Sensor and Simulation Notes

Note 300

16 December 1986

Use of Modified Pole Series for Characterizing the Surface Response of Scatterers

Carl E. Baum
Air Force Weapons Laboratory

Abstract

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In efficiently characterizing the surface response of electromagnetic scatterers one often uses the singularity expansion method (SEM) whether from scattering calculations or from experimental data. This is particularly efficient at intermediate and lower frequencies (or intermediate and later times). This note discusses another technique to speed up convergence, particularly at lower frequencies. Defining a set of quasi-static modes, these can be incorporated into SEM via modified pole series.

singularity expansion method (SEM), scattering, magnetic fields

I. Introduction

In experimental characterization of the surface response of electromagnetic scatterers one often uses CW illumination in an anechoic chamber [2]. This data can in turn be used to obtain SEM parameters (natural frequencies, natural modes, and coupling coefficients) provided sufficient samples for surface position, direction of incidence, and polarization are available [9, 10, 11]. One also needs a sufficiently wide band of frequencies for this purpose.

At low frequencies (small compared to first natural frequency) quasi-static concepts apply. One can define a small set of quasi-static modes which are quite adequate for characterizing the response in this region of frequencies. One might then separately measure the quasi-static response, perhaps in a facility specially designed for this purpose [4].

The quasi-static modes can be combined with the SEM response to give what might be termed a modified pole series [5, 14]. This form of expansion is developed and discussed herein.

II. Quasi-Static Modes

As a first part of this development consider the response of some scatterer, approximated as perfectly conducting, at zero frequency. This leads to what is usually referred to as the static or quasi-static response. As a practical matter what is important is that the object dimensions be small compared to the radian wavelength λ of the electromagnetic fields. It is well known that the scattered far fields under such conditions are dominated by the induced dipole moments (electric and magnetic) through polarizability tensors [15 (section 1.4.1.4)]. Here however, we are concerned by the surface current and charge densities. Let us then construct an appropriate set of quasi-static modes for the surface-current and surface-charge densities. For this purpose let us consider the perfectly conducting scatterer with surface S and of finite size (contained within a sphere of finite radius) as illustrated in fig. 2.1. The outward-pointing unit normal vector is designated as $\vec{1}_S(\vec{r}_s)$ where $\vec{r}_s$ is a position on S.

A. Surface-charge-density modes

Consider the special problem that there is a quasi-static incident electric field in, say, the x direction. Let the surface-charge density be designated as $\rho_x^{(0)}(\vec{r}_s)$ in normalized form, i.e., if the x component of the incident electric field is E_x then

$$E_x \Rightarrow \varepsilon_0 E_x \rho_{s_x}^{(0)}(\vec{r}_s) \tag{2.1}$$

as the response surface-charge density. Similarly there are quasi-static modes for response to both y and z components of the incident electric field. Note that in this form the quasi-static surface-charge-density modes are dimensionless.

The surface-charge density at zero frequency (s = 0) is then written as

$$\tilde{\rho}_s(\vec{r}_s,0) = \varepsilon_0 \vec{E} \cdot \vec{\rho}_s^{(0)}(\vec{r}_s) \tag{2.2}$$

