Sensor and Simulation Notes

Note XXV
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# The Multiple Moebius Strip Loop

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## Abstract

The moebius strip loop is generalized to give mutli-turn loops of varying numbers of turns for measuring magnetic fields. In the lower orders of complexity such devices are practical to construct.

## I. Introduction

In a previous note a technique was described whereby we could azimuthally distribute the signal inputs to loops and/or construct multi-turn loops which can be treated as single turn loops.[1] In this note there is described another technique for constructing multi-turn loops which can be treated as single turn loops. This technique is a generalization of the moebius strip loop concept discussed in another note.[2]

Unlike the technique in Sensor and Simulation Note XXIII, which lends itself to axial distribution of the signal inputs in a cylindrical loop design, this type of loop, the multiple moebius strip loop, has a toroidal type of structure. However, if desired, a combination of two or more such toroidal structures in parallel on a common axis to achieve a cylindrical structure can be accomplished. Azimuthal distribution of the signal inputs can also be combined with the multiple moebius strip loop.[3]

This note describes the multiple moebius strip loop, giving examples of the lower orders of such devices. Unfortunately, for higher orders (and degrees of complexity) this type of loop becomes increasingly more difficult to construct. However, there may not be too much difficulty associated with the first few orders of such devices.

## II. Cable Configuration

Before considering the multiple moebius strip loop, let us consider the things which go to make up such a loop. These are the transmission lines and the manner in which the transmission lines are interconnected. The former collectively form a cable bundle. The latter is accomplished by moebius strip gaps.

Figure 1 shows the cross section of the cable bundle which is used in the multiple moebius strip loop. In figure 1A there is the first order cable bundle, simple coax, which has two independent conductors. (We might consider a zeroeth order case as a single wire.) This coax has an impedance, $Z_1$, and the coax shield (outer conductor) is denoted by $S_1$. As indicated in figure 1B the second order cable bundle is constructed by placing a second shield around a coax and placing a second coax alongside it, electrically connecting the two shields. This gives four independent conductors in the cross section for the second order cable bundle. In a similar manner we can construct third and fourth order cable bundles. As illustrated in figure 1E we can construct the cable bundle of order M+1 from two cable bundles of Mth order and an additional shield, $S_{M+1}$, which surrounds one of the bundles and is electrically connected to the outermost shields of the other bundle. Thus, in going from a cable bundle of order M to one of order M+1 we have exactly doubled the number of independent conductors. Since the first order cable bundle has two independent conductors, then the number, N, of independent conductors in an Mth order cable bundle is

$$N = 2^M \tag{1}$$

As will be seen later, N is also the number of turns in the Mth order multiple moebius strip loop.



1. Lt Carl E. Baum, Sensor and Simulation Note XXIII, A Technique for the Distribution of Signal Inputs to Loops, July 1966.
2. Lt Carl E. Baum, Sensor and Simulation Note VII, Characteristics of the Moebius Strip Loop, Dec 1964.
3. See figure 4 of ref. 1 for an example with the single (or first order) moebius strip loop.

