Sensor and Simulation Notes

Note XXIX 29

28 October 1966

The Influence of Radiation and Conductivity on $\dot{B}$ Loop Design

1/Lt Carl E. Baum

Air Force Weapons Laboratory

Abstract

The air conductivity and the nuclear radiation are significant factors to be considered in the design of $\dot{B}$ loops for close-in EMP measurements. The maximum air conductivity and the desired upper frequency response limit the allowable size of the loop. The nuclear radiation introduces noise signals which can be significant at high radiation intensities.

I. Introduction.

Some of the frequency response characteristics of a $\dot{B}$ loop in the presence of negligible air conductivity are discussed in a previous note.[1] The purpose of the present note is to discuss, in a general way, the influence of the air conductivity on the response of a $\dot{B}$ loop and the noise signals generated in such a loop by the nuclear radiation. Other notes consider the effects of the air conductivity and the nuclear radiation on electric field dipole design.[2,3] The influence of these phenomena on $\dot{B}$ loop design, however, is somewhat different.

Figure 1 illustrates two types of loop geometry: toroidal and cylindrical. These geometries can be used for various numbers of loop turns, but for simplicity in this discussion only single turn loops are considered. Although the discussion applies to various loop geometries, the cylindrical loop (as in figure 1B) is used for the calculations.

II. Air Conductivity Related Problems.

The desired response of a $\dot{B}$ loop is based on Faraday's law (one of Maxwell's equations) which in integral form is

$$\oint \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot d\vec{S} \qquad (1)$$

This reduces to the simpler form for the $\dot{B}$ loop as

$$V = \dot{B}A_{eff} \qquad (2)$$

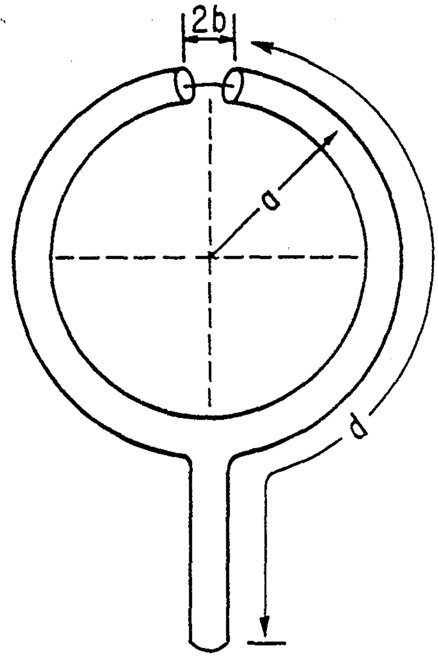
where V is the signal from the loop, $\dot{B}$ is the component of the time rate of change of the magnetic field, $\vec{B}$, which is being measured, and A_{eff} is the effective area of the loop, a constant of proportionality.[4] For the single turn loops of figure 1

$$A_{eff} = \pi a^2 \qquad (3)$$

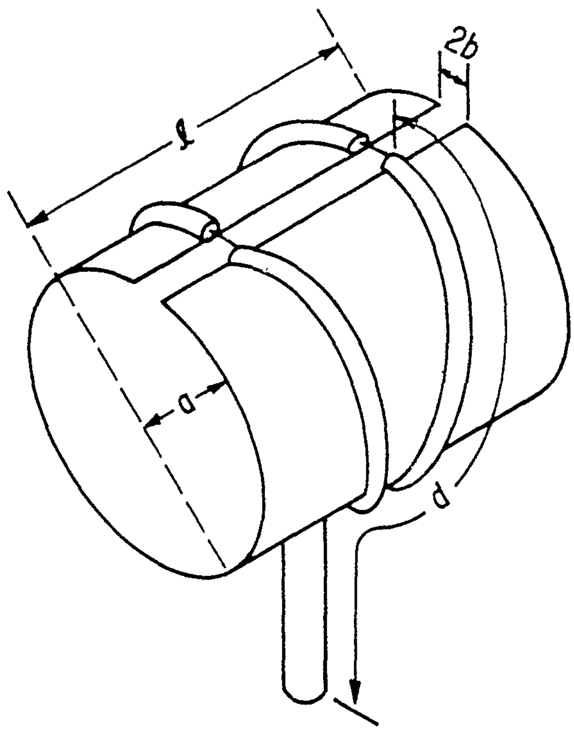
and the component of the magnetic field being measured is the one perpendicular to the circular cross section of area πa^2. For loops of N turns the effective area is $N\pi a^2$ when the same geometries are considered.

