Sensor and Simulation Notes

Note 321

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Radiation of Impulse-Like Transient Fields

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Abstract

This paper considers the radiation of narrow pulses (impulse-like) from nnas. Beginning with the aperture formulation for antennas, the properties of a pulse radiated from such an aperture focused at infinity are discussed. This is then specialized to the case of a conical TEM feed structure combined with a reflector.

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aperture antennas, pulsed radiation

I. Introduction

The radiation of a narrow beam of electromagnetic fields at high frequencies is an important problem in antenna theory. Various types of antennas involving reflectors, lenses, phased arrays, etc., have been used for this purpose. Considering the radiation of transient pulses a new set of problems is encountered due to the broad band of frequencies involved, at least for some kinds of transients. In this paper the desired radiated pulse is a narrow impulse, subject to the usual restriction on the radiation of low frequencies by finite size antennas with finite energy available.

Beginning with an aperture formulation for the antenna radiation, the tangential electric field on the aperture is specified as a plane wave appropriate to focusing the aperture at ∞ in some specified direction $\vec{1}_o$. For a step-function aperture illumination this leads to an impulse-like waveform, the width of which becomes very narrow as the observer direction $\vec{1}_r$ approaches $\vec{1}_o$. However, the aperture model is limited in its application to the low-frequency behavior of certain kinds of antennas (such as lenses and reflectors).

An appropriate type of antenna for radiating such a pulse is a parabolic reflector fed by a conical TEM wave launcher. Such a launcher supports a step-like TEM wave on two or more conical conductors leading from some apex to the edge of the reflector. The waveform radiated from such an antenna is discussed, including some effects of the feed.

II. Step Response of an Aperture Antenna Focused at Infinity in the Far-Field Approximation

Consider an antenna as modeled by an aperture on the z=0 plane (S) as shown in Figure 2.1. Let the source coordinates on this plane be x′, y′ with the specified tangential electric field

$$\vec{E}_t(x',y';t) = \overleftrightarrow{1}_z \cdot \vec{E}(x',y',0;t) = \left(E_x(x',y',0;t),\ E_y(x',y',0;t),\ 0\right)$$

$$\overleftrightarrow{1}_z \equiv \overleftrightarrow{1} - \vec{1}_z\,\vec{1}_z\ ,\ \overleftrightarrow{1} = \vec{1}_x\,\vec{1}_x + \vec{1}_y\,\vec{1}_y + \vec{1}_z\,\vec{1}_z \tag{2.1}$$

We have various definitions

$$R \equiv |\vec{r}-\vec{r}\,'| = \left[(x-x')^2 + (y-y')^2 + z^2\right]^{\frac{1}{2}}$$

$$\vec{1}_R \equiv \frac{\vec{r}-\vec{r}\,'}{R} = \frac{x-x'}{R}\,\vec{1}_x + \frac{y-y'}{R}\,\vec{1}_y + \frac{z}{R}\,\vec{1}_z$$

$$\vec{r}\,' \equiv (x',y',0) \equiv \text{source coordinates}$$

$$\vec{r} \equiv (x,y,z) \equiv \text{observer coordinates}$$

$$s \equiv \text{Laplace-transform (two-sided) variable} \equiv \text{complex frequency}$$

$$\gamma \equiv \frac{s}{c} \equiv \text{free-space propagation constant} \tag{2.2}$$

$$\sim\ \equiv \text{Laplace transform (two-sided)}$$

$$c \equiv \frac{1}{\sqrt{\mu_o \epsilon_o}} \equiv \text{speed of light}$$

$$Z_o \equiv \sqrt{\frac{\mu_o}{\epsilon_o}} \equiv \text{free-space wave impedance}$$

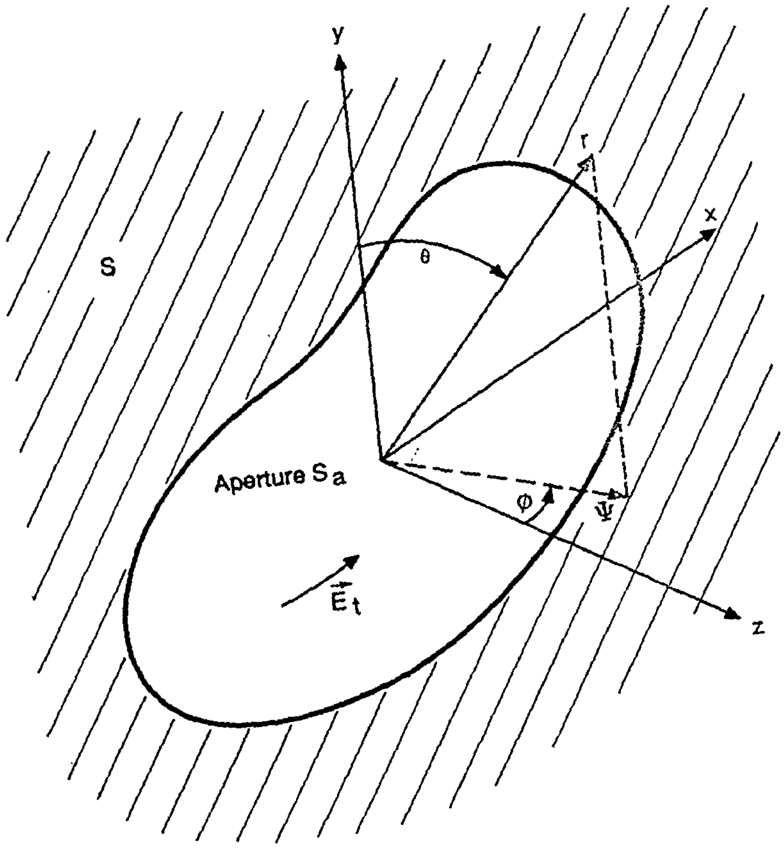

Figure 2.1. Aperture Antenna with Specified Tangential Electric Field

Then as developed in [10,18] the fields for z > 0 are

$$\tilde{\vec{E}}(\vec{r},s) = \frac{1}{2\pi} \int_S \frac{\gamma R+1}{R^2} \left[\left[\vec{1}_z \times \tilde{\vec{E}}_t(\vec{r}\,',s)\right] \times \vec{1}_R\right] e^{-\gamma R}\, dS' \tag{2.3}$$

$$\tilde{\vec{H}}(\vec{r},s) = \frac{-1}{2\pi\mu_o s} \left\{\nabla \times \int_{S'} \frac{\gamma R+1}{R^2} \left[\left[\vec{1}_z \times \tilde{\vec{E}}_t(\vec{r}\,',s)\right] \times \vec{1}_R\right] e^{-\gamma R}\, dS'\right\}$$

In time domain one has the formulas formed by replacing

Laplace (complex frequency) domain <----> time domain

$$\begin{array}{ccc} e^{-\gamma R}\ \tilde{\vec{E}}_t(\vec{r}\,',s) & \longleftrightarrow & \vec{E}_t\left(\vec{r}\,',\ t - \frac{R}{c}\right) \\ s & \longleftrightarrow & \frac{\partial}{\partial t} \\ s^{-1} & \longleftrightarrow & \int dt \end{array} \tag{2.4}$$

The reader can note that related problems can also be formulated in terms of currents on the aperture [13].

