Sensor and Simulation Notes 33
Note XXXIII
4 February 1967

Two Types of Vertical Current Density Sensors

Capt Carl E. Baum
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Abstract

Two types of sensors for measuring the vertical component of the total current density are discussed. One of these directly samples the current density while the other inductively couples to it. Some of the advantages and limitations of each are discussed.

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Two types of sensors for measuring the vertical component of the total current density are discussed. One of these directly samples the current density while the other inductively couples to it. Some of the advantages and limitations of each are discussed.

I. Introduction

There are many physical parameters associated with the nuclear electromagnetic pulse (either real or simulated) which we may wish to measure. Such parameters may include electric fields, magnetic fields, gamma flux densities, and various other quantities. A device which interacts with one of these physical quantities to produce a second physical quantity (such as an analog electrical signal) that can be related back to the first quantity we call a sensor. To be meaningful, the sensor must produce the second physical quantity in response primarily to the first quantity. The sensor response to other physical parameters (considering the degree to which they are present) should be small compared to the response to the quantity of interest.

Consider now those physical parameters, pertinent to the nuclear electromagnetic pulse, which are related by Maxwell's equations of the form

$$\nabla \times \vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \tag{1}$$

and

$$\nabla \times \vec{H} = \vec{J}_c + \left(\sigma + \epsilon\frac{\partial}{\partial t}\right)\vec{E} \tag{2}$$

where μ and ϵ are assumed independent of time. The Compton current density, $\vec{J}_c$, the electric field, $\vec{E}$, the magnetic field, $\vec{H}$, the permittivity, ϵ, the permeability, μ, and the conductivity, σ, are explicitly identified in these equations. One might design sensors which respond to any of the above parameters. In the case of vector quantities the sensor might respond to one component of the vector or to some combination of the components (such as magnitude or direction). Ideally a sensor could be designed to respond to any desired quantitatively definable combination of these parameters. Other parameters can be derived from those in equations (1) and (2) by performing various mathematical operations on them such as integration or differentiation with respect to the spatial coordinates or time. Voltage and current are examples of such derived parameters.

In measurements of the nuclear electromagnetic pulse, components of $\vec{E}$ and $\vec{H}$ and their derivatives are normally considered. It may be desirable, however, to measure other electromagnetic parameters to obtain a better understanding of the various important physical processes. Since these parameters are related through Maxwell's equations, the results of the various measurements can be compared with each other for consistency as a check on the validity of the measurements. Besides components of the electric and magnetic fields other interesting possibilities might be components of the curls of the electric and magnetic fields. Equation (1) shows that the curl of the electric field is the negative time rate of change of the magnetic field, $\dot{\vec{B}}$. This relation, Faraday's law, is the principle of the operation of a $\dot{B}$ loop, a commonly used device. Thus, the curl of the electric field is not a basically different quantity to measure.

Look, however, at equation (2) for the curl of the magnetic field. The right side of this can be called a total current density which we define as

$$\vec{J}_t = \vec{J}_c + \left(\sigma + \epsilon\frac{\partial}{\partial t}\right)\vec{E} \tag{3}$$

In cases where $\vec{J}_c$ and σ are both zero we are left with the displacement current density, $\epsilon \frac{\partial \vec{E}}{\partial t}$. In this case a measurement of the curl of the magnetic field would be a measurement of the time derivative of the electric field. In a more general case there is also the Compton current density, $\vec{J}_c$, and the conduction current density, $\sigma\vec{E}$, such that the curl of the magnetic field is distinct from the electric field. There are several interesting parameters in equation (3) which might be measured, including the Compton current density, the conductivity, and the conduction current density. In this note we consider sensors to measure the total current density, or equivalently the curl of the magnetic field. Specifically we consider sensors which measure the vertical component of the total current density at the soil or water surface.