Equation (2), however, has certain restrictions based upon the assumptions required to derive it from equation (1). First, there must be a frequency independent magnetic field distribution or a locally uniform magnetic field in the immediate vicinity of the loop to which we can relate the sensor output. Likewise, the magnetic field distribution linking the loop should be frequency independent so that there is a well defined surface integral as in equation (1).

1. Lt Carl E. Baum, Sensor and Simulation Note VIII, Maximizing Frequency Response of a $\dot{B}$ Loop, Dec. 1964.
2. Lt Carl E. Baum, Sensor and Simulation Note XV, Radiation and Conductivity Constraints on the Design of a Dipole Electric Field Sensor, Feb. 1965.
3. Lt Carl E. Baum, Sensor and Simulation Note XXVI, The Influence of Finite Soil and Water Conductivity on Close-in Surface Electric Field Measurements, Sept. 1966.
4. Units are rationalized MKSA unless otherwise specified.



A. TOROIDAL LOOP



B. CYLINDRICAL LOOP

FIGURE 1. EXAMPLES OF LOOPS

The distance over which the field changes significantly in the air must then be larger than the characteristic dimensions of the loop. For the case of negligible air conductivity this characteristic distance is of the order of $\lambda\!\!\!^{-}$, the radian wave length, where

$$\lambda\!\!\!^{-} = \frac{c}{\omega} \tag{4}$$

and where c is the speed of light and ω is any radian frequency of interest. For the case of greater interest in this note (in which $\sigma >> \omega\varepsilon$), this characteristic distance is of the order of δ, the skin depth, where

$$\delta = \sqrt{\frac{2}{\omega\mu_o\sigma}} \tag{5}$$

Define

$$\delta_o = \sqrt{\frac{2}{\omega_o\mu_o\sigma_o}} \tag{6}$$

where ω_o is the maximum frequency of interest and σ_o is the maximum air conductivity during the magnetic field pulse. Then, if we consider a magnetic field "propagating" perpendicular to the loop axis (so that the magnetic field is perpendicular to the circular cross section) and if we make $a << \delta_o$, the magnetic field will be locally uniform for all frequencies of interest throughout the magnetic field pulse. Since the local environment is part of the source region, the magnetic field is actually locally generated. The sensor even occupies part of the source region.

Actually this skin depth concept is somewhat approximate for two reasons. First, the air conductivity is changing with time, restricting somewhat the use of frequency domain concepts. Second, the air conductivity is a function of the electric field which in turn is a function of both time and position. This also restricts the use of frequency domain concepts. The use that we are making of this concept, however, is quite limited. Since the skin depth is inversely proportional to both frequency and conductivity, the maximum air conductivity gives a minimum skin depth which corresponds to a maximum frequency of interest. Within the frequency range of interest the magnetic field is uniform over distances of the order of δ_o or even larger. This minimum skin depth does not give the quantitative variation of the magnetic field with position. It may be used, however, as a criterion in sensor design. Consider the largest allowable value of the appropriate sensor dimensions as around δ_o for a certain upper frequency response at a certain maximum air conductivity level.

