A,B+C)
DPJ2=DPJ2(IN1,IN2,Y,ZN,A,B+C)

DPJ1=DPJ1(IN1,IN2,Y,ZN,A+B+C)

250 CONTINUE

IF(Y.GT.C) GO TO 300

IF(Y.GT.C) GO TO 300

100 CONTINUE

C

THIS IS THE NEXT COMMENT CARD

NSO=NN*NN
NSO=NSO+4

NSO=NSO+4

N=NN+2

K=1
J=0
DO 325 I=1,NSQ
    J=J+1
    AA(I)=AA(J,K)
    IF(J.EQ.N) K=K+1
    IF(J.EQ.N) J=0
325 CONTINUE
PRINT 1002,N
IF(N.LT.30) PRINT 1001,(AA(I),I=1,NSQ)
PRINT 1003
PRINT 1001,(HR(I),I=1,N)
CALL 4AT5OLV(N,3,IX,TR1,AA(0,RR),XX)
PRINT 1004
PRINT 1001,(XX(I),I=1,N)
DO 350 I=1,10000
350 AA(I)=0.0
DO 360 I=1,100
BB(I)=0.0
360 CONTINUE
C ALSO REMOVE THE FOLLOWING CARDS TO CHANGE PROGRAM
GO TO 366
370 CONTINUE
DO 365 I=1,100
365 XX(I)=0.0
366 CONTINUE
NPT=0
ERR=0.0
DO 1600 L1=1,LL
    Y=FLOAT(L1-1)*C/FLOAT(N1-1)
    IF(Y.GT.C) Y=C+FLOAT(N1-1)
    DO 1600 L2=1,LL
        X=FLOAT(L2-1)*C/FLOAT(N1-1)
        IF(X.GT.C) X=C+FLOAT(N2)
        IF(Y.GT.X) GO TO 1600
        IF(Y.GT.C) GO TO 1700
        ZP=R+SORT(C^2-Y^2)
        ZN=R-SORT(C^2-Y^2)
        GO TO 2200
1600 CONTINUE
1700 ZP=R
ZN=R
2200 CONTINUE
NPT=NPT+1
NSP=0
IN1=0
IN2=1
ERR1=ERR2=0.0
IF(Y.LE.C) ERR1=P1C(X,ZN,A,B,C,AC)-P1C(Y,ZN,A,B,C,AC)
    IF(Y.LE.C) ERR2=P1C(X,ZP,A,B,C,AC)-P1C(Y,ZP,A,B,C,AC)
DO 1500 I=1,NN
\[
\begin{align*}
\text{IN1} &= \text{IN1} + 1 \\
\text{IN2} &= \text{IN1} \times \text{NSTEP} \\
\text{IF} (\text{IN2} \leq \text{NSTEP}) & \quad \text{IN1} = 1 \\
\text{IF} (\text{IN2} \leq \text{NSTEP}) & \quad \text{IN2} = \text{IN1} \times \text{NSTEP} \\
\text{IF} (\text{Y} \leq \text{ST}) & \quad \text{GO TO 1400} \\
\text{ER1} &= \text{ER1} + \text{PI1} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \times \text{XX} (1) \times \text{ERR} \\
\text{ER2} &= \text{ER2} + \text{PI2} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \times \text{XX} (1) \times \text{NSTEP} + \text{ERR} \\
\text{CONTINUE} & \quad \text{GO TO 1400} \\
\text{ERR1} &= \text{ERR1} + \text{PI1} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \times \text{XX} (1) \times \text{PI2} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \\
\text{ERR2} &= \text{ERR2} + \text{PI1} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \times \text{XX} (1) \times \text{PI2} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \\
\text{CONTINUE} & \quad \text{GO TO 1400} \\
\text{ER1S} &= \text{ER1S} \times \text{PI1} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \times \text{XX} (1) \times \text{PI2} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \\
\text{CONTINUE} & \quad \text{GO TO 1400} \\
\text{ER1S} &= \text{ER1S} / 2.0 \\
\text{PRINT} 1005, \text{ER1S} \\
\text{PRINT} 1006, \text{AC} \\
\text{FORMAT} (\text{H}, \text{H}) = \text{ERROR} = \text{ERR1S} \times \text{PI1} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \times \text{XX} (1) \times \text{PI2} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \\
\text{CONTINUE} & \quad \text{GO TO 1400} \\
\text{FUNCTION} \text{PI1} (\text{IN1}, \text{IN2}, \text{X}, \text{Y}, \text{Z}, \text{A}, \text{B}, \text{C}) \\
\text{N1} &= \text{IN1} - 1 \\
\text{N2} &= \text{IN2} - 1 \\
\text{IF} (\text{N1} + \text{N2} \leq 0) & \quad \text{GO TO 100} \\
\text{PI} &= \text{ACOS} (-1.0) \\
\text{FACT} &= \text{PI} \times \text{SIN} (\text{FLOAT} (\text{IN1}) \times \text{A} / \text{FACT}) \\
\text{PI1} &= \text{ACOS} (\text{PI} \times \text{FLOAT} (\text{IN1}) \times \text{X} / \text{A}) \times \text{COS} (\text{PI} \times \text{FLOAT} (\text{IN2}) \times \text{Y} / \text{A}) \times \text{SINHFACT \times Z) \\
\text{TEMP} &= \text{N1} \\
\text{N1} &= \text{N2} \\
\text{N2} &= \text{TEMP} \\
\text{PI1} &= \text{ACOS} (\text{PI} \times \text{FLOAT} (\text{IN1}) \times \text{X} / \text{A}) \times \text{COS} (\text{PI} \times \text{FLOAT} (\text{IN2}) \times \text{Y} / \text{A}) \times \text{SINHFACT \times Z) \\
\text{N1} &= \text{N2} \\
\text{N2} &= \text{TEMP} \\
\text{PI1} &= \text{PI1} / 2.0 \\
\text{CONTINUE} & \quad \text{GO TO 100} \\
\text{PI1} &= \text{PI1} / 2.0 \\
\text{CONTINUE} & \quad \text{GO TO 100} \\
\text{END}
\end{align*}
\]
FUNCTION P12(N1, N2, X, Y, Z, A, B, C)
    PI = ACOS(-1.0)
    N1 = N1 - 1
    N2 = N2 - 1
    FACT = EXP(N1) * (FACT * (FLOAT(N1) + 2 * (FLOAT(N2) + 2))
    P12 = COS(P1 * FLOAT(N1) * X / A) + COS(P1 * FLOAT(N2) * Y / A) * EXP(FACT * (B - Z))
    TEMP = N1
    N1 = N2
    N2 = TEMP
    PI2 = PI2 * PI2
    TEMP = PI2 / 2.0
    N1 = N2
    N2 = PI2
    RETURN
END

