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Sensor and Simulation Notes

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AN ANISOTROPIC LENS FOR TRANSITIONING PLANE WAVES BETWEEN MEDIA OF DIFFERENT PERMITTIVITIES

A. P. Stone
University of New Mexico
Albuquerque, NM 87131

and

C. E. Baum
Air Force Weapons Laboratory
Kirtland Air Force Base, NM 87117

ABSTRACT

In this paper a lens is specified for transitioning plane waves between media of specified permittivities. It is desired to have the plane wave in the second medium propagate normal to the assumed plane boundary of that medium. The results for the case of normal incidence are then generalized to the case of non-normal incidence. In particular, the conditions of transit time conservation and impedance matching are related to the Brewster angle.

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Keywords: plane waves, transit time, impedance

1 Introduction

One of the possible approaches to lens design for transient or broadband electromagnetic waves involves differential impedance matching and transit-time conservation. One seeks to transition plane waves, ideally with no distortion or reflection, between two TEM transmission lines. The design of such lenses is specified by giving the material properties and shape of the lens. These properties, given usually by the permittivity tensor, $\overleftrightarrow{\epsilon}$, and the permeability tensor, $\overleftrightarrow{\mu}$, may vary from point to point within the lens, but are assumed to be independent of frequency. (The conductivity is assumed to be zero.) In order to specify the material properties and geometry of a lens one generally has to solve an initial value problem which arises from enforcing certain physical principles. That is, at a lens/transmission line boundary, impedances must be matched differentially and a wave in the lens should go into an inhomogeneous TEM plane wave in the adjacent region. Thus the travel time for waves following different paths should be equal. For example, a lens may be specified to transition TEM waves between two cylindrical coaxial waveguides of different size (see [1]). Other examples have appeared in the literature ([2], [3]).

In this paper we consider a particularly simple geometry in which an anisotropic lens is specified for the transitioning of plane waves between media of different permittivities. Unlike the lenses discussed in earlier results, the impedance matching and transit time conservation requirements have not lead to the same system of differential equations. It is interesting to note that in the differential geometry examples to date, we have a plane or spherical wave (incident on or leaving the lens) which is normally incident on the boundary between regions of different permittivities and for permeabilities.

Of course, in the case of non-normal incidence we can have the situation where a TEM wave propagates with a TM polarization, and the wave can pass through the boundary with no reflection. That is, the angle of incidence is the Brewster angle, ψ_B, which can be calculated if the properties of the media are known. For a discussion of the Brewster angle phenomenon, see [4]. In Section 2 of this paper we discuss briefly this situation, and show that enforcing the requirements of differential impedance matching and transit-time conservation at boundaries of regions of different permittivities leads to the Brewster angle condition.

In Section 3 of this paper the main results are presented for the case of a plane wave propagating in region I, normally incident on a boundary between I and a lens region L, through L and on into a second region II. The regions I, L, and II have respective permittivities $\epsilon_1, \epsilon_\ell$, and ϵ_2. The permittivity and permeability of free space are denoted by ϵ_0 and μ_0, respectively. All regions will be assumed to have the same permeability, which we will take as μ_0. The requirements of continuity of impedances and transit-time conservation are to hold at all boundaries. The shape of the lens region and its permittivity are the design objectives, and the results appear in Section 3.

Finally, in Section 4 of this paper some further remarks are made on the case of non-normal incidence. The case of normal incidence generalizes to the case of non-normal incidence. This result then admits the possibility of constructing an array of lenses.

2 Differential Transit-Time and Impedance Matching for Plane Wave Propagating From One Uniform Isotropic Dielectric Medium into Another

Let us consider as in Figure 2.1 two regions, I and II, of permittivities ϵ_1 and ϵ_2, respectively. A plane boundary separates these regions. If a wave in region I is incident on this boundary at the angle ψ_B, and if this angle is the Brewster angle, then we have the well-known