By this last restriction we have reduced the wave equation to a Laplace or Poisson type equation for our calculations. This is a quasi-static case. With this limitation on the skin depth we can consider the equations for the electric field, $\vec{E}$, and magnetic field, $\vec{B}$, separately. Specifically, the magnetic field is not significantly affected by the local air conductivity and thus it is not significantly affected by the locally distorted air conductivity. This is in contrast to the electric field dipole in which the nonlinear air conductivity affects its basic sensitivity (effective height) since conductivity enters directly into the equation for the quasi-static local electric field distribution.[5] This difference between the electric and magnetic fields is seen in two of Maxwell's equations which we write in the forms

5. For a discussion of this case see references 2 and 3.

$$\nabla\cdot[(\sigma+\epsilon\frac{\partial}{\partial t})\vec{E}+\vec{J}_c] = 0 \qquad (7)$$

and

$$\nabla\cdot\vec{B} = 0 \qquad (8)$$

where $\vec{J}_c$ is the Compton current density. The electric and magnetic fields are not coupled (in these equations) without introducing other Maxwell's equations. In equation (7) the electric field is directly coupled with the conductivity. The conductivity does not appear in equation (8) for the magnetic field. In this quasi-static case the permeability, μ, has a similar role for the magnetic field as the conductivity or permittivity for the electric field. The permeability appears in the relation

$$\vec{B} = \mu\vec{H} \qquad (9)$$

where for air μ is just the permeability of free space, μ_o, for all practical purposes unless we deliberately introduce some material in the vicinity of the sensor with a significantly different permeability.

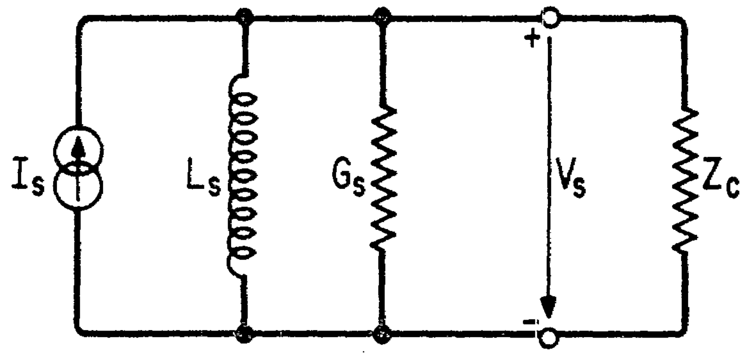
The magnetic field does, however, depend on the current density in the form

$$\nabla\times\vec{H} = (\sigma+\epsilon\frac{\partial}{\partial t})\vec{E} + \vec{J}_c \qquad (10)$$

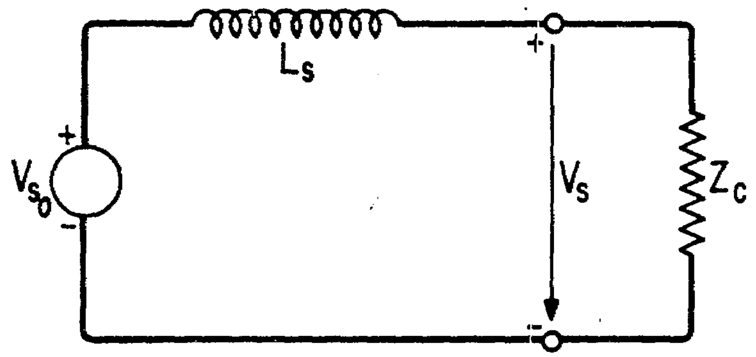
Thus, for the case of interest in which $\sigma\gg\omega\epsilon$, a type of derivative of the magnetic field with respect to position is proportional to the current density. It therefore depends on both the local distortions in the conductivity and the Compton current density. On the other hand, the magnetic field can be thought of as an integral of this positional derivative over a distance of the order of the approximate skin depth. Then, as long as these distortions in the current density distribution are of the same order as or less than the undisturbed current density distribution and occur over distances small compared to the skin depth, their significance is also small. This is not to say that there are not cases in which the distortions could significantly contribute to the sensor signal, particularly if the sensor is making a measurement of a non-principal magnetic field component (which is ideally zero). Another such case might be one in which the sensor makes a measurement at many skin depths or diffusion depths away from an assymmetry in the source distribution (the ground or water surface for a surface burst) which produces the magnetic field. Returning to figure 1, note that the electric field distortion may extend over distances larger than the sensor size because of the presence of associated conducting and/or insulating objects such as signal cables both above and below the ground or water surface.[6] Clearly it is desirable to minimize the effect of any accompanying magnetic field distortions both by minimizing the distortions and by the judicious use of symmetry in the design of the sensor and related equipment so as to minimize the coupling of the magnetic field distortions into the loop.

