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LATE-TIME UNDERGROUND EMP SIMULATOR MODEL

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A quasistatic late-time underground EMP simulator model is described. The model calculates current and voltage distributions in the volume of the simulator as well as an average impedance of the simulator.

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# I. INTRODUCTION

An underground EMP simulator, DISCUS, has been proposed to
study the effects of EMP on underground components such as buried
antennas, power lines, and structures. The current conceptual
design of the simulator is depicted in Figure 1. The simulator
consists of two sets of electrodes placed in the ground and a set
of generators with ground plates on the surface between these
electrodes. The generators are discharged in a timed sequence to
produce a traveling wave between the electrodes. The late time
field is driven by an additional generator connected between the
ground rods.

This report addresses the calculation only at late times when
the generators are discharging at low enough rates to allow a
quasistatic solution of the fields and current distribution in
the ground. To implement this method a step-wise solution in
time is sought. At any time, $t = n\Delta t$, we assume that the poten-
tials on the various electrodes of the simulator are known from a
previous calculation. During the present time step, these
impressed potentials, to good approximation, produce a steady-
state current distribution in the ground and on the test object.
This current distribution and the effective impedance of the simu-
lator are calculated using the code described herein. Knowing
this impedance allows one to calculate potentials at time
$t = (n+1)\Delta t$ from the driving circuit equations. The procedure is
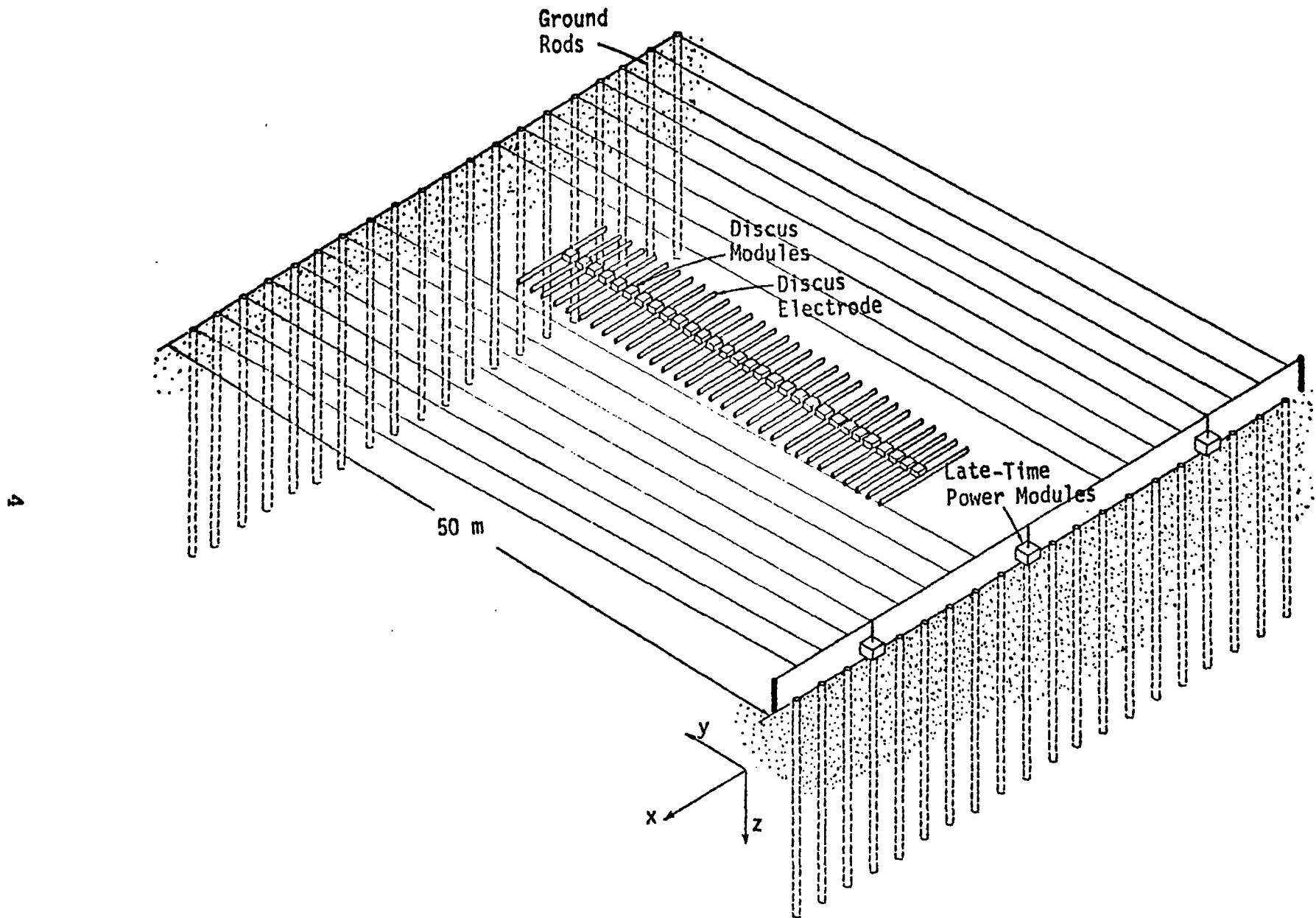then repeated. For linear cases in which the simulator impedance