In a previous paper [10] our concern was maximizing the fields at a given frequency and particular observer position ($\vec{r}_o$) with a specified maximum magnitude of the tangential electric field on S. This leads to the concept of an aperture with a uniform magnitude, single direction tangential electric field focused at the single position $\vec{r}_o$. Here our emphasis is quite different.

Consider the far fields defined by letting $r \to \infty$ with $|s|$ bounded (i.e. not allowed to go to ∞). Then introduce the coordinates (as in Figure 2.1).

$$\begin{array}{ll} x = \Psi \cos(\phi) & , \quad y = \Psi \sin(\phi) \\ z = r \cos(\theta) & , \quad \Psi = r \sin(\theta) \\ x = r \sin(\theta) \cos(\phi) & , \quad y = r \sin(\theta) \sin(\phi) \end{array} \tag{2.5}$$

Let us define some preferred direction $\vec{1}_o$ for radiating fields from the aperture. This also specifies angles θ_o, ϕ_o with

$$\vec{1}_r(\theta,\phi) = \frac{\vec{r}}{r} = \vec{1}_x \sin(\theta)\cos(\phi) + \vec{1}_y \sin(\theta)\sin(\phi) + \vec{1}_z \cos(\theta) \tag{2.6}$$

$$\vec{1}_o \equiv \vec{1}_r(\theta_o,\phi_o) = \vec{1}_x \sin(\theta_o)\cos(\phi_o) + \vec{1}_y \sin(\theta_o)\sin(\phi_o) + \vec{1}_z \cos(\theta_o)$$

Let the aperture be focused at ∞ in the direction $\vec{1}_o$. As in [10] let $\vec{r}_o$ be defined as $r\vec{1}_o$ with now $r \to \infty$, so we set

$$\begin{aligned} \vec{E}_t(\vec{r}',s) &\equiv e^{-\gamma\vec{1}_o\cdot\vec{r}'} E_o \tilde{f}(s)\, \vec{g}(\vec{r}') \quad \text{as} \quad r \to \infty \\ \tilde{f}(s) &\equiv \text{waveform on aperture} \\ \vec{g}(\vec{r}) &\equiv \text{aperture spatial distribution} \end{aligned} \tag{2.7}$$

In the direction $\vec{1}_o$ every elementary part of the aperture is phased so as to add constructively at ∞. For other directions $\vec{1}_r$ the effect in time domain will be to spread out the pulse. Note the separation into a spatial factor $\vec{g}(\vec{r}')$ (real), a frequency function $\tilde{f}(s)$ and a scale factor E_o (real).

In (2.3) we now have for $r \to \infty$

$$R = r - \vec{1}_r \cdot \vec{r}' + O(r^{-1})$$

$$\vec{1}_R = \vec{1}_r + O(r^{-1})$$

$$\left[\vec{1}_z \times \tilde{\vec{E}}_t(\vec{r}',s)\right] \times \vec{1}_R = \left[\left(\vec{1}_z \cdot \vec{1}_r\right) \overleftrightarrow{1} - \vec{1}_z\, \vec{1}_r\right] \cdot \tilde{\vec{E}}_t(\vec{r}',s) + O(r^{-1})$$

$$\tilde{\vec{E}}(\vec{r},s) = \tilde{\vec{E}}_f(\vec{r},s) + O\left(\frac{e^{-\gamma r}}{r^2}\right) \tag{2.8}$$

$$\tilde{\vec{E}}_f(\vec{r},s) = \frac{se^{-\gamma r}}{2\pi c r} \left[\left(1_z \cdot \vec{1}_r\right) \overleftrightarrow{1} - \vec{1}_z\, \vec{1}_r\right] \cdot \int_S e^{\gamma\vec{1}_r\cdot\vec{r}'}\, \tilde{\vec{E}}_t(\vec{r}',s)\, dS'$$

$$\tilde{\vec{H}}(\vec{r},s) = \tilde{\vec{H}}_f(\vec{r},s) + O\left(\frac{e^{-\gamma r}}{r^2}\right)$$

$$\tilde{\vec{H}}_f(\vec{r},s) = \frac{1}{Z_o} \vec{1}_r \times \tilde{\vec{E}}_f(\vec{r},s)$$

Note that the asymptotic expansion for large r is not uniform with respect to s, particularly if we take $\vec{1}_r$ as $\vec{1}_o$ and let $|s| \to \infty$. This can be treated as a special case.

Considering $\tilde{\vec{E}}_f$ this can be conveniently decomposed after applying the form in (2.7) as

$$\tilde{\vec{E}}_f(\vec{r},s) = e^{-\gamma r} \frac{E_o A}{2\pi c r} \tilde{\vec{F}}\left(\vec{1}_r, s\right)$$

$$A \equiv \text{aperture area} \tag{2.9}$$

$$\tilde{\vec{F}}\left(\vec{1}_r, s\right) \equiv s\tilde{f}(s)\, \tilde{\vec{F}}_a\left(\vec{1}_r, s\right) \equiv \text{far-field waveform function}$$

$$\tilde{\vec{F}}_a\left(\vec{1}_r, s\right) \equiv \frac{1}{A}\left[\left(\vec{1}_z \cdot \vec{1}_r\right) \overset{\leftrightarrow}{1} - \vec{1}_z\, \vec{1}_r\right] \cdot \int_S e^{\gamma[\vec{1}_r - \vec{1}_o]\cdot\vec{r}\,'}\, \vec{g}(\vec{r}\,')\, dS'$$

$$\equiv \text{aperture function}$$

Note that if $\tilde{f}(s)$ is $1/s$, (i.e. a unit step in time domain) then the aperture function is like a delta function in time, except spread out by a factor depending on the nearness of $\vec{1}_r$ to $\vec{1}_o$.