Considering now that the $\dot{B}$ loop is in a locally uniform magnetic field, develop the equivalent circuit of such a sensor as in figure 2. In particular

6. See reference 3.



A. NORTON EQUIVALENT



B. LOW FREQUENCY THÉVENIN EQUIVALENT

FIGURE 2. EQUIVALENT CIRCUITS FOR SIGNAL

use the cylindrical loop as in figure 1B. For $\ell >> a$ the short circuit current, I_s, due to the magnetic field is approximately

$$I_s \approx H\ell \tag{11}$$

(To derive this physically short the loop gap and apply the boundary condition in which the magnetic field (H) equals the surface current density on a perfect conductor and ignore fringing fields at the ends of the cylinder.) This is shown in the Norton equivalent circuit of figure 2A. Next, consider the admittances (or impedances) which load the loop gap. Ignoring momentarily the signal cable load, the loop inductance is

$$L_s \approx \mu_o \frac{\pi a^2}{\ell} \tag{12}$$

There is also a conductance

$$G_s \approx \sigma \frac{a\ell}{b} \tag{13}$$

due to the air conductivity which acts to short the gap. In this equation σ is the value of the air conductivity near the loop gap. This conductance is very approximate and roughly represents an upper limit. For this sensor to have a response proportional to $\dot{B}$, it is necessary that the inductance contribute the dominant admittance at the loop gap. Taking a limiting case by equating the magnitudes of these two admittances gives

$$\frac{\ell}{\omega\mu_o\pi a^2} \approx \sigma \frac{a\ell}{b} \tag{14}$$

or

$$\delta = \sqrt{\frac{2}{\omega\mu_o\sigma}} \approx a\sqrt{2\pi \frac{a}{b}} \tag{15}$$

Thus, if b/a is not too small, and if the skin depth is of the order of the loop dimensions or larger for the maximum air conductivity and for all frequencies of interest, then the inductance dominates the conductance for all frequencies of interest. The skin depth limitation for magnetic field uniformity in the vicinity of the loop is also the limitation for the frequency response of the $\dot{B}$ loop.

With the limitation that the shorting conductance be much less than the magnitude of the inductive admittance for frequencies of interest it does not matter that the air conductivity is time varying and a function of the electric field. The conductance is restricted by design to be insignificant. Any plasma sheath of lower conductivity which might form adjacent to the loop structure is also insignificant; it simply lowers the conductance, if anything. We might even think of purposely reducing this shorting conductance by the use of insulators both inside and outside the loop structure, thus lowering the required ratio of the minimum skin depth (of interest) to the loop radius for operation as a $\dot{B}$ loop. This limitation, however, is still present in the requirement for a uniform magnetic field in the vicinity of the loop. Insulators might help in another way by minimizing the possibility of an electrical breakdown (or by reducing adverse effects of such a breakdown) which might occur in the ionized air at positions on or near the loop structure with large electric fields.

With appropriate frequency restrictions (as above) we can then ignore G_s and have the simpler Thévenin equivalent circuit as in figure 2B. The open circuit voltage is just

$$V_{s_o} = \dot{B}A_{eff} \tag{16}$$

where A_{eff} is given by equation (3). Including now the cable resistive impedance, Z_c, gives another limitation on the $\dot{B}$ loop response. For V_s, the signal transmitted down the cable, to equal V_{s_o}, Z_c must be much greater than ωL_s for frequencies of interest. This gives another upper frequency response as

$$\omega_1 = \frac{Z_c}{L_s} \tag{17}$$

The actual upper frequency response of the $\dot{B}$ loop is the smaller of ω_1 (as above) and ω_o (the upper frequency response from the skin depth limitation). If maximum sensitivity is desired for a given upper frequency response then the loop area is fixed by the skin depth limitation in the conducting air. The frequency, ω_1, can be made the order of ω_o if

$$Z_c \simeq \frac{1}{G_s} \tag{18}$$

This may be accomplished by increasing ℓ (as in equation (13)) if we wish Z_c to be some convenient number (such as 50 Ω or 100 Ω). By further increasing ℓ (and decreasing L_s) ω_1 can be made even higher; but beyond some point, this would gain little for the overall frequency response.