Consider the spherical and cylindrical coordinate systems of figure 1A with the (x,y) plane taken as the ground or water surface and with the nuclear source placed at the origin of the coordinate systems, or more generally, somewhere in the air on the positive z axis. With the usual assumptions of sources and geometry independent of ϕ Maxwell's equations simplify somewhat. In spherical coordinates (ρ,θ,ϕ) the components of the total current density are of the form

$$J_{t_\rho} = \frac{1}{\rho\sin(\theta)} \frac{\partial}{\partial\theta} (\sin(\theta)H_\phi) \tag{4}$$

and

$$J_{t_\theta} = -\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho H_\phi) \tag{5}$$

In cylindrical coordinates (r,ϕ,z) the components are

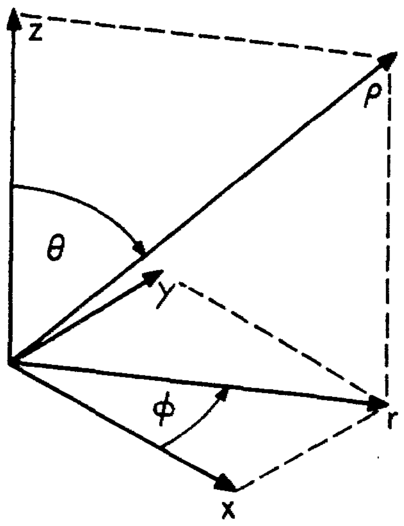
$$J_{t_r} = -\frac{\partial H_\phi}{\partial z} \tag{6}$$

and

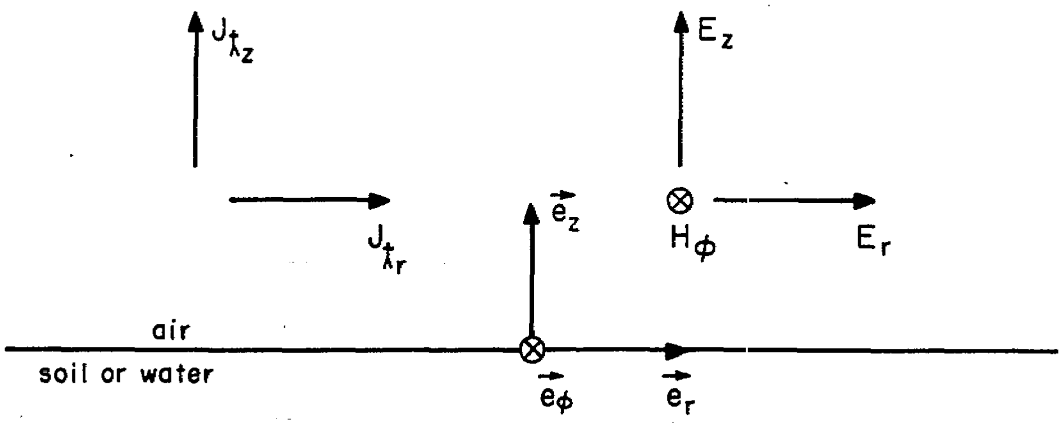
$$J_{t_z} = \frac{1}{r}\frac{\partial}{\partial r}(rH_\phi) = \frac{H_\phi}{r} + \frac{\partial H_\phi}{\partial r} \tag{7}$$

The field components near the ground or water surface together with the unit vectors for the three cylindrical coordinates are illustrated in figure 1B.

Then consider sensors which respond to the vertical component, J_{t_z}, of the total current density at the ground or water surface ($z = 0$). There are two general kinds of such devices discussed in this note. One type of sensor interrupts the flow of the total current density and samples it directly. The second type of sensor inductively couples to the total current density; or it can be thought of as measuring the curl of the magnetic field. Some sensors of the second type are called Rogowski coils. Each of these types of sensor has different advantages as well as different limitations.



A. Spherical and Cylindrical Coordinates



$\vec{e}_\phi$ and H_ϕ are pointing into the page.

B. Cylindrical Coordinates with Electromagnetic Quantities

Fig. 1. COORDINATE SYSTEMS

II. Current Sampling Vertical Current Density Sensor

First consider the kind of sensor which directly samples the vertical current density at the ground or water surface. An example of such a device is given in figure 2A. Basically, this sensor intercepts the total current density over a certain area, A, and generates a voltage, V, proportional to the current by passing the current through a resistive load. The total current into the sensor is