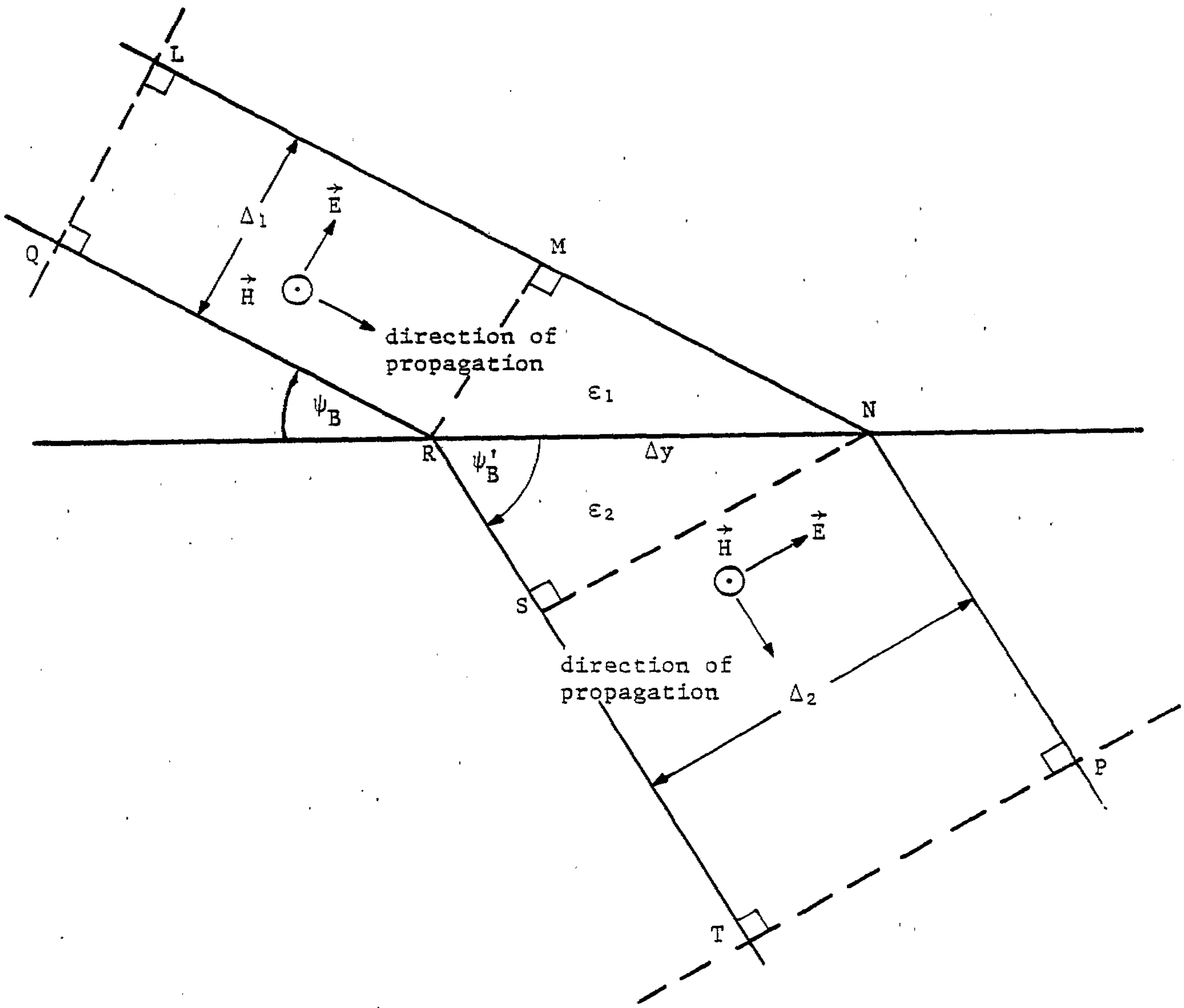

Figure 2.1: Matching Differential Transit Times and Impedances at an Interface Between Two Homogeneous Isotropic Dielectric Media

formula

$$\tan(\psi_B) = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\,. \tag{2.1}$$

The regions I and II are assumed to have the same permeability μ. In region II we also have

$$\cot(\psi'_B) = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \tag{2.2}$$

and consequently $\psi_B + \psi'_B = \pi/2$.

Now if the conditions of transit-time conservation and continuity of impedances are enforced at the boundary, we will find that the Brewster angle conditions (2.1) and (2.2) must hold as a consequence. The details are as follows.

First, we investigate the condition that differential travel times for waves following different paths must be equal. Thus in Figure 2.1 the travel times along the straight line paths LMNP and QRST must be equal. From the geometry it is clear that the time along MN must equal the time along RS. Thus we have

$$\begin{aligned} t_1 &= \sqrt{\mu\epsilon_1}(\overline{MN}) \\ t_2 &= \sqrt{\mu\epsilon_2}(\overline{RS})\,. \end{aligned} \tag{2.3}$$

Now since,

$$\begin{aligned} \overline{MN} &= (\Delta y)\cos(\psi_B) \\ \overline{RS} &= (\Delta y)\cos(\psi'_B)\,, \end{aligned} \tag{2.4}$$

the condition that $t_1 = t_2$ implies

$$\frac{\cos(\psi'_B)}{\cos(\psi_B)} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\,. \tag{2.5}$$

Since we must also have impedances matched differentially across the boundary, we also have

$$\Delta Z_1 = \Delta_1\sqrt{\frac{\mu_0}{\epsilon_1}} \tag{2.6}$$

$$\Delta Z_2 = \Delta_2 \sqrt{\frac{\mu_0}{\epsilon_2}}$$

where

$$\Delta_1 = (\Delta y)(\sin(\psi_B)) \tag{2.7}$$
$$\Delta_2 = (\Delta y)(\sin(\psi'_B)) .$$

Impedance continuity across the boundary must then imply

$$\frac{\sin(\psi_B)}{\sin(\psi'_B)} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} . \tag{2.8}$$

Hence the requirements of transit-time conservation (2.5) and impedance matching (2.8) imply

$$\frac{\cos(\psi'_B)}{\cos(\psi_B)} = \frac{\sin(\psi_B)}{\sin(\psi'_B)} . \tag{2.9}$$

Since (2.9) implies

$$\sin(2\psi'_B) = \sin(2\psi_B) , \tag{2.10}$$

we find among the many solutions of (2.10) the following important cases. First, if $\psi_B = \psi'_B$, then $\epsilon_1 = \epsilon_2$ and we obtain the special case of uniform media. Second, if we have $\psi_B + \psi'_B = \pi/2$, we have the Brewster angle condition with the usual formulae

$$\tan(\psi_B) = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \tag{2.11}$$
$$\cot(\psi'_B) = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

resulting. Thus, if $\epsilon_1 \neq \epsilon_2$, then the Brewster angle conditions (2.11) are a consequence of the travel-time and impedance-matching conditions.

3 Statement of the Problem and Its Solution

Let us consider the geometry as indicated in Figure 3.1. Consider a line located at P, which can be taken as the z axis in a rectangular coordinate system (x, y, z). The positive