Consider now the simple case that the aperture spatial distribution is taken as uniform over the aperture (S_a) of finite size with a constant polarization $\vec{1}_a$ (real unit vector) as

$$\vec{g}(\vec{r}\,') = \begin{Bmatrix} \vec{1}_a & \text{for} & \vec{r}\,' \in S_a \\ 0 & \text{for} & \vec{r}\,' \notin S_a \end{Bmatrix}$$

$$\vec{1}_a \cdot \vec{1}_z = 0 \tag{2.10}$$

$$\int_S \vec{g}(\vec{r}\,')\, dS' = A\, \vec{1}_a$$

The aperture function then reduces to

$$\tilde{\vec{F}}_a\left(\vec{1}_r,s\right) = \left[\left(\vec{1}_z \cdot \vec{1}_r\right)\vec{1}_a - \left(\vec{1}_r \cdot \vec{1}_a\right)\vec{1}_z\right]\tilde{f}_a\left(\vec{1}_r,s\right)$$

$$= -\vec{1}_r \times \left[\vec{1}_z \times \vec{1}_a\right]\tilde{f}_a\left(\vec{1}_r,s\right)$$

$$\tilde{f}_a\left(\vec{1}_r,s\right) \equiv \frac{1}{A}\int_{S_a} e^{\gamma[\vec{1}_r-\vec{1}_o]\cdot\vec{r}'}\,dS' \tag{2.11}$$

$\equiv$ scalar aperture function

We have the special case (focus at the observer)

$$\vec{1}_r = \vec{1}_o$$

$$\tilde{f}_a\left(\vec{1}_r,s\right) = \frac{1}{A}\int_{S_a} e^{\gamma[\vec{1}_r-\vec{1}_o]\cdot\vec{r}'}\,dS' = 1 \tag{2.12}$$

$$\left[\left(\vec{1}_z \cdot \vec{1}_r\right)\vec{1}_a - \left(\vec{1}_r \cdot \vec{1}_a\right)\vec{1}_z\right] = -\vec{1}_r \times \left[\vec{1}_z \times \vec{1}_a\right]$$

$\equiv$ angular function (including polarization)

In this special case the waveform in (2.9) has the form $s\tilde{f}(s)$, i.e. a time differentiator. For example if $\tilde{f}(s)$ is taken as $1/s$ or a unit step in time domain, this special case then gives a delta function for the far field. As discussed later this is not a true delta function (which would imply infinite energy flux density) but has some non-zero width.

Extending the analysis let the aperture be circular (radius a) and introduce cylindrical coordinates on the aperture

$$dS' = dx'\,dy' = \Psi'\,d\Psi'\,d\phi' \tag{2.13}$$

Then we have

$$\tilde{f}_a\left(\vec{1}_r, s\right) = \frac{1}{\pi a^2} \int_o^a \int_o^{2\pi} e^{\gamma\Psi' [\vec{1}_r - \vec{1}_o] \cdot \vec{1}_{\Psi'}} \Psi' \, d\phi' \, d\Psi' \tag{2.14}$$

In this integral the range of ϕ' is over the unit circle and it makes no difference where it is begun. Consider that angle at which $\vec{1}_{\Psi'}$ is parallel to $\overleftrightarrow{1}_z \cdot [\vec{1}_r - \vec{1}_o]$ (i.e. the projection on S) as the starting point of the integration (say with new angle ϕ''). Then we have [16]

$$\begin{aligned} I_o(q) &= \frac{1}{2\pi} \int_o^{2\pi} e^{q \cos(\phi'')} \, d\phi'' \\ q &= \gamma\Psi' \left| \overleftrightarrow{1}_z \cdot \left[\vec{1}_r - \vec{1}_o\right]\right| \\ \int_o^{2\pi} e^{\gamma\Psi' [\vec{1}_r - \vec{1}_o] \cdot \vec{1}_{\Psi'}} \, d\phi' &= 2\pi \, I_o(q) \end{aligned} \tag{2.15}$$

Substituting in (2.14) gives [16]

$$\begin{aligned} \tilde{f}_a\left(\vec{1}_r, s\right) &= \frac{2}{(\gamma a)^2} \left[\left|\overleftrightarrow{1}_z \cdot \left[\vec{1}_r - \vec{1}_o\right]\right|\right]^{-2} \int_o^{q_a} q \, I_o(q) \, dq \\ &= \frac{2}{(\gamma a)^2} \left[\left|\overleftrightarrow{1}_z \cdot \left[\vec{1}_r - \vec{1}_o\right]\right|\right]^{-2} q_a \, I_1(q_a) \\ q_a &= \gamma a \left|\overleftrightarrow{1}_z \cdot \left[\vec{1}_r - \vec{1}_o\right]\right| \end{aligned} \tag{2.16}$$

$$\tilde{f}_a\left(\vec{1}_r, s\right) = 2\left[\gamma a \left|\overleftrightarrow{1}_z \cdot \left[\vec{1}_r - \vec{1}_o\right]\right|\right]^{-1} I_1\left(\gamma a \left|\overleftrightarrow{1}_z \cdot \left[\vec{1}_r - \vec{1}_o\right]\right|\right)$$

$$\rightarrow 1 \quad \text{as} \quad q_a \rightarrow 0$$

An alternate form is found as

$$\gamma = jk \;,\; k = \frac{\omega}{c}$$

$$\tilde{f}_a\left(\vec{1}_r, j\omega\right) = \frac{1}{A}\int_{S_a} e^{jk[\vec{1}_r - \vec{1}_o]\cdot\vec{r}'}\, dS' = 2\left[ka\left|\overleftrightarrow{1}_z\cdot\left[\vec{1}_r - \vec{1}_o\right]\right|\right]^{-1} J_1\left(ka\left|\overleftrightarrow{1}_z\cdot\left[\vec{1}_r - \vec{1}_o\right]\right|\right) \tag{2.17}$$