$$I_t = - AJ_{t_z} = - \pi a^2 J_{t_z} \tag{8}$$

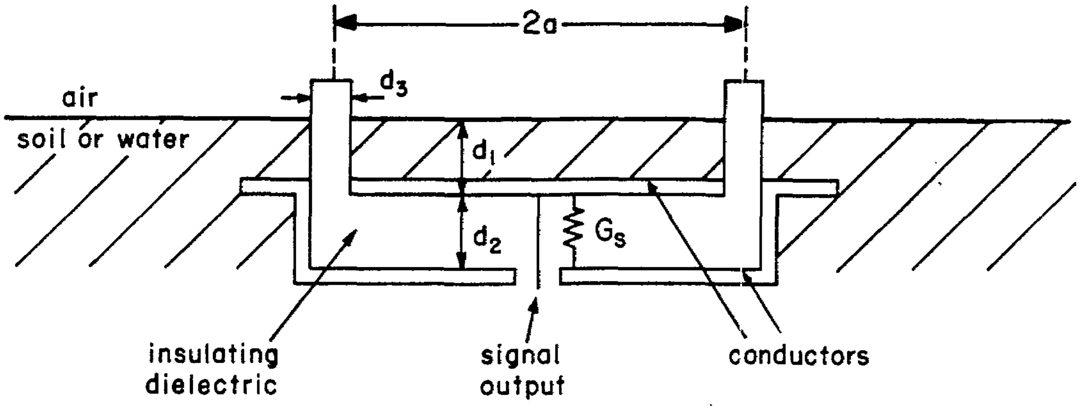
where, for this example, we have let the area over which the current is intercepted be a circle of radius, a. Ideally this current passes through a conductance, G, composed of perhaps both a conductance due to a signal cable and a shunt conductance, G_s, to raise the total conductance. This gives a signal voltage of

$$V = \frac{I_t}{G} = - \frac{\pi a^2 J_{t_z}}{G} \tag{9}$$

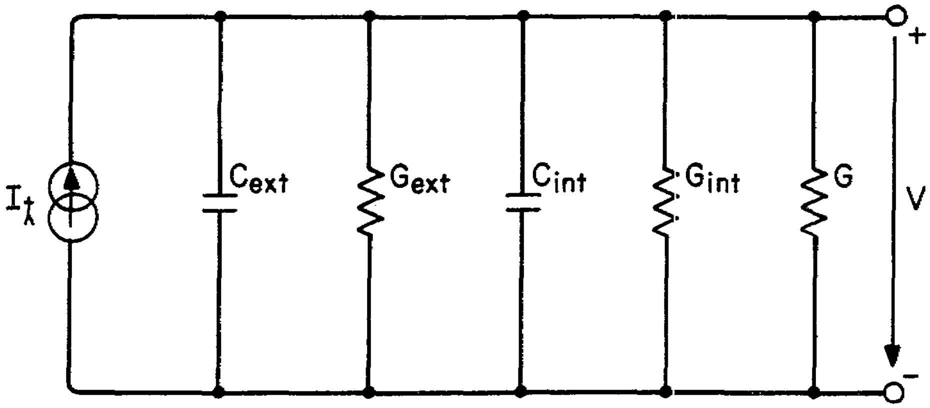
There are many possible ways to arrange the signal cable output and shunt conductance including the simple manner illustrated in figure 2A. Perhaps the shunting conductance and/or the signal cable load could be distributed around the current interception area in some manner to improve the frequency response of the device. Note that there is also an electrode in the sensor which is in electrical contact with the soil or water to complete the current flow path for I_t.

For proper operation there are certain restrictions on the admittances associated with the sensor. Consider the approximate equivalent circuit of figure 2B. There are capacitances associated with the sensor, C_{ext} and C_{int}, roughly associated with electric fields above the ground or water surface (external to the sensor) and associated with electric fields inside the sensor, respectively. Considering the load conductance, G, as the only conductance of significance, there is a time constant, $(C_{ext}+C_{int})/G$, which limits the upper frequency response of the device. The upper frequency response can be improved, then, both by decreasing the capacitance (for instance by increasing the plate spacing, d_2) and by increasing the load conductance. With the presence of ionizing nuclear radiation there are other admittances to consider. The air can be highly conducting, introducing a conductance, G_{ext}, which is rapidly varying with time. For this time varying conductance to have insignificant effect on the sensor performance it is necessary that $G \gg G_{ext}$ for times of interest. Also there may be a conductance, G_{int}, introduced inside the sensor due to the nuclear radiation. This can be made small compared to the maximum G_{ext} through the use in the sensor of insulating dielectrics which have minimum conductivity under radiation. Briefly, G must be the dominant admittance for all frequencies of interest for proper sensor performance.