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Figure 1. Conceptual DISCUS simulator design with late-time drive.

is independent of the driving potentials, the steady-state ground current code needs to be run only one time for a given geometrical configuration. The calculation is general and does not depend on the particular circuit of a given simulator driver. The only restriction is that the late-time fields resulting from the driver can be treated by a quasistatic calculation. Results can be calculated for a given driver circuit configuration by using the results of the model described here in conjunction with one of the standard circuit analysis codes. Such calculations would provide valuable design information for the DISCUS simulator.

Section II enumerates some conditions that must be met to ensure the validity of the quasistatic approach. The formulation of the problem and the treatment of thin wires in the mesh are given in Sections III and IV, respectively. Finally, some typical DISCUS calculations are presented in Section V. MKS units will be used throughout.

## II. CONDITIONS FOR THE VALIDITY OF THE QUASISTATIC SOLUTION

In this section some necessary conditions for the validity of the quasistatic approach to obtain a late-time model of the DISCUS simulator are given.

Steady-state currents are not attainable until the initial large pulse has passed through the simulator. This propagation time is

$$t_p = \frac{L}{\sqrt{\mu\varepsilon}} \tag{1}$$

where L is the length of the simulator and where $\mu$ and $\varepsilon$ are the electrical properties of the ground. If significant reflections from the ground electrodes occur, then the quasistatic approach will not be valid until after several times $t_p$. As an illustration, we choose L = 50 m, $\mu = \mu_o$, $\varepsilon = 10 \varepsilon_o$, then $t_p = 5.3 \times 10^{-7}$ sec.

Another necessary condition is that steady-state conditions can occur only after times greater than the relaxation time of the medium given by

$$t_r = \frac{\kappa\varepsilon_o}{\sigma} \tag{2}$$

where $\kappa$ and $\sigma$ are respectively the dielectric constant and conductivity of the ground. As an example, we take $\kappa = 10$ and $\sigma = 10^{-3}$, then $t_r = 8.85 \times 10^{-8}$ sec.

The problem will reach steady state only when the time dependent terms in Maxwell's equations become small compared to the spatial dependent terms. A convenient way of expressing this is to use the continuity equation. We observe that we must have

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$$\nabla \cdot \vec{j} \gg \frac{\partial \rho}{\partial t} \qquad (3)$$

Finally, we note that for the capacitors of the driving system to discharge in a quasi-steady state fashion, the change in voltage of the capacitor during a time step must be small compared to the total voltage across the capacitor, i.e.,

$$\frac{dQ}{C} = \frac{Idt}{C} \ll \frac{Q}{C} \qquad (4)$$

where Q is the charge on the capacitor, I is the current flowing in a circuit from one plate to the other, and C is the capacitance.

## III. FORMULATION OF THE PROBLEM

### 1. DIFFERENTIAL EQUATIONS

For time independent problems, Maxwell's equations reduce to

$$\nabla \cdot D = \rho \tag{5}$$

$$\nabla \cdot B = 0 \tag{6}$$

$$\nabla \times E = 0 \tag{7}$$

$$\nabla \times H = \vec{j} \tag{8}$$

Equation 3 implies that the electric field is conservative, i.e.,

$$\oint \vec{E} \cdot d\vec{\ell} = 0 \tag{9}$$

along any closed path. Then, $\vec{E}$ is derivable from a scalar potential

$$\vec{E} = -\nabla\Phi \tag{10}$$

For steady-state problems, the continuity equation becomes

$$\nabla \cdot \vec{j} = 0 \tag{11}$$

We assume that in the ground all currents are conduction currents,

$$\vec{j} = \sigma\vec{E} \tag{12}$$

Further, we assume the following properties of matter

$$\vec{D} = \kappa\varepsilon_o\vec{E} \tag{13}$$

$$\vec{B} = \mu\vec{H} \tag{14}$$

The problem domain will be divided into several regions.
In any given region we assume that $\sigma$, $\kappa$, and $\mu$ are independent
of both space and time.