There are then two limitations on the response of a $\dot{B}$ loop in this nonlinear, time varying conducting medium. First, the skin depth for the maximum conductivity and highest frequency of interest should be of the same order as or larger than the appropriate loop dimension. Second, the upper frequency of interest should be less than the frequency response determined by the signal cable load and the loop inductance. An important point to note is that even though the air conductivity is nonlinear and time varying these effects do not enter into the basic low frequency sensitivity of the loop. They do, however, influence the upper frequency response of the loop. A frequency domain analysis of the loop response which, of necessity, assumes a linear, time independent air conductivity will therefore obtain the correct low frequency response but will not accurately determine the high frequency rolloff due to the air conductivity because frequency response doesn't really apply to this case. However, this kind of analysis does allow us to assign an approximate upper frequency response to the loop and allows us to compare various loop designs for their frequency response characteristics. We plan to use this type of analysis in the future on the cylindrical loop, including the use of insulators to improve the frequency response.

III. Nuclear Radiation Related Problems.

Besides making the air conducting, the nuclear radiation also affects the $\dot{B}$ loop measurement by transporting charge into, out of, and throughout the sensor. This produces spurious signals from the sensor, or noise as far as the measurement is concerned. Consider primarily γ-ray effects. For arbitrarily assumed 2 MeV γ rays (unidirectional) there is a Compton current density in the air (ignoring signs) of[7]

$$J_c \simeq 2.1 \times 10^{-8}\, \gamma \tag{19}$$

where γ is in roentgens/sec. For materials other than air this relationship still gives the right order of the current density. This then is also the order of Compton current density in the sensor structure.

Consider again the cylindrical loop structure of figure 1B with the loop gap electrically shorted along the full length of the structure. Express the short circuit current across the loop gap, attributable to the charge transport in the loop structure by the γ rays, in the form

$$I_{c_s} = f_s\, 2a\ell\, J_c \tag{20}$$

Here f_s represents a fraction (not necessarily less than one) of the total Compton current over a loop cross section of area, $2a\ell$, (perpendicular to the radiation direction of travel). The cylindrical loop axis is taken as perpendicular to a line from the radiation source for this discussion. These quantities (f_s and I_{c_s}) are intended to account for only the Compton current noise from the loop structure. Any such noise generated in signal cables is considered below.

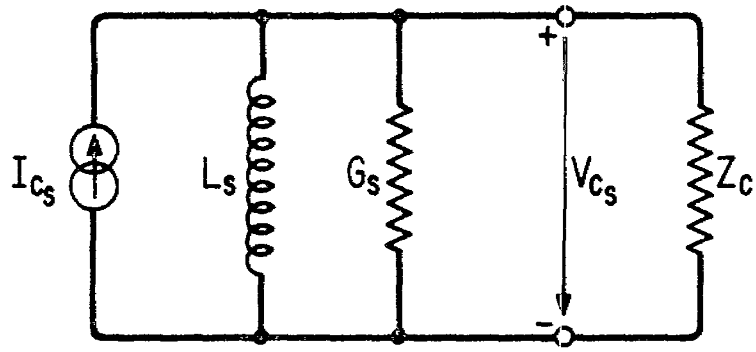
Figure 3A shows this noise current source, I_{c_s}, in an equivalent circuit. Note that the admittance with this noise short circuit current should be the same as the admittance considered with the signal short circuit current if nonlinearities are ignored. The equivalent circuits of figures 2A and 3A are then the same except for the different current sources. Using equations (11) and (20) we can relate this noise to the signal as

$$\frac{V_{c_s}}{V_s} = \frac{I_{c_s}}{I_s} = f_s\, 2a\, \frac{J_c}{H} \tag{21}$$