As the air conductivity becomes large enough to approach the soil or water conductivity the radial electric field, E_r, at the ground or water surface approaches, in magnitude, the vertical electric field, E_z,



A. Cross Section



B. Equivalent Circuit

Fig. 2. CURRENT SAMPLING VERTICAL CURRENT DENSITY SENSOR

in the air at the same location.[1] This radial electric field is locally distorted by the sensor conductors and insulators near the surface. Since the air conductivity is a function of the magnitude of the electric field, this conductivity is locally distorted by the distortion of E_r. This in turn distorts E_z and J_{t_z}. Perhaps some way can be found to sample the vertical component of the total current density without distorting the horizontal component at the same location. If not, this type of sensor may be limited in application to cases in which the air conductivity is much less than the soil or water conductivity. Some of the problems with this current sampling type of sensor are similar to those encountered in sensors for the electric field in the conducting air.[2,3]

Even with the air conductivity much less than the ground or water conductivity one should be careful that the vertical component of the total current density is not distorted. There may be various boundary layer effects (such as electron depletion layers) at the interface of the air with the soil or water. These effects should be maintained in the vicinity of the sensor or the vertical current density may be distorted giving an error in a measurement. Typical conductors which may be used for the sensor may have somewhat different boundary layer effects (compared to soil or water) at an interface with the conducting air. Thus, it may be desirable to use the same soil or water for that part of the sensor which intercepts the vertical total current density for a measurement. This feature is included in figure 2A. This sampling electrode is also separated from the soil or water medium by an insulating dielectric of thickness, d_3, to avoid a shorting conductance by such a path. This insulator perturbs the electric field distribution in its immediate vicinity, but as long as $d_3 << a$ this distortion affects only a small part of the electrode area, πa^2, and should have a correspondingly small effect on the sensor response.

The Compton current density, $\vec{J}_c$, which is produced by high energy photons such as gamma rays, composes part of the total current density. The vertical component, J_{c_z}, should be included in the measured current density. There may be problems, however, associated with nuclear radiation transport in the sensor itself. The nuclear radiation may transport charge between the sampling electrode which intercepts the current density and the rest of the sensor body. This may add to or subtract from the desired signal. The ground or water may also be sources of gamma rays which in turn produce Compton currents (and ionization).[4] The sampling electrode should play the same role as the surrounding soil or water in the radiation transport. Again, the upper part of the electrode should be the same as the surrounding soil or water medium, but for this problem it should be several mean free paths

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[1] Lt Carl E. Baum, EMP Theoretical Note XIX, A Technique for the Approximate Solution of EMP Fields from a Surface Burst in the Vicinity of an Air-Ground or an Air-Water Interface, Sept. 1966.

[2] Lt Carl E. Baum, Sensor and Simulation Note XV, Radiation and Conductivity Constraints on the Design of a Dipole Electric Field Sensor, Feb. 1965.

[3] Lt Carl E. Baum, Sensor and Simulation Note XXVI, The Influence of Finite Soil and Water Conductivity on Close-in Surface Electric Field Measurements, Sept. 1966.

[4] Lt Richard R. Schaefer, EMP Theoretical Note XIV, Later Time Sources of EMP, Feb. 1966.

thick to the significant nuclear radiation to act an an adequate radiation source for the air above it. Making the electrode thickness, d_1, thick should also help in shielding the lower parts of the sensor from the nuclear radiation.

III. Inductively Coupled Vertical Current Density Sensor

Now consider another kind of sensor which measures the vertical component of the total current density. An example of this type of sensor is a Rogowski coil of radius, a, which encompasses an area, A (see figure 3A). The loop area, per turn, is A' and a number of turns, N, is multiplied by A' to give an effective area NA' for the device. To determine the signal voltage from the device, first consider the total current through the Rogowski coil which is given by equation (8). The average magnetic field linking the turns of the Rogowski coil, due to the current through the coil, is just

$$H' = \frac{I_{t_z}}{2\pi a} = -\frac{a}{2} J_{t_z} \qquad (10)$$

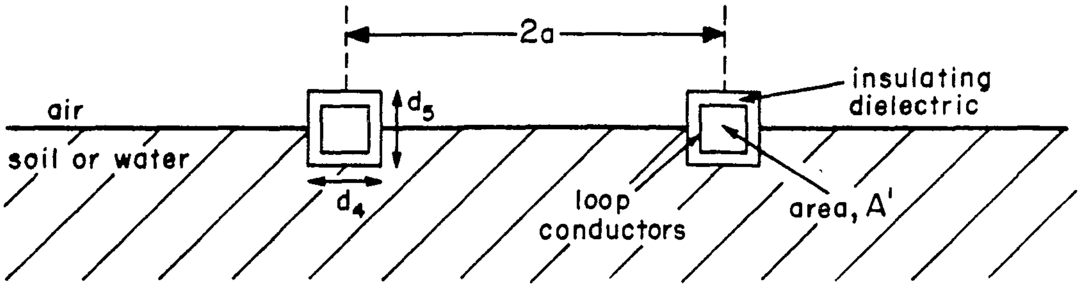
The signal voltage from the sensor is then