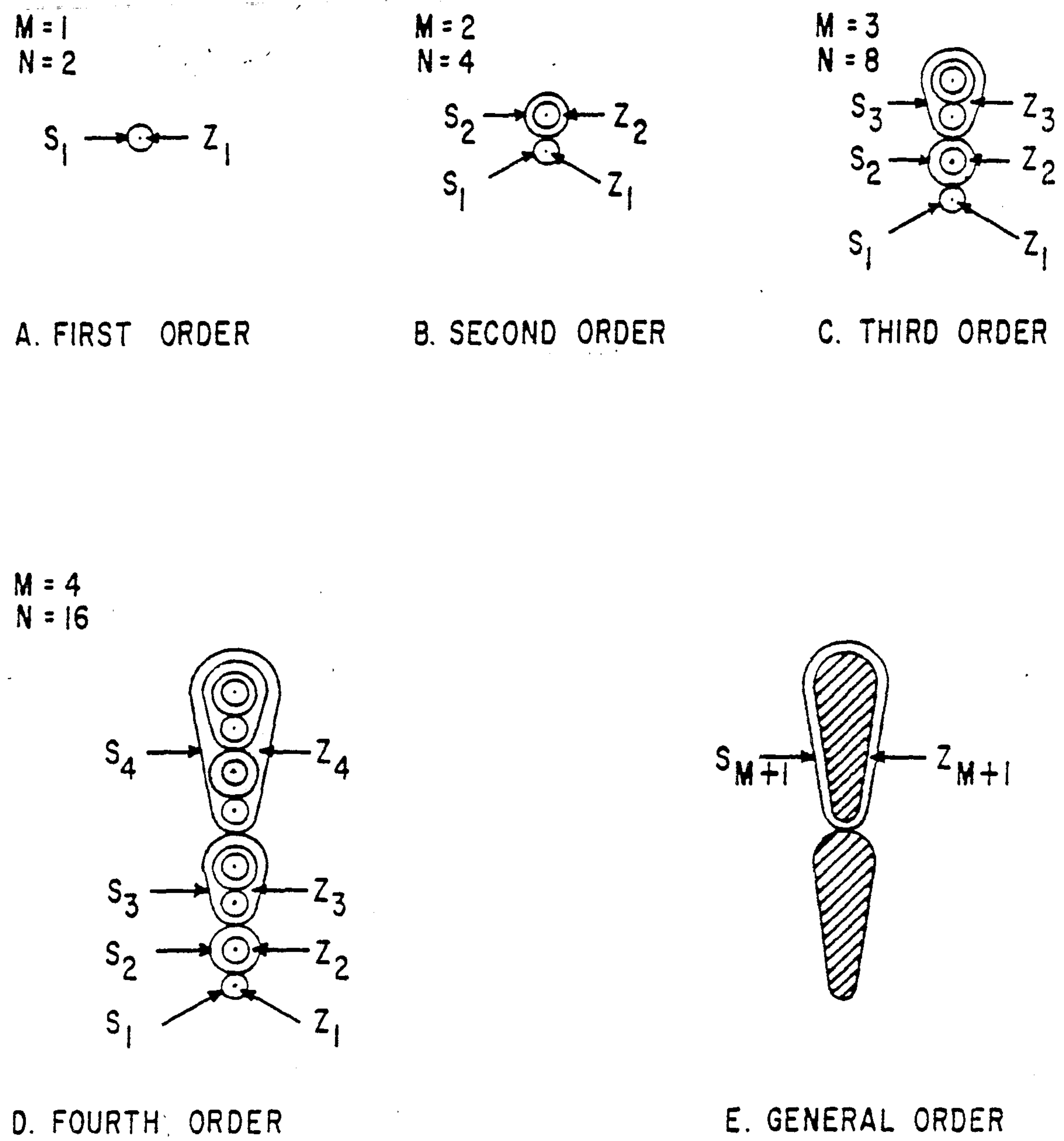



FIGURE 1 CABLE BUNDLE CONFIGURATION

Returning to figure 1A, note that the transmission line consisting of the coaxial shield and the center conductor has a characteristic pulse impedance, $Z_1$. Likewise in figure 1B there is an impedance, $Z_2$, for the transmission line consisting of the shield, $S_2$, and the coaxial shield inside it. In the general case (figure 1E) there is an impedance, $Z_{M+1}$, for the transmission line consisting of the shield, $S_{M+1}$, and the outermost conductors of the cable bundle (of order M) inside the shield. For $Z_3$ and higher order impedances, difficulty may arise in choosing the dimensions for a given impedance because of the lack of circular geometry. However, it is not necessary that the same relative positioning inside the cable bundles be maintained as illustrated. The positioning can be rearranged (and even the conductor cross sections can be changed to achieve convenient geometries) for electrical and/or mechanical convenience.

The interconnection of the transmission lines, in order to make an N turn loop, is accomplished by means of moebius strip gaps as in figure 2. Figure 2A shows a moebius strip gap in coax as illustrated in previous notes.[4] This is generalized in figure 2B for a cable bundle of order M+1 (as in figure 1E). In this case the relative positions of the two cable bundles of order M are interchanged (without touching each other) in passing from one side of the gap in the shield ($S_{M+1}$) to the other side. A voltage, V, across the gap will propagate signals each of amplitude, V, but opposite polarities from the gap down both transmission lines of impedance, $Z_{M+1}$, formed by the shields ($S_{M+1}$) and the cable bundles of order M inside these shields. The impedance which the moebius strip gap presents to the signal is 0.5 $Z_{M+1}$.