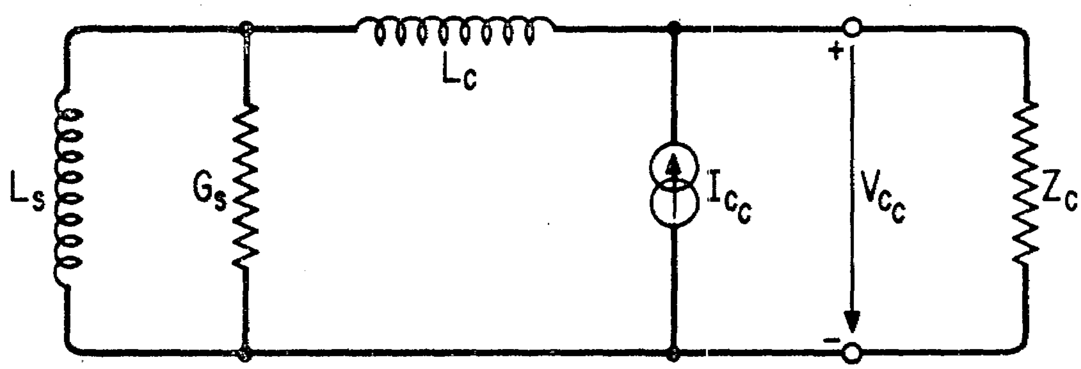
Note that for frequencies such that L_s provides the dominant admittance so that the loop is operating in the $\dot{B}$ mode, not only does the loop differentiate the magnetic field, it also differentiates this Compton current noise.

If we think of the magnetic field, H, as roughly J_c times an appropriate skin depth, δ, or diffusion depth, then as long as $\delta \gg f_s\, 2a$ the signal dominates this Compton current noise.[8] The fractional current,

7. Lt Carl E. Baum, EMP Theoretical Note XIX, A Technique for the Approximate Solution of EMP Fields from a Surface Burst in the Vicinity of an Air-Ground or an Air-Water Interface, Sept. 1966.
8. For a discussion of this rough approximation for the magnetic field see reference 7.



A. NOISE FROM LOOP STRUCTURE



B. NOISE FROM ASSOCIATED CABLES

FIGURE 3. EQUIVALENT CIRCUITS FOR COMPTON CURRENT NOISE

f_s, should be a function of various loop parameters such as loop radius, materials used, etc. Minimizing the loop radius should decrease the noise-to-signal ratio as in equation (21), but this would apply only to this particular noise source. It may be advantageous to increase a to increase the signal so that it dominates over other noise sources. Perhaps also f_s can be minimized by appropriate designs for the loop structure. There are also limitations here in that the loop structure has other functions to perform, e.g., it may be desirable to include insulators in the structure to minimize the effects associated with the air conductivity.

Besides in the loop structure, Compton current noise can be produced in the associated signal cables. Typical coaxial cables have sensitivities of 10^{-12} to 10^{-11} coulombs/roentgen per meter of cable for something like the fission γ-ray spectrum.[9] Except for use in the loop structure itself, however, we might expect to use twinax signal cable which might have somewhat different radiation sensitivity characteristics than these. Assuming a differential sensor design, represent the differential Compton current noise signal generated in the signal cables as

$$I_{c_c} = f_c J_c d \tag{22}$$

where f_c has a dimension of meters and where d is the typical length of cable exposed the radiation flux (as in figure 1). Beyond a certain point the cable is assumed to end (with proper termination) or to be adequately shielded by something such as soil or water.

This current source is placed in an equivalent circuit in figure 3B but note the addition of an extra circuit element, an inductance. The previously considered current sources drive the signal cable which has a characteristic impedance, Z_c. The noise current generated in the cables, however, drives the cable in one direction (toward the recording instrumentation) and also drives an unterminated cable in the other direction. This unterminated cable, in series with the sensor inductance and conductance combination, presents an inductance for low frequencies. By low frequencies we mean frequencies both for which the magnitude of the sensor impedance is much less than the characteristic cable impedance and which have periods much longer than the transit time of the current back to the sensor. Since this current source is distributed throughout the exposed signal cable, take t_r as the average transit time to the loop gap giving an inductance

