Sensor and Simulation Notes
Note 212
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A Figure of Merit for Transit-Time-Limited Time-Derivative Electromagnetic Field Sensors

Carl E. Baum
Air Force Weapons Laboratory

Abstract

This note derives a sensitivity-bandwidth figure of merit for comparing the performance of sensors designed to measure the time derivative of $\vec{D}$ or $\vec{B}$. This figure of merit is a dimensionless number combining sensitivity and upper frequency response (bandwidth) in a form equivalent area times the square of bandwidth. The larger the figure of merit Λ, the more efficient is the design. This figure of merit is appropriate where sensor size is not a factor, but sensitivity and bandwidth are of primary concern.

I. Introduction

In designing sensors for the time derivative of electromagnetic fields one would like to have quantitative parameters for deciding what are the best designs. Previous notes[2,5] have discussed the concept of equivalent volume V_{eq} and a figure of merit based on the ratio V_{eq}/V_g where V_g is an appropriate geometric volume in which the sensor is to be enclosed. Such a figure of merit is a measure of how efficiently the sensor fills the specified geometrical volume. Based on the usual electromagnetic scaling procedures this dimensionless figure of merit is independent of size for a given sensor design shape. This figure of merit is appropriate where enclosing volume is a constraint on the sensor design. For defining this equivalent-volume figure of merit the sensor was assumed electrically small for all frequencies of interest and the characteristic L/R time for loops and RC time for electric dipoles was assumed large compared to transit times on the sensor geometry.

This note defines a different figure of merit based on a different type of design constraint. Let us assume that the sensor can be any size or shape for measuring electromagnetic fields in an incident plane wave propagating in the $\vec{1}_1$ direction with unit vectors $\vec{1}_2$ and $\vec{1}_3$ for polarizations as illustrated in figure 1.1 and with orthogonality conditions

$$\vec{1}_1 \times \vec{1}_2 = \vec{1}_3 , \quad \vec{1}_2 \times \vec{1}_3 = \vec{1}_1 , \quad \vec{1}_3 \times \vec{1}_1 = \vec{1}_2$$

$$\vec{1}_n \cdot \vec{1}_m = \delta_{n,m} , \quad n,m=1,2,3 \text{ (orthogonal unit vectors)} \tag{1.1}$$

Let the surrounding medium be free space with permittivity ϵ_o and permeability μ_o for which we have

$$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}} , \quad c = \frac{1}{\sqrt{\mu_o \epsilon_o}} , \quad \gamma = \frac{s}{c} \tag{1.2}$$

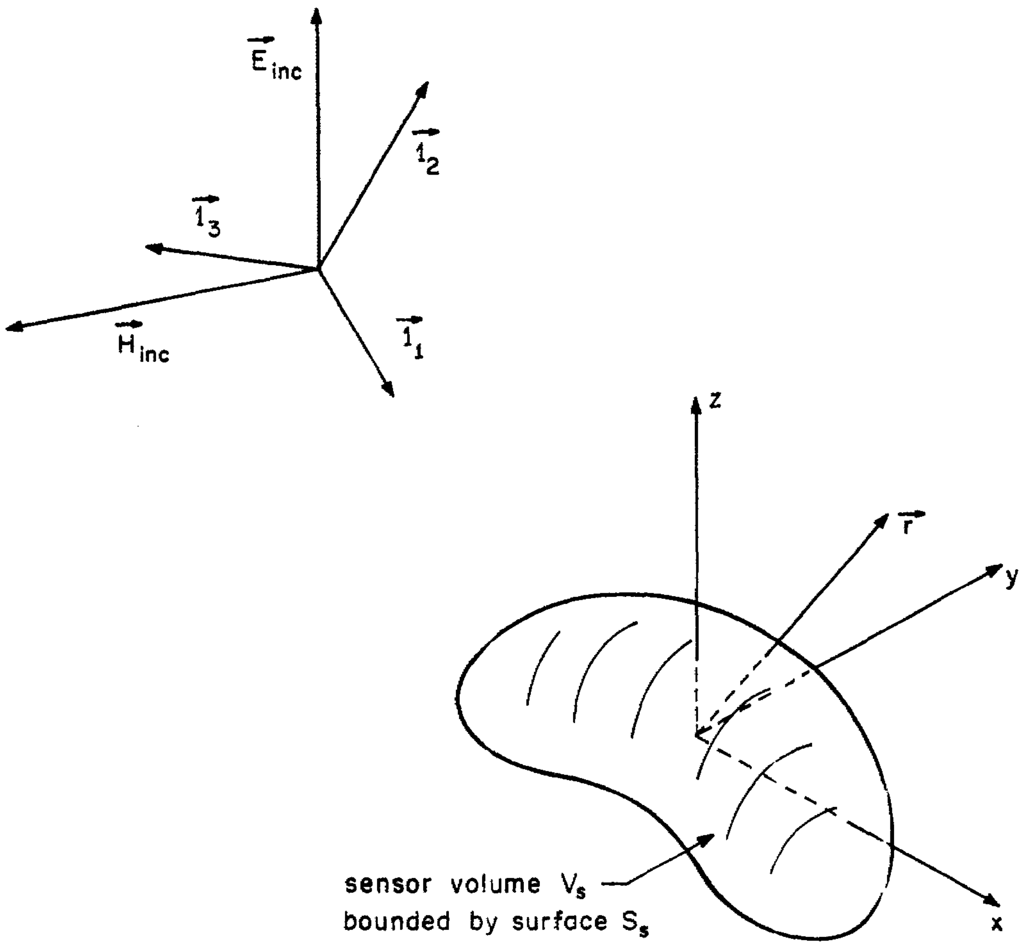


Figure I.I. Electromagnetic Field Sensor in Free Space Illuminated by a Uniform Plane Wave where s is the Laplace transform variable (two sided) corresponding to time dependence e^{st} for CW purposes. Our incident plane wave is then described by