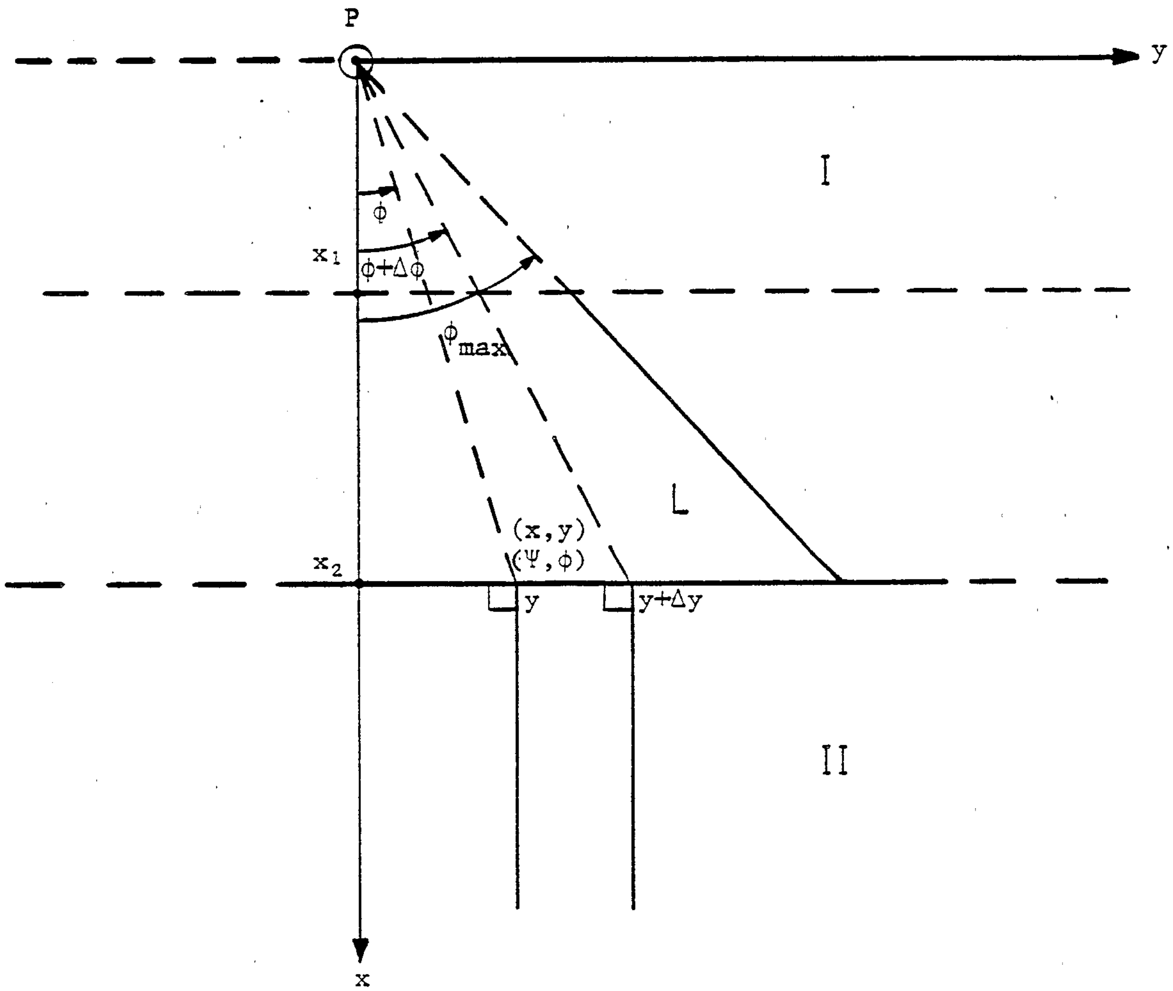

Figure 3.1: Transit Time and Differential Impedance Matching from Lens to Homogeneous Isotropic Dielectric Medium at Normal Incidence

direction of the x-axis is downward, and the positive direction of the y-axis is to the right. The line $x = x_1$ (to be discussed latter) forms the upper boundary of the lens region L while the line $x = x_2$ forms the lower boundary of the lens and is the boundary between regions L and II, whose respective permittivities are ϵ_ℓ and ϵ_2. At any point on the boundary line $x = x_2$ we can express the coordinates (x_2, y_2) in terms of polar coordinates (Ψ_2, ϕ_2) as

$$\begin{aligned} x_2 &= \Psi_2 \cos(\phi_2) \\ y_2 &= \Psi_2 \sin(\phi_2) \,. \end{aligned} \tag{3.1}$$

A plane wave (as shown later) is to propagate in a normal direction in region I and on into region L and then into II. We wish to specify the permittivity of region L as well as the shape of the lens region.

We assume that the permeability, $\mu = \mu_0$, is the same in all regions and try a solution of the form

$$\frac{\epsilon_\ell}{\epsilon_2} = f(\Psi)g(\phi) \tag{3.2}$$

where $f(\Psi)$ and $g(\phi)$ are functions which are to be determined. Now since we require continuity of impedances across the boundary $x = x_2$ between the lens region L and region II, we compute changes in impedances as follows. First, for the lens region we have

$$(\Delta Z)_\ell = \sqrt{\frac{\mu_0}{\epsilon_\ell}}\Psi(\Delta\phi) \,. \tag{3.3}$$

In the lens region L surfaces of constant ϕ are perfect conductors (with only $E_\phi \neq 0$). Hence as Ψ varies, $(\Delta Z)_\ell$ should be a constant for constant ϕ, and so the substitution of (3.2) into (3.3) yields

$$(\Delta Z)_\ell = \sqrt{\frac{\mu_0}{\epsilon_2 f(\Psi)g(\phi)}}\Psi(\Delta\phi) \,. \tag{3.4}$$

Thus, since $(\Delta Z)_\ell$ is not a function of Ψ in the (dispersionless) lens region L, we must have

$$f(\Psi) = (\Psi/\Psi_0)^2 \tag{3.5}$$

for some constant Ψ_0. Thus (3.4) becomes

$$(\Delta Z)_\ell = \sqrt{\frac{\mu_0}{\epsilon_2 g(\phi)}}\Psi_0 \Delta\phi \ . \tag{3.6}$$

Next, in region II the change in impedance is given by

$$(\Delta Z)_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}\Delta y_2 \tag{3.7}$$

and since we must have continuity of differential impedances across the boundary we obtain from (3.6) and (3.7) (using $\phi = \phi_2$ on the boundary)

$$\frac{\Delta y_2}{\Delta \phi_2} = \Psi_0 \frac{1}{\sqrt{g(\phi_2)}} \ . \tag{3.8}$$