In time domain the scalar aperture function from (2.11) is

$$f_a\left(\vec{1}_r, t\right) = \frac{1}{A}\int_{S_a} \delta\left(t + \frac{1}{c}\left[\vec{1}_r - \vec{1}_o\right]\cdot\vec{r}'\right) dS' \tag{2.18}$$

This applies for the plane-wave specification of an arbitrary aperture shape in the form of (2.7) and (2.10). For step-function excitation f_a gives a response as an impulse broadened by the aperture dimensions. One can find the waveform in (2.18) by integrating over the aperture. The integration is rather straightforward for any aperture shape, being over that portion of the aperture for each t such that the argument of the delta function is near zero. This is a line formed by the wavefront of the step-function excitation on the aperture. Note that the integral is over two-dimensional space (not time) which brings a factor of $c/|\overleftrightarrow{1}_z \cdot [\vec{1}_r - \vec{1}_o]|$ in going from time to space. Then for general aperture shapes we have

$$f_a\left(\vec{1}_r, t\right) = \frac{c}{A\left|\overleftrightarrow{1}_z \cdot \left[\vec{1}_r - \vec{1}_o\right]\right|}\left\{\begin{matrix}\text{"width" of } S_a \text{ at} \\ \text{wavefront on } S_a\end{matrix}\right\} \tag{2.19}$$

i.e. the only contributions come from those $\vec{r}' \in S_a$ with

$$\left[\vec{1}_r - \vec{1}_o\right]\cdot\vec{r}' = -\,ct \tag{2.20}$$

This is a line parallel to $\overleftrightarrow{1}_z \cdot [\vec{1}_r - \vec{1}_o]$ on S_a. As t progresses this line sweeps across S_a (at a speed $\geq$ c). The waveform is zero for t such that this line is outside S_a, the limits of this giving the pulse width. The length of the line on S_a gives the pulse amplitude (times some other factors).

For a circular aperture (radius a) this becomes

$$f_a\left(\vec{1}_r, t\right) = \frac{1}{\pi a T}\left\{2a\left[1 - \left(\frac{t}{T}\right)^2\right]\left[u(t+T) - u(t-T)\right]\right\}$$

$$= \frac{2}{\pi T}\left[1 - \left(\frac{t}{T}\right)^2\right]\left[u(t+T) - u(t-T)\right] \tag{2.21}$$

$$T = \frac{a}{c}\left|\vec{1}_z \cdot \left[\vec{1}_r - \vec{1}_o\right]\right|$$

$$2T = \text{pulse width}$$

This can also be found by taking the inverse transform of (2.16), such as from standard tables [15 (Vol. 1, p. 138, No. 14)]. Note that for simplicity (greater symmetry) the pulse is centered on t=0 instead of beginning here.

Considering the properties of this waveform as in (2.19) for the general case, or (2.21) for the circular aperture note the variation of the pulse as $\vec{1}_r$ (the observer direction) is changed. At $\vec{1}_o$ the pulse has zero width and infinite amplitude in the far-field approximation which is not accurate in this extreme case. As $\vec{1}_r$ goes away from $\vec{1}_o$ the pulse width increases, but the amplitude is finite and proportional to the reciprocal of the pulse width. This is seen in the calculation of the impulse (i.e. the "area" under the waveform) as

$$\int_{-\infty}^{\infty} f_a\left(\vec{1}_r, t\right)\, dt = \int_{-\infty}^{\infty} \frac{1}{A}\int_{S_a} \delta\left(t + \frac{1}{c}\left[\vec{1}_r - \vec{1}_o\right]\cdot \vec{r}'\right)\, dS'\, dt = \frac{1}{A}\int_{S_a} dS' = 1 \tag{2.22}$$

Note that the complete time integral is the same as the transform evaluated at zero frequency, giving an alternate way to get the same result. So the impulse of the scalar part of the aperture function is not a function of $\vec{1}_r$, i.e. is conserved with respect to angle. This holds for arbitrary aperture shapes.

Viewed in a frequency-domain sense the spectral content is not a function of $\vec{1}_r$ at low frequencies (wavelengths large compared to aperture dimensions). The spectral content is flat out to some frequency which is higher as $\vec{1}_r$ approaches $\vec{1}_o$ (i.e. as a characteristic time such as T becomes small). So the beam width for such a transient pulse depends on how small are the times of interest or how high are the frequencies of interest. Such parameters may be given by other considerations such as attainable risetimes of a real pulse, or desired resolution of (say) scattered signals.

In terms of the vector aperture function the impulse is

$$\int_{-\infty}^{\infty} \vec{F}_a\left(\vec{1}_r, t\right) dt = \tilde{\vec{F}}_a\left(\vec{1}_r, 0\right) = \left(\vec{1}_z \cdot \vec{1}_r\right) \vec{1}_a - \left[\vec{1}_r \cdot \vec{1}_a\right] \vec{1}_z = -\vec{1}_r \times \left[\vec{1}_z \times \vec{1}_a\right] \tag{2.23}$$

So the impulse of the radiated waveform is a slowly varying function of angle, the same vector function which multiplies the scalar waveform function which we have been considering. It is interesting to note that this result is independent of $\vec{1}_o$, the direction for which the aperture is focused at infinity. This special direction $\vec{1}_o$ is significant only for frequencies sufficiently high that wavelengths are smaller than the aperture dimensions.

Consider the far field from a magnetic dipole located at the origin [6] given by

$$\tilde{\vec{E}}_{m_f}(\vec{r}, s) = \frac{e^{-\gamma r}}{4\pi r} \frac{\mu_o}{c} s^2 \vec{1}_r \times \tilde{\vec{m}}(s)$$

$$\tilde{E}_{m_f}(\vec{r}, t) = \frac{\mu_o}{4\pi c r} \vec{1}_r \times \left[\frac{\partial^2}{\partial t^2} \vec{m}\left(t - \frac{r}{c}\right)\right] \tag{2.24}$$

Comparing this to the far field for the aperture antenna, then (2.9) and (2.11) can be written as

$$\tilde{\vec{E}}_f(\vec{r}, s) = \frac{e^{-\gamma r}}{4\pi r} \frac{\mu_o}{c} s^2 \left[\vec{1}_r \times \tilde{\vec{m}}_a(s)\right] \tilde{f}_a\left(\vec{1}_r, s\right)$$

$$\tilde{\vec{m}}_a(s) = -\frac{2E_o A}{\mu_o} \frac{\tilde{f}(s)}{s} \tag{2.25}$$

$\equiv$ effective magnetic dipole moment
(for low frequencies)

Taking $\tilde{f}(s)$ as $1/s$ for the step excited aperture gives

$$\tilde{\vec{m}}_a(s) = -\frac{2E_oA}{\mu_o}\frac{1}{s^2} \quad , \quad \vec{m}_a(t) = -\frac{2E_oA}{\mu_o} t\, u(t) \tag{2.26}$$

which in time domain is called a ramp function.