$$\vec{E}_{inc}(\vec{r},t) = E_o\left\{f_2\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right)\vec{1}_2 + f_3\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right)\vec{1}_3\right\}$$

$$\vec{H}_{inc}(\vec{r},t) = \frac{1}{Z_o}\vec{1}_1 \times \vec{E}_{inc}(\vec{r},t)$$

$$= H_o\left\{f_2\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right)\vec{1}_3 - f_3\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right)\vec{1}_2\right\} \tag{1.3}$$

$$E_o = Z_o H_o$$

or in Laplace form by

$$\tilde{\vec{E}}_{inc}(\vec{r},s) = E_o\left\{\tilde{f}_2(s)\vec{1}_2 + \tilde{f}_3(s)\vec{1}_3\right\}e^{-\gamma \vec{1}_1 \cdot \vec{r}}$$

$$\tilde{\vec{H}}_{inc}(\vec{r},s) = H_o\left\{\tilde{f}_2(s)\vec{1}_3 - \tilde{f}_3(s)\vec{1}_2\right\}e^{-\gamma \vec{1}_1 \cdot \vec{r}} \tag{1.4}$$

where the tilde ~ over a quantity indicates the Laplace transform as

$$\tilde{f}(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt \;,\quad f(t) = \frac{1}{2\pi i}\int_{\Omega_o - i\infty}^{\Omega_o + i\infty}\tilde{f}(s)e^{st}ds$$

$$s = \Omega + i\omega \tag{1.5}$$

with Ω_o to the right of any singularities in the complex s plane. For present purposes constrain the incident wave to have a single polarization $\vec{1}_e$ for electric field and $\vec{1}_h$ for magnetic field as

$$\vec{1}_e \times \vec{1}_h = \vec{1}_1 , \quad \vec{1}_e \cdot \vec{1}_h = 0$$

$$\tilde{\vec{E}}_{inc}(\vec{r},s) = E_o \tilde{f}(s) e^{-\gamma \vec{1}_1 \cdot \vec{r}} \vec{1}_e \tag{1.6}$$

$$\tilde{\vec{H}}_{inc}(\vec{r},s) = H_o \tilde{f}(s) e^{-\gamma \vec{1}_1 \cdot \vec{r}} \vec{1}_h$$

In this note we first review the characterization of the sensor response under the assumption that the sensor is electrically small. Then we introduce the high frequency limitation as the characteristic frequency or time for which the ideal time derivative behavior becomes invalid. Considering the power delivered to the load at this maximum frequency appropriately normalized to the power in the incident wave and accounting for the time derivative nature of the response an appropriate dimensionless figure of merit is defined.

The sensors of concern are designed to measure a broadband transient pulse and often drive a coaxial or twinaxial transmission line of characteristic impedance Z_c which is approximately frequency independent. We then assume that the load is purely resistive and frequency independent and denote its value by Z_c. In common practice Z_c is 50 ohms (coax for single ended outputs) or 100 ohms (twinax or two coax in series for differential output), although other constant resistive impedances are possible.

II. Electrically Small Antennas

By electrically small antennas (or scatterers) is meant that the maximum linear dimension is small compared to the radian wavelength $\lambda\!\!\!^{-}$ (or $|\gamma|^{-1}$ in the complex frequency sense). More strictly it means that the first terms in the low frequency expansion (around s = 0) are adequate to describe the open circuit voltage and short circuit current from the antenna. For electric and magnetic dipole sensors these have well-known forms.

A. Electric dipole sensors

An electric dipole is constructed by two separated conductors[7] (perhaps containing some impedance loading). This can transmit an electric dipole field at large distances and low frequencies by its electric dipole moment. As a sensor (receiving antenna) its induced electric dipole moment is of concern and leads to a concept of an equivalent length (height) and an equivalent area both of which are constant vectors. In free space its impedance at low frequencies is described by a capacitance, This leads to equivalent circuits as in figure 2.1.

Such a sensor has basic parameters at low frequencies

C sensor capacitance

$\vec{\ell}_{e_{eq}}$ equivalent length (height)