FUNCTION DP11(N1, N2, X, Y, Z, A, R, C)
    DP11 = PI1(N1, N2, X, Y, Z, A, R, C) - PI1(N1, N2, X, Y, Z, A, B, C) / (10 * B)
    RETURN
END

FUNCTION DP12(N1, N2, X, Y, Z, A, B, C)
    DP12 = P12(N1, N2, X, Y, Z, A, B, C) - P12(N1 + N2, X, Y, Z, A, B, C) / (10 * B)
    RETURN
END

FUNCTION COSH(X)
    COSH = (EXP(X) + EXP(-X)) / 2.0
    RETURN
END

FUNCTION SINH(X)
    SINH = (EXP(X) - EXP(-X)) / 2.0
    RETURN
END

FUNCTION PIC(X, Y, A, B, C, AC)
    PI = 3.14159
    T1 = COSH(P10(X) / A) - COS(P10(X) / A)
    T2 = COSH(P10(Y) / A) - COS(P10(Y) / A)
    PIC = AC * LOG(T1 / T2)
    RETURN
END

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192
BRPHI #1

PROGRAM BRPHI (OUTPUT, FILEPL)
DIMENSION A(51,101), P(6), CUI(20)
CHANGE MADE TO NORMALIZE B AND MAKE A VARIABLE

NV=20
CUI(1)=.01
CUI(2)=.05
DO 900 KK=3, NV
CUI(KK)=CUI(KK-1)*.95
900 CONTINUE
DO 981 KK=1, NV
CUI(KK)=CUI(KK)*2
981 CONTINUE

C=1.
B=1.

DO 400 NN=1,4
DO 400 NNN=1,5
Z=(FLOAT(NNN)-1.0)/4.0
Z=AY*Z
AY=FLOAT(NNN)
PRINT 101, AY
101 FORMAT (1H1, 6E12, 1X)
XMAX=4.0*AY
DX=XMAX/100
YMAX=AY
DY=YMAX/50
Y=2.*AY-DY
DY=YMAX/25
Y=AY-DY
DO 20 J=1,51
Y=Y+DY
X=DX
DO 10 I=1,101
X=X+DX
500 CALL PHI(A,Y,B,X+Y,PH)
   APH=PH
      CALL PHI(A,Y,B,X+Z,PH)
      RPH=PH
      CALL PHI(A,Y,2*B,X+Y,PH)
      CPH=PH
      CALL PHI(A,Y,2*B,X+Z,PH)
      PH=(PH*APH*RPH*CPH)/4.
      A(J+I)=PH
10 CONTINUE
200 FORMAT(1X,F10.8)
100 FORMAT(1X,11E12.4/I)
20 CONTINUE

12=1
171=12+10
DO 302 K=1,9
  DO 301 J=1,51
301 CONTINUE
12=12+11
171=12+10
P=INT 303
303 CONTINUE
300 FORMAT(1X,11E12.4)
V=101
M=51
P(1)=0.
   P(2)=DX*100.
   P(3)=-AY
   P(4)=AY
   P(5)=.8
   P(6)=P(5)
   J=R
   I=K
   CALL BRUIT(A,M,N,P,CU,NV,I,J)
400 CONTINUE
400 CONTINUE
STOP
END

NOLIST
SUBROUTINE BRUIT(A,M,N,P,CU,NV,I,J)
DIMENSION A(M,N),CU(NV),P(7)
COMON /GDX/,NX,K,SAVEX(S100),SAVEY(S100)
CALL D4 (P(1),P(2),P(3),P(4),I,J,P(5),P(6),1,SAVEX,SAVEY,1,-1)
DY=(P(4)-P(3))/FLOAT(M-1)
DX=(P(2)-P(1))/FLOAT(NN-1)
NONM1=NM-1
NONN1=NN-1
DO 54 I=1,NV

K=0
DO 5 N=1,NN
DO 5 M=1,NONH1
IF ((A(M,N)-CU(I))*A(M+1,N)-CU(I)),GT,0.) GO TO 5
K=K+1
IF (K,GT,500) GO TO 20
SAVEX(K)=FLOAT(N-1)*DX*P(1)
SAVEY(K)=FLOAT(N-1)*DY*P(3)+(DY/(A(M+1,N)-A(M,N)))*CU(I)-A(M,N)
CONTINUE
DO 10 M=1,WH
DO 10 N=1,NONH1
IF ((A(M,N)-CU(I))*A(M+1,N)-CU(I)),GT,0.) GO TO 10
K=K+1
IF (K,GT,500) GO TO 20
SAVEX(K)=FLOAT(N-1)*DX*P(1)+DX/(A(M+1,N)-A(M,N)))*CU(I)-A(M,N)
SAVEY(K)=FLOAT(N-1)*DY*P(3)
CONTINUE
15 FORMAT (1HN/1H0.20X+F10.6,9H CONTOUR/33X,13HNO. OF Pts. =+14)
IF (.LT.,4) GO TO 45
GO TO 30
CONTINUE
20 Format (1X,23'=*** ARRAY OVERFLOW FOR 'F10.6,9H CONTOUR')
30 CALL ORDER (P)
DO 40 MISUM=1,3
LOAD=0
DO 35 JOY=1,K
IF (SAVEX(JOY).GE.,1,05) GO TO 35
LOAD=LOAD+1
SAVEX(LOAD)=SAVEX(JOY)
SAVEY(LOAD)=SAVEY(JOY)
CONTINUE
35 FORMAT (1X,23'=*** ARRAY OVERFLOW FOR 'F10.6,9H CONTOUR')
40 CALL ORDER (P)
CONTINUE
45 CALL PLOT(FLOAT(I)*P(5),3.6,9..-3)
RETURN
END
SUBROUTINE ORDER (P)
COMMON /GOOP/ DX, DY, 1PLOT, SAVEX(500), SAVEY(500)
DIMENSION PX(500), PY(500) P(7)
V= MAXP
CALL SORT (SAVEX, I PLOT, SAVEY)
MAX=0
TEMPX=SAVEX(1)
TEMPY = SAVEY(I)

IFD = 0

IFD = 0

IP = 1

GO TO 10

5

IP = 2

I = J HOLD

PX(I) = SAVEX(I)

PY(I) = SAVEY(I)

10

PX(IP) = SAVEX(I)

PY(IP) = SAVEY(I)

15

KOUNT = 0

DO 50 J = 1, IPLOT

IF (SAVEX(J) GT 1.5) GO TO 50

SX = SAVEX(J) - SAVEX(I)