In time domain we also have

$$\vec{E}_f(\vec{r},t) = \frac{1}{4\pi r}\frac{\mu_o}{c}\vec{1}_r \times \left\{\left[\frac{\partial^2}{\partial t^2}\vec{m}_a\left(t - \frac{r}{c}\right)\right] \circ f_a\left(1_r, t\right)\right\} \tag{2.27}$$

$\circ \equiv$ convolution with respect to time

So the general result (for plane-wave aperture illumination) takes the form of separate factors. There is an effective magnetic-dipole term which is modified at high frequencies by a scalar aperture function which gives a more and more concentrated beam as frequency is increased. In time domain this scalar aperture function gives an angular-dependent pulse width for step illumination of the aperture. For more general cases there is a temporal convolution of these two terms. For example, a non-zero risetime for the aperture illumination gives a lower bound on the pulse width in the far field.

III. Peculiarities Near Focal Direction

As discussed in the previous section, the far-field approximation has difficulty as $|s| \to \infty$, particularly near $\vec{1}_r = \vec{1}_o$. This is seen in the fact that in this case the scalar aperture function becomes (from (2.18)).

$$f_a\left(\vec{1}_o, t\right) = \delta(t) \tag{3.1}$$

Now for finite (even though large) r, this is not exact in that it represents infinite energy flux density, even integrated over a small region near the direction of propagation. This is basically a problem concerning the order of two limits: $r \to \infty$ and $|s| \to \infty$.

This is understood in the context of an "electromagnetic missile" [12,14] for which the energy flux density and energy integrated over some small cross section is finite and falls off slower than r^{-2}. Consider the aperture antenna in Figure 2.1 with plane-wave illumination at direction $\vec{1}_o$ as in (2.7) and (2.10). For some specified $\vec{1}_o$ consider the projection of the aperture as a cylinder (not necessarily circular) parallel to this line. An observer within this cylinder sees an electric field which is initially a plane wave with amplitude undiminished from the source. This is obtained by setting the tangential aperture field $E_o \vec{1}_a$ equal to the tangential part of the field at the observer as

$$\begin{gathered} \overleftrightarrow{1}_z \cdot \tilde{\vec{E}}(\vec{r}, t) = E_o \, \vec{1}_a \, u\left(t - \frac{r}{c}\right) \\ \vec{1}_o \cdot \tilde{\vec{E}}\left(r\vec{1}_o, t\right) = 0 \\ \text{for} \quad t - \frac{r}{c} < \Delta t \end{gathered} \tag{3.2}$$

where Δt is some time at which the observer first sees a signal from the boundary of the aperture. This Δt is maximized if the observer is approximately centered within the aperture projection. For convenience the aperture "center" is taken as $\vec{r} = \vec{0}$ so that such an observer can be taken on the $r\vec{1}_o$ line. Note that as in Section 2 the aperture field is taken as a step function in time.

For a constant polarization $\vec{1}_a$ of the tangential fields on the aperture the form of (2.3) allows us to write

$$\vec{E}\left(r\vec{1}_o,t\right) = \chi \left[-\vec{1}_o \times \left[\vec{1}_z \times \vec{1}_a\right]\right] E_o\, u\left(t - \frac{r}{c}\right)$$

$$= \chi \left[\left(\vec{1}_z \cdot \vec{1}_o\right) \vec{1}_a - \left(\vec{1}_o \cdot \vec{1}_a\right) \vec{1}_z\right] E_o\, u\left(t - \frac{r}{c}\right) \tag{3.3}$$

$$\text{for} \quad t - \frac{r}{c} < \Delta t$$

where χ is some scalar constant to be determined. Applying (3.2) gives

$$\vec{1}_a = \chi\, \vec{1}_z \cdot \left[\left(\vec{1}_z \cdot \vec{1}_o\right) \vec{1}_a - \left(\vec{1}_o \cdot \vec{1}_a\right) \vec{1}_z\right] = \chi \left(\vec{1}_z \cdot \vec{1}_o\right) \vec{1}_a$$

$$\chi = \left(\vec{1}_z \cdot \vec{1}_o\right)^{-1} \tag{3.4}$$

$$\vec{E}\left(r\vec{1}_o,t\right) = \left[\vec{1}_a - \frac{\vec{1}_o \cdot \vec{1}_a}{\vec{1}_z \cdot \vec{1}_o}\, \vec{1}_z\right] E_o\, u\left(t - \frac{r}{c}\right) \quad \text{for} \quad t - \frac{r}{c} < \Delta t$$

For $\vec{1}_o$ chosen normal to S we have the simple result

$$\vec{1}_o = \vec{1}_z$$

$$\vec{E}\left(z\vec{1}_z,t\right) = \vec{1}_a\, E_o\, u\left(t - \frac{z}{c}\right) \quad \text{for} \quad t - \frac{z}{c} < \Delta t \tag{3.5}$$

Let the closest distance of the observer to the aperture projection be designated as b. Then we have as $r \to \infty$

$$\Delta t = \frac{1}{c}\,\frac{b^2}{2r} + O(r^{-2}) \tag{3.6}$$

as the differential time to first detect the boundary of the aperture. Define the partial impulse of this wave as the integral up to the time the aperture boundary is first detected, giving

$$\begin{aligned}\int_{-\infty}^{\frac{r}{c}+\Delta t} \vec{E}\left(r\vec{1}_o,t\right) dt &= \left[\vec{1}_a - \frac{\vec{1}_o \cdot \vec{1}_a}{\vec{1}_z \cdot \vec{1}_o} \vec{1}_z\right] E_o \, \Delta t \\ &= \left[\vec{1}_a - \frac{\vec{1}_o \cdot \vec{1}_a}{\vec{1}_z \cdot \vec{1}_o} \vec{1}_z\right] \frac{1}{c} \frac{b^2}{2r} E_o + O(r^{-2}) \quad \text{as} \quad r \to \infty\end{aligned} \tag{3.7}$$

Note that this partial impulse falls off as r^{-1}, a far-field-like behavior. However, the power density during this early portion of the pulse does not fall off with distance. The associated energy flux for this portion of the pulse is

$$\begin{aligned}\int_{-\infty}^{\frac{r}{c}+\Delta t} \frac{1}{Z_o} \left|\vec{E}\left(r\vec{1}_o,t\right)\right|^2 dt &= \left[1 + \left(\frac{\vec{1}_o \cdot \vec{1}_a}{\vec{1}_z \cdot \vec{1}_o}\right)^2\right] \frac{E_o^2}{Z_o} \Delta t \\ &= \frac{\mu_o \, b^2}{2r} E_o^2 + O(r^{-2}) \quad \text{as} \quad r \to \infty\end{aligned} \tag{3.8}$$

which falls off like r^{-1} (an "electromagnetic missile") instead of the usual r^{-2} (for bounded $|s|$).