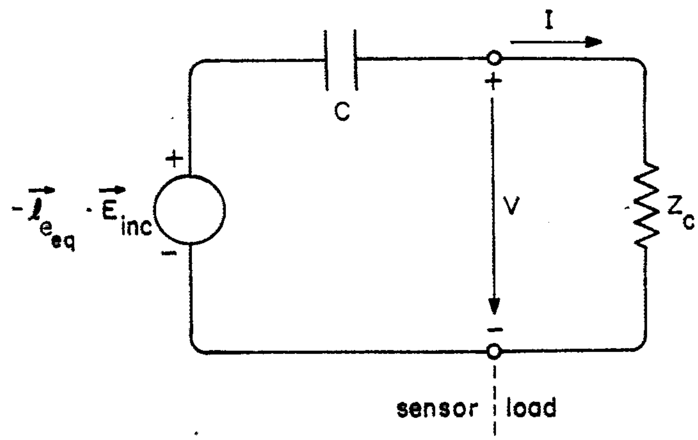
$\vec{A}_{e_{eq}}$ equivalent area

which are related by

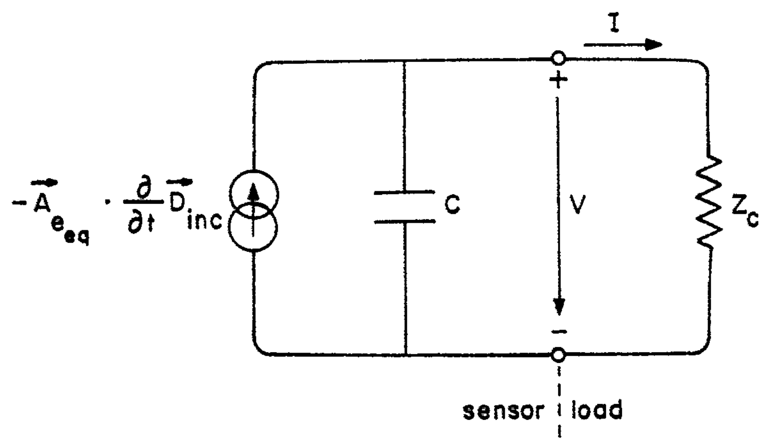
$$\vec{A}_{e_{eq}} = \frac{C}{\epsilon_0}\vec{\ell}_{e_{eq}}$$

$$A_{e_{eq}} = \frac{C}{\epsilon_0}\ell_{e_{eq}} \qquad (2.1)$$

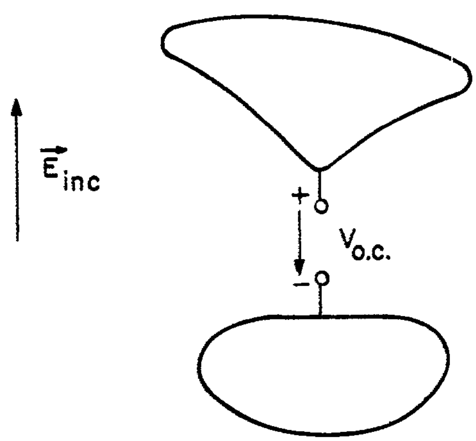
$$A_{e_{eq}} \equiv \left|\vec{A}_{e_{eq}}\right| , \quad \ell_{e_{eq}} \equiv \left|\vec{\ell}_{e_{eq}}\right|$$



A. Thevenin equivalent circuit



B. Norton equivalent circuit



C. Electric dipole sensor

Figure 2.1. Electrically Small Electric Dipole Sensor in Free Space

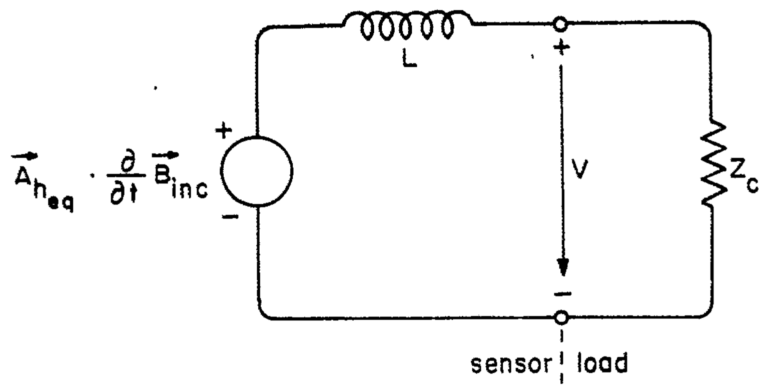
We also have (with fields evaluated at the sensor "location" $\vec{r} = \vec{0}$)

$$
\begin{aligned}
V_{o.c.} &= - \vec{\ell}_{e_{eq}} \cdot \vec{E}_{inc} && \text{open circuit voltage} \\
I_{s.c.} &= - \vec{A}_{e_{eq}} \cdot \frac{\partial}{\partial t} \vec{D}_{inc} && \text{short circuit current} \\
\vec{D}_{inc} &= \epsilon_o \vec{E}_{inc} && \\
\tilde{V} &= \tilde{V}_{o.c.} \frac{sCZ_c}{1 + sCZ_c} && \text{voltage into load} \\
\tilde{I} &= \tilde{I}_{s.c.} \frac{1}{1 + sCZ_c} && \text{current into load} \\
\frac{\tilde{V}_{o.c.}}{\tilde{I}_{s.c.}} &= \frac{1}{sC} && \text{source impedance} \\
\frac{\tilde{V}}{\tilde{I}} &= Z_c && \text{load impedance} \\
V_{e_{eq}} &= \frac{\epsilon_o}{C} \vec{A}_{e_{eq}} \cdot \vec{A}_{e_{eq}} = \frac{C}{\epsilon_o} \vec{\ell}_{e_{eq}} \cdot \vec{\ell}_{e_{eq}} = \vec{A}_{e_{eq}} \cdot \vec{\ell}_{e_{eq}} && \text{equivalent volume}
\end{aligned}
\tag{2.2}
$$

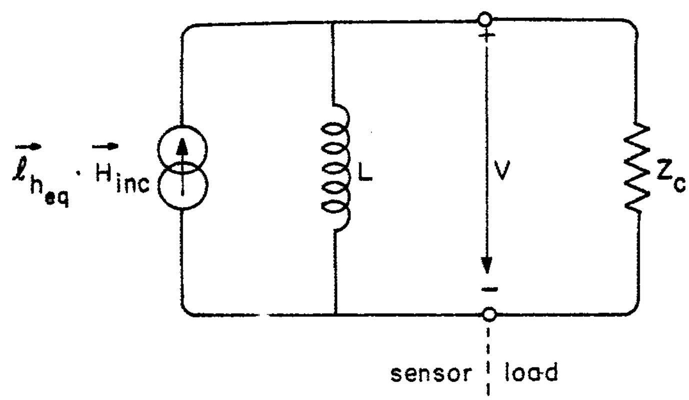
B. Magnetic dipole sensors (loops)

A magnetic dipole is constructed by a conducting loop[7] (perhaps with some parallel impedance loading). This can give a magnetic dipole field at large distances and low frequencies by its magnetic dipole moment. As a sensor its induced magnetic dipole moment leads to the concepts of vector equivalent length and equivalent area. The low frequency impedance is an inductance. Equivalent circuits are given in Figure 2.2.