SY = SAVEY(J) - SAVEY(I)

DEL = SX * SY + SY

IF (DEL.LT.1.2) GO TO 50

KEEP = 0

IF (DEL.GT.12) GO TO 30

IF (J, FO, IPLOT) GO TO 30

K = J + 6

KK = MINO(KK, IPLOT)

DELT = DEL

KEEP = 1

IF (K, FO, IPLOT) GO TO 30

K = J + 1

DO 25 K = KR, KK

SXC = SAVEX(K) - SAVEX(I)

SYC = SAVEY(K) - SAVEY(I)

DELC = SXC * SXC + SYC

IF (DELC.LT.1.E-12) GO TO 25

IF (DELC-DELT) 20, 25, 25

DELT = DELC

KEEP = K

20

CONTINUE

30

IF (KEEP) 40, 15, 75

35

IF (IFD) 45, 46, 45

50

J HOLD = J

45

IFD = IFD + 1

50

KOUNT = KOUNT + 1

IF (KOUNT.EQ.1.PLOT, AND. IFD.NE.0) GO TO 55

IF (IP.GT.4) CALL D4(I1, D1, P(1), D2, P(3), D3, P(5), P(6), IP, PX, PY, Z, -1)

IF (K6, FO, 1) GO TO 100

IF (IP.FO, IPLOT) GO TO 95

1MAX = 1MAX + 1

IF (MAX.LT.(I1PLOT+5)) GO TO 15

TSX = PX(P) - TMPX

TSY = PY(P) - TEMPY
15 \[ M = N / 2 \]

IF \( M + L \leq 1 \) RETURN

\[ K = N - M \]

DO 25 \( J = 1 + K \)

\[ I = J \]

20 \[ I = I + M \]

IF \( \text{KEY}(1) \leq \text{KEY}(14) \) GO TO 25

\[ T = \text{KEY}(1) \]

\[ \text{KEY}(1) = \text{KEY}(14) \]

\[ \text{KEY}(1) = \text{KEY}(14) \]

\[ \text{KEY}(1) = T \]

\[ I = 1 - M \]

IF \( 1.0 \leq 1 \) GO TO 20

CONTINUE

END

SUBROUTINE D4 (XMIN, XMAX, YMIN, YMAX, IL, IH, SX, SY, NPTS, X, Y, KIND, LAST)

DIMENSION X(NPTS), Y(NPTS)

DATA IFT, JFT, FX, FY, 4HF6, 1.4HF6, 2.06, 0.59/

IF (KIND = 1) \( 1.0, 1.5, 5 \)

5 IF (KIND = 2) 10, 20, 10

RETURN

15 IF (1.0 = 2) 20, 25, 20

20 CALL PLOTS (T, I, B, 10)

10 = ?

25 CALL PLOT (FX, FY, 3)

\[ \text{REAL} = IH \]

\[ \text{REAL} = IL \]

\[ \text{SCALE} = (XMAX - XMIN) / \text{REAL} \]

\[ \text{SCALE} = (YMAX - YMIN) / \text{REAL} \]

\[ \text{SCALE} = \text{SCALE} / \text{SCALE} \]

\[ \text{SCALE} = \text{SCALE} / \text{SCALE} \]

DO 5 \( R = 1.4 \)

GO TO (30, 35, 40, 35, 1, 1)

30 \[ \text{VF} = \text{IH} \]

GO TO 40

35 \[ \text{NN} = \text{IL} + 1 \]

40 \[ \text{DO} 5 \( R = 1.4 \)

\[ \text{GO TO} (45, 55, 45, 75) \]

45 \[ \text{A} = \text{REAL} \]

CALL PLOT (-1.5, FX, R*SY, FY*2)

CALL PLOT (FX, R*SY, FY*2)

YNUM = R*SCALEY*YMIN

IF (AR(YNUM, LE, E-10) YNUM = 0)

RR = (REAL - 1) * SY - 0.3

CALL NUMBER (-0.6, FX, RR, FY, 10, YNUM, 0, JFT)
CALL PLOT (FX+R*S*SY+FY*3)
      IF (N-NN) 50*A*5,50
      GO TO 85
50      CALL PLOT (FX+REALN*S*SY+FY*2)
      GO TO 85
55      R=REALN-1.
      X=R*SY+FY*2
      CALL PLOT (REALN*S*X,REALH*S*Y+FY*2)
      IF (N-NN) 60*A*5,60
60      CALL PLOT (REALN*S*X+REALH*S*Y+FY*2)
      GO TO 85
65      R=REALN-1.
      X=R*S*X+F*X,05+F*Y*2
      CALL PLOT (R*S*X+F*X,05+F*Y*2)
      IF (N-NN) 70*A*5,70
70      CALL PLOT (REALN*S*X+(RR-1)*S*Y+FY*2)
      GO TO 85
75      R=REALN-1.
      X=R*S*X+F*X,05+F*Y*2
      CALL PLOT (R*S*X+F*X,05+F*Y*2)
      XNUM=R*SCALFX*XMIN
      IF (ABS(XNUM).LE.1.E-10) XNUM=0.
      QQ=R*S*X-.25
      CALL NUMBER (QQ+FY,-.25+FY*10,XNUM,0.,1,FT)
      CALL PLOT (R*S*X+F*X,3)
      IF (N-NN) 80*A*5,80
80      CALL PLOT ((R-1)*SX+F*X,FY*2)
85      CONTINUE
90      CALL PLOT ((X(1)-XMIN)*RSCLFX*X+FY,Y(1)-YMIN)*RSCLFY*SY+FY*3)
      DO 95 I=1,NP1
      XX=(X(I)-XMIN)*RSCLFX
      YY=(Y(I)-YMIN)*RSCLFY
      CALL PLOT (XX*FX,YY*SY,FY*2)
      IF (LAST.LT.0) RETURN
      CALL PLOT ((RFALL+3.)*SX+F*X,FY,-3)
      RETURN
END

SUBROUTINE PHI(A,X,Y,PH)
      P1=ACOS(-1.,)
      X1=(X+B)*P1/A
      CO SH1=EXP(X1)+EXP(-X1)*.5
      X2=(X-B)*P1/A
      CO SH2=EXP(X2)+EXP(-X2)*.5
      CO S1=CO S(P1*Y/4)
      IF (COSH2-COS1).LE.0.001 RETURN
      X1=(COSH1-COS1)/(COSH2-COS1)
      PH=0.0
      IF (X.EQ.0.0) RETURN
      PH=ACOS(-1.,)
      X1=(X+B)*P1/A
      CO SH1=EXP(X1)+EXP(-X1)*.5
      X2=(X-B)*P1/A
      CO SH2=EXP(X2)+EXP(-X2)*.5
      CO S1=CO S(P1*Y/4)
      IF (COSH2-COS1).LE.0.001 RETURN
      X1=(COSH1-COS1)/(COSH2-COS1)
```
PH = A / (2. * PI * O) * ALOG(X1)
RETURN
END
```

```
**BRPHI #2**