$$V = NA'\mu_o \frac{\partial H'}{\partial t} = - \mu_o NA' \frac{a}{2} \frac{\partial J_{t_z}}{\partial t} \qquad (11)$$

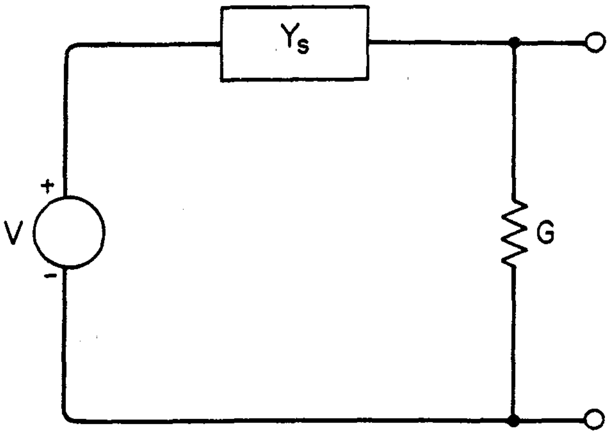
where μ_o is the assumed permeability of the sensor materials. The signs for H' and V are chosen abritrarily. Note that the Rogowski coil is designed such that H' links it with the same polarity uniformly around the circumference, $2\pi a$.

One advantage of a Rogowski coil for this type of sensor is its symmetry. It can be rotated about a vertical axis without affecting its sensitivity. Thus, it only has to be oriented with respect to the vertical axis to measure the vertical component of the total current density. However, there might be other ways to design this inductively coupled type of sensor. From equation (7) we can relate J_{t_z} to a radial derivative of the magnetic field. Perhaps, then, two loops at different r's with their outputs differenced can be used for this type of sensor. The sensitivities of the two loops might even be made slightly different so that the difference in the signals can be used for the radial derivative of rH_ϕ. A disadvantage of this two-loop approach is that it relies on the assumption of symmetry, i.e., that H_ϕ is the only magnetic field component, independent of ϕ. Without this symmetry assumption the vertical component of $\nabla x\vec{H}$ has a more general form. A Rogowski coil type of sensor is not based on this symmetry assumption and thus may be a better approach for measuring J_{t_z} in some cases.

A rough equivalent circuit of an inductively coupled vertical current density sensor is given in figure 3B. The signal voltage may drive a load admittance, G, associated with a signal cable, plus a series admittance associated with the sensor. This sensor admittance, Y_s, is basically due to an inductance, L, but as frequency is increased capacitance and/or conductance also become significant. This type of device is a modification of a B loop and its response characteristics are similar.



A. Cross Section



B. Equivalent Circuit

Fig. 3. INDUCTIVELY COUPLED VERTICAL CURRENT DENSITY SENSOR

As such, some of the considerations for $\dot{B}$ loop response apply here.[5] One may even improve the sensor's response by encapsulating it in a dielectric which is much less conducting than the air in the presence of the nuclear radiation.[6] While we have considered this type of current density sensor with signal output proportional to the time derivative of J_{t_z} (as in equation(11)), it can also be made to have an output proportional to J_{t_z}. In this latter case the time constant, LG, should be made much larger than times of interest. For an output as in equation (11) this same time constant should be much smaller than times of interest.