where the vector surface-charge-density mode is

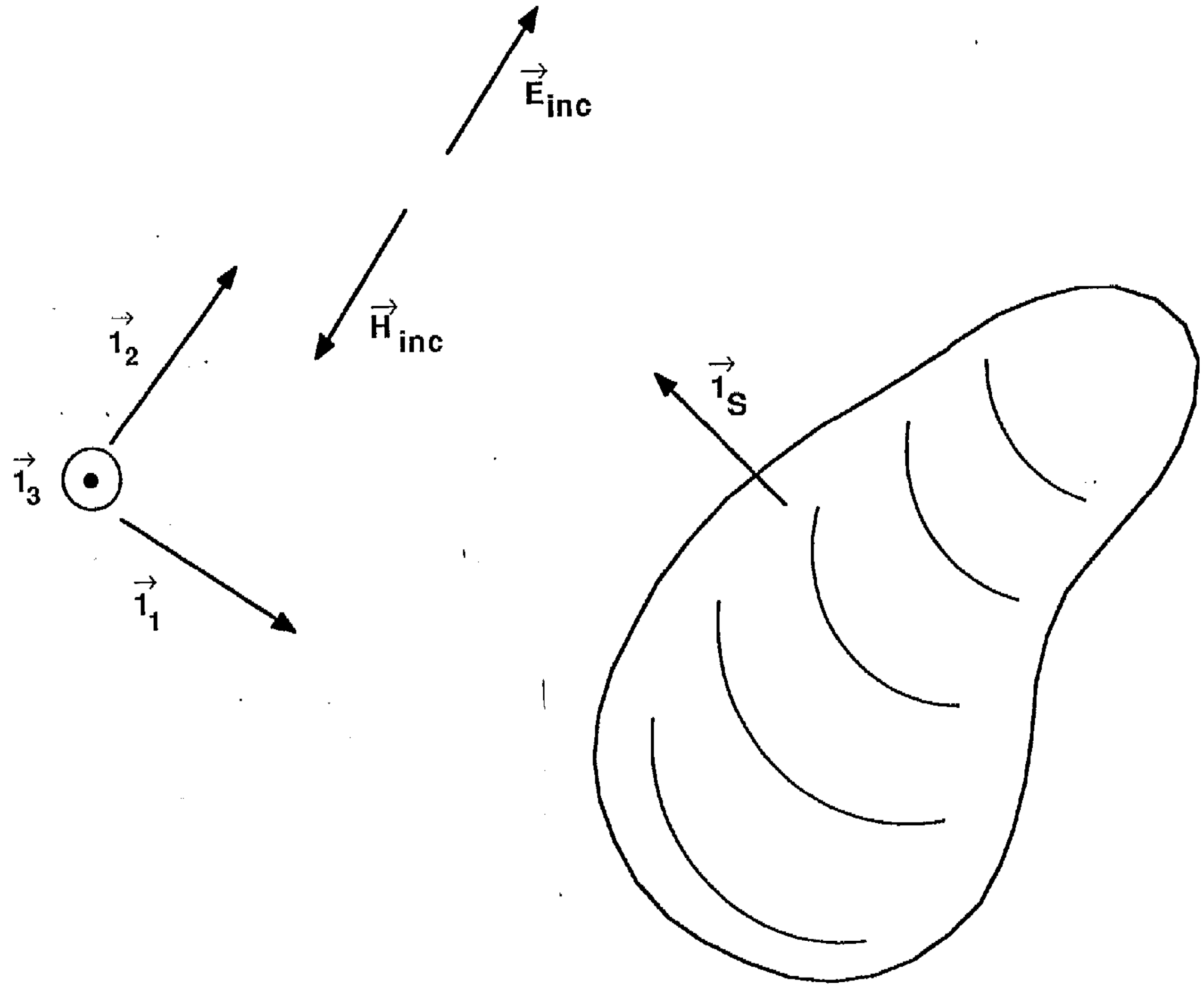

Figure 2.1 Perfectly-Conducting Finite-Sized Scatterer.

$$\vec{\rho}_s^{(0)}(\vec{r}_s) = \vec{1}_x \rho_{s_x}^{(0)}(\vec{r}_s) + \vec{1}_y \rho_{s_y}^{(0)}(\vec{r}_s) + \vec{1}_z \rho_{s_z}^{(0)}(\vec{r}_s) \tag{2.3}$$

The direction of the quasi-static incident electric field is designated by the unit vector $\vec{1}_e$, so that

$$\vec{E} = E_0 \vec{1}_e \tag{2.4}$$

Then the zero-freauency surface-charge density is

$$\rho_s(\vec{r}_s, 0) = \varepsilon_0 E_0 \vec{1}_e \cdot \vec{\rho}_s^{(0)}(\vec{r}_s) \tag{2.5}$$

Note that $\rho_x^{(0)}$, $\rho_y^{(0)}$, $\rho_z^{(0)}$ can all be separately calculated and/or measured in appropriate experiments. If the scatterer of interest has various symmetry properties, these can be used to define the center of coordinates and align the coordinate axes to cast the quasi-static surface-charge-density modes into some simplest canonical form.

B. Surface-current-density modes

Let there now be a quasi-static magnetic field in, say, the x direction. Let the surface-current density be designated as $\vec{j}_{s_x}^{(0)}(\vec{r}_s)$ in normalized form, i.e. if the x component of the incident magnetic field is H_x then

$$H_x \Rightarrow H_x \vec{j}_{s_x}^{(0)}(\vec{r}_s) \tag{2.6}$$

as the response surface-current density. Similarly there are quasi-static modes for response to both y and z components of the incident magnetic field. Again the surface-current-density modes are dimensionless.