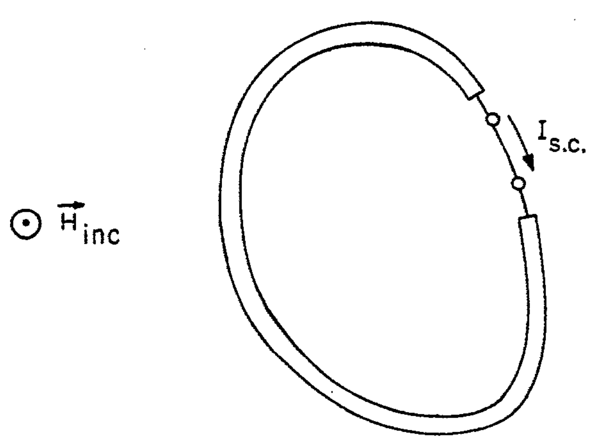
This type of sensor has basic low-frequency parameters



A. Thevenin equivalent circuit



B. Norton equivalent circuit



C. Magnetic dipole sensor (loop)

Figure 2.2. Electrically Small Magnetic Dipole Sensor in Free Space

L	sensor inductance
$\vec{\ell}_{h_{eq}}$	equivalent length
$\vec{A}_{h_{eq}}$	equivalent area

which are related by

$$\vec{A}_{h_{eq}} = \frac{L}{\mu_o} \vec{\ell}_{h_{eq}}$$

$$A_{h_{eq}} = \frac{L}{\mu_o} \ell_{h_{eq}} \tag{2.3}$$

$$A_{h_{eq}} \equiv \left| \vec{A}_{h_{eq}} \right| , \quad \ell_{h_{eq}} \equiv \left| \vec{\ell}_{h_{eq}} \right|$$

We also have (with fields evaluated at the sensor "location" $\vec{r} = \vec{0}$)

$$V_{o.c.} = \vec{A}_{h_{eq}} \cdot \frac{\partial}{\partial t} \vec{B}_{inc} \qquad \text{open circuit voltage}$$

$$I_{s.c.} = \vec{\ell}_{h_{eq}} \cdot \vec{H}_{inc} \qquad \text{short circuit current}$$

$$\vec{B}_{inc} = \mu_o \vec{H}_{inc}$$

$$\tilde{V} = \tilde{V}_{o.c.} \frac{1}{1 + s\frac{L}{Z_c}} \qquad \text{voltage into load} \tag{2.4}$$

$$\tilde{I} = \tilde{I}_{s.c.} \frac{s\frac{L}{Z_c}}{1 + s\frac{L}{Z_c}} \qquad \text{current into load}$$

$$\frac{\tilde{V}_{o.c.}}{\tilde{I}_{s.c.}} = sL \qquad \text{source impedance}$$

$$\frac{\tilde{V}}{\tilde{I}} = Z_c \qquad \text{load impedance}$$

$$V_{h_{eq}} = \frac{\mu_o}{L} \vec{A}_{h_{eq}} \cdot \vec{A}_{h_{eq}} = \frac{L}{\mu_o} \vec{\ell}_{h_{eq}} \cdot \vec{\ell}_{h_{eq}} = \vec{A}_{h_{eq}} \cdot \vec{\ell}_{h_{eq}}$$

equivalent volume

III. High Frequency Limitation

Assuming that the sensor is desired to have a time-derivative behavior then we need to determine some maximum frequency $\omega_c = 2\pi f_c$ for which this time-derivative behavior is followed. This is the sensor bandwidth for a time-derivative sensor. Corresponding to the characteristic time which might be defined in terms of ω_c or might be defined directly in terms of a time domain measurement or calculation.

For electric dipole sensors the time derivative behavior is associated with the short circuit current characterized by an equivalent area $\vec{A}_{e_{eq}}$. For magnetic dipole sensors the time derivative behavior is associated with the open circuit voltage characterized by an equivalent area $\vec{A}_{h_{eq}}$. Note that these equivalent areas are constant vectors by definition; they are vector coefficients in the asymptotic forms of the low frequency response.

In summary then we have the ideal sensor behavior for time-derivative operation

$$I_{ideal} = -\vec{A}_{e_{eq}} \cdot \frac{\partial}{\partial t}\vec{D}_{inc} \qquad \text{electric dipole sensor}$$

$$V_{ideal} = \vec{A}_{h_{eq}} \cdot \frac{\partial}{\partial t}\vec{B}_{inc} \qquad \text{magnetic dipole sensor} \tag{3.1}$$

or in frequency domain

$$\tilde{I}_{ideal} = -s\vec{A}_{e_{eq}} \cdot \tilde{\vec{D}}_{inc} \qquad \text{electric dipole sensor}$$

$$\tilde{V}_{ideal} = s\vec{A}_{h_{eq}} \cdot \tilde{\vec{B}}_{inc} \qquad \text{magnetic dipole sensor} \tag{3.2}$$

We are then interested in characteristic frequencies or times for which V and I begin to deviate from the ideal form above.