Continuing on to figure 2C we can see how the moebius strip gaps, internal to the cable bundle, are utilized. (The dotted lines are the innermost conductors, simple wires.) This structure appears at the bottom of the loop structure (opposite from the signal introduction position on the loop). Comparing the gap structure with the cable cross section we see that the bottom coax is split to drive a twinax for the signal exit from the loop. The cable bundles inside the outermost shields have a moebius strip gap in them. From above, these moebius strip gaps have an effective impedance of $Z_M/2$, where M is the order of the cable bundle which has the moebius strip gap. With equal and opposite voltages assumed arriving at the gap inside the shield, $S_{M+1}$, surrounding this bundle, then $Z_{M+1}$ must be half of this gap impedance to terminate the signals without reflections. Thus, we constrain that

$$Z_{M+1} = \frac{Z_M}{4} \tag{2}$$

or that

$$Z_M = \left(\frac{1}{4}\right)^{M-1} Z_1 = \left(\frac{1}{2}\right)^{2(M-1)} Z_1 \tag{3}$$

Here a limitation exists in that for large M, $Z_M$ can become impractically small, limiting the useful range of M.

As can be seen, these cable bundles and gap structures can get rather complex for large M. However, the first two orders require only coaxial and triaxial cable

4. See references 1 and 2.

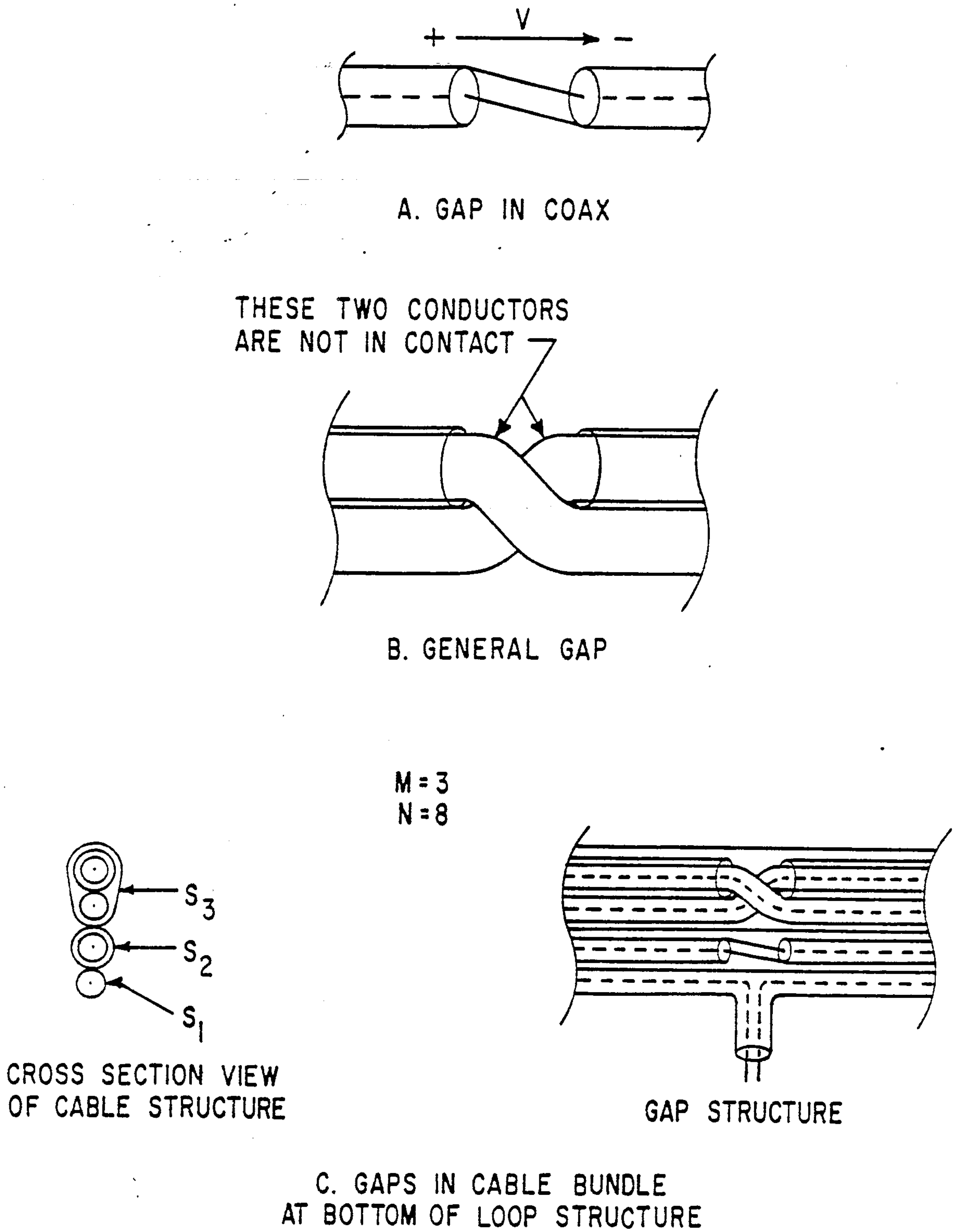



FIGURE 2 MOEBIUS STRIP GAP CONFIGURATION

and thus at least these two cases seem rather practical.