Combining Equations 11 and 12 yields

$$\nabla \cdot \vec{E} = 0 \qquad (15)$$

in regions where $\sigma$ is constant.  This combined with Equations 5 and
13 indicates that

$$\rho = 0 \qquad (16)$$

in any volume in which $\epsilon$ is constant.  Surface charges may, however,
appear on boundary surfaces.  Combining Equations 10 and 15 yields

$$\nabla^2 \phi = 0 \qquad (17)$$

Thus the field and current distributions are derivable from
Laplace's equation.

Cartesian coordinates will be employed.  Then

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \qquad (18)$$

Use the following finite different approximations

$$\left. \frac{\partial \phi}{\partial x} \right|_{i+\frac{1}{2},j,k} = \frac{1}{\Delta x} \left[ \phi_{i+1,j,k} - \phi_{i,j,k} \right] \qquad (19)$$

$$\left. \frac{\partial^2 \phi}{\partial x^2} \right|_{i,j,k} = \frac{1}{\Delta x^2} \left[ \phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k} \right] \qquad (20)$$

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Similar equations are used for y and z. We write

$$\nabla^2\Phi = \frac{1}{\Delta x^2}[\Phi_{i+1,j,k} - 2\Phi_{i,j,k} + \Phi_{i-1,j,k}]$$

$$+ \frac{1}{\Delta y^2}[\Phi_{i,j+1,k} - 2\Phi_{i,j,k} + \Phi_{i,j-1,k}]$$

$$+ \frac{1}{\Delta z^2}[\Phi_{i,j,k+1} - 2\Phi_{i,j,k} + \Phi_{i,j,k-1}]$$

$$= 0 \tag{21}$$

Solve this for $\Phi_{i,j,k}$

$$\Phi_{i,j,k} = \frac{1}{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}\left[\frac{1}{2\Delta x^2}\left(\Phi_{i+1,j,k} + \Phi_{i-1,j,k}\right)\right.$$

$$\left. + \frac{1}{2\Delta y^2}\left(\Phi_{i,j+1,k} + \Phi_{i,j-1,k}\right) + \frac{1}{2\Delta z^2}\left(\Phi_{i,j,k+1} + \Phi_{i,j,k-1}\right)\right] \tag{22}$$

This equation, together with the boundary conditions, is solved by an iterative technique for $\Phi$. Subsequently, $\vec{E}$ and $\vec{j}$ are obtained from finite difference approximations of Equations 10 and 12.

2. BOUNDARY CONDITIONS

At the metallic surfaces of the simulator, the potentials are assumed to be known from a solution of the driving circuit equations. These potentials are set as boundary conditions.

At the air-ground interface, z = 0, it is assumed that the normal component of current vanishes. Thus

$$\lim_{z \to 0} \vec{E}_z = 0$$

This is equivalent to setting

$$\lim_{z \to 0} \frac{\partial \phi}{\partial z} = 0 \tag{23}$$

The boundary condition at the outer surface of the mesh should be that the potential vanishes at infinity. We have instead enclosed the calculational volume of earth in both dielectric and perfectly conducting boxes. Fortunately, these boundary conditions have little effect on the impedance of the simulator or on currents on test objects if the artificial boxes are sufficiently large. These boundary conditions ensure that the divergence of the current is zero.

To conserve computer storage and computational time, symmetries in the problem are exploited. Effectively, only one-fourth of the problem domain must be calculated. This results from a vertical plane of symmetry along the axis of the simulator and a vertical plane perpendicular to this one passing through the center of the simulator on which $\vec{j}$ is continuous and antisymmetric.

# IV. TREATMENT OF CONDUCTING WIRES

Thin, uninsulated wires in a finite difference mesh have been treated by Granzow (Ref. 1). His treatment was for time dependent problems using transmission line theory. For the present steady-state problem with a highly conducting bare wire the treatment becomes much simpler.

Near the wire the electric field is easily obtained from Gauss' Law.

$$E = \frac{1}{2\pi\epsilon} \frac{\lambda}{r} \tag{24}$$

where

$$\lambda = q/\ell \tag{25}$$

is the charge per unit length on the wire. Thus if a is the radius of the wire, the field at a is related to the field at radius r as

$$E_a = \frac{r}{a} E_r \tag{26}$$

The current entering a small length, $\Delta\ell$, of wire is limited by the conductivity of the earth. Thus

$$\Delta I = 2\pi a \Delta\ell \sigma_g E_a \tag{27}$$

Combining this with Equation 26 gives

$$\frac{\Delta I}{\Delta\ell} = 2\pi r \sigma_g E_r \tag{28}$$

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1. Granzow, K. D., <u>Methods for Treating Boundaries in Transient Electromagnetic Solutions Applied to the A3D Code</u>, Dikewood Industries, Inc., DC-TN-1290-13, June 1978.