One possible definition of the bandwidth is

$$\omega_c = 2\pi f_c = \frac{1}{t_c} = \begin{cases} \dfrac{1}{CZ_c} & \text{electric dipole sensor} \\[2ex] \dfrac{Z_c}{L} & \text{magnetic dipole sensor} \end{cases} \tag{3.3}$$

where these correspond to frequencies that the magnitude of the response is $1/\sqrt{2}$ times the ideal magnitude. This is based on the equivalent circuit and is a very natural definition for cases that $t_c \gg$ transit times on the sensor, i.e. for cases that the sensor is still electrically small at this characteristic frequency.

However, as has been discussed previously,[1] maximizing the equivalent area for a given upper frequency limit implies increasing the sensor size until transit time limits are of the same order as circuit relaxation time limits (CZ_c or L/Z_c). Hence a sensor with maximum bandwidth for a given sensitivity (equivalent area) will not be able to directly use the definitions in equations 3.3.

It would be desirable to have a definition of ω_c and/or t_c which can be directly measured in CW and/or transient experiments. Let us mention a few possible definitions. For CW purposes (or from transformed time-domain data) one might define ω_c as the first frequency for which the response magnitude deviated from its ideal form by some fractional amount ξ, i.e.

$$\left| \frac{\tilde{V}(i\omega_c)}{\tilde{V}_{ideal}(i\omega_c)} \right| - 1 \le \xi \text{ for } \omega \le \omega_c \tag{3.4}$$

Note that voltage or current can be used interchangeably since they are related by the constant impedance Z_c. Since the sensor response is a function of the directions of incidence $\vec{1}_1$ and field directions $\vec{1}_2$ and $\vec{1}_3$, and since for a non-electrically-small sensor $\tilde{V}(i\omega)$ does not in general have the simple dot-product angular dependence (equations 2.2 and 2.4) as does $\tilde{V}_{ideal}(i\omega)$, then this limitation may have been imposed in an upper bound sense over all directions of incidence and field directions. Another approach might be to define a weighted average of the left side of equation 3.4 (averaged over directions of incidence and/or field directions) and require this weighted average to be less than or equal to ξ.

In time domain one might excite the sensor by an ideal type of transient wave, say a step function of near zero rise time. Integrating the sensor output gives a step-like waveform, the early portion of which can be used to define t_c. If there is no significant overshoot or other pronounced oscillatory behavior one might define some kind of rise time such as the usual 10% to 90% or some other form (0% to 50%, etc.). If this is taken as some Δt then one might define t_c in any of several ways such as

$$\begin{aligned} t_c &\equiv \Delta t \\ t_c &\equiv \max(\Delta t) \qquad \text{by varying } \vec{1}_1\text{, etc.} \\ t_c &\equiv \operatorname{avg}(\Delta t) \qquad \text{with appropriate weighting} \end{aligned} \tag{3.5}$$

Having define ω_c and/or t_c this can be converted to characteristic length ℓ_c by

$$\ell_c = c\,t_c \tag{3.6}$$

and/or

$$\ell_c = \frac{c}{\omega_c} \tag{3.7}$$

This parameter ℓ_c will be used in defining the figure of merit.

Note that there are many possible detailed ways to define ω_c, t_c, and ℓ_c. One should be specific in the definition. To distinguish one choice from another one might use subscripts, for example ℓ_{10-90} based on 10% to 90% rise time, or $\ell_{0.3}$ based on a deviation of the response magnitude of 30%, etc.

IV. Figure of Merit

The figure of merit, which might be referred to as a sensitivity-bandwidth figure of merit, is defined in a manner such that a larger value of this parameter means a larger bandwidth for a given sensitivity, or conversely a larger sensitivity for a given bandwidth. The figure of merit should combine the sensitivity and bandwidth in a manner which gives a number independent of size of the sensor, i.e. in scaling the sensor dimensions this number should not change. This indicates that the sensitivity which is an area and the bandwidth will combine like sensitivity times bandwidth squared, i.e. like $A_{eq}\omega_c^2$ or $A_{eq}t_c^{-2}$.

A. Definition of figure of merit

Taking the incident wave as defined in equations 1.6 consider the ideal voltage normalized to the incident electric field multiplied by the normalized bandwidth as the definition of a voltage figure of merit Λ_V, specifically

$$\Lambda_V = \left| \frac{\tilde{V}_{ideal}(i\omega_c)}{E_o\tilde{f}(i\omega_c)} \frac{i\omega_c}{c} \right| = \left| \frac{\tilde{V}_{ideal}(i\omega_c)}{E_o\tilde{f}(i\omega_c)} \right| \frac{1}{\ell_c} \tag{4.1}$$

For this definition the orientation of the field (electric or magnetic depending on sensor type) is taken parallel to $\vec{A}_{eq}$ to maximize the result, and where the ideal voltage is evaluated at $s = i\omega_c$ to maximize the result. Similarly define a current figure of merit Λ_I with the current normalized to the incident magnetic field as

$$\Lambda_I = \left| \frac{\tilde{I}_{ideal}(i\omega_c)}{H_o\tilde{f}(i\omega_c)} \frac{i\omega_c}{c} \right| = \left| \frac{\tilde{I}_{ideal}(i\omega_c)}{H_o\tilde{f}(i\omega)} \right| \frac{1}{\ell_c} \tag{4.2}$$

These definitions both apply to both electric and magnetic dipole sensors.