However on the boundary x_2 is a constant and so y_2 is a function only of ϕ_2, since $y_2 = x_2 \cdot \tan(\phi_2)$. Hence we must also have

$$\frac{dy_2}{d\phi_2} = x_2 \sec^2(\phi_2) \ . \tag{3.9}$$

Thus a comparison of (3.8) and (3.9) yields the result

$$g(\phi_2) = \left(\frac{\Psi_0}{x_2}\right)^2 \cos^4(\phi_2) \ . \tag{3.10}$$

If we now choose the arbitrary constant Ψ_0 as x_2 then we have

$$g(\phi_2) = \cos^4(\phi_2) \ . \tag{3.11}$$

This result then requires that the form of the function g be

$$g(\phi) = \cos^4(\phi) \ . \tag{3.12}$$

Hence the functions f and g appearing in (3.2) have been determined, with

$$f(\Psi) = (\Psi/x_2)^2 \tag{3.13}$$

and

$$g(\phi) = \cos^4(\phi) \,. \tag{3.14}$$

Thus equation (3.2) assumes the form

$$\epsilon_\ell = \epsilon_2 \left(\frac{\Psi}{x_2}\right)^2 \cos^4(\phi) = \epsilon_2 \left(\frac{x}{x_2}\right)^2 \cos^2(\phi) \,. \tag{3.15}$$

Equation (3.15) and its various alternative forms will be used to specify the lens design.

Note that the only condition that has been imposed up to this point is the differential impedance-matching condition. We now consider the transit-time requirement. If we consider some x_0 such that

$$0 < x_1 < x_0 < x_2 \tag{3.16}$$

where the lines $x = x_1$ and $x = x_2$ define the boundary lines for our lens (region L), then the point (x_0, y_0) is within the lens. We seek the wave velocity relative to the x-coordinate (i.e., the reciprocal of local differential transit time). In the Ψ direction the velocity v is

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_\ell}} = c\sqrt{\frac{\epsilon_0}{\epsilon_\ell}} = \frac{c}{\sqrt{\epsilon_{r_\ell}}} \,, \; \epsilon_{r_\ell} \equiv \frac{\epsilon_\ell}{\epsilon_0} \,. \tag{3.17}$$

However, the wave is slower in terms of x by a factor of $\cos(\phi)$ and so

$$v_x = v\cos(\phi) = \frac{c\cos(\phi)}{\sqrt{\epsilon_{r_\ell}}} \tag{3.18}$$

or

$$v_x = \frac{c}{\sqrt{\epsilon_{r_\ell}}} \left(\frac{x_2}{\Psi}\right) \frac{1}{\cos(\phi)} \,. \tag{3.19}$$

Now on a surface of constant $x = x_0$, we have $\psi\cos(\phi) = x_0$ and so

$$v_x = \frac{c}{\sqrt{\epsilon_{r_\ell}}} \frac{x_2}{x_0} \,. \tag{3.20}$$

Hence the velocity is proportional to $1/x$ and hence is a function only of x. Thus any surface of constant x is a wavefront or surface of constant phase. Note x can be chosen arbitrarily in $x_1 \leq x \leq x_2$ including both upper and lower boundaries.

Let us next consider the upper boundary $x = x_1$ of the lens. Clearly (see Figure 3.1) $\Delta y_1 \cdot \cos(\phi_1) = \Psi_1 \Delta\phi_1$, while the change in impedance, ΔZ, in the lens is proportional to $(\Psi_\ell \Delta\phi_\ell)/\sqrt{\epsilon_\ell}$. But

$$\frac{\Psi_\ell \Delta\phi_\ell}{\sqrt{\epsilon_\ell}} = \left(\frac{x_2}{x_1}\right) \sec(\phi_1) \cdot \Psi_1 \frac{\Delta\phi_1}{\sqrt{\epsilon_1}} \tag{3.21}$$

$$= \left(\frac{x_2}{x_1}\right) \frac{\Delta y_1}{\sqrt{\epsilon_1}} . \tag{3.22}$$

At normal incidence (i.e., $y_1 = 0$ and $\phi_1 = 0$) we must have

$$\frac{\epsilon_\ell}{\epsilon_1} = \left(\frac{x_1}{x_2}\right)^2 \tag{3.23}$$

and hence ΔZ is proportional to $(\Delta y_1)/\sqrt{\epsilon_1}$ as required. The form of the equations for the upper boundary is the same as the form for the lower boundary.

We now turn to the problem of determining the spatial limits of our lens. The notation remains as before and the geometry is as in Figure 3.1. The permittivity in the lens, ϵ_ℓ, is given as in Equation (3.15) in various forms. Thus we have, from the conditions on the lower boundary,

$$\begin{aligned} \epsilon_\ell &= \epsilon_2 \left(\frac{\Psi}{x_2}\right)^2 \cos^4(\phi) \\ &= \epsilon_2 \left(\frac{x}{x_2}\right)^2 \cos^2(\phi) \\ &= \epsilon_2 \left(\frac{x}{x_2}\right)^2 \frac{x^2}{x^2 + y^2} \end{aligned} \tag{3.24}$$

while from the conditions on the upper boundary

$$\begin{aligned} \epsilon_\ell &= \epsilon_1 \left(\frac{\Psi}{x_1}\right)^2 \cos^4(\phi) \\ &= \epsilon_1 \left(\frac{x}{x_1}\right)^2 \cos^2(\phi) . \end{aligned} \tag{3.25}$$

Thus we have the same form for both boundaries (x_1 and ϵ_1 versus x_2 and ϵ_2).

Now we must have $\epsilon_\ell \geq \epsilon_0$ everywhere. Clearly ϵ_ℓ is minimized in the lens region for minimum x and maximum y. Hence we may put $x = x_1$, $\phi = \phi_{max}$ and obtain from (3.24) and (3.25)

$$\epsilon_{\ell_{min}} = \epsilon_2 \left(\frac{x_1}{x_2}\right)^2 \cos^2(\phi_{max}) = \epsilon_1 \cos^2(\phi_{max}) \,. \tag{3.26}$$

Hence we need

$$\frac{\epsilon_1}{\epsilon_0} \geq \sec^2(\phi_{max}) \geq 1 \tag{3.27}$$

and so we may want to keep ϕ_{max} small if we want to have ϵ_1 near ϵ_0. Note also that

$$\frac{y_{1_{max}}}{y_{2_{max}}} = \frac{x_1}{x_2} \tag{3.28}$$

which shows that for given $y_{2_{max}}$ that $y_{1_{max}}$ can be no less than $(x_1/x_2)y_{2_{max}}$. Hence the upper boundary, $x = x_1$, of the lens cannot be extended upward to the line $x = 0$ (i.e., to the z-axis).