The surface-current density at zero frequency is then written as

$$\tilde{\vec{J}}_s(\vec{r}_s, 0) = \vec{H} \cdot \overleftrightarrow{j}_s^{(0)}(\vec{r}_s) \tag{2.7}$$

where the dyadic surface-current-density mode is

$$\overleftrightarrow{j}_s^{(0)}(\vec{r}_s) = \vec{1}_x \vec{j}_{s_x}^{(0)}(\vec{r}_s) + \vec{1}_y \vec{j}_{s_y}^{(0)}(\vec{r}_s) + \vec{1}_z \vec{j}_{s_z}^{(0)}(\vec{r}_s) \tag{2.8}$$

The direction of the quasi-static incident magnetic field is designated by the unit vector $\vec{1}_h$, so that

$$\vec{H} = H_o \vec{1}_h \tag{2.9}$$

Then the zero-frequency surface-current density is

$$\tilde{\vec{J}}_s(\vec{r}_s,0) = H_o \vec{1}_h \cdot \overleftrightarrow{j}_s^{(0)}(\vec{r}_s) \tag{2.10}$$

An alternate formulation is to consider the surface magnetic field and define modes as

$$\vec{h}_{\substack{x\\y\\z}}^{(0)}(\vec{r}_s) = -\vec{1}_S(\vec{r}_s) \times \vec{j}_{s_{\substack{x\\y\\z}}}^{(0)}(\vec{r}_s), \quad \vec{j}_{s_{\substack{x\\y\\z}}}^{(0)}(\vec{r}_s) = \vec{1}_S(\vec{r}_s) \times \vec{h}_{\substack{x\\y\\z}}^{(0)}(\vec{r}_s) \tag{2.11}$$

where $\vec{1}_S$ is the unit normal vector (outward pointing) on S as in fig. 2.1. Defining

$$\overleftrightarrow{h}^{(0)}(\vec{r}_s) = \vec{1}_x \vec{h}_x^{(0)}(\vec{r}_s) + \vec{1}_y \vec{h}_y^{(0)}(\vec{r}_s) + \vec{1}_z \vec{h}_z^{(0)}(\vec{r}_s) \tag{2.12}$$

we have

$$\overleftrightarrow{h}^{(0)}(\vec{r}_s) = -\vec{1}_S(\vec{r}_s) \times \overleftrightarrow{j}_s^{(0)}(\vec{r}_s), \quad \overleftrightarrow{j}_s^{(0)}(\vec{r}_s) = \vec{1}_S(\vec{r}_s) \times \overleftrightarrow{h}^{(0)}(\vec{r}_s) \tag{2.13}$$

In this form the quasi-static surface magnetic field is

$$\tilde{\vec{H}}(\vec{r}_s,0) = H_o \vec{1}_h \cdot \overleftrightarrow{h}^{(0)}(\vec{r}_s) \tag{2.14}$$

where the magnetic field is defined just outside S.

Note that one should be careful concerning $\vec{J}_s$. If S is degenerate in the sense that some portion is collapsed so that both sides are "outside" then (2.11) must be interpreted as for the surface-current density on one side, not the sheet current density involving both sides.

C. Some comments

While one is typically interested in the response of the scatterer to an incident plane wave, the results of this section do not require such. In a plane wave $\vec{1}_e$ and $\vec{1}_h$ are orthogonal, but these results are more general.

On S the equation of continuity is

$$\nabla_s \cdot \tilde{\vec{J}}_s(\vec{r}_s, s) = -s\tilde{\rho}_s(\vec{r}_s, s) \tag{2.15}$$

With $\tilde{\rho}_s$ bounded as the complex frequency $s \to 0$ then

$$\nabla_s \cdot \tilde{\vec{J}}_s(\vec{r}_s, 0) = 0 \tag{2.16}$$

implying

$$\nabla_s \cdot \vec{j}^{(0)}_{s_{\substack{x\\y\\z}}}(\vec{r}_s) = 0 \tag{2.17}$$

Thus the quasi-static surface-current-density modes are divergenceless or solenoidal, and thus not in effect the same modes as the quasi-static surface-charge-density modes.