Note that the result is independent of the radius of the wire. This follows because the surface area of the wire increases as r, but the electric field decreases as 1/r. Equation 28 is integrated along the wire to obtain the current on the wire as a function of distance from the center of the wire.

## V. TYPICAL DISCUS CALCULATIONS

A parameter study was performed with the code to determine the sensitivity of the current in an underground test cable to various design parameters of the simulator. The base case for the study was: simulator length--50 m, simulator width--25 m, simulator rod depth--30 m, ground conductivity--1 mho/m, voltage across simulator--1 V/m (50 V).

Figure 2 shows a typical distribution of $j_y$ as a function of space. In Figure 3, other parameters were held constant and the simulator width was varied. Figure 4 gives the variation of the current in the wire as the length of the simulator rods are changed. Figure 5 illustrates the change in wire current as the length of the wire varies. Finally, Figure 6 shows the change in wire current with wire depth, other parameters being held constant. In all of the figures, $I_w$ is the current in the test cable and $I_{tot}$ is the total current flowing in the simulator, i.e., the sum of the current in the cable and the current in the ground.

Note that the calculations were all done for unit conductivity in the ground and for unit voltage across the simulator. The reason for this is that for a given simulator geometry, and a highly conducting cable the problem is linear in the ground conductivity and the simulator voltage. Thus one can obtain results for any ground conductivity or simulator voltage by multiplying values from the figures by the conductivities or voltages.
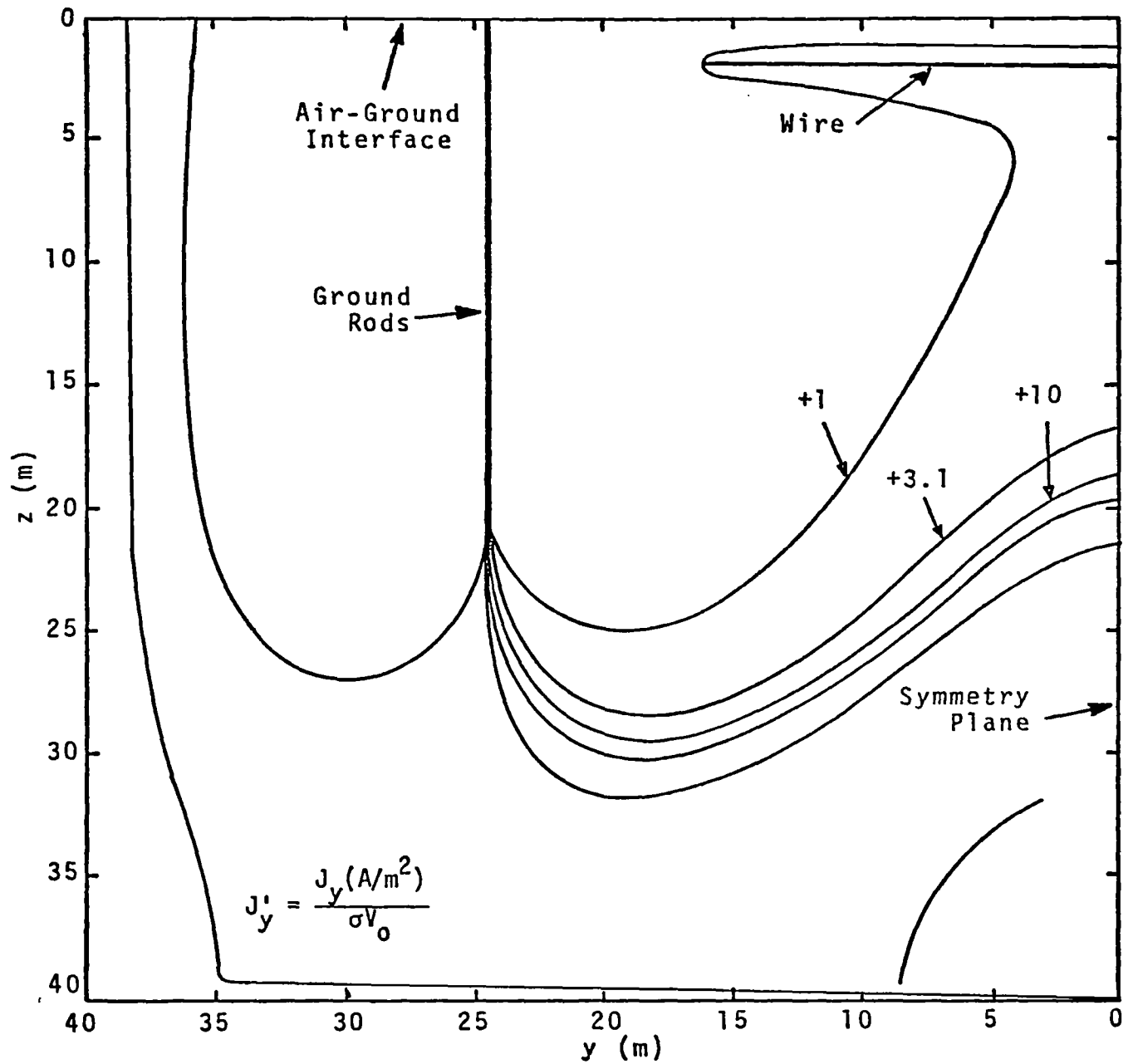
Figure 2. Contour plot in the y-z plane (Fig. 1) of a typical y component of current distribution (amps).
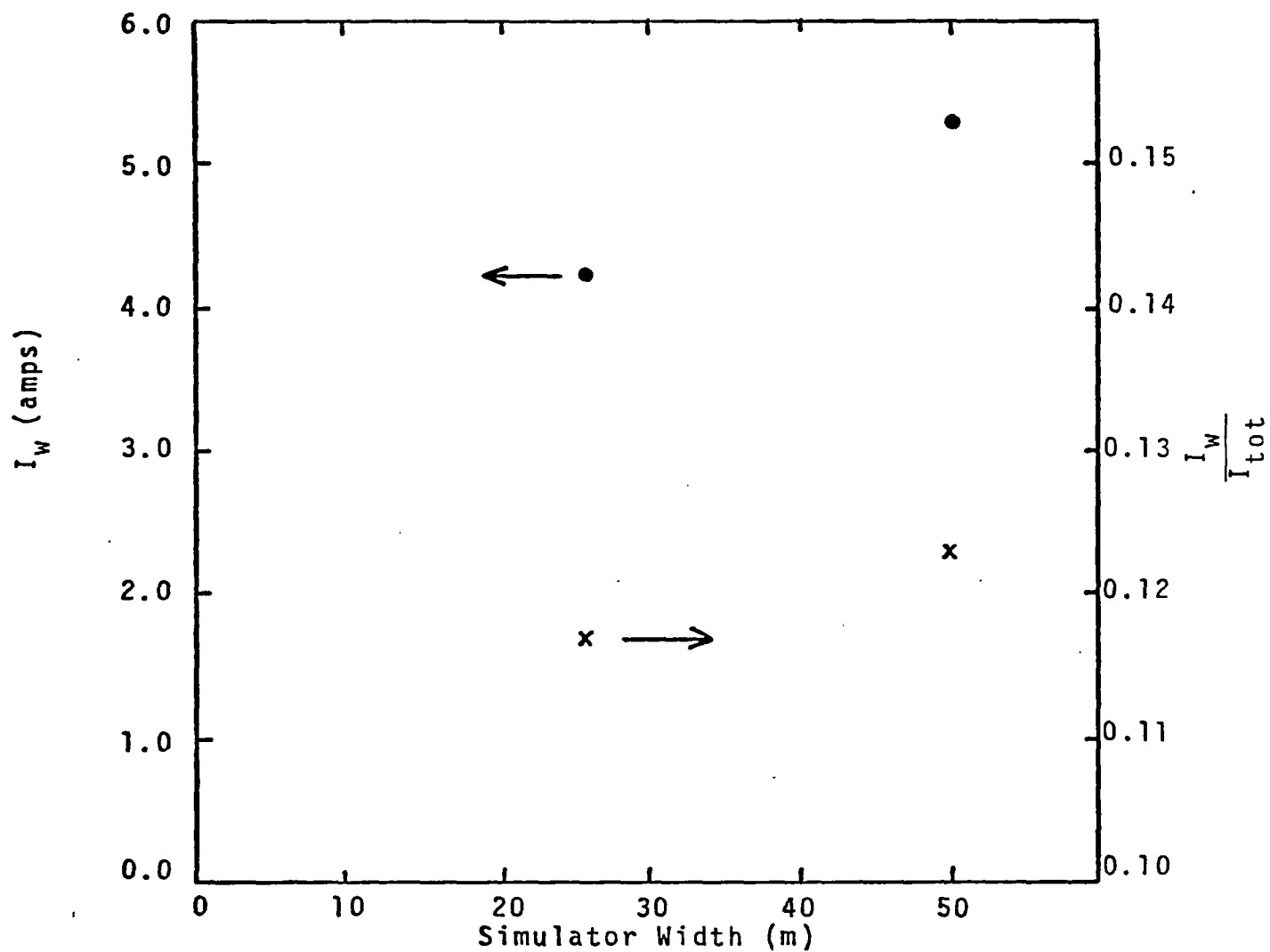
Figure 3.   Variation of current in wire as a function
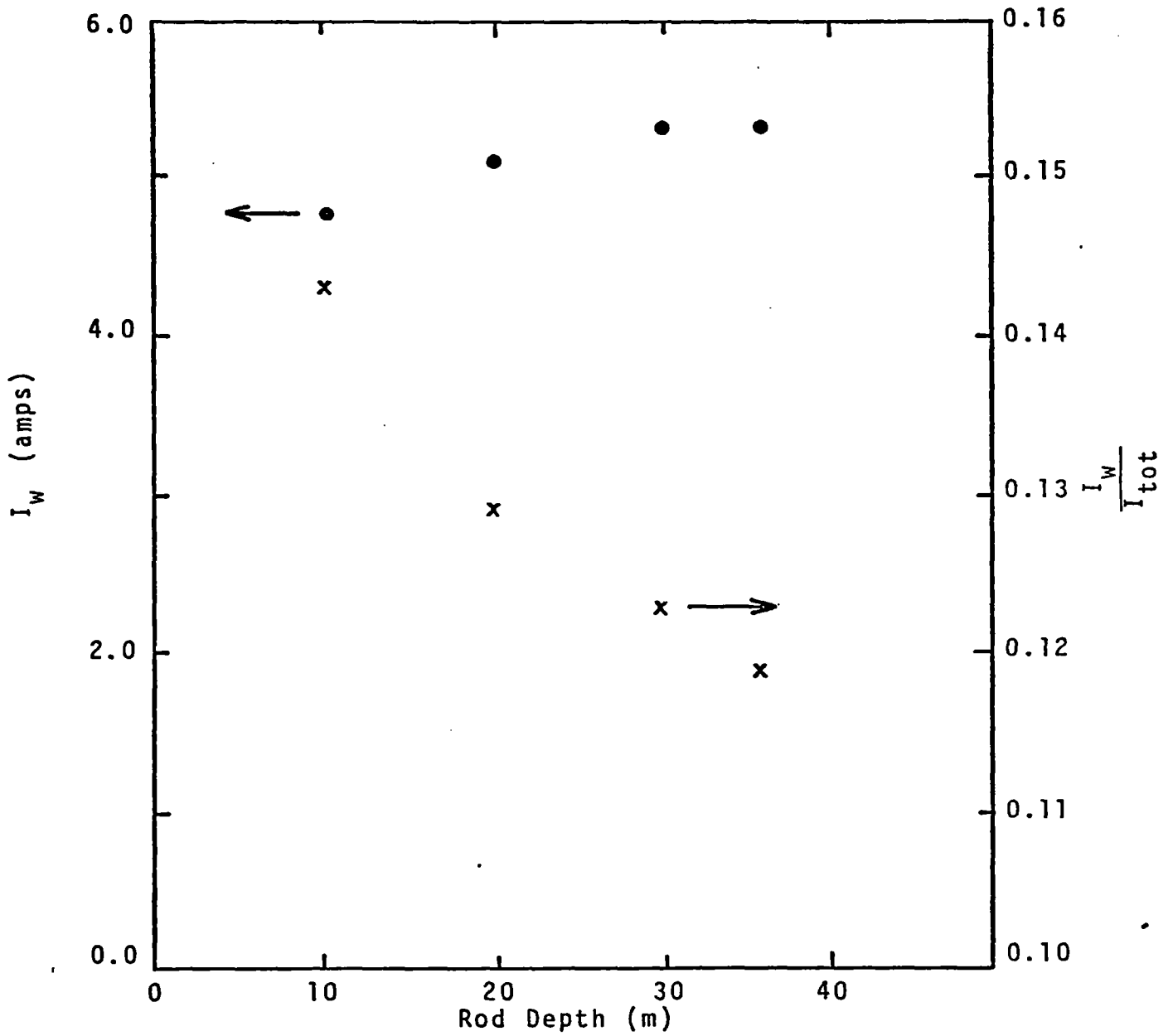of simulator width.