As the air conductivity approaches the soil or water conductivity, making E_r significant compared to E_z at the interface between the two media, there may be problems with the sensor distorting E_r and thereby also distorting J_{t_z}. With this inductively coupled device, however, it is not necessary that the sensor conductors be in direct contact with either medium. Such a feature is, however, required of the current sampling sensor. Perhaps, then, one can avoid significantly distorting E_r with the inductively coupled sensor. Suppose that the sensor is separated from the conducting media by a material which is comparatively a good insulator. If this insulator is thick enough, the sensor body does not short E_r across the entire extent of the sensor, at least for frequencies low enough that the capacitance across the insulator does not provide a significant current path. A two loop type of sensor might be better than a Rogowski coil in this regard in that the two loop signal cables might provide a higher impedance than the Rogowski coil body. This shorting effect might be further reduced by putting chokes around the signal cables, thereby breaking a short circuit between the two loops, at least for some of the higher frequencies. Even with the insulators the electric field distribution is distorted near the sensor body (which includes the insulators). Referring to figure 3A, the dimensions of a cross section, d_4 and d_5, should be much less than a, which we can regard as the radius of a Rogowski coil or as half the separation of two loops. The electric field distortion then covers a small part of the region inside the Rogowski coil or between the two loops. It is this region where the J_{t_z} of interest flows. Perhaps, then, it is possible to avoid to some extent some of the electric field distortion problems by using some kind of an inductively coupled sensor.

The inductively coupled sensor also tends to avoid problems associated with boundary layer effects between the conducting air and the lower medium. The area enclosed by the Rogowski coil (or between two loops) is not changed by the presence of the sensor. Ideally, J_{t_z} flows between the air and lower medium without passing into or out of the sensor. The sensor couples to the magnetic field associated with this current density. Since the sensor leaves the lower medium intact the nuclear radiation transport into and out of the lower medium, in the region enclosed by the sensor, should not be altered, except

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5. Lt Carl E. Baum, Sensor and Simulation Note XXIX, The Influence of Radiation and Conductivity on $\dot{B}$ Loop Design, Oct. 1966.
6. Capt Carl E. Baum, Sensor and Simulation Note XXX, The Single-Gap Cylindrical Loop in Non-Conducting and Conducting Media, Jan. 1967.

perhaps close to the sensor body. In some cases, this feature of leaving the area undisturbed, through which the current density of interest flows, may then be an advantage for the inductively coupled type of sensor.

There may be some problem with pickup of the principal magnetic field for the inductively coupled sensor. For simplicity consider the device consisting of two loops at different r's. Each responds to its local magnetic field. To determine J_{t_z} one must take the difference of the two signals, thus determining the derivative of rH_ϕ with respect to r. In some cases (for longer times), H_ϕ may change over distances comparable to those over which the nuclear radiation changes, say a γ-ray mean free path of a few hundred meters. The difference between the signals from two loops a meter or so apart is then quite small compared to the individual signals. This could present a significant problem in common mode rejection. For shorter times H_ϕ may change in much shorter distances, thus reducing the problem. One might separate the loops much farther apart to increase the differential signal but this lowers the frequency response of the sensor by increasing the transit time for the fields between the two loops. For changes in times less than this transit time J_{t_z} is not, in general, uniform between the two loops. A Rogowski coil may have this same problem as the two loops because of problems in putting sufficient accuracy into the symmetry of the device. Typically a Rogowski coil is used to measure a current through it in cases in which the magnetic field of interest is associated only with this current and not with a current density distributed over a comparatively large region. Perhaps a Rogowski coil type device can be made to reject some of the unwanted magnetic field such that it does not even link the turns of the coil. In any case, this type of common mode signal is a disadvantage not suffered by the current sampling type of sensor.

IV. Summary

It may be desirable, then, to measure other physical quantities besides electric and magnetic fields associated with the nuclear electromagnetic pulse. One such quantity is the total current density, or equivalently, the curl of the magnetic field. Specifically, it may be possible to measure the vertical component of the total current density at the ground or water surface.

Two types of sensors to measure this vertical current density are one which directly samples the current density, and one which inductively couples to it. Each of these sensors has certain advantages and disadvantages compared to the other. The current sampling sensor has a signal output proportional to the current density, even for low frequencies, while the inductively coupled sensor has an output proportional to the time derivative of the current density, at least for sufficiently low frequencies. The inductively coupled sensor may have a significant problem with common mode signals. On the other hand, the inductively coupled sensor may have less of an electric field distortion problem at high air conductivity levels than the current sampling sensor. The area, through which the current density being measured flows, is ideally left undisturbed by the current sampling sensor. Perhaps these sensors are somewhat complementary in that they might each be best used for different regimes of time, air conductivity, and conductivity of the lower medium. In some cases, we may have two somewhat independent methods of measuring the same quantity, the vertical total current density.