```plaintext
PROGRAM BRPHI (INPUT, OUTPUT, FILMPL)
C
***** SERIES SOLUTION --- COUNTOUR MAPS ******
C
5 CONTINUE

DIMENSION A(51,101) ,P(6), CU(20) ,X1(100), X2(100)
EQUIVALENCE (VO,VO)

999 FORMAT (IS,6F10.5)
      IF (NX.EQ.0) GO TO 500
      KK=NX
      IF (DIM.EQ.0) GO TO 777
      VX=0
      DO 776 I=1,NX
      NX=I+NX
      CONTINUE

READ 88A (X1('N'),N=1,NX)
PRINT 88A (X1('N'),N=1,NX)
READ 88A (X2('N'),N=1,NX)
PRINT 88A (X2('N'),N=1,NX)

N=20
CU(1) = 0.1
CU(2) = 0.5
DO R80 KK=3,NV
CU(KK) = CU(KK-1) * 0.05
CONTINUE

DO R81 KK=1,NV
CU(KK) = CU(KK) * 0.5
CONTINUE
NDIM=5*DIM
```

---

201
DO 400 NNN=1,NDiM
Z=AY*FLOAT(NNN) -1.0)/4.0
PRINT 101,AY
101 FORMAT(1H1,16=9,FP0.8,/) 
X4AX=2.*AY
DX=X4AX/100.
Y4AX=AY
DY=Y4AX/25.
X=DX
DO 20 J=1,51
Y=AY-DY
X=X-0X
DO 10 I=1,101
IF(J,J.E.26) GO TO 50
A(J, I)=A(52-J, I)
GO TO 10
50 CALL PHI(NX,AY,B,C,Y,X,Z,X1,X2,PH,VO,DIM )
A(J, I)=PH+PIC(Y,X,AY,B,C+AC)/VO
A(J, I)=A(J, I)* DIM*PIC(Z*X,AY,B+C+AC)/VO
CONTINUE
10 CONTINUE
200 FORMAT(1X1E20.8)
100 FORMAT(1X111E12.4/) 
70 CONTINUE
IZ=1
IZ1=IZ+10
DO 302 K=1,9
DO 301 J=1,51
301 CONTINUE
IZ=IZ+11
IZ=IZ+10
PRINT 303
302 CONTINUE
303 FORMAT(/)
300 FORMAT(1X111E12.4)
N=101
M=51
P(1)=0.
P(2)=DX*100.
P(3)=AY
P(4)=AY
P(5)=8
P(6)=P(5)
J=8
I=7
CALL RRUT(A+M+IP,CU+NV+1, J)
40 CONTINUE
40 CONTINUE
700 CONTINUE
FUNCTION PI11(N1,N2,X,Y,Z,A,B,C)

N1 = N1 - 1
N2 = N2 - 1

IF((N1+N2).F0.0) GO TO 100

P1 = ACOS(-1.0)

FACT = PI*SORT((FLOAT(N1))**2+((FLOAT(N2))**2))

FACT = FACT/A

PP1 = COS(P1*FLOAT(N1)*X/A) * COS(P1*FLOAT(N2)*Y/A) * SINH(FACT*Z)

1/SINH(FACT*A)

IF((N1+N2).FQ.0) "GO TO 100"

PP1 = PI/2.0

TEMP = N1

PP1 = PP1/2.0

CONTINUE

PP1 = Z/B

200 CONTINUE

N2 = N2 + 1

N1 = N1 + 1

PI11 = PP1

RETURN

END

FUNCTION PI12(N1,N2,X,Y,Z,A,B,C)

N1 = N1 - 1
N2 = N2 - 1

FACT = PI*SORT((FLOAT(N1))**2+((FLOAT(N2))**2))

FACT = FACT/A

PP2 = COS(P1*FLOAT(N1)*X/A) * COS(P1*FLOAT(N2)*Y/A) * EXP(FACT*(B-Z))

TEMP = N1

N1 = N2
N2 = TEMP

PP2 = COS(P1*FLOAT(N1)*X/A) * COS(P1*FLOAT(N2)*Y/A) * EXP(FACT*(B-Z))

1*PP2

PP2 = PP2/2.0

TEMP = N1

N1 = N2
N2 = TEMP

PP2 = PP2/2.0

CONTINUE

200 CONTINUE

N2 = N2 + 1

N1 = N1 + 1

PI12 = PP2

RETURN

END
FUNCTION PIC(X,Y,A,B,C)
PIC=0.0
PI=3.14159
T1=COSH(P1*(Y+B)/A)-COS(P1*X/A)
T2=COSH(P1*(Y-B)/A)-COS(P1*X/A)
IF(ABS(T2).LT.0.001)GO TO 100
PIC=AC+ALOG(T1/T2)
100 RETURN
END
FUNCTION PI1(X,Y,A,B,C)
PI=3.14159
AN=FLOAT(N)
FACT=(AN-1.0)*PI/A
IF(AN.FQ.1.0) GO TO 100
PI1=SINH(FACT*X)/COSH(FACT*Y)*COS(FACT*X)/SINH(FACT*Y)
GO TO 200
100 PI1=Y/A
200 CONTINUE
RETURN
END
FUNCTION PI2(N,X,Y,A,B,C)
PI=4.159
AN=FLOAT(N)
FACT=(AN-1.0)*PI/A
IF(AN.FQ.1.0) GO TO 100
PI2=EXP(FACT*(B-Y))*COS(FACT*X)
RETURN
END
FUNCTION COSH(X)    COSH=(EXP(X)+EXP(-X))/2.0
RETURN
END
FUNCTION SINH(X)     SINH=(EXP(X)-EXP(-X))/2.0
RETURN
END
NOLIST
SUBROUTINE BRIT(A,H4,N4,NP,CU,NN,IL,J)
DIMENSION A(MM,NN), CU(INV), P(7)
COMMON /GOOP/ DX, DY, K, SAVEX(500), SAVEY(500)
CALL 04(P(1),P(2),P(3),P(4),IL,J,P(5),P(6),1,SAVEX,SAVEY,1,-1)
Dy=(P(4)-P(3))/FLOAT(MM-1)
Dx=(P(2)-P(1))/FLOAT(NN-1)
NOMM=MM-1
NON4=NN-1
DO 45 I=1,NN
Dy=(P(4)-P(3))/FLOAT(MM-1)
45 CONTINUE
DO 44 K=0,NN
Dy=(P(4)-P(3))/FLOAT(MM-1)
44 CONTINUE
IF ((A(H4,NP)-CU(I))*A(H4,NP-NP)-CU(I)),GT,0.0) GO TO 5
ENDIF
205
IF (K > 500) GO TO 20

SAVEX(K) = FLOAT(N-1)*DX*P(1)
SAVEY(K) = FLOAT(M-1)*DY*P(3) + (DY/(A(M+1,N) - A(M,N))) * (CU(I) - A(H,N))
CONTINUE

DO 10 M=1,MM
DO 10 N=1,NN
IF ((A(M,N) - C(I))' (A(N,N+1) - C(I))*) GT.0) GO TO 10

K=K+1
IF (K > 500) GO TO 20
SAVEX(K) = FLOAT(N-1)*DX*P(1) + (DX/(A(M,N+1) - A(M,N))) * (CU(I) - A(H,N))
SAVEY(K) = FLOAT(M-1)*DY*P(3)
CONTINUE

PRINT 15, CU(I), K
FORMAT (1HO/1H0,2X,F10.6,9H CONTOUR/33X,13HNO. OF PTS. = *14)
IF (K < 4) GO TO 45
GO TO 30

PRINT 25, CU(I)
FORMAT (1X,23*ARRAY OVERFLOW FOR F10.6 CONTOUR)
CALL ORDER (P)
DO 40 J=1,4,15
LOAD=0
DO 35 JOY=1,K
IF (SAVEX(JOY) .GE. 1.05) GO TO 35
LOAD=LOAD+1
SAVEX(LOAD)=SAVEX(JOY)
SAVEY(LOAD)=SAVEY(JOY)
CONTINUE
IF (LOAD.LT.5) GO TO 45
K=LOAD
CONTINUE
CALL ORDER (P)
CALL PLOT(FLOAT(I)) * (5)*3 .60,-3)
RETURN
END

COMMON /GO00/ DX, DY, I, PLOT, SAVEX(500), SAVEY(500)
DIMENSION PX(500), PY(500), P(7)
V = DX*DX*DY*DY
CALL SORT (SAVEX, I, PLOT, SAVEY)
I=1
TEMP=SAVEX(I)
TEMP=SAVEY(I)
K=0
IF(K=0)
I=1
IP=1
GO TO 10
IP=2
I=JHOLD
PX(I)=SAVEX(I)
PY(I)=SAVEY(I)
10 PX(IP)=SAVEX(I)
PY(IP)=SAVEY(I)
KOUNT=0
DO 50 J=1,I.PLOT
  IF (SAVEX(J).LT.1.E-12) GO TO 50
  Sx=SAVEX(J)-SAVEX(I)
  Sy=SAVEY(J)-SAVEY(I)
  DEL=Sx*Sx*Sy*Sy
  IF (DEL.LT.1.E-12) GO TO 50
  KEEP=0
