Sensor and Simulation Notes

Note 330

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General Properties of Antennas

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Abstract

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Abstract

In antenna design there are some fundamental relationships based on reciprocity. The equivalence of antenna pattern in transmission and reception is well known. Less well known is the time-derivative relationship going from reception to transmission. These relationships are derived here and expressed in various useful forms. Electric and magnetic dipoles are given special consideration, and the combined form constructed as a terminated TEM transmission line (the BTW antenna) is discussed for its transmission and reception properties.

antennas, TEM waves, dipole antennas

I. Introduction

There are many kinds of antennas for various applications, such as communications, radar, measurements, EMP (electromagnetic pulse) simulation, etc. Some are intended to operate over narrow bands of frequencies, while others are intended to radiate/receive transient pulses. All of these have certain general properties as discussed in this paper.

Assuming the use of reciprocal media there are fundamental reciprocity relations between two antennas, and between the transmission and reception properties of the same antenna. Here these are exhibited in various useful forms involving open-circuit voltages, short-circuit currents, and wave (scattering) parameters.

An important special class of antennas is that of dipoles, electric and magnetic. Most antennas behave as such for frequencies sufficiently low that the antennas are electrically small. Besides being of interest themselves, ideal dipoles are also useful in establishing the reciprocity relation between transmission and reception for more general antennas. A special kind of dipole combining electric and magnetic properties is also discussed in both transmission and reception.

II. Antenna Parameters

Consider some antenna comprised of linear reciprocal material near the coordinate origin $\vec{r} = \vec{0}$. It has a port (terminal pair) with voltage V and current I with conventions of positive power into the antenna. As a transmitter we have the radiated far field as

$$\tilde{\vec{E}}_f(\vec{r},s) = \frac{e^{-\gamma r}}{r}\ \tilde{\vec{F}}_V\left(\vec{1}_r,s\right)\ \tilde{V}(s)$$

$$= \frac{e^{-\gamma r}}{r}\ \tilde{\vec{F}}_I\left(\vec{1}_r,s\right)\ \tilde{I}(s)$$

$$= \frac{e^{-\gamma r}}{r}\ \tilde{\vec{F}}_w\left(\vec{1}_r,s\right)\ \tilde{V}_s(s) \equiv \text{far field (transmitted)}$$

$$\tilde{Z}_{in}(s) = \frac{\tilde{V}(s)}{\tilde{I}(s)} \equiv \text{input impedance} = \tilde{Y}_{in}^{-1}(s)$$

$$\tilde{\vec{F}}_I\left(\vec{1}_r,s\right) = \tilde{Z}_{in}(s)\ \tilde{\vec{F}}_V\left(\vec{1}_r,s\right)$$

$$\tilde{V}_s(s) = \frac{\tilde{Z}_{in}(s)+\tilde{Z}_s(s)}{\tilde{Z}_{in}(s)}\ \tilde{V}(s) \equiv \text{source voltage}$$

$$\tilde{Z}_s(s) \equiv \text{source impedance}$$

$$\tilde{\vec{F}}_w\left(\vec{1}_r,s\right) = \frac{\tilde{Z}_{in}(s)}{\tilde{Z}_{in}(s)+\tilde{Z}_s(s)}\ \tilde{\vec{F}}_V\left(\vec{1}_r,s\right)$$

$$\vec{1}_r \equiv \frac{\vec{r}}{|\vec{r}|} \equiv \text{radiation direction}$$

$$\tilde{\vec{F}}_V\left(\vec{1}_r,s\right)\cdot\vec{1}_r = 0 \quad \text{(transverse far field)} \qquad (2.1)$$

It is important to note that this is a far-field form which requires $r \to \infty$ for bounded $|s|$. As discussed in [7], for finite $r = |\vec{r}|$ one should limit the highest frequency (or fast changes in temporal characteristics) for such results to be valid. Recognizing this limitation let us proceed with this far-field form.