## III. Loop Configuration

Now consider the multiple moebius strip loop structure. In figure 3 we have the single (or first order) moebius strip loop where $Z_1$ is arbitrarily taken as 50 $\Omega$ for illustration.[5] This is an effective two turn loop driving a resistive load, $Z_L$, of 100 $\Omega$. In general for the multiple moebius strip loop

$$Z_L = 2Z_1 \qquad (4)$$

In the single moebius strip loop (and the general case as well) we maintain symmetry in the loop structure to make the transit times between the signal transition positions identical for both transmission lines involved. The impedance relationship of equation (4) is necessary if we desire that the signals (equal and opposite voltages), which simultaneously arrive at the twinax exit from the loop structure, be impedance matched into the twinax with no reflections. The single moebius strip loop can be considered as a one turn toroidal loop with an effective resistance at the gap of 25 $\Omega$ (i.e., 0.5 $Z_1$), the voltage produced at the gap being doubled at the output.

Having the first order device, let us generalize it to arbitrary order by a process of induction as illustrated in figure 4. In figure 4A we have the Mth order device, showing a moebius strip gap (of the general kind, as in figure 2B) at the top of the structure and a twinax to remove the signal at the bottom of the structure. The details of the cable bundle and gap structure are absent because this represents the general case. The loop structure, excluding the various connections at top and bottom, consists of a cable bundle as in figure 1, bent into a circular shape. The electrically continuous shield of the cable bundle makes it externally behave as a single conductor. This continuous electrical conductor is the actual electrical structure of the loop presented to an incident electromagnetic wave, at least for frequencies high enough that fields do not penetrate the conductors. The gap structure at the bottom of the loop is like that illustrated in figure 2C in which all the gaps are inside the outermost shield. The only signal entry to the loop is at the moebius strip gap at the top. This gap, can be represented by a gap impedance, $Z_g$, as

$$Z_g = \frac{Z_M}{2} \qquad (5)$$

since at the gap there is a parallel combination of two transmission lines, each of impedance, $Z_M$.

Let us take this Mth order moebius strip loop and transform it into one of order M+1. First, as in figure 4B, add a shield (of order M+1) around the upper half of the loop structure. We then have two transmission lines of equal transit times and equal impedances, $Z_{M+1}$, which transmit signals from the two introduction points (at A and B) up to the moebius strip gap now inside this shield. For signals of equal and opposite voltage reaching this upper moebius strip gap, the impedance relationship of equation (2) must hold for the signals to be impedance matched at this gap. In this configuration the signal inputs have been azimuthally distributed as in a previous note.[6]