Comparing to the results in Section 2 the complete impulse from (2.9) and (2.23) is

$$\begin{aligned}\int_{-\infty}^{\infty} \vec{E}_f(\vec{r},t) \, dt = \tilde{\vec{E}}_f(\vec{r},0) &= \left\{-\vec{1}_r \times \left[\vec{1}_z \times \vec{1}_a\right]\right\} \frac{E_o A}{2\pi c r} \\ &= \left[\left(\vec{1}_z \cdot \vec{1}_o\right) \vec{1}_a - \left(\vec{1}_o \cdot \vec{1}_a\right) \vec{1}_z\right] \frac{E_o A}{2\pi c r}\end{aligned} \tag{3.9}$$

For a circular aperture (radius a) this is

$$\int_{-\infty}^{\infty} \vec{E}_f(\vec{r},t)\, dt = \left[\left(\vec{1}_z \cdot \vec{1}_o\right) \vec{1}_a - \left(\vec{1}_o \cdot \vec{1}_a\right) \vec{1}_z\right] \frac{E_o\, a^2}{2cr} \tag{3.10}$$

Comparing this to (3.7) for the partial impulse we see that these are closely related. For the case of focusing in the $\vec{1}_z$ direction we have them equal since

$$\begin{aligned} \vec{1}_o &= \vec{1}_z \\ b &= a \end{aligned} \tag{3.11}$$

Note this equality is for the case that the observer is precisely on the z axis so that the distance a is the shortest distance (i.e. the only distance) to the aperture projection.

Since the impulse only relies on the transform at zero frequency, for which the far-field expansion is valid, then (3.9) is correct, even for $\vec{1}_r$ near $\vec{1}_o$. So the impulse of even the "electromagnetic missile" should be included in this answer. The partial impulse as in (3.7) should be regarded as just that (i.e. partial). So as $\vec{1}_r$ approaches $\vec{1}_o$ the waveform does not become a delta function but has some non-zero width bounded below by Δt. The waveform, when integrated, must be given by (3.9).

IV. Limitations of Aperture Model

While the aperture model in Section 2 gives some important properties of certain kids of antennas, it has limitations. In particular outside S_a, but still on S, the tangential electric field is constrained to be zero. As indicated in Figure 2.1 this outer region may be a conducting surface on which surface currents are allowed to flow to maintain the boundary condition.

One can postulate various ways to synthesize the electric field distribution on S_a. Thinking of a magnetic frill one might construct an array of magnetic dipoles (loops) with axes parallel to $\vec{1}_a$. While this would require no currents on S outside S_a there would be waves with opposite tangential electric field propagating in both directions away from S (+z and -z). These magnetic dipoles, except for varying turn-on times (to focus in the direction $\vec{1}_o$) would have linearly increasing magnetic dipole moments (as in (2.26)) which corresponds to linearly increasing current with constant voltage (L dI/dt) for a power linearly increasing with time. This points out another limitation. A pulser with finite energy can at best give a non-zero late-time magnetic (or electric) dipole moment [6]. So the assumptions of the aperture antenna model need to be modified to allow for this late-time and low-frequency restriction.

Another way to synthesize the electric-field distribution on S is by driving currents in the direction $\vec{1}_a$ over S_a and letting the currents return on conductors on S outside S_a. In this case step-function-like voltage sources on S_a will deliver currents into an inductive load (inductive for low frequencies). This gives the same problem of a linearly increasing current at late times unless the late-time equivalent magnetic dipole moment is bounded (modifying the waveform). In this example there are outgoing waves on both sides of S (+z and -z), but with equal tangential electric fields on S.

If one allows tangential electric field to spread out on S other types of antennas can be synthesized. As an example one might have a TEM horn, i.e. a conical transmission line on which a spherical TEM wave (non dispersive) can propagate with arbitrary time-domain waveform from a source near the apex of the two (or more) conical conductors in a simple medium such as free space [2,18]. At the open end of the horn one can define the aperture plane. Of course, the wave on S is a spherically expanding wave with a radius of curvature given by the length of the horn, so the wave is not focused at infinity, but has a beam spread related to the angular opening of the horn (at high frequencies). In addition, the discontinuity at the end of the horn produces a scattered (non TEM) field on S in addition to the incident spherical TEM wave.

This can be improved as in Figure 4.1 by including a lens in the mouth of the TEM horn. This lens should convert the incoming fast-rising spherical wave into a similar plane wave on S_a, not necessarily on all of S. One can have an ideal (reflectionless and dispersionless) TEM lens (e.g. based on bispherical coordinates) [4], or an approximate type of lens which gives only small perturbations [3]. The aperture field is in general not uniform, but $\vec{1}_a$ can be regarded as some average direction for the electric field, here taken parallel to $\vec{1}_x$. There is also the scattered (reflected) field due to the truncation of the guiding conductors at the aperture plane. This scattered field, however, contributes negligibly to the fast-rising portion of the aperture fields. At late time (low frequencies) the antenna behaves as an electric dipole. If the voltage on the antenna is bounded and oscillations die out (perhaps due to the inclusion of some damping resistance, say in the source at the apex) then the late-time electric dipole moment $\vec{p}(\infty)$ governs the low-frequency behavior and makes the radiated waveform have net-zero time integral [5].