Both Λ_V and Λ_I are dimensionless figures of merit but they have a certain deficiency. If one were to take the sensor output into impedance Z_c and introduce an ideal 1 to N turn transformer and change the load impedance to $N^2 Z_c$ on the transformer secondary the voltage would be increased by a factor of N. This would increase Λ_V by a factor of N since the load on the sensor has remained Z_c and hence the upper bandwidth has remained ω_c. Similarly the current and Λ_I would be decreased by a factor of N. To make the figure of merit independent of this type of change to the sensor output define a figure of merit as

$$\begin{aligned}
\Lambda &= \left[\Lambda_V \Lambda_I\right]^{1/2} \\
&= \left| \frac{\tilde{V}_{ideal}(i\omega_c)}{E_o \tilde{f}(i\omega_c)} \frac{\tilde{I}_{ideal}(i\omega_c)}{H_o \tilde{f}(i\omega_c)} \right|^{1/2} \frac{1}{\ell_c} \\
&= \left(\frac{Z_o}{Z_c}\right)^{1/2} \left| \frac{\tilde{V}_{ideal}(i\omega_c)}{E_o \tilde{f}(i\omega_c)} \right| \frac{1}{\ell_c} \\
&= \left(\frac{Z_c}{Z_c}\right)^{1/2} \left| \frac{\tilde{I}_{ideal}(i\omega_c)}{H_o \tilde{f}(i\omega_c)} \right| \frac{1}{\ell_c}
\end{aligned} \tag{4.3}$$

From another point of view this figure of merit combines voltage and current in the form of the square root of power. Power is conserved on passing through an ideal transformer. This figure of merit is then in the form of $(\text{power})^{1/2}$ times bandwidth.

B. Electric dipole sensor

Applying these results to an electric dipole sensor the ideal voltage and current from section 3 together with the incident wave give

$$\Lambda_V = \frac{Z_c}{Z_o} A_{e_{eq}} \ell_c^{-2} = \frac{Z_c}{Z_o} A_{e_{eq}} \left(\frac{\omega_c}{c}\right)^2$$

$$\Lambda_I = A_{e_{eq}} \ell_c^{-2} = A_{e_{eq}} \left(\frac{\omega_c}{c}\right)^2 \tag{4.4}$$

$$\Lambda = \left(\frac{Z_c}{Z_o}\right)^{1/2} A_{e_{eq}} \ell_c^{-2} = \left(\frac{Z_c}{Z_o}\right)^{1/2} A_{e_{eq}} \left(\frac{\omega_c}{c}\right)^2$$

The high-frequency figure of merit Λ is then proportional to $\sqrt{Z_c/Z_o}$ for an electric dipole sensor. The equivalent area and characteristic length form a term $A_{e_{eq}} \ell_c^{-2}$ which reappear in the same form in the magnetic dipole sensor.

C. Magnetic dipole sensor (loop)

Applying equations 4.1 through 4.3 to a magnetic dipole sensor gives

$$\Lambda_V = A_{h_{eq}} \ell_c^{-2} = A_{h_{eq}} \left(\frac{\omega_c}{c}\right)^2$$

$$\Lambda_I = \frac{Z_o}{Z_c} A_{h_{eq}} \ell_c^{-2} = \frac{Z_o}{Z_c} A_{h_{eq}} \left(\frac{\omega_c}{c}\right)^2 \tag{4.5}$$

$$\Lambda = \left(\frac{Z_o}{Z_c}\right)^{1/2} A_{h_{eq}} \ell_c^{-2} = \left(\frac{Z_o}{Z_c}\right)^{1/2} A_{h_{eq}} \left(\frac{\omega_c}{c}\right)^2$$

The magnetic dipole sensor then has Λ proportional to $\sqrt{Z_o/Z_c}$, the reciprocal of the factor appearing in the case of the electric dipole sensor. It is this square root of an impedance ratio which makes the figures of merit for the electric and magnetic dipole sensors comparable. Note that $A_{e_{eq}}$ and $A_{h_{eq}}$ are not physically the same since they relate different fields to different circuit quantities (volts, current).

V. Extension to Sensors on Ground Planes

The previous discussion has centered around electromagnetic sensors in free space spaced away from other objects. Often it is desired to mount such sensors on conducting ground planes for measurement of the surface fields, or equivalently of the surface current and charge densities. For this purpose the ground plane is assumed to be approximately flat, at least in the vicinity of the sensor.

Consider a sensor with an electromagnetic symmetry plane[8] located in free space as illustrated in figure 5.1. In such a situation it measures fields in some incident wave $\vec{E}_{inc}$, $\vec{H}_{inc}$. It is convenient to define mirror quantities $\vec{E}_{inc_m}$ and $\vec{H}_{inc_m}$ via a reflection dyad as

$$\begin{aligned}
\overleftrightarrow{R} &= \overleftrightarrow{1} - \vec{1}_n\vec{1}_n \\
\vec{r}_m &= \overleftrightarrow{R} \cdot \vec{r} \\
\vec{E}_{inc_m}(\vec{r}_m, t) &= \overleftrightarrow{R} \cdot \vec{E}_{inc}(\vec{r}, t) \\
\vec{H}_{inc_m}(\vec{r}_m, t) &= -\overleftrightarrow{R} \cdot \vec{H}_{inc}(\vec{r}, t)
\end{aligned} \tag{5.1}$$

This formalism can be extended to all electromagnetic quantities.[8] Here $\overleftrightarrow{1}$ is the unit dyad (identity) and $\vec{1}_n$ is a unit vector normal to the symmetry plane.