Figure 4.  Wire current as a function of simulator
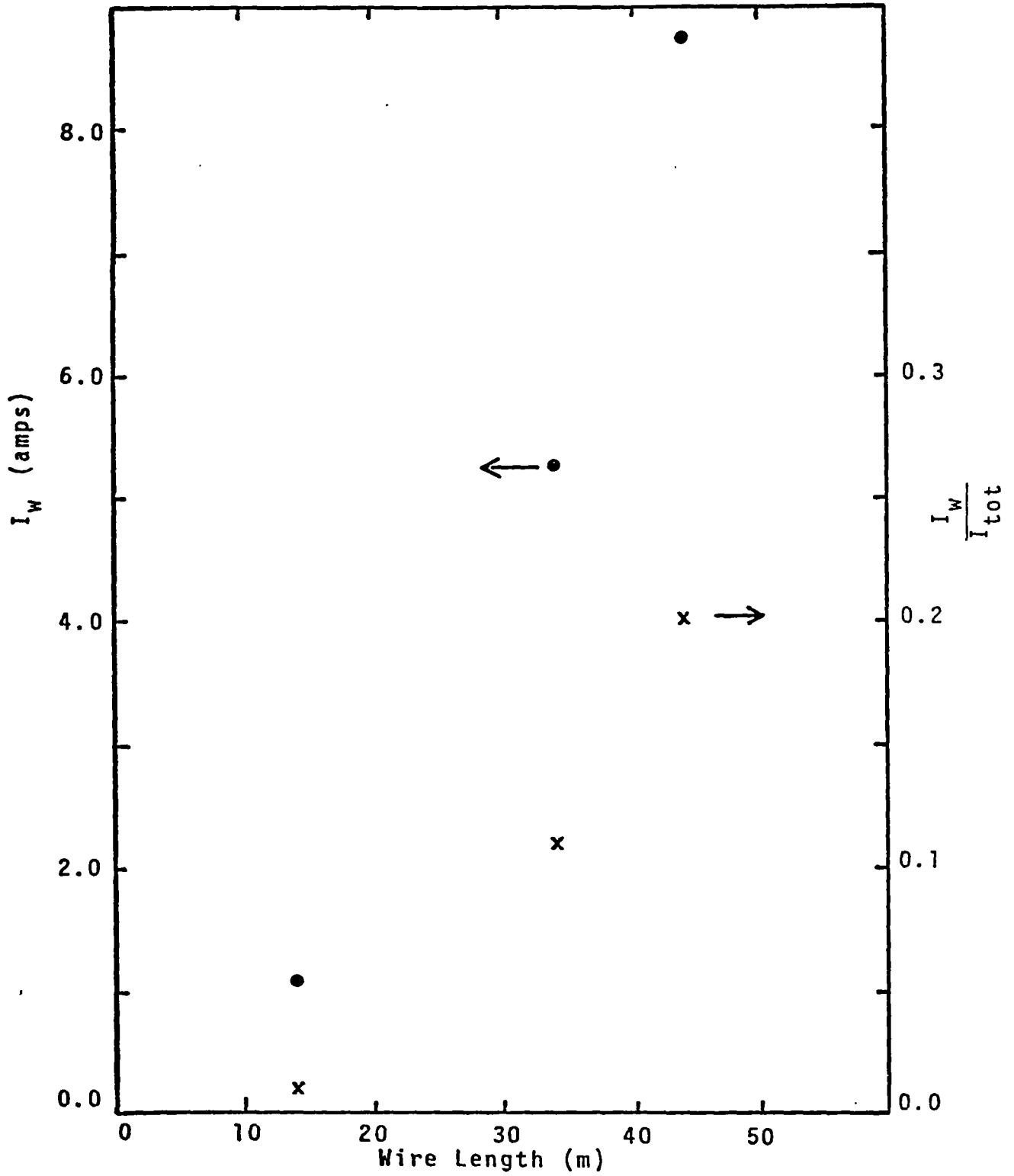rod depth.