5. See reference 2 for a discussion of this device.

6. See reference 1.

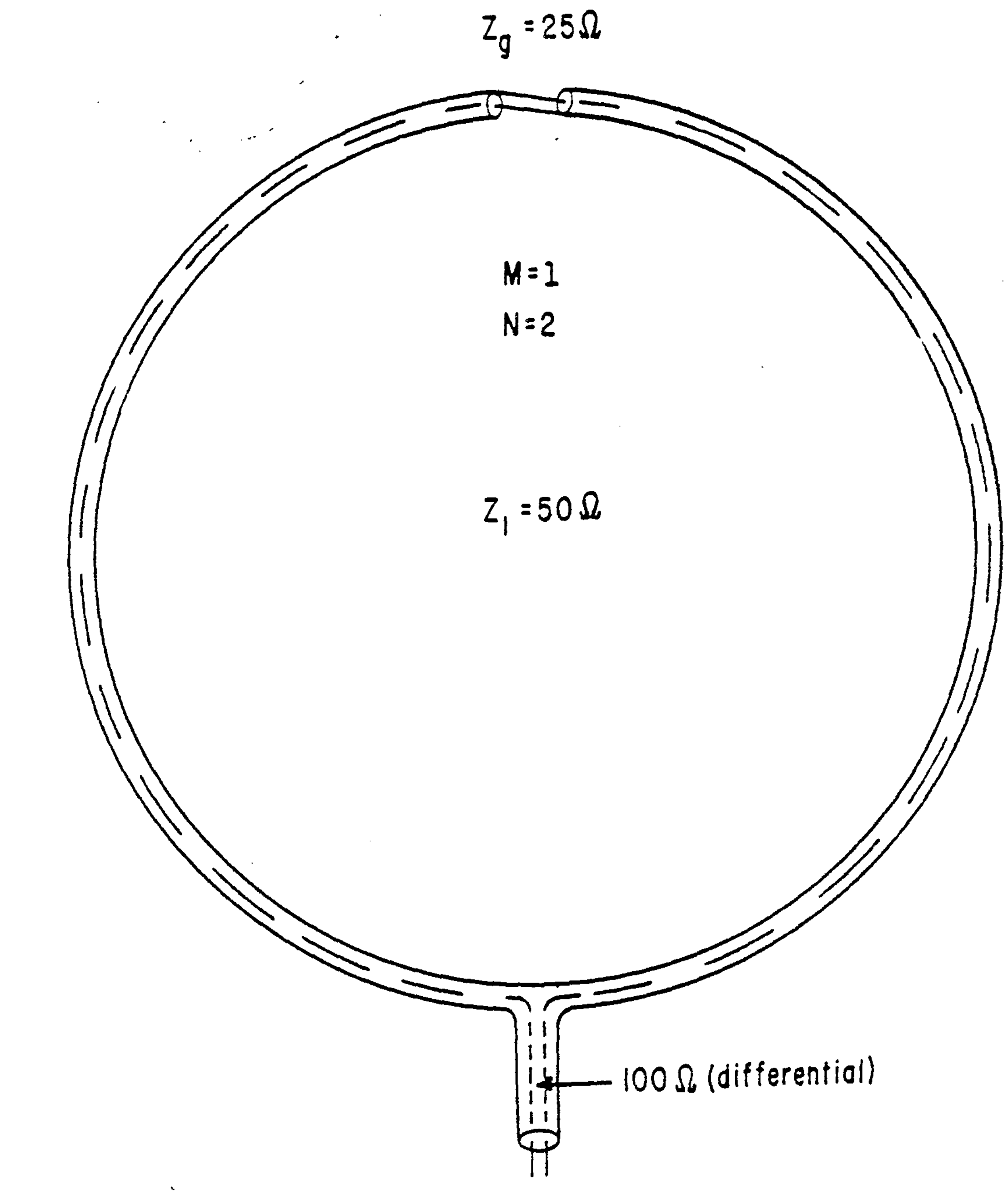
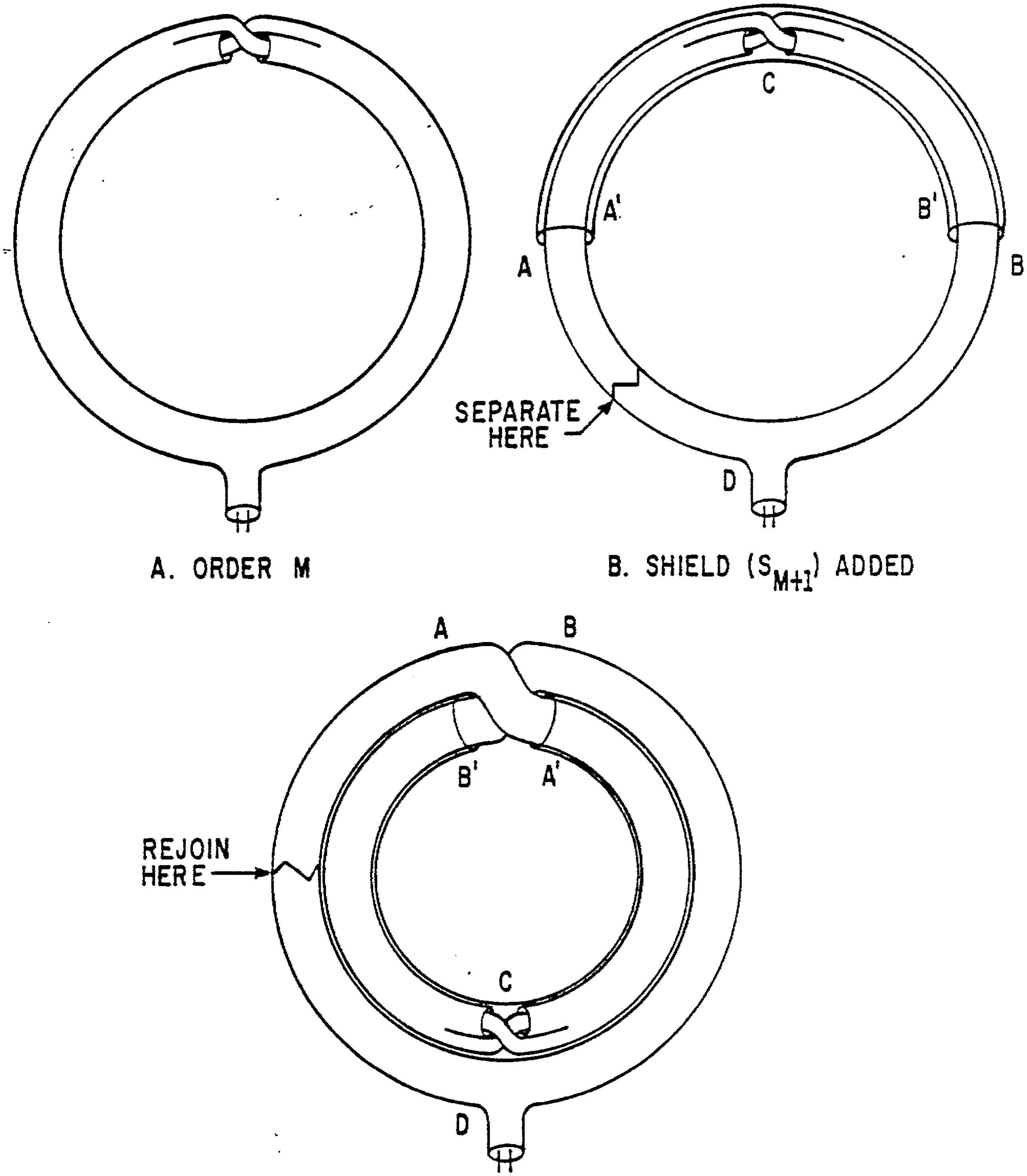