Finally, to obtain contours of constant ϵ_ℓ, consider Equations (3.24). If we normalize ϵ_ℓ by dividing by ϵ_2, then

$$\begin{aligned} \epsilon_n &\equiv \frac{\epsilon_\ell}{\epsilon_2} = \Psi_n^2 \cdot \cos^4(\phi) \\ \Psi_n &\equiv \Psi/x_2 \,. \end{aligned} \tag{3.29}$$

If values of ϵ_n are chosen, contour plots may be obtained. These are shown in Figure 3.2 for values of $\epsilon_n = 0.25$, 0.50, 1.0, 2.0, and 4.0. It should be noted that at any point on a contour of constant ϵ_n the slope is

$$\frac{dy}{dx} = \frac{x^2 + 2y^2}{xy} \tag{3.30}$$

This result is obtained by differentiation of ϵ_n in (3.29) with the substitution of rectangular coordinates from (3.24). Moreover, differentiation of (3.30) yields

$$\frac{d^2y}{dx^2} = \frac{x(2y^2 - x^2)(x^2 + y^2)}{(xy)^3} \tag{3.31}$$

and hence we have inflection points on the contours of constant ϵ_n at those points where the contours intersect the line whose equation is $y = x/\sqrt{2}$ (i.e., the ray $\phi = 35.3°$).

——— ——— ———: denotes perfectly conducting sheets (typical)

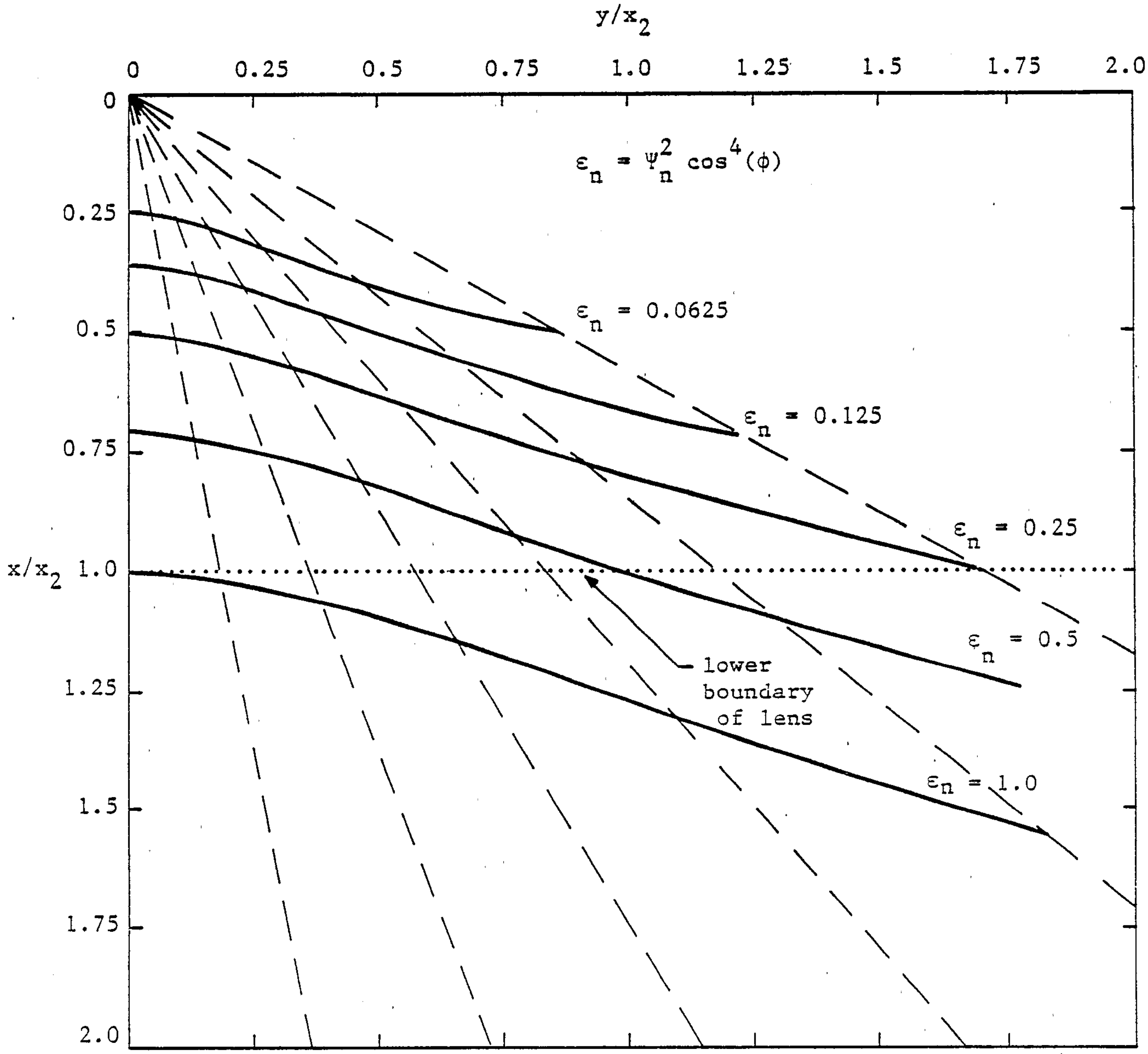

Figure 3.2: Contours of Constant Normalized Permittivity in Lens

In Figure 3.3 the shape of such a lens is shown. Equal ϵ_n contours are superimposed on the lens diagram. The lens may or may not be symmetrical about the x-axis, and perfectly conducting sheets are inserted on some set of surfaces of constant ϕ sufficiently close to make their spacing electrically small.