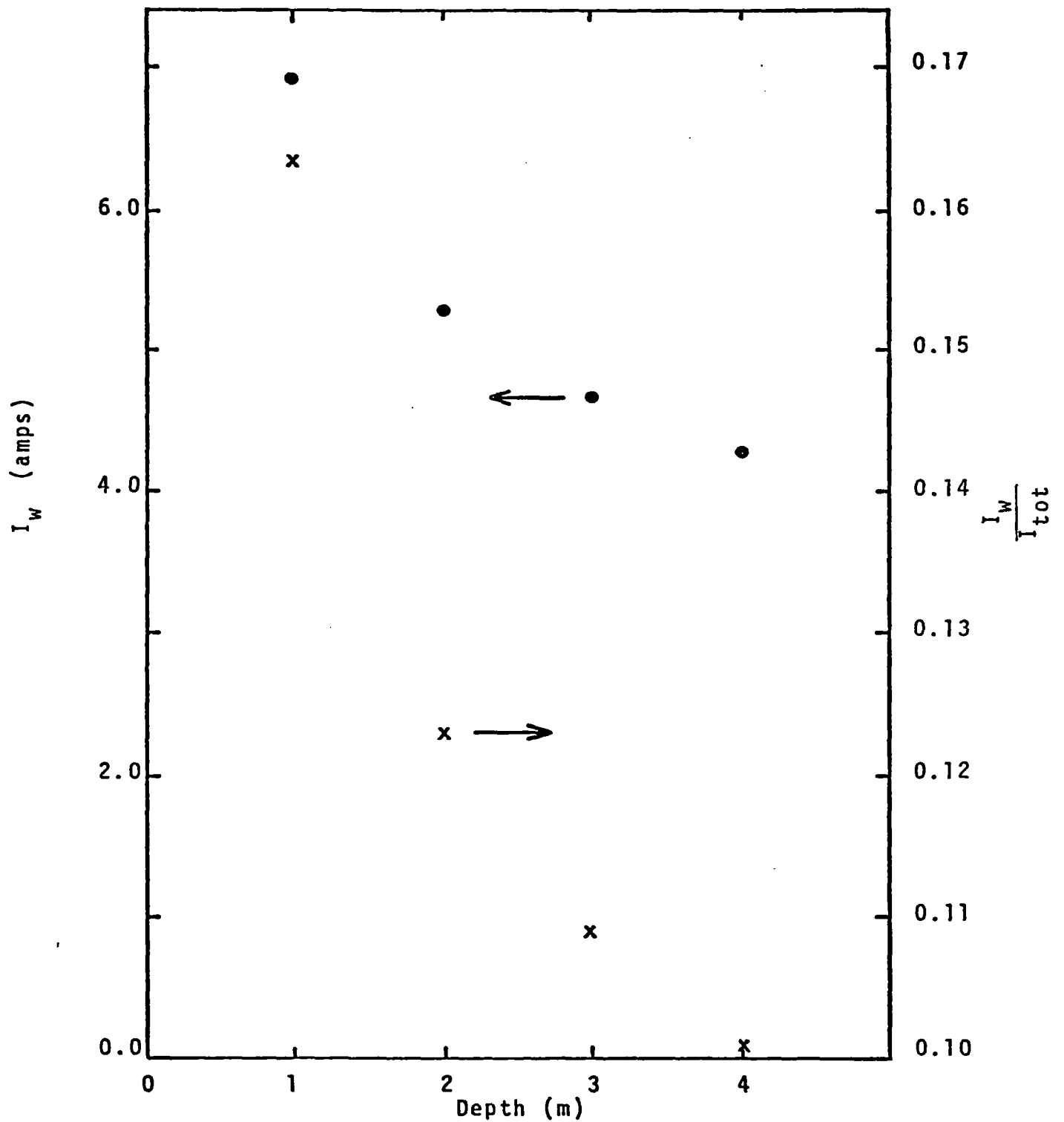Figure 5.   Current in wire as a function of
            wire length.