  IF (DEL.GT.V) GO TO 30
  KK=J+6
  DELT=DEL
  KEEP=-1
  IF (J.FQ.I.PLOT) GO TO 30
  K=J+1
  DO 25 K=KR, KK
  SXC=SAVEX(K)-SAVEX(I)
  SYC=SAVEY(K)-SAVEY(I)
  DELC=SXC*SXC*SYC*SYC
  IF (DEL.LT.1.E-12) GO TO 25
  IF (DEL.LT.25*25) GO TO 25
  DELT=DECL
  KEEP=K
  CONTINUE
25 IF (KEEP) A0, 75, 75
30 IF (IFD) 45, 40, 45
40 JHOLD=J
45 IFD=IFD+1
50 KOUNT=KOUNT+1
  IF (COUNT.EQ.I.PLOT.AND.IFD.NE.0) GO TO 55
  IF (IP.GT.4) CALL 04(P(1)+P(3)+M+P(5)+P(6), IP, PX, PY, Z-1)
  IF (IP.EQ.1) GO TO 100
  IF (IP.EQ.PLOT) GO TO 95
  IF (IP.EQ.1) GO TO 95
  IP=IP+1
  IF (IP.EQ.PLOT) GO TO 95
  IF (I.MAX.LT.I.PLOT+5) GO TO 15
  TX=PX(IP)-TEMPX
  TSX=TX*TX
  TSY=PY(IP)-TEMPY
  DEL=TSX*TSX*TSY*TSY
  IF (DEL.LT.V) GO TO 110
  CONTINUE
  IF (I.MAX.LT.0) GO TO 60
  C PRINT 105, (PY(L), PY(L)+L, L=1, IP)
  SAVEX(IP)=SAVEX(IP)+10..
IF (IP.GT.4) CALL D4 (P(1),D,P(3),D,H,4,P(5),P(6),IP,PX,PY,2,-1)
60 IF (IP.LE.4) CALL D4 (P(1),D,P(3),D,H,4,P(5),P(6),IP,PX,PY,2,-1)
65 K6=0
TSX=PX(IP)-TEMPX
TSY=PY(IP)-TEMPY
DEL=TSX*TSX+TSY*TSY
IF (DEL.GT.V) GO TO 100
PX(IP)=TEMPX
PY(IP)=TEMPY
GO TO 100
70 K6=1
GO TO 5
75 J=KEEP
80 IP=IP+1
PX(IP)=SAVFX(IP)
PY(IP)=SAVFY(IP)
SAVEX(IP)=SAVEX(IP)+1.E6
IF (J=JHOLD) 90,85,90
85 IFD=0
JHOLD=0
90 J=J
IF (IP=IPL advantages) 15,95,95
95 TSX=PX(IP)-TEMPX
TSY=PY(IP)-TEMPY
DEL=TSX*TSX+TSY*TSY
IF (DEL.LE.V) GO TO 110
100 CONTINUE
105 FORMAT (10x,7HFROM 75/(15x,2E15.5,I4))
110 CALL D4 (P(1),D,P(3),D,H,4,P(5),P(6),IP,PX,PY,2,-1)
RETURN
110 CALL D4 (P(1),D,P(3),D,H,4,P(5),P(6),IP,PX,PY,2,-1)
RETURN
END
SUBROUTINE SORT (KEY,NUM,KEY1)
INTEGER KEY(NU4),T,KEY1(NU4),T2
5 IF (NUM.LT.2) RETURN
I=1
10 I=I+1
IF (I.LE.NUM) GO TO 10
4=I-1
15 W=M/2
IF (W.LT.1) RETURN
K=NUM-M
GO TO 25
J=1
I=J
20 M=I+M
SRT
IF (KEY(I).LE.KEY(I-1)) GO TO 25
T2=KEY(I)
T=KEY(I)
KEY(I)=KEY(I-1)
KEY(I-1)=T2
KEY(I)=T
IF (T.GE.11) GO TO 20
CONTINUE
END

SUBROUTINE D (X, Y, XMIN, XMAX, YMIN, YMAX, IL, IH, SX, SY, NPTS, X, Y, KIND, LAST)
DIMENSION X(NPTS), Y(NPTS)
DATA IFT, FJT, FX, FY, IHF6, JHF6, IH, IH6, 0.6, 0.59/

IF (KIND-1) I=15.5
IF (KIND=2) I=90, 10
RETURN
IF (I-2) 20, 25, 20
CALL PLOTS (T, T, 10)
I0=T
CALL PLOTS (FX, FY, 3)
REALH=1H
REALL=1L
SCALEX=(XMAX-XMIN)/REALH
SCALEY=(YMAX-YMIN)/REALL
RSCALEX=1./SCALEX
RSCALEY=1./SCALEY
DO AS I=1, 4
GO TO (30, 35, 50, 55, 75, 75)
30 N=IH+1
GO TO 40
35 NN=IL+1
40 DO AS N=1, NN
REALN=N
GO TO (45, 55, 55, 75, 75)
45 R=REALN-1.
CALL PLOT (-.5*FX, R*SY*FY, 2)
CALL PLOT (FX, R*SY*FY, 2)
YNUM=R*SCALEY*YMIN
IF (AR*YMIN).LE.1.E-10) YNUM=0.
RR=REALN-1.*SY/-0.3
CALL NUMBER (.-5.*FX, RR*FY, 10, YNUM, 0, *, JFT)
CALL PLOT (FX, R*SY*FY, 2)
IF (N=NN) 50, 50, 50
CALL PLOT (FX, REALN*SY*FY, 2)
GO TO 85
55 R=REALN-1.
RR=REALN+.05
CALL PLOT (R*SX+FX, R*SY+FY, 2)  
CALL PLOT (R*SX+FX, REALH*SY+FY, 2)  
IF (V=NN) 60, 65  
60 CALL PLOT (REALN*SX+FX, REALH*SY+FY, 2)  
GO TO 85  
65 R=RFALL-.05  
RR=REALH-REALN+1.  
CALL PLOT (R*SX+FX, RR*SY+FY, 2)  
IF (V=NN) 70, 85  
70 CALL PLOT (REALN*SX+FX, (RR-1.)*SY+FY, 2)  
GO TO 85  
75 R=REALN-RR  
CALL PLOT (R*SX+FX, 0.5*SY+FY, 2)  
CALL PLOT (R*SX+FX, FY, 2)  
XNUM=R*SCAXL*XMIN  
IF (XMIN+R*SY, .25)  
RR=R*SX-.25  
CALL NUMBER (R*FY, .25*FY, 10*XNUM, 0, IFT)  
CALL PLOT (R*SX+FX, FY, 3)  
IF (V=NN) 80, 90  
80 CALL PLOT (R-1.)*SX+FX, FY, 2  
85 CONTINUE  
90 CALL PLOT ((X(1)-XMIN)*SCAXL*X+FX, (Y(1)-YMIN)*SCALY*SY+FY, 3)  
DO 95 I=1, NPTS  
XX=X(1)-XMIN*SCAXL  
YY=Y(1)-YMIN*SCALY  
95 CALL PLOT (XX*SX+FX, YY*SY+FY, 2)  
IF (LAST, LT, 0) RETURN  
CALL PLOT ((RFALL+3.)*SX+FX, FY, -3)  
RETURN  
END  
LIBRARY  
PLT000029  
LAST  
00000000000000000000000000000000  
10000000000000000000000000000000  
L00000000000000000000000000000000
ETRA #1

PROGRAM ETRA (INPUT, OUTPUT, FILMPL)
DIMENSION P(1), XPTS(1000), YPTS(1001)
1000 FORMAT (9(5x,F7.3))
1001 FORMAT (5x,10H,10H,CONTAINED,10H,ESCAPED,10H)
1003 FORMAT (AF7.3,413)
1004 FORMAT (10H, 1=1000, 3(5x,F7.3))
1005 FORMAT (33H, NUMBER OF ESCAPED ELECTRONS = 13)
1006 FORMAT (33H, NUMBER OF CONTAINED ELECTRONS = 13)
1007 FORMAT (11x,9^9,11x,9^9,9^9,9^9,9^9,9^9,9^9,9^9,9^9,9^9)
1008 FORMAT (5x,F7.3,213)
90 READ 1003 A, R, C, DT, ZMAX, ENG, THETA, RETA, NX, NY, NB, VT
IF (A,F0.0,0) GO TO 170
READ 1003, RH1, RH2, AA, RB, CC, XSHIFT, YSHIFT, START
RHO=RHO/2.0
START=AA
BB=R^2.0
AA=A
CC=C
XSHIFT=C
YSHIFT=A
ABET=RETA
ATET=THETA
DO 165 MR=1,N4
DO 165 MT=1,NT
VE=0
VC=0
RETA=ATET*FLOAT(MR-1)
THETA=ATET*FLOAT(MT-1)
THETA=THETA+START+.1745
IF (THETA.F0.0,0,0,0) GO TO 165
DI=3.14159
R=1.0
```plaintext
X4AX=ZMAX