4 Non-Normal Incidence Revisited

Let us consider further the case of non-normal incidence. We assume, as in Figure 4.1, that a plane boundary $x = x_2$ separates regions I and II of permittivities ϵ_1 and ϵ_2. We assume further that both regions have the same permeability μ_0. For a wave specified to have a velocity v_i (phase velocity) along the boundary of the lower region (of permittivity ϵ_2), v_2 will make an angle ψ_2 with the boundary, and we must have

$$\frac{v_2}{v_i} = \cos(\psi_2) \leq 1 \tag{4.1}$$

where $v_2 = (\mu_0\epsilon_2)^{-1/2} \leq v_i$. Hence if v_i and ϵ_2 are specified, both v_2 and ψ_2 are determined.

Now in the upper region I, whose permittivity is ϵ_1, we must have

$$\frac{v_1}{v_i} = \cos(\psi_1) \leq 1 \tag{4.2}$$

and hence

$$\frac{\cos(\psi_2)}{\cos(\psi_1)} = \frac{v_2}{v_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\,. \tag{4.3}$$

Thus specifying v_i and ϵ_2 determines v_2 and ψ_2, and if ψ_2 is a Brewster angle (which is implied by impedance continuity and transit-time conservation as shown in Section 2), then ψ_1 and hence v_1 and ϵ_1 are determined. Equation (4.3) then becomes

$$\tan(\psi_1) = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\,, \tag{4.4}$$

as expected.

— — —: denotes contours of constant permittivity (typical)
.........: denotes perfectly conducting sheets (typical)