Figure 6.  Wire current vs. wire depth.

Some generalizations about the characteristics of late-time quasistatic fields in a DISCUS type simulator can be drawn from the calculations depicted in the curves. In each case discussed, it is assumed that all parameters in the problem remain constant except those mentioned. The conclusions are: 1) $I_w$ and $I_w/I_{tot}$ increase gradually with increased simulator width, 2) $I_w$ increases slowly with rod depth and finally saturates (in contrast, $I_w/I_{tot}$ decreases with increased rod depth), 3) both $I_w$ and $I_w/I_{tot}$ increase with increased wire length, and 4) $I_w$ and $I_w/I_{tot}$ decrease with increased depth of cable burial. The rate of decrease is much more rapid near the surface of the earth.

# VI. CONCLUSIONS

A model has been developed to predict late-time fields in the DISCUS EMP simulator. The model is quite general and can be used to calculate fields for a variety of driver configurations and simulator geometries. The model is relatively inexpensive to run. It can, therefore, be a useful tool in designing the DISCUS simulator before its actual construction. This is particularly true of the simulator geometry.

Examples of the types of results which can be obtained using this model were presented in this report. These results included current distributions in the soil around conductors in (or near) the test volume, the ratio of current driven on test objects to the total current output of the source module, and the effective late-time impedance of the simulator which would serve as the load for the source module. Effects of modifications of simulator or test object parameters on the current distributions can be easily determined using the model.