IF(MX,GT,5) GO TO 12

DO 10 I=1,25

YPST(I)=B

XPTS(I)=(2.0*C/25.0)*FLOAT(I)-1.0)
YPTS(I)=(2.0*A/25.0)*.86603*FLOAT(I)-1.0)

XPTS(I+50)=(2.0*C/7.0*A*.86603-1.00*FLOAT(I)-1.0)
YPTS(I+50)=8.0

DO 10 I=1,50

XPTS(I)=2.0*A/25.0)*.86603*FLOAT(I))
YPTS(I)=YSHIF*.5*CC*(2.5-FLOAT(I))/24.5

CONTINUE

CALL DRAW4(0.0,4.0,0.0,5.0,8.10,100,XPTS,YPTS,1,1)

IF(RH02.EQ.0) GO TO 13

DO 9 I=1,50

XPTS(I)=XSHIF*YSHIF*.86603-CC*(2.5-FLOAT(I))/25.5

CONTINUE

CALL DRAW4(0.0,4.0,0.0,5.0,8.10,50,XPTS,YPTS,2,1)

9 CONTINUE

CALL DRAW4(0.0,4.0,0.0,5.0,8.10,50,XPTS,YPTS,2,1)

CONTINUE

DO 11 I=1,25

XPTS(I)=(2.0*A/25.0)*.86603-CC*(2.5-FLOAT(I))/25.0

CONTINUE

CALL DRAW4(0.0,4.0,0.0,5.0,8.10,25,XPTS,YPTS,2,1)

CONTINUE

PRINT 1007

PRINT 100A,3+1,200.0A

DO 160 K=1,NX

Y=(FLOAT(KK)-.5)/FLOAT(NY)

Y=Y*2.0*A

X=(FLOAT(K)+.5)/FLOAT(NX)

X=x*2.0*C

Z=0.0

V=SQRT(2.0*ENG)

VZ=V*COS(THETA)

VX=V*SIN(THETA)*COS(BETA)

VY=V*SIN(THETA)*SIN(BETA)