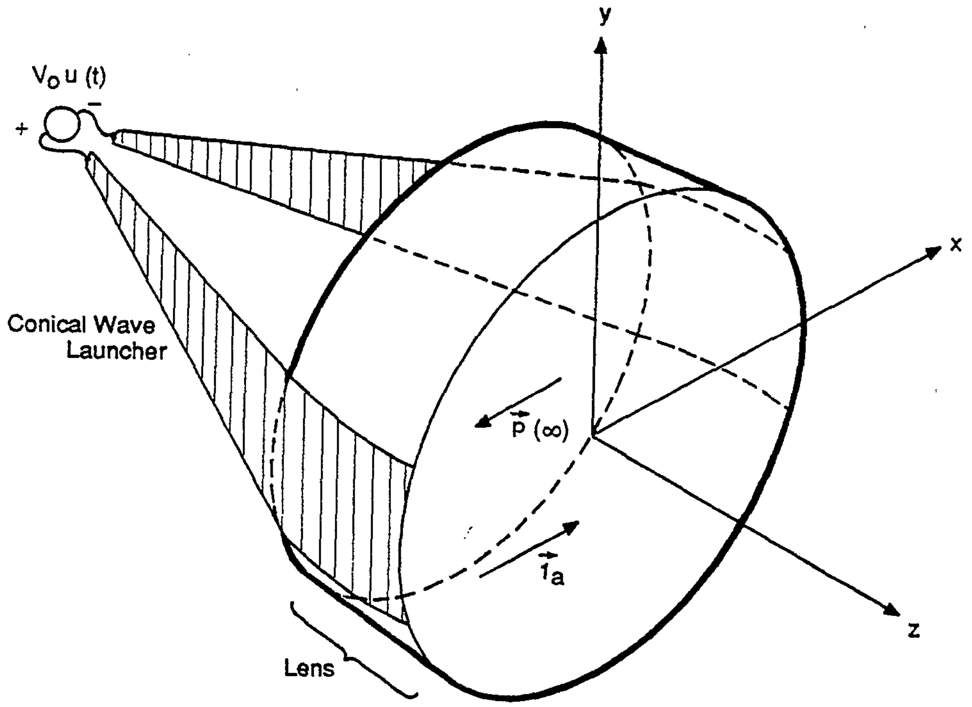

Figure 4.1. TEM Horn and Lens as Wave Launcher

V. Launching TEM Waves into Reflector Antennas

The more common way to focus waves at infinity by impressing a plane wave on an aperture plane is through the use of a reflector antenna [17]. This is often a conducting dish shaped as a paraboloid. This is often fed by a horn on the end of an H-mode waveguide, or by a small antenna at a place (focus in reception) that makes the spherical wave be reflected into a plane wave by the paraboloid. This feed may be on the axis of revolution, or may be offset. Dual offset systems involving a second reflector are also used. Furthermore, this feed is designed to operate over narrow bands of frequency as contrasted to the present considerations.

Then let us combine the conical TEM wave launcher (step excited) with a parabolic reflector as illustrated in Figure 5.1. This functions in the same way as the usual reflector antenna except that the feed waveform can be made an approximate step function. Note that the feed can be offset as in Figure 5.1 so as to reduce aperture blockage. The beam (direction $\vec{1}_o$) can be steered by rotating either the reflector or the feed (with some pivot arrangement at the reflector) or both. One may also include some absorber material to reduced the high frequencies radiated from the wave-launcher apex in the focal direction $\vec{1}_o$.

The conical wave launcher will have some characteristic impedance Z_c (frequency independent) [7,8]. This conical system can consist of two flat strips or even several strips, including approximation by wires. One might have two wires or say four wires with an optimal spacing ratio to optimize the field uniformity launched into the reflector [1]. Note that the fields on the conical transmission line between the conductors have the average direction as indicated by $\vec{E}_{launch}$ in Figure 5.1B, here taken as the $-\vec{1}_x$ direction. On the outside of the launch conductors the fields have the opposite direction. By terminating the launcher conductors on the edge of the reflector these external fields are not reflected. This is advantageous in that these external fields are not reflected in a manner which would subtract from the fields polarized in the $\vec{1}_a$ direction (opposite $\vec{E}_{launch}$) and radiated in the $\vec{1}_o$ direction. By appropriate design of the wave-launcher conductors (and reflector edge (boundary) shape) one can try to optimize the fields radiated from the reflector for a given voltage, power, etc. on the feed.