Let us assume that the symmetry plane is replaced by a conducting plane. This forces the fields to be antisymmetric[8] so as to enforce zero tangential $\vec{E}$ on the plane, i.e.

$$\begin{aligned}
\vec{E}_{inc_{as}}(\vec{r}, t) &= \frac{1}{2}\left[\vec{E}_{inc}(\vec{r}, t) - \vec{E}_{inc_m}(\vec{r}, t)\right] \\
\vec{H}_{inc_{as}}(\vec{r}, t) &= \frac{1}{2}\left[\vec{H}_{inc}(\vec{r}, t) - \vec{H}_{inc_m}(\vec{r}, t)\right]
\end{aligned} \tag{5.2}$$

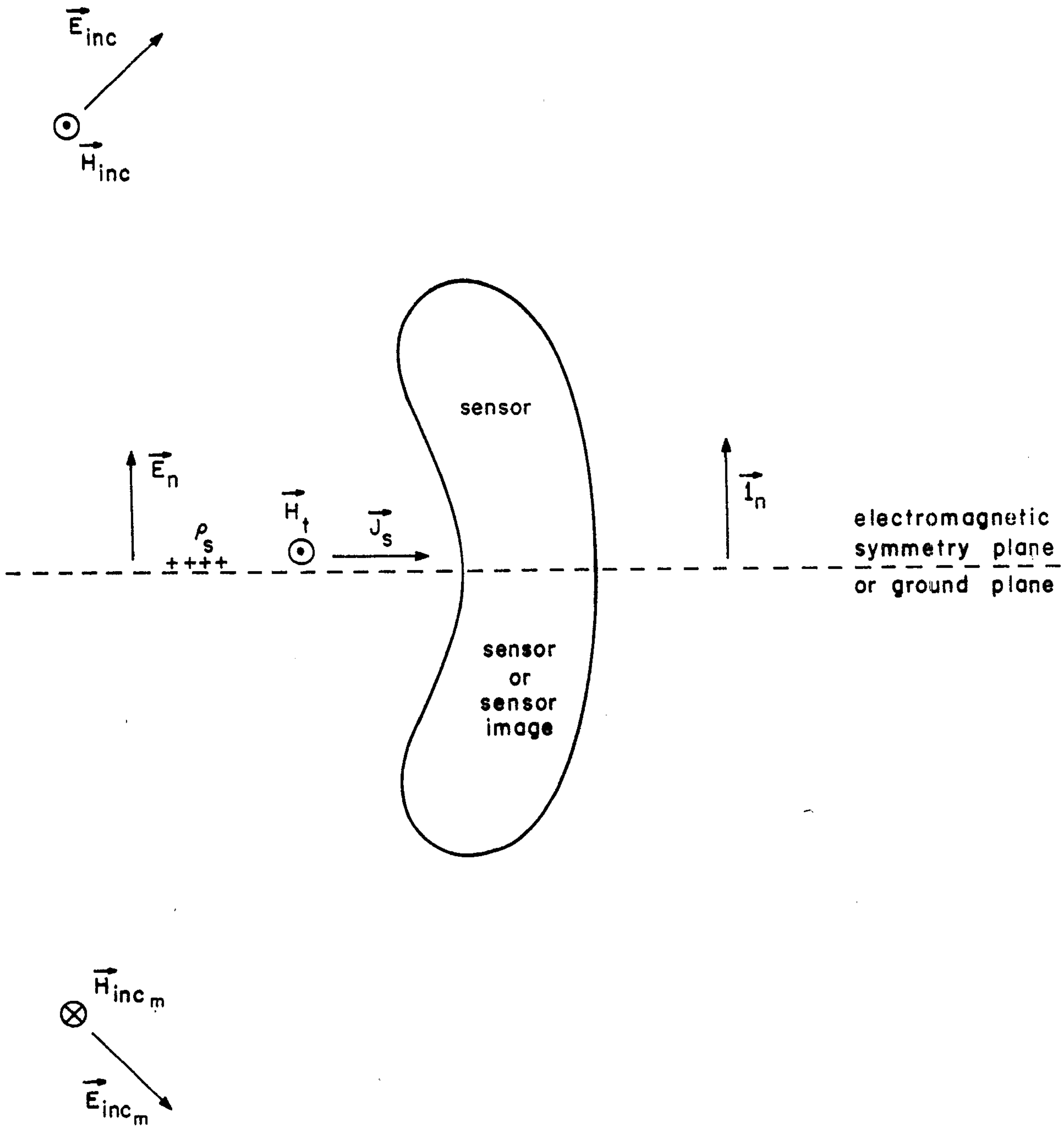


Figure 5.1. Electromagnetic Field Sensor with Symmetry Plane or Ground Plane

On or near the ground plane the fields are $\vec{E}_n$ normal to the plane and $\vec{H}_t$ parallel to the plane, i.e.

$$\vec{1}_n \times \vec{E}_n = \vec{0}$$
$$\vec{H}_t \cdot \vec{1}_n = 0 \tag{5.3}$$

Hence only antisymmetric field distributions need be considered[9] (including the image in principle). A sensor for use on such a ground plane is then appropriate only for measuring the antisymmetric quantities, $\vec{E}_n$ and $\vec{H}_t$, there. Equivalently it is used for measuring surface current and charge densities through

$$\rho_s = \epsilon_o \vec{E}_n \cdot \vec{1}_n$$
$$\vec{J}_s = \vec{1}_n \times \vec{H}_t \tag{5.4}$$

Note that in the presence of a ground plane one may consider incident and reflected fields as separate quantities and refer measurements to them. More generally since ground planes are often small (say the surface of an aircraft) it is convenient to refer all measurement to ρ_s and $\vec{J}_s$, or $\vec{E}_n$ and $\vec{H}_t$, as the resulting quantities at the measurement location in the absence of the sensor. This is the convention adopted here.