J=J+1

100 CONTINUE

J=J+1
```
I = I + 1
CALL FIELD(A*H*C*HO1*X+Y+Z, EEX, EEY, EEZ)
CALL FIELD(A+B*C*HO2*X+Y+Z, ESHIF, EYSHIF, EYSHIF, EYSHIF)
TERM1 = RH01*(1.0/A)*(1.0/C)
TERM2 = RH02*(1.0/AA)*(1.0/CC)
TERM3 = 1.0/(P+PJ*(TERM1+TERM2))

EX = RH01*TERM3*EY+RH02*TERM3*EY
EY = RH01*TERM3*EX+RH02*TERM3*EY
EZ = RH01*TERM3*EY+RH02*TERM3*EZ

X = X + VX*NT + EX*0.5*(DT*2)
Y = Y + VY*NT + FY*0.5*(DT*2)
Z = Z + VZ*NT + FZ*0.5*(DT*2)

VX = VX + EX*NT
VY = VY + EY*NT
VZ = VZ + EZ*NT

P(3J-1) = X
P(3J) = Y
P(3J+1) = Z

XPTS(I) = X + Y*8603
YPTS(I) = Y + Z*5

IF(XPTS(I) > 4.0) XPTS(I) = 4.0
IF(YPTS(I) > 5.0) YPTS(I) = 5.0

C
IF(J, EQ, 3) PRINT 1000
IF(J, EQ, 3) J = 0
IF(I, EQ, 1000) PRINT 1004
IF(I > 1000) 110, 150, 150
110 IF(Z-%MAX) 130, 130, 120
120 CONTINUE
NE = NE + 1
GO TO 150
130 IF(Z > 140) 140, 140, 100
140 CONTINUE
NC = NC + 1
150 CONTINUE
IF(NX > 5) GO TO 160
IF(NX > 5) GO TO 164
CALL DRAW(4.0, 4.0, 0.0, 5.0, 5.0, 8.10, 1, XPTS, YPTS, 2, 1)
164 CONTINUE
CALL DRAW(4.0, 4.0, 0.0, 5.0, 8.10, 1, 0.0, 0.0, 2, 1)
PRINT 1006, NC
PRINT 1005, NE
165 CONTINUE
GO TO 90
170 CONTINUE
STOP
END
SUBROUTINE FIELD(A, B, C, RH01, X, Y, Z, EX, EY, EZ)
REAL N1, N2
IF (RH0.ED,0,0) GO TO 100

PI=ACOS(-1.0)
D1=(COSH(P1*(Z-B)/A)-COS(P1*Y/A))
N1=(COSH(P1*(Z+B)/A)-COS(P1*Y/A))
D2=(COSH(P1*(Z-B)/C)-COS(P1*X/C))
N2=(COSH(P1*(Z+B)/C)-COS(P1*X/C))
TERM=1.0
EX=TERM*(PI/C)*((1.0/N2)-(1.0/((D2))))*SIN(P1*X/C)
EY=TERM*(PI/A)*((1.0/N1)-(1.0/((D1))))*SIN(P1*Y/A)
EZ1=(PI/A)*((SINH(P1*(Z-B)/A))/N1-(SINH(P1*(Z-B)/A))/((D1))
EZ2=(PI/C)*((SINH(P1*(Z-B)/C))/N2-(SINH(P1*(Z-B)/C))/((D2))
EZ=TERM*(EZ1+EZ2)

100 CONTINUE
RETURN
END

FUNCTION COSH(X)
COSH=(EXP(X)*EXP(-X))/2.0
RETURN
END
FUNCTION SINH(X)
SINH=(EXP(X)-EXP(-X))/2.0
RETURN

SUBROUTINE PHAK4 (XMIN, XMAX, YMIN, YMAX, IL, IH, NPTS, X, Y, KIND, LAST) 49
COMMON /DU/ DU
COMMON /K2/ SX, SY, FX, FY
COMMON /NTS/ X(NPTS), Y(NPTS)
DATA JFT, JFT, FY, SX, SY/4HFA.3,4HFA.2,0,6,0,59,0,80/
XMIN=XMAX-XMIN
YMAX=YMAX-YMIN
REAL=IL
REAL=IL
SCALEX=XY/REFULL
SCALEY=Y/REFULL
SCALEX=1./SCALEX
SCALEY=1./SCALEY
IF (KIND=1) 10,20,5
IF (KIND=2) 10,110,10
PRINT 15, KIND
FORMAT (4H THE KIND OF GRAPH ASKED FOR IS IN ERROR, 8A2)
RETURN
10 IF (10-2) 25, 30, 25
25 CALL PLOTS (TQ, T9, 10)
10=2
30 CALL PLOT (FX, FY, 3)
DO 90 I=1, 4
GO TO (35, 40, 75, 40), 1
35 NN=1H+1

RETURN
90 IF (10-2) 25, 30, 25
CALL PLOT (XX, SX, FX, FY, 3)
CALL PLOT (XX, SX, FX, REALH*SY, FY, 2)

XX=(X(1)-XMIN)*RSCALX
YY=(Y(1)-YMIN)*RSCALY
CALL PLOT (XX, SX, FX, YY, SY, FY, 3)
DO 115 I=1,NPTS
XX=(X(I)-XMIN)*RSCALX
YY=(Y(I)-YMIN)*RSCALY
CALL PLOT (XX, SX, FX, YY, SY, FY, 2)
CONTINUE
115 IF (LAST) RETURN
120 RETURN
CALL SYMBOL (6, 7, 0, 14, 0, 0, 0, 13)
CALL PLOT (R, SX, FX, FY, -3)
RETURN
END
PROGRAM ETRA((input, output, file))
DIMENSION P(4), XPTS(1001), YPTS(1001), X(100), Y(100)
EQUIVALENCE (VO*V0)
1000 FORMAT(9(5X,F7.3))
1001 FORMAT(5X*10HCONTAINED, 5X+F7.3, 5X+F7.3, 5X+F7.3)
1002 FORMAT(5X*10HSCAPED, 5X+F7.3, 5X+F7.3, 5X+F7.3)
1003 FORMAT(5H/4.3)
1004 FORMAT(11H1=1000, 3(5X+F7.3))