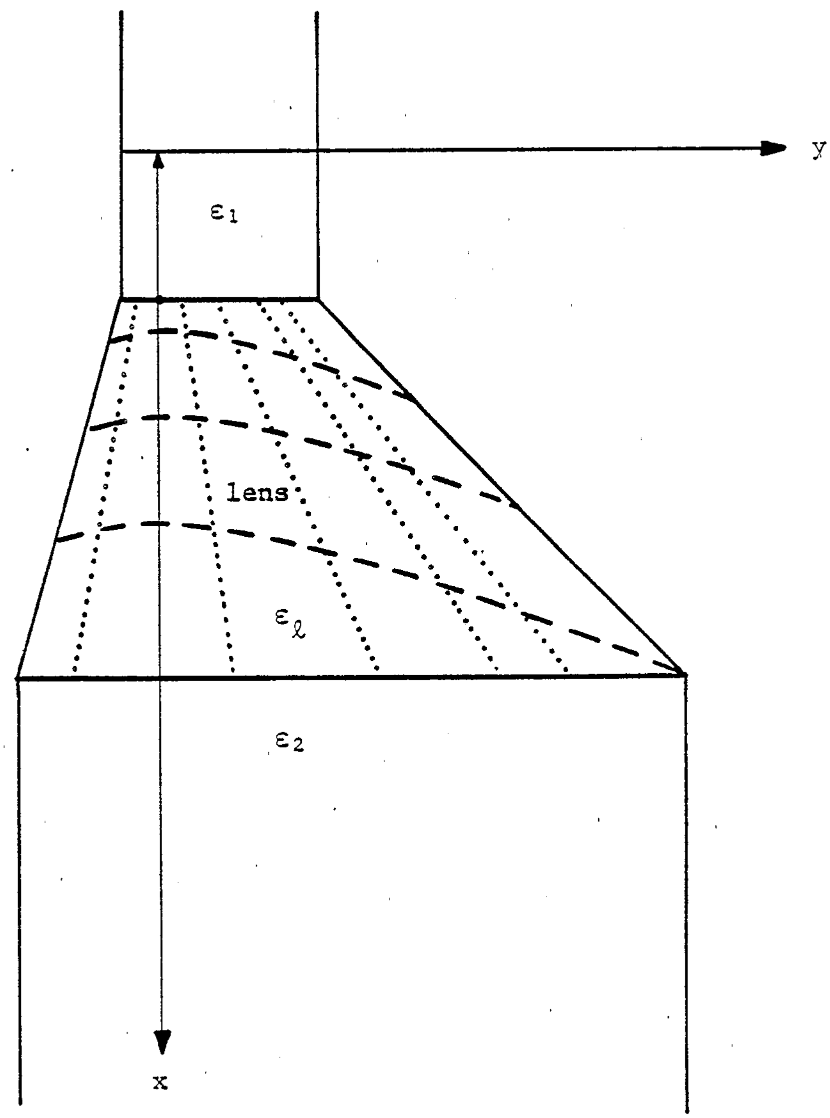

Figure 3.3: Possible Lens Geometry

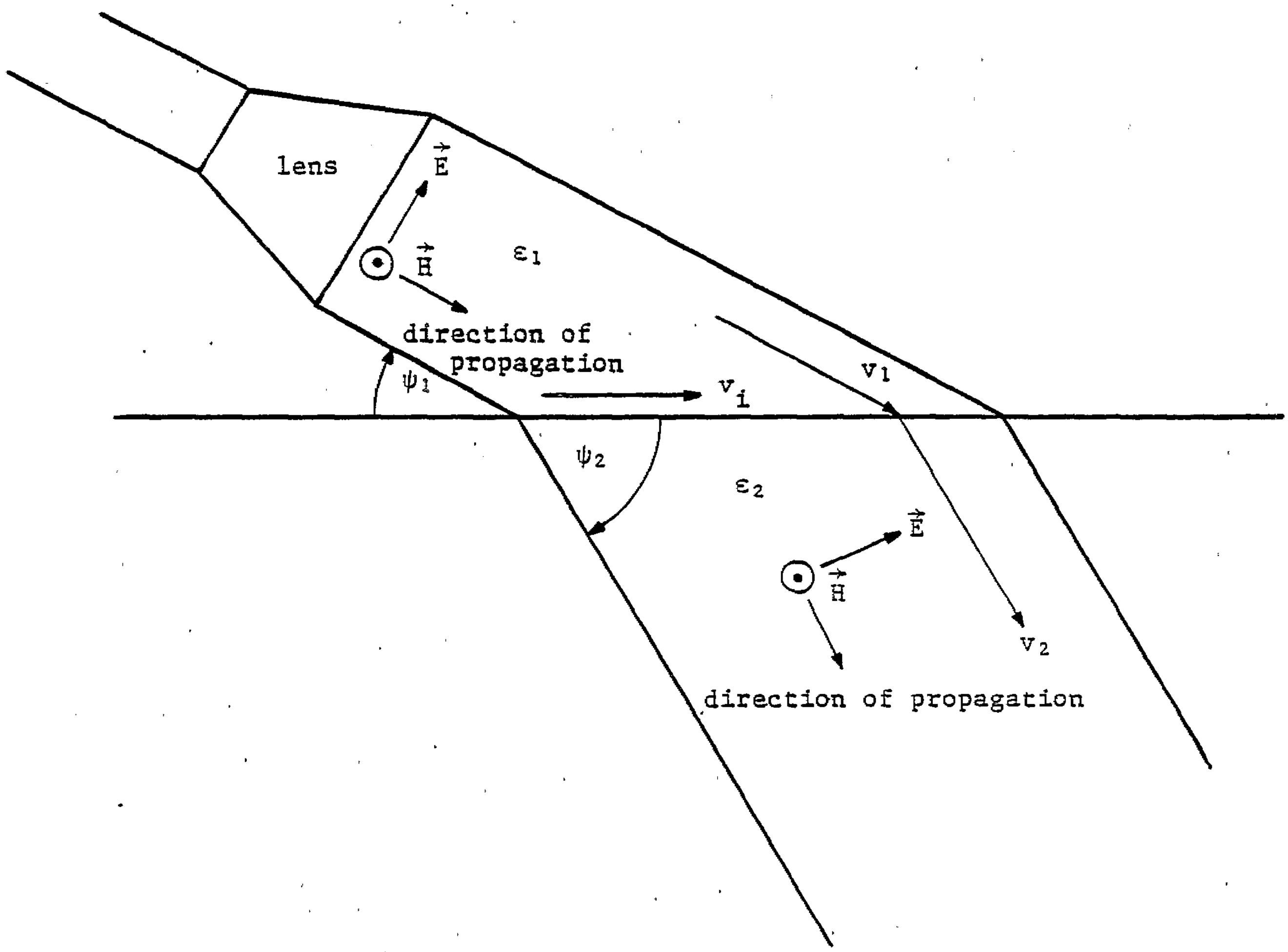

Figure 4.1: Use of Lens to Launch Wave for Non Normal Incidence at Interface Between Two Homogeneous Isotropic Dielectric Media

Let us consider some numerical examples. For example, if a relative value of ϵ_2 is specified, say $\epsilon_2 = 10\epsilon_0$, and if $v_i = c$, then

$$\begin{aligned}
\cos(\psi_2) &= \frac{v_2}{c} = \left(\frac{\epsilon_0}{\epsilon_2}\right)^{1/2} = \frac{1}{\sqrt{10}} \\
\sin(\psi_1) &= \frac{1}{\sqrt{10}} \\
\tan(\psi_1) &= \frac{1}{3}
\end{aligned} \tag{4.5}$$

and hence $\psi_1 = 71.6°$, $\psi_2 = 18.4°$, and $\epsilon_1 = \frac{10}{9}\epsilon_0$. On the other hand, if we choose $\epsilon_2 = 4\epsilon_0$, with $v_i = c$, then

$$\begin{aligned}
\cos(\psi_2) &= \frac{1}{2} \\
\sin(\psi_1) &= \frac{1}{2} \\
\tan(\psi_1) &= \frac{1}{\sqrt{3}}
\end{aligned} \tag{4.6}$$

and hence $\psi_1 = 30°$, $\psi_2 = 60°$, and $\epsilon_1 = \frac{4\epsilon_0}{3}$. The examples are included in Table 4.1 which shows that a reduction in the chosen value of ϵ_2, with $v_i = c$, will yield a slight increase in the value of ϵ_1, with $\epsilon_1 \geq \epsilon_0$. Thus in the Brewster angle case, a reduction in the chosen value of ϵ_2, with $v_i = c$, results in a slight increase in the value of ϵ_1, with $\epsilon_1 \geq \epsilon_0$.

Let us now summarize the implications of the above analysis. First of all, if we assume that our wave in the lower region is to have certain prescribed properties (i.e., non-normal propagation) so that its direction is governed by a known phase velocity v_i along the interface between the regions I and II, and that the permittivity of the lower region is specified, then the velocity v_2 and angle ψ_2 are determined. Secondly, if the upper region is a uniform medium, then (as discussed in Section 2) the impedance and transit-time requirements will lead to the Brewster angle condition in which the permittivity ϵ_1 of the upper region emerges as part of our solution. Thus the previous solution (Section 3) generalizes to the case of non-normal incidence through a simple rotation of coordinates

ϵ_2	$\cos(\psi_2)$	ψ_2	$\sin(\psi_1)$	ψ_1	ϵ_1
$10\epsilon_0$	$\frac{1}{\sqrt{10}}$	71.6°	$\frac{1}{\sqrt{10}}$	18.4°	$\frac{10\epsilon_0}{9}$
$9\epsilon_0$	$\frac{1}{3}$	70.5°	$\frac{1}{3}$	19.5°	$\frac{9\epsilon_0}{8}$
$6\epsilon_0$	$\frac{1}{\sqrt{6}}$	65.9°	$\frac{1}{\sqrt{6}}$	24.1°	$\frac{6\epsilon_0}{5}$
$4\epsilon_0$	$\frac{1}{2}$	60°	$\frac{1}{2}$	30°	$\frac{4\epsilon_0}{3}$

Table 4.1: Permittivities and Brewster Angles for the Case of $v_i = c$

by the Brewster angle. In this event we can now assume we have a plane wave in the upper region, and this wave is launched so that it is normally incident on an interface.