In a previous paper [3] the question of possibly dividing the surface current density according to the surface divergence (giving ρ_s) and the normal component of the surface curl (giving equivalent magnetic current density k) has been considered. This has been suggested as a means of categorizing eigenmodes and natural modes [9]. In the present context since the quasi-static surface-current-density modes have zero divergence and are thus magnetic modes with non-zero normal surface curl. The surface-current-density modes corresponding to non-zero $\tilde{\rho}_s(\vec{r}_s, 0)$ must have zero amplitude at $s = 0$ and are not the quasi-static surface-current-density modes considered here. Viewed another way the quasi-static incident electric and magnetic fields are independent of each other, and so the amplitudes of the $\rho_s^{(0)}$ modes and $\vec{j}_s^{(0)}$ modes are unrelated to one another.

III. Modified Pole Series

In order to assure a more rapid convergence of an SEM expansion (in terms of poles) at low frequencies it may be advantageous in some cases to first pull out the zero-frequency (quasi-static) term, which is of course the exact result at zero frequency. If $\tilde{g}(s)$ is a meromorphic function then we may expand it as

$$\tilde{g}(s) = \sum_{\alpha} R_{\alpha}(s - s_{\alpha})^{-1} + \text{entire function} \tag{3.1}$$

where only first-order poles are assumed for this simple analysis.

Now an alternate representation, assuming $\tilde{g}(s)$ is analytic near $s = 0$, is

$$\tilde{g}(s) = \tilde{g}(0) + \sum_{\alpha} R_{\alpha}[(s - s_{\alpha})^{-1} + s_{\alpha}^{-1}] + \text{entire function} \tag{3.2}$$

In this form any remaining entire function must also be zero at $s = 0$. Note that the inverse transform of $\tilde{g}(0)$ (a constant) is just a delta function $\tilde{g}(0)\delta(t)$, so such a term has been removed from the remaining entire function.

This form of an SEM representation has proven useful in representing the input admittance (with $\tilde{g}(0) = 0$) of a thin-wire antenna [1, 7, 8]. In general this is a very useful form for synthesizing equivalent circuits in which the elementary circuit modules corresponding to poles should have either zero admittance or zero impedance at $s = 0$ [5, 14]. Note that (3.2) is not appropriate if there is a pole at $s = 0$. As discussed in [5, 14] there are simple asymptotic forms of admittance and impedance of various types of finite-size antennas (in free space) limited to a positive constant times s^{-1}, 1, or s. Similar behavior applies near $s = \infty$, being just a positive constant in typical cases. This severely restricts any allowable entire function for the case of antenna impedance or admittance.

A more general form of (3.2) involves a time shift as [6, 13, 14]

$$\tilde{g}(s) = e^{-st_0}\tilde{g}(0) + \sum_{\alpha} e^{(s_\alpha - s)t_0} R_\alpha[(s - s_\alpha)^{-1} + s_\alpha^{-1}]$$

$$+ \text{ entire function} \tag{3.3}$$

This form can also be applied in (3.1) without the $\tilde{g}(0)$ term pulled out. In time domain (3.3) gives

$$g(t) = \tilde{g}(0)\,\delta(t - t_0) + \sum_{\alpha} e^{s_\alpha t_0} R_\alpha[e^{s_\alpha(t - t_0)} u(t - t_0) + s_\alpha^{-1}\delta(t - t_0)]$$

$$+ \text{ entire function (transformed)}$$

$$t_0 \equiv \text{ turn-on time} \tag{3.4}$$

Note that in this form the residues have not changed since if $s = s_\alpha$ the corresponding exponential is one.

This shifted form is not appropriate for input admittance or impedance since passivity requires that current and voltage begin simultaneously at the port. However, it is quite appropriate for representing the response of a scatterer to an incident field. Clearly t_0 should be at least as early as the response (say surface current density) begins, so as to minimize the necessity for an additional entire-function contribution. Note, of course, that $\tilde{g}(s)$ can be a vector or dyadic function, and depend on spatial coordinates as well as s (or t). In this context t_0 can depend on coordinates on S. While this establishes how late we might choose t_0 there is another question concerning how early we might choose t_0; this question is discussed in [12] based on the minimum t_0 allowed such that the pole series converges to a finite value for all times. If in addition we require t_0 to be chosen independent of angle of incidence and polarization, then one is led to a concept of a minimum circumscribing sphere containing the scatterer, the center of this sphere being the origin of coordinates, or the position the leading edge of the incident wave passes at t = 0 [12]. In this context t_0 is -a/c where a is the radius of this sphere. However, this result is limited to a class of scatterers for which 2a is the maximum linear dimension of the scatterer. While this establishes some limits on t_0 it does not in itself necessarily eliminate an additional entire function.