FIGURE 3 SINGLE (OR FIRST ORDER) MOEBIUS STRIP LOOP




A. ORDER M

B. SHIELD ($S_{M+1}$) ADDED

C. ORDER M+1

FIGURE 4 INCREASING THE ORDER
OF THE MULTIPLE MOEBIUS STRIP LOOP BY ONE

Continuing the transformation we sever the cable bundle as indicated in figure 4B. Then we bend the loop structure into a tighter circle, doubling it up as indicated in figure 4C, and we rejoin the severed cable bundle, rejoining each conductor to the one from which it was separated. The reason for severing the cable bundle at all is to avoid a twist in the cable bundle. The letters which label corresponding positions on the two loop structures in figures 4B and 4C can be used to follow this transformation. The added shield is then electrically bonded to the adjacent cable bundle. We are then left with a loop structure with a continuous conducting shield with only one externally visible gap. The cable bundle forming the loop structure is now of order M+1. The external moebius strip gap is again at the top of the loop structure and has an impedance of 0.5 $Z_{M+1}$. The moebius strip gap at the top of the Mth order loop structure has been relocated to the bottom of the new loop structure, joining the other moebius strip gaps there (as in figure 2C). The two multiple moebius strip loops are then of the same form.

In the process of transforming to the next higher order multiple moebius strip loop (from figure 4B to figure 4C) we have doubled the number of turns in the loop. Each previous turn has become two turns. Since the single moebius strip loop has two turns, then the number of turns, N, in the Mth order loop is just that given by equation (1). Also, since the single moebius strip loop has one moebius strip gap and increasing the order of the loop by one adds another such gap, then an Mth order loop has M moebius strip gaps, only one of these being externally visible. All the internal gaps are at the bottom of the loop structure. Combining equations (3), (4), and (5) we have

$$Z_g = \left(\frac{1}{2}\right)^{2M} Z_L \tag{6}$$

relating the impedance at the external loop gap, the impedance the loop drives, and the order of the loop. In terms of the number of turns in the loop we have, using equation (1),

$$Z_g = N^{-2} Z_L \tag{7}$$

Since the signal is impedance matched in going from the gap of order M+1 into the gap of order M (and since M represents the general case) then the signal is matched from the external gap through all the internal gaps to the loop output without reflections. We have a voltage multiplication factor of N which is made up of voltage doublings at each of the internal moebius strip gaps (M-1 of them) and a doubling at the transition to the signal output twinax for a total of M voltage doublings. This voltage multiplication by N is consistent with the change in impedance by a factor of $N^2$ (equation (7)) so that power is conserved in a signal propagated from the external moebius strip gap to the signal exit.