1005 FORMAT(11HNUMBER OF ESCAPED ELECTRONS = I3)
1006 FORMAT(13HNUMBER OF CONTAINED ELECTRONS = I3)
1007 FORMAT(11X*A111X1*0*11X1*C01Y*01*0*0*0*0*010*010*0*0*0*0*)
1008 FORMAT(8(5X+F7.3)+13)
P1=ACOS(-1.0)
90 HEAU 1003*K1*AA
HEAU 1003*(A-B-C+2*MAX+ENG)*H*E*TA*X*NY*Y*NN
IF (A*E*U=0.0) GO TO 170
HEAU 1010*CHAD+V0*NN
1010 FORMAT(2F7.3+13)
IF (3X*E*U=1.0) GO TO 90
DIM=1,0
IF (INT(X1)=1) DIM=0,0
AA=V1*AA/(PIC(CHA*D+1.0)*PI+PIC(A+B+A+CHAD+1.0))
NN=NN
IF (NY=0,0) GO TO 777
777 CONTINUE
76 NN=1*NN
777 CONTINUE
HEAU 1011*(X1(1)+11*NN)
PRINT 1011*(X2(1)+11*NN)
HEAU 1011*(X2(1)+11*NN)
PRINT 1011*(X2(1)+11*NN)
PRINT 1011*(X2(1)+11*NN)
PRINT 1011*P3*0.0*1.0*DIM)*AA*B/A*X2(1)
NN=KK
80 CONTINUE
ABE=EBETA
ATE=HEETA
U0 LO5 WB=1*NN
U0 LO5 MT=1*NN
NE=U
NN=NN=0
1011 FORMAT(7G11.4)
NC=U
ABE=ABET+FLXAT(4U-1)
ATE=ATE+FLXAT(4T-1)
IF ((ATE=EQ.0.0),A,(ABE,GT,0)) GO TO 162
PI=J*14159
H=1*U
XMAX=ZMAX
IF (NX,GT,5) GO TO 12
IF (MT,GT,1) CALL OMA4O (0.0,0.0,0.0,0.0,9.0,10.0,0.0,0.0,0.0,0.0,0.0,0.0)
IF (NY,GT,1) GO TO 14
UO 11=1*25
YPTS(1) **=
XPTS(11)=F(D*U+C/25.0)*F(D*XAT(1)-1.0)
XPTS(1*25)=F(D*U+C/25.0)*F(D*XAT(1)-1.0)
YPTS(1*25)=F(D*U+C/25.0)*F(D*XAT(1)-1.0)
217
APTS(1+50) = c.0*V+c.0+a.0+V*H-6603*(c.0*G/24.0)*(FLUA1(1)-1.0)
YP1(1+50) = y+a
XPTS(1+50) = (c.0+a/25.0)*(g.0+FLUA1(1))
YP1(1+50) = y+(A/25.0)*(g.0+FLUA1(1))

10 CONTINUE
CALL UHA*W(0,0,0,0,0,0,0,0,0,0,0,0,XPTS,ypt5,-1)
UO = 11
I=1+25
XPTS(1) = 1.0+86603*(FLUA1(1)-1.0)/25.0
YP1(1) = (FLUA1(1)-1.0)/25.0

11 CONTINUE
CALL UHA*W(0,0,0,0,0,0,0,0,0,0,0,0,XPTS,ypt5,-1)

14 CONTINUE
P'INIT 100/
P'INIT 1000+AH+B+C+DI, MAX+ENG+THEIA+DETA+NX+NY
P'INIT 1000+A+CRAD+V0+Y
ENG=ENG*P'IN
UO = 100
KK=1+NY
UO = 100
K=1+NX
I=0
YES=NO
IF (NY,LE,1,1) Y=0.0
X=(FLUA1(K)-5.0)/FLUA1(NX)
X=2.0*C
Z=0.0
V=8.0*(ENG)
Z=V*X*COS(I)*2
Y=V*X*SIN(I)*2
Y=V*X*SIN(I)*2
J=0

100 CONTINUE
J=J+1
L=I+1
CALL FIELU(X+D+C
X=VX+DT+VX+0.5*(V+*Z)
Y=TV*DT+TV+0.5*(Y+*Z)
Z=TV*DT+TV+0.5*(Z+*Z)
VA=VA+Tv*V
Y=V*X*Y+V
V=V*X*Y+V
Y=X*Y+V
Y=X*Y+V
Y=X*Y+V
YWINC=Y
XWENCE=X
P(3*J-3+1)=X
P(3*J-3)=Y
P(3*J-3)=Z
XPTS(1) = X+Y*0.6603
YP5(1) = Z*Y*0.5
IF (XPTS(1)) = GT,0.0
IF (XPTS(1)) = GT,0.0
IF (XPTS(1)) = GT,0.0
IF (XPTS(1)) = GT,0.0
200 IF (XWENCE+YWENCE+ZWENCE) = GT,0.0

218
Read Input Data

Stop yes
A=0?

yes
NN=0?

no
Find Line Charge Solution Coefficient

no

yes
NN=5?

Cal. Error Matrix Element

no
1st Col.?

yes
Cal. Column Matrix Element

Loop Over Indicies

EMPEF #2

Loop Over Coord.

223

Call Matsolv

Calculate Squared Error

Loop Over Coord.
Set Up Initial Parameters

Calculate Potential

Loop Over Coordinates

Set New Parameters

Call Brute

DO For all Cases

Stop
Start

Read Parameters

Stop

yes

NX=0?

no

yes

DIM=0?

no

Find 3 Dimension Index Limits

Read Coefficients

Calculate Potentials

Loop Over 2 Dims.

Call Brute

Loop Over 3rd Dim

BRPHI #2
Read Input

Draw Grids

Cal. Initial Position & Velocity

Cal. New Position & Velocity

Z ≤ C

Or Zmax?

yes

no

Has e Hit Wire?

yes

no

Loop Over Initial Positions

Print Output

Loop Over Initial Angles

Stop