IV. The Incident Wave

In section 2 it was pointed out that the quasi-static modes were excited in proportion to the quasi-static electric and magnetic fields (for surface charge denisty and surface current density respectively). This is quite general for various forms of incident fields.

For various applications the incident fields are in the form of a plane wave. Referring to fig. 2.1 we have a set of orthogonal unit vertors as

$$\vec{1}_1 \times \vec{1}_2 = \vec{1}_3\,, \quad \vec{1}_2 \times \vec{1}_3 = \vec{1}_1\,, \quad \vec{1}_3 \times \vec{1}_1 = \vec{1}_2 \tag{4.1}$$

where

$$\begin{aligned} \vec{1}_1 &\equiv \text{direction of incidence} \\ \vec{1}_p &= \vec{1}_2 \text{ or } \vec{1}_3 \text{ (or some combination)} \\ &\equiv \text{direction of polarization} \\ &\quad\text{(of the incident electric field)} \end{aligned} \tag{4.2}$$

An incident plane wave then takes the form

$$\begin{aligned} \vec{E}^{(inc)}(\vec{r},t) &= E_2 f_2(t - \vec{1}_1\cdot\vec{r}/c)\,\vec{1}_2 + E_3 f_3(t - \vec{1}_1\cdot\vec{r}/c)\,\vec{1}_3 \\ \vec{H}^{(inc)}(\vec{r},t) &= \vec{1}_1 \times \vec{E}^{(inc)}(\vec{r},t) \\ &= \frac{1}{Z_o}\left[E_2 f_2(t - \vec{1}_1\cdot\vec{r}/c)\,\vec{1}_3 - E_3 f_3(t - \vec{1}_1\cdot\vec{r}/c)\,\vec{1}_2\right] \end{aligned} \tag{4.3}$$

or in frequency domain

$$\begin{aligned} \tilde{\vec{E}}^{(inc)}(\vec{r},s) &= E_2 e^{-s\vec{1}_1\cdot\vec{r}/c}\,\tilde{f}_2(s)\vec{1}_2 + E_3 e^{-s\vec{1}_1\cdot\vec{r}/c}\,\tilde{f}_3(s)\vec{1}_3 \\ \tilde{\vec{H}}^{(inc)}(\vec{r},s) &= \frac{1}{Z_o}\left[E_2 e^{-s\vec{1}_1\cdot\vec{r}/c}\,\tilde{f}_2(s)\,\vec{1}_3 - E_3 e^{-s\vec{1}_1\cdot\vec{r}/c}\,\tilde{f}_3(s)\,\vec{1}_2\right] \end{aligned} \tag{4.4}$$

where f_2 and f_3 are waveforms and E_2 and E_3 are scaling constants with dimensions V/m.

Note that as $s \to 0$ the case of a plane wave gives

$$\tilde{\vec{E}}^{(inc)}(\vec{0}, 0) = E_2 \tilde{f}_2(0)\vec{1}_2 + E_3 \tilde{f}_3(0)\vec{1}_3$$
$$= E_0 \vec{1}_e$$

$$\tilde{\vec{H}}^{(inc)}(\vec{0}, 0) = \frac{1}{Z_0}[E_2 \tilde{f}_2(0)\vec{1}_3 - E_3 \tilde{f}_3(0)\vec{1}_2]$$
$$= H_0 \vec{1}_h \qquad (4.5)$$

From this $\vec{1}_e$ and $\vec{1}_h$ can be determined. In an experimental situation the incident wave can be controlled such that, say

$$\vec{1}_e = \vec{1}_2, \; E_3 = 0, \; E_0 = E_2 \tilde{f}_2(0)$$

$$\vec{1}_h = \vec{1}_3, \; H_0 = \frac{E_2}{Z_0} f_2(0) = \frac{E_0}{Z_0} \qquad (4.6)$$

Thus the quasi-static fields in section 2 can be easily related to some assumed plane wave.