Figure 3 shows the simple single moebius strip loop. Figures 5 and 6 show the double (or second order) and triple (or third order) moebius strip loops, respectively, with all the conductors shown. The dotted lines are the innermost conductors, the center conductors of the coaxes. Starting at one of the two inner conductors of the twinax we can follow the conductors around the loop structure arriving at the other twinax inner conductor and verify that these loops have the number of turns indicated. For the triple moebius strip loop this may require perseverence due to the complexity of the structure. For the impedances shown, based on an arbitrary $Z_1$ of 50 Ω, we can see in figure 6 that $Z_3$ is rather small and perhaps difficult to achieve. Hence, the triple moebius

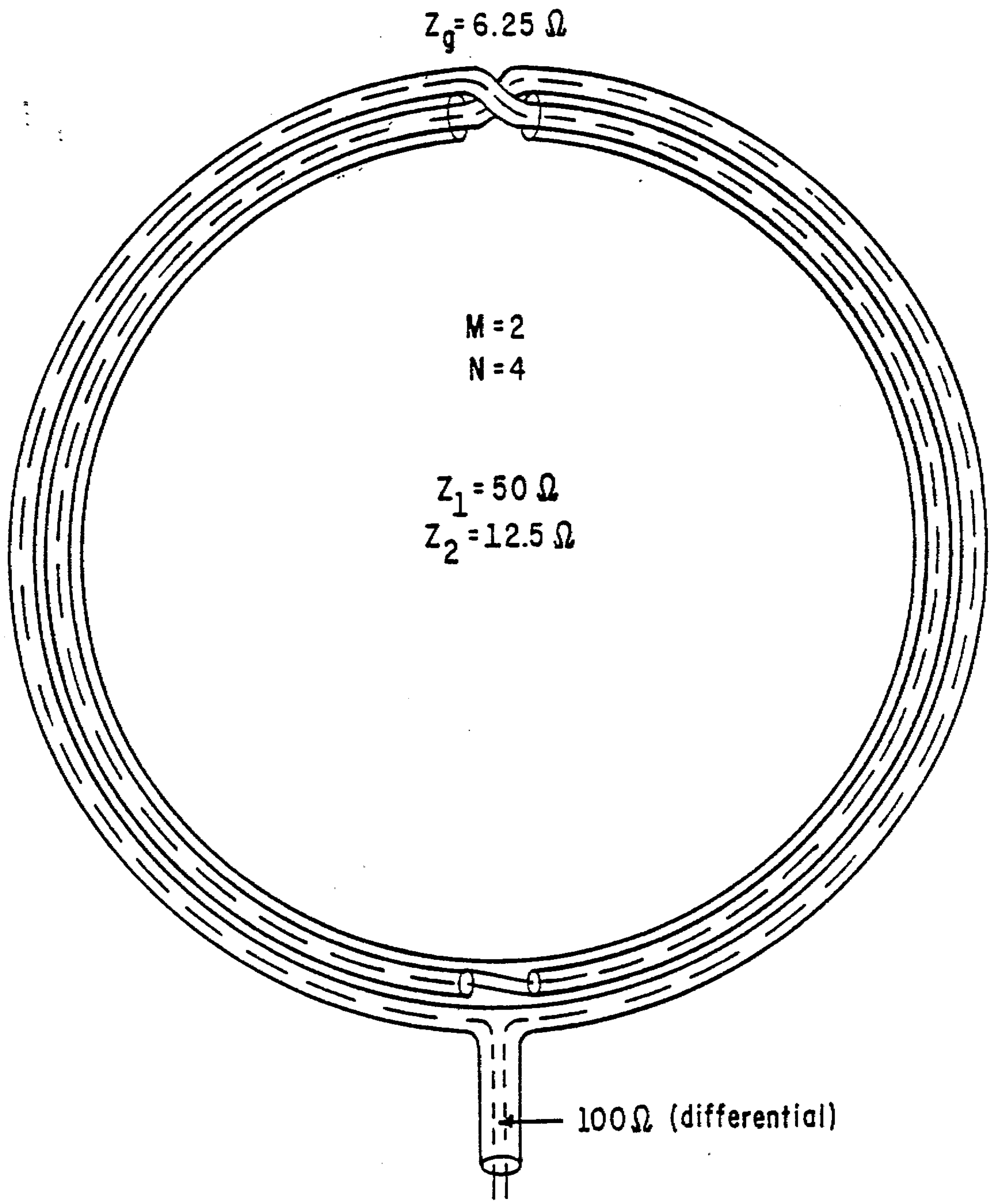



FIGURE 5 DOUBLE (OR SECOND ORDER) MOEBIUS STRIP LOOP

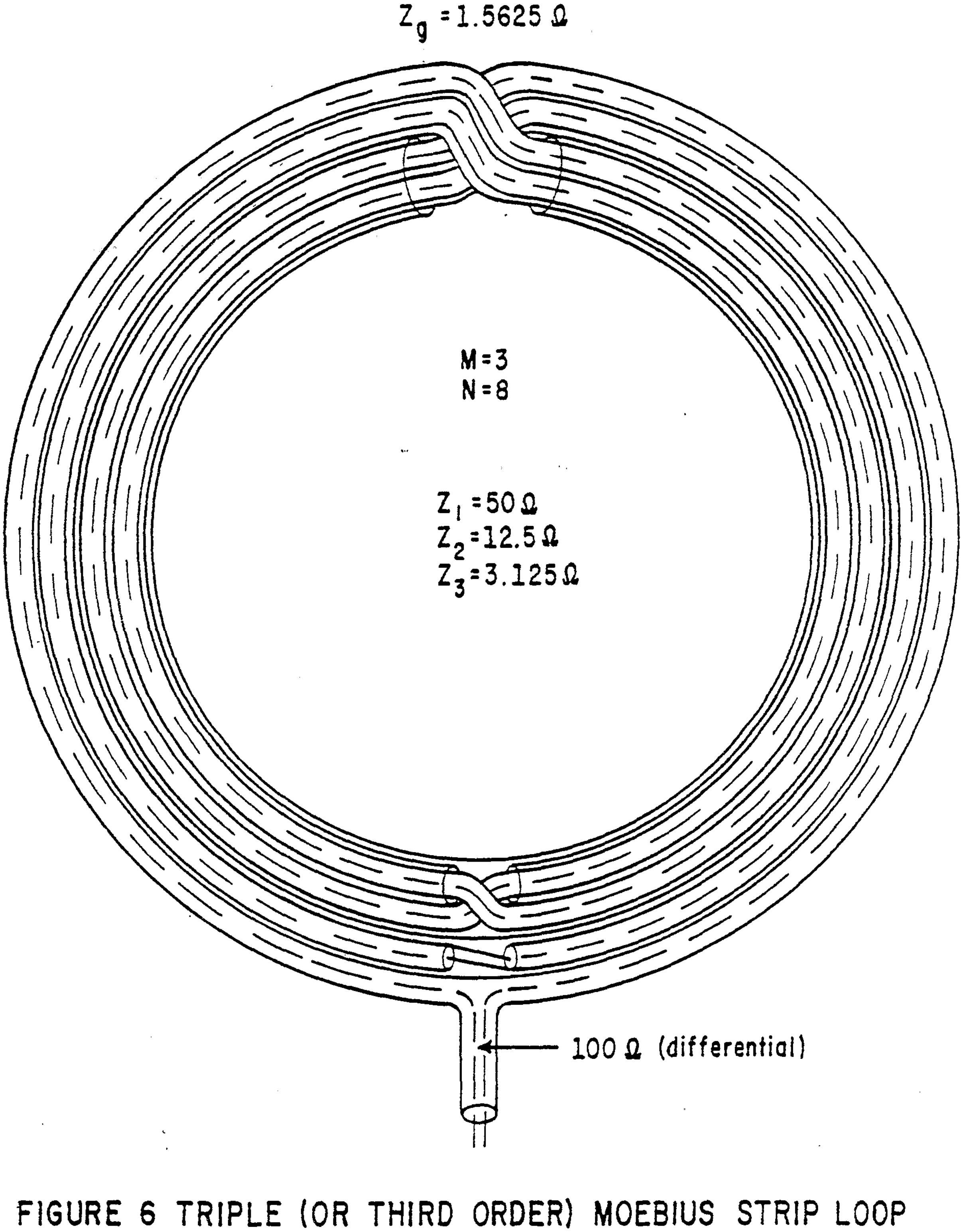



FIGURE 6 TRIPLE (OR THIRD ORDER) MOEBIUS STRIP LOOP

strip loop may be difficult to construct. However, the double moebius strip loop looks rather practical. The technique used to construct the loop of order M+1 from that of order M is only illustrative and not necessarily the most desirable way to construct a given order multiple moebius strip loop.

## IV. Summary

We then have a technique for making multi-turn loops. The multiple moebius strip loop allows us to make a multi-turn loop which presents the structure of a one turn loop to an incident electromagnetic wave. Since we are limited to an approximately toroidal geometry (maximizing the inductance for a given loop radius) we might use this technique for self integrating loops to measure the magnetic field, H.

For the higher orders of this device the structure becomes rather complicated and the impedances become impractically small. However, at least the single and double moebius strip loops are rather practical. To how much higher orders we could practically go is difficult to say.