$$L_c = t_r Z_c \tag{23}$$

If βc is the propagation velocity in the cable (where c is the speed of light in vacuum), then

$$L_c \approx \frac{d}{2\beta c} Z_c \tag{24}$$

Ignoring G_s and assuming frequencies low enough that $\omega[L_c+L_s] \ll Z_c$, we can then relate the Compton current noise from the cables to the magnetic field signal. This noise signal from the cables also drives an inductance and is differentiated. It is related to the signal as

$$\frac{V_{c_c}}{V_s} = \frac{j\omega[L_c+L_s]\, I_{c_s}}{j\omega L_s I_s} = \left[\frac{L_c}{L_s} + 1\right] \frac{f_c J_c d}{H \ell} \tag{25}$$

9. W. C. Anderson and L. P. Hocker, EG&G Report S-343-R, Project Bass Drum Cable Hardening Studies, Sept. 1966.

Or, substituting for the inductances,

$$\frac{V_{c_c}}{V_s} \approx \left[\frac{\dfrac{dZ_c}{2\beta c}}{\dfrac{\mu_o \pi a^2}{\ell}} + 1 \right] \frac{f_c J_c d}{H \ell} \tag{26}$$

If L_s is minimized to improve the frequency response, then L_c could be significantly larger than L_s. In this case, the noise-to-signal ratio is proportional to d^2/a^2. This indicates that the height of the sensor above a shielding medium (soil, water, etc.) should be minimized and the sensor radius maximized to reduce the noise-to-signal ratio. This is in contrast to the results for the Compton current noise in the loop structure which indicate that a small loop radius is desirable. In order to reduce the noise from the cable, perhaps the cable sensitivity (to the radiation) can also be minimized. Looking at these two Compton current noise sources, I_{c_s} and I_{c_c}, it would seem that the latter, because of the added cable inductance in the equivalent circuit, could be the more troublesome in some cases, particularly if the sensor has a significant length of cable exposed.

There may be other side effects of the radiation interaction, such as common mode radiation noise signals and conductivity in the insulators which are used in the sensor structure and associated cables, that are not considered here. While the above discussion is based on γ-ray interaction, it also applies qualitatively for X rays with perhaps additional attention paid to using low atomic number materials to minimize photoelectric cross sections. Neutrons may also interact with the sensor and associated equipment through processes such as (n, γ) and (n, p) reactions. The γ rays from the first process can interact as above while the protons from the second process may represent another noise current.

Based on the relative size of the magnetic field signal and the radiation signal (as in equation (21)), a satisfactorily small noise-to-signal ratio should be attainable. This should also hold regarding noise from the signal cable if it is not too long. If high frequencies and high-γ-ray intensities are considered, however, then the approximate skin depth can be rather small, of the order of practical loop sizes, so that the magnetic field signal is of the same order as the Compton current noise (if we assume f_s to be of order one in equation (20)). In such a case care should be taken to minimize the Compton current noise to an acceptable level.

IV. Summary.

The air conductivity and nuclear radiation significantly influence close-in loop measurements of $\dot{B}$. The maximum air conductivity limits the loop radius to the order of a skin depth or less at the highest frequency of interest. Below this frequency the air conductivity does not significantly enter into the loop response. Thus, for such a loop the nonlinear and time varying character of the air conductivity is insignificant. However, sensor associated equipment such as cables in the air medium should generally be limited to the same dimensions to avoid magnetic field distortions which may couple into the loop. We may be able to minimize the conductivity related effects by the use of insulators with the loop structure. Also, the cable impedance which loads the loop can be chosen, together with the loop inductance, to give a frequency response of the order of the skin depth limitation.

The nuclear radiation introduces noise signals into the sensor structure and the associated signal cables. This does not appear as too serious a problem in many cases. Some improvement can be gained by reducing to a minimum the amount of unshielded signal cable, such as above a ground or water surface. For high radiation intensities and high frequencies, however, such that the approximate skin depth in the air becomes of the order of the loop size, this could be a significant problem. Such problems can perhaps be reduced to acceptable levels through appropriate designs for the sensor and related equipment.