V. Response on Scatterer Surface

A. Surface-current-density representation

Applying the modified pole series as in (3.3) to the surface-current-density representation using class-1 coupling coefficients gives [13].

$$\tilde{\vec{J}}_s(\vec{r}_s,s) = H_o\vec{1}_h \cdot \vec{\vec{j}}_s^{(0)}(\vec{r}_s)$$

$$+ \sum_{\alpha,p} \tilde{F}_p(s_\alpha)\, N_\alpha(\vec{1}_1,\vec{1}_p)\, \vec{j}_{s_\alpha}(\vec{r}_s)\, e^{(s_\alpha - s)\,t_o} \{[s - s_\alpha]^{-1} + s_\alpha^{-1}\} \tag{5.1}$$

$$\eta_\alpha(\vec{1}_1,\vec{1}_p) = \frac{\langle \vec{j}s_\alpha(\vec{r}_s);\ \vec{E}^{(inc)}(\vec{r}_s \cdot s_\alpha)\rangle}{\langle \vec{j}_{s_\alpha}(\vec{r}_s);\ \frac{\partial}{\partial s}\tilde{\vec{\vec{Z}}}(\vec{r}_s,\vec{r}_s^{\,'};s)\Big|_{s=s_\alpha};\ \vec{j}_{s_\alpha}(\vec{r}_s^{\,'})\rangle}$$

The detailed formula for calculating the coupling coefficient from an integral eauation (as above) is not needed if one is determining η_α from empirical data, as in an anechoic chamber [2]. In this case the η_α are merely empirical parameters, as is t_o, the turn-on time for an optimal fit to the data which may involve various $\vec{1}_1$, $\vec{1}_p$, and $\vec{r}_s$ samples [9].

This form can also be carried into time domain as in (3.4).

B. Surface-charge-density natural modes

From the continuity equation (2.15) surface-charge-density natural modes can be related to surface-current-density natural modes via

$$\rho_{s_\alpha}(\vec{r}_s) = -\frac{c}{s_\alpha}\nabla_s \cdot \vec{j}_{s_\alpha}(\vec{r}_s) \tag{5.2}$$

Including the speed of light c the ρ_{s_α} modes have the same dimensions as the $\vec{j}_{s_\alpha}$ modes. As discussed in [3] not all $\vec{j}_{s_\alpha}$ modes need have associated ρ_{s_α} modes, but may be basically H modes with a non-zero normal component of the surface curl. This parameter (k_s) could also be expanded in natural modes as above.

C. Surface-charge-density representation

Again applying the modified pole series as in (3.3) to the surface-charge-density representation gives

$$\tilde{\rho}_s(\vec{r}_s,s) = \epsilon_0 E_0 \vec{1}_e \cdot \vec{\rho}_s^{\,(0)}(\vec{r}_s)$$

$$+ \sum_{\alpha,p} \tilde{f}_p(s_\alpha)\eta_\alpha(\vec{1}_1,\vec{1}_p)\,\frac{1}{c}\,\rho_{s_\alpha}(\vec{r}_s)\; e^{(s_\alpha - s)t_0}\{[s - s_\alpha]^{-1} + s_\alpha^{-1}\} \qquad (5.3)$$

Note that while the quasi-static part is quite separate from the corresponding part of the surface current density, the poles are quite related between current and charge via the continuity equation (2.15).

Similarly this form can be carried into time domain as in (3.4).

VI. Some Comments

In fitting data on the surface response of scatterers in frequency domain ($s = j\omega$) one is often confronted with limited frequency ranges. If the region of concern is the resonance region and lower, then one would like to tailor the representation to most efficiently fit this. The quasi-static response might be measured separately [4]. This can in turn be combined with more conventional data (as from an anechoic chamber [2]). At least at the lower frequencies this should be an efficient procedure since this type of modified pole series is exact as $s \to 0$.

As one goes up in frequency, successive poles enter into the data. There is still the question of t_0. This is a complex question theoretically [12]. However, t_0 is another parameter which can be chosen for a best fit to the data. One should note, however, that the best choice for one set of excitation conditions is not necessarily the best choice for another such set. One would like to choose t_0 based on some optimum coordinate center, and independent of observation position and direction of incidence to the extent feasible.

While this is an efficient representation in frequency domain, it is not necessarily optimal in time domain. As in (3.4) there is a series of delta functions to contend with. However, one can use the SEM parameters obtained from using the present representation in any other SEM representation (say, the usual poles) as seems most accurate for the problem at hand.

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