Thus we can construct, as shown in Figure 4.2, an array of lenses. The solid lines shown correspond to metal sheets, and common boundaries (indicated by solid lines with cross-hatching) can be dispensed with. The angles displayed correspond to a value of $\epsilon_2 = 4\epsilon_0$.

5 Summary

For the case of a plane wave propagating in the x-direction and normally incident on a boundary $x = x_2$ between media of different permittivities we have the lens geometry and medium specified by Equations (3.24) through (3.25). Note that as one goes away from the symmetry plane (Figure 3.2), the values of ϵ_n decrease. The contour plots of Figure 3.3 indicate the lens shape for various ϵ_n. Moreover, we cannot bring the lens down to a line source (see (3.28)). The case of normal incidence is then shown to be applicable, at least for a certain range of parameters, to the case of non-normal incidence.

: boundaries that can be removed without disturbing the wave

$\varepsilon_1 = \frac{4}{3}\varepsilon_o$	$\psi_1 = 30°$
$\varepsilon_2 = 4\varepsilon_o$	$\Psi_2 = 60°$

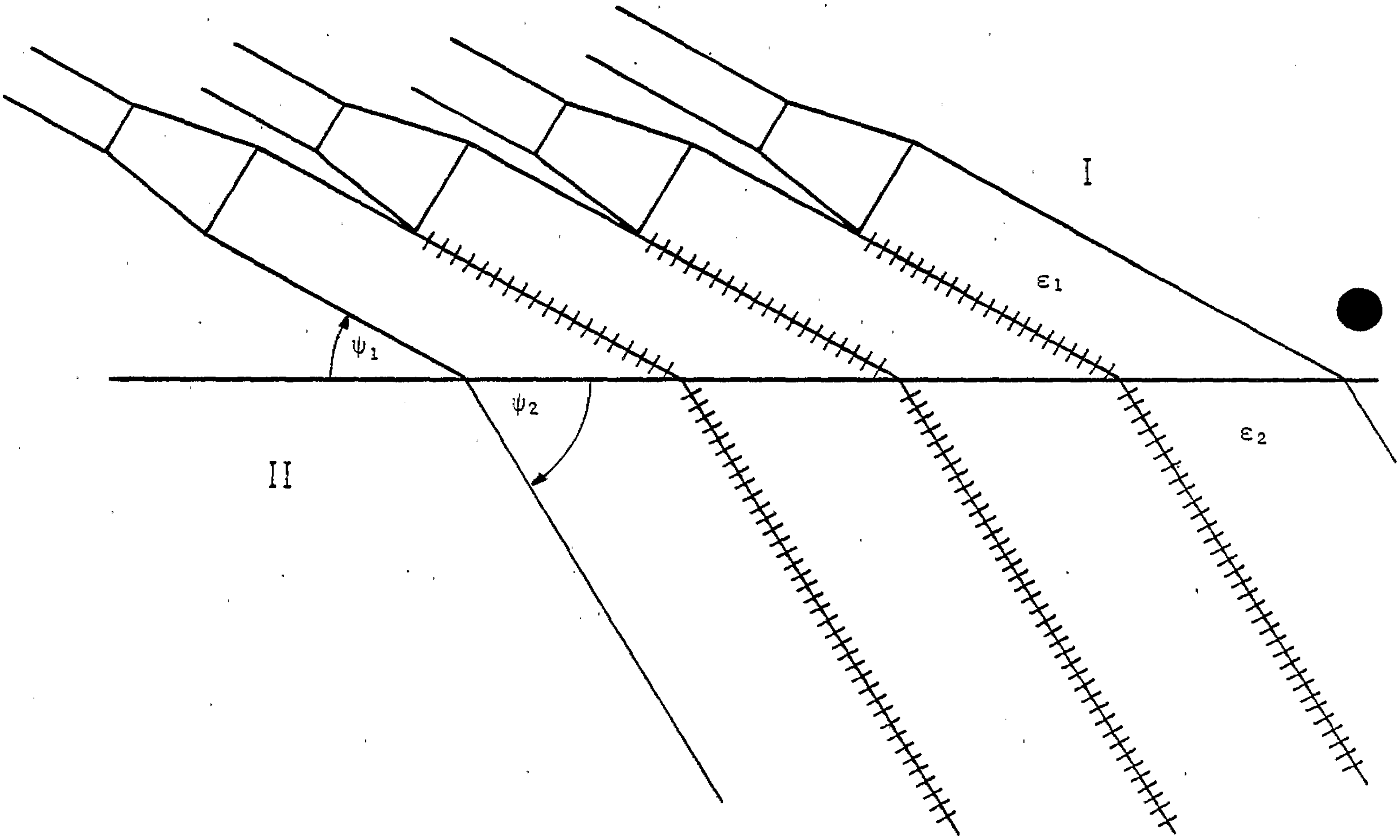

Figure 4.2: Lens Array for Launching Wave in One Uniform Isotropic Dielectric Medium for Propagation into Second Such Medium

References

[1] T. C. Mo, C. H. Papas, and C. E. Baum, "Differential geometry scaling method for electromagnetic field and its applications to coaxial waveguide junctions", SSN169, March 1973 (also as a paper by the same authors under the title "General scaling method for electromagnetic field with applications to a matching problem", J. Math. Physics, vol. 14, pp. 479-483, April 1973).

[2] C. E. Baum, "A scaling technique for the design of idealized electromagnetic lens", SSN64, August 1968, Kirtland Air Force Base, Albuquerque, New Mexico 87117.

[3] A. P. Stone and C. E. Baum, "An anisotropic lens for launching TEM waves on a conducting circular conical system," SSN285, June 1984, Kirtland Air Force Base, Albuquerque, New Mexico 87117 (also in shorter form in Electromagnetics, vol. 5, no. 1, pp. 35-62).

[4] J. D. Jackson, "Classical Electrodynamics", 2nd ed., J. Wiley & Sons, Inc., New York, 1975 (p. 282).