In taking a particular type of sensor design and cutting it in half to mount on a ground plane some of the electrical parameters are changed. If the original output is differential (say $100\,\Omega$) then the halved sensor is made to drive half the impedance (say $50\,\Omega$) neglecting introduction of transformers. In the process V is halved with I remaining the same. The equivalent area is then halved for magnetic dipole sensors but remains the same for electric dipole sensors. This is summarized in table 5.1. Note that the directions of the vector sensitivities for ground plane application are constrained by

$$\vec{1}_n \times \vec{A}_{e_{eq}} = \vec{0} \, , \quad \vec{1}_n \times \vec{\ell}_{e_{eq}} = \vec{0}$$

$$\vec{1}_n \cdot \vec{A}_{h_{eq}} = 0 \, , \quad \vec{1}_n \cdot \vec{\ell}_{h_{eq}} = 0 \qquad (5.5)$$

The upper frequency response ω_c has remained unchanged in conversion from free space to ground plane since the linear dimensions are not altered except to remove half the sensor. The sensitivity-bandwidth figure of merit (equations 4.4 and 4.5) is then altered as indicated in table 5.2. Note the factor of $1/\sqrt{2}$ reduction in Λ which applies to both electric and magnetic dipole sensors. One should be careful when comparing sensor designs whether the free space or ground plane parameters are being used. The conversion between the two is quite simple.

electric dipole sensor	C (ground plane)	=	2C (free space)
	Z_c (ground plane)	=	$\frac{1}{2} Z_c$ (free space)
	CZ_c (ground plane)	=	CZ_c (free space)
	$\vec{A}_{e_{eq}}$ (ground plane)	=	$\vec{A}_{e_{eq}}$ (free space)
	$\vec{\ell}_{e_{eq}}$ (ground plane)	=	$\frac{1}{2} \vec{\ell}_{e_{eq}}$ (free space)
	$V_{e_{eq}}$ (ground plane)	=	$\frac{1}{2} V_{e_{eq}}$ (free space)
magnetic dipole sensor	L (ground plane)	=	$\frac{1}{2}$L (free space)
	Z_c (ground plane)	=	$\frac{1}{2} Z_c$ (free space)
	L/Z_c (ground plane)	=	L/Z_c (free space)
	$\vec{A}_{h_{eq}}$ (ground plane)	=	$\frac{1}{2} \vec{A}_{h_{eq}}$ (free space)
	$\vec{\ell}_{h_{eq}}$ (ground plane)	=	$\vec{\ell}_{h_{eq}}$ (free space)
	$V_{h_{eq}}$ (ground plane)	=	$\frac{1}{2} V_{h_{eq}}$ (free space)

Table 5.1. Change in low-frequency sensor parameters in conversion from differential free-space snesor to ground plane version

$$\omega_c \text{ (ground plane)} = \omega_c \text{ (free space)}$$

$$t_c \text{ (ground plane)} = t_c \text{ (free space)}$$

$$\ell_c \text{ (ground plane)} = \ell_c \text{ (free space)}$$

$$\Lambda \text{ (ground plane)} = \frac{1}{\sqrt{2}} \Lambda \text{ (free space)}$$

Table 5.2. Change in figure-of-merit parameters in conversion to ground plane sensor

VI. Summary

By defining a dimensionless sensitivity-bandwidth figure of merit Λ for electric and magnetic sensors (operated in time-derivative manner) based on power delivered to a constant resistive load at the sensor's maximum frequency response the performance of various sensor designs can be compared on a common basis. Several sensor designs for high bandwidth with given sensitivity have been realized in various specific models with various sizes (sensitivities) and application (free space, ground plane). These are listed in the Electromagnetic Pulse Sensor Handbook.[10] An important example is the basic MGL (multi-gap loop) design[3] for measuring $\partial\vec{B}/\partial t$. Taking the MGL-1 data of $A_{h_{eq}} = 0.1 m^2$, measured 10% to 90% rise time of about 3.0 ns as t_c, and load impedance of $Z_c = 100\,\Omega$, gives a figure of merit (free space) of $\Lambda_{10-90} \simeq 0.24$. Another example is the basic HSD (hollow spherical dipole) design[6] for measuring $\partial\vec{D}/\partial t$. Taking the HSD-2 data of $A_{e_{eq}} = 0.1\,m^2$, measured 10% to 90% rise time of about 2.7 ns as t_c, and load impedance of $Z_c = 100\,\Omega$, gives a figure of merit (free space) of $\Lambda_{10-90} \simeq 0.079$. A new $\partial\vec{D}/\partial t$ design is the ACD (asymptotic conical dipole) sensor[4] which has an increased figure of merit, approaching that of the MGL design.

Having defined the sensitivity-bandwidth figure of merit Λ for electromagnetic sensors one can ask some fundamental questions about optimal sensor design. For example, what is the best way to define ℓ_c (from ω_c or t_c). This may require some detailed understanding of the high-frequency behavior of transit-time-limited time-derivative electromagnetic field sensors. For a given definition of ℓ_c what is the theoretical maximum Λ? This would give some idea of how close existing designs approach the optimum performance and indicate for future designs when the optimum performance was being approached. For this purpose one might consider an idealized spherical sensor as a resistive spherical shell and assume all the power deposited in the shell associated with the lowest E and H modes were available to drive Z_c of electric and magnetic dipole sensors respectively. Such calculations might even suggest sensor designs with larger values of Λ.

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