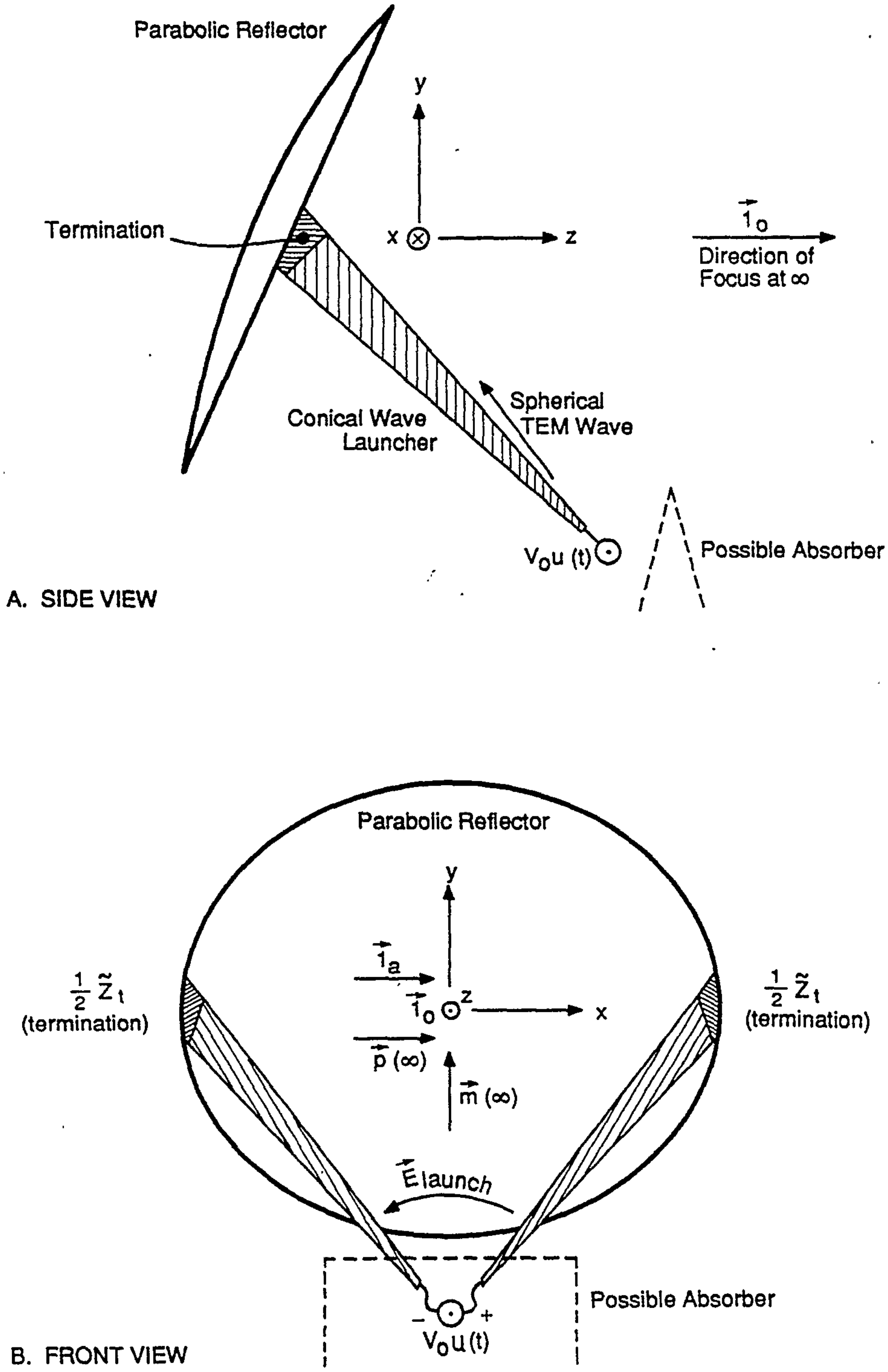

Figure 5.1. Conical TEM Wave Launcher Feeding Reflector

Note that at late times or low frequencies the antenna is dominated by electric and/or magnetic dipole moments [5,6], indicated by $\vec{p}_\infty$ and $\vec{m}_\infty$ in Figure 5.1B. The magnetic dipole moment is associated with current flowing around the loop comprised of feed and reflector. An electric dipole is produced by some termination consisting of two impedances, each $Z_t/2$ in series at the connection of the feed conductors to the reflector. One might choose

$$\tilde{Z}_t = Z_c \tag{5.1}$$

which terminates the feed (at low frequencies) in its characteristic impedance. The electric and magnetic dipoles then combine to maximize the low-frequency radiation in a direction opposite to the wave-launcher direction [6,9,11]. This choice should significantly reduce oscillations at late time on the antenna. Other more sophisticated choices for $\tilde{Z}_t$ are possible, including a resistance at low frequencies, but perhaps a lower impedance at high frequencies.

Consider now the radiated waveform as illustrated in Figure 5.2. First there is a comparatively low amplitude (negative) step associated with the spherical wave launched from the feed apex. This can have reduced high frequencies (a slowed risetime) by use of blocking absorbers. Then there is the large narrow reflector impulse (positive) for the observer near the main beam direction ($\vec{1}_r$ near $\vec{1}_o$). This is followed by a complicated waveform return region. Here the waveform returns to zero, but in such a manner that the total time integral (or area) of the waveform is zero, even given non-zero late-time electric and/or magnetic dipole moments [5]. If these dipole moments are allowed to decay to zero (as eventually they must for practical pulsers) then the radiated low frequencies are further reduced. The details of the return of the waveform to zero are strongly influenced by the choice of $\tilde{Z}_t$, such as for removing oscillations.

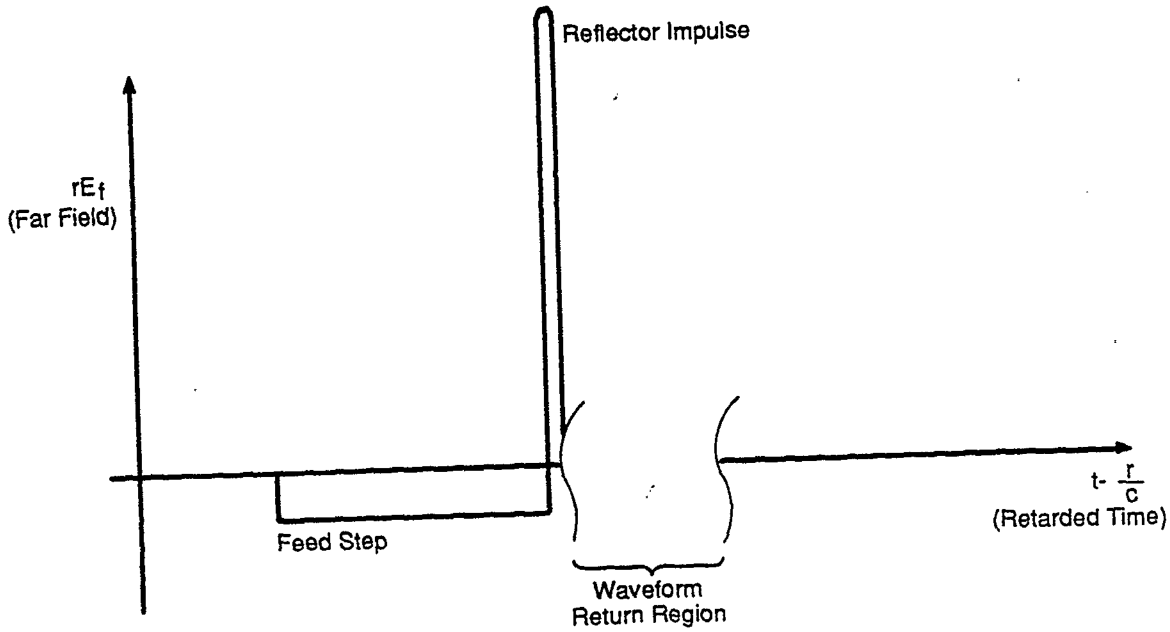

Figure 5.2. Canonical Waveform

VI. Concluding Remarks

While there are analytic models appropriate to the early-time and low-frequency behavior of antennas for radiating impulse-like fields, the intermediate-frequency regime is not so simple in form. Numerous details of launcher, terminator, reflector, etc. need to be optimized. This paper gives some of the principal features of such antennas and outlines how the whole thing can be put together.

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