As a receiver consider an incident plane wave (asymptotic form of a far field) as

$$\tilde{\vec{E}}_i(\vec{r},s) = \tilde{\vec{E}}_o(s)\ e^{-\gamma \vec{1}_i \cdot \vec{r}}$$

$$\tilde{\vec{E}}_o(s) \cdot \vec{1}_i = 0$$

$$\vec{1}_i \equiv \text{direction of incidence} \tag{2.2}$$

Receiving this wave defines

$$\tilde{V}_{o.c.}(s) \equiv \tilde{\vec{h}}_V\left(\vec{1}_i,s\right) \cdot \tilde{\vec{E}}_o(s) = \text{open circuit voltage}$$

$$\tilde{I}_{s.c.}(s) \equiv \tilde{\vec{h}}_I\left(\vec{1}_i,s\right) \cdot \tilde{\vec{E}}_o(s) = \text{short circuit current}$$

$$\tilde{V}_L(s) \equiv \tilde{\vec{h}}_w\left(\vec{1}_i,s\right) \cdot \tilde{\vec{E}}_o(s) = \text{voltage into impedance } \tilde{Z}_L(s) \text{ loading antenna port} \tag{2.3}$$

The traditional effective height is $\tilde{\vec{h}}_V$. The other related vectors are

$$\tilde{\vec{h}}_I\left(\vec{1}_i,s\right) = -\tilde{Y}_{in}(s)\ \tilde{\vec{h}}_V\left(\vec{1}_i,s\right)$$

$$\tilde{\vec{h}}_w\left(\vec{1}_i,s\right) = \frac{\tilde{Z}_L(s)}{\tilde{Z}_{in}(s)+\tilde{Z}_L(s)}\ \tilde{\vec{h}}_V\left(\vec{1}_i,s\right) \tag{2.4}$$

noting again that positive current is for power <u>into</u> the antenna port.

Now consider the characteristics of the input impedance or

$$\tilde{Y}_{in}(s) \equiv \frac{1}{\tilde{Z}_{in}(s)} \equiv \text{input admittance} \tag{2.5}$$

As discussed in [13] one can make a pole/zero expansion of this function. The leading term might be inductive, capacitive, or even resistive depending on the particular antenna type being considered, this being characteristic of electrically small antennas, such as considered in Section III. So let us write

$$\tilde{Z}_{in}(s) = \tilde{Z}_\ell(s)\ \frac{\prod_{\nu=1}^{\infty}\left(1 - \frac{s}{s_{z_\nu}}\right)}{\prod_{\alpha=1}^{\infty}\left(1 - \frac{s}{s_{p_\alpha}}\right)}$$

$$s_{p_\alpha} = \text{poles of } \tilde{Z}_{in}(s) = \text{zeros of } \tilde{Y}_{in}(s)$$

$$\neq 0$$

$$s_{z_v} = \text{ zeros of } \tilde{Z}_{in}(s) = \text{ poles of } \tilde{Y}_{in}(s)$$

$$\neq 0 \tag{2.6}$$

where the low-frequency behavior can be summarized in the possibilities

$$\tilde{Z}_{\ell}(s) = \begin{cases} sL & \text{for inductance (parallel) across input (loop or magnetic dipole)} \\ \dfrac{1}{sC} & \text{for capacitance (series) across input (electric dipole)} \\ R & \text{for resistance across input (parallel to capacitance, series to inductance)} \end{cases} \tag{2.7}$$

Note the poles and zeros occur in conjugate pairs in the left-half s plane, except of course for those on the $-\Omega$ axis. The terms in (2.6) should be included in order of ascending $|s_{p_\alpha}|$ and $|s_{z_v}|$, conjugate pairs being included together in the successive approximations to maintain the conjugate symmetry at each stage.

One can ask if these cover the various possibilities. Early considerations thought of this as some kind of circuit model. Commensurate with this one can establish $\tilde{Z}_{in}(s)$ and $\tilde{Y}_{in}(s)$ as positive real (p.r.) functions with

$$\tilde{Z}_{in}(s^*) = \tilde{Z}^*_{in}(s)$$

$$\text{Re}\left[\tilde{Z}_{in}(s)\right] \geq 0 \text{ for } \text{Re}[s] \geq 0 \text{ (RHP)}$$

$$-\frac{\pi}{2} \leq \arg\left[\tilde{Z}_{in}(s)\right] \leq \frac{\pi}{2} \text{ in RHP}$$

$$\tilde{Z}_{in}(s) \sim As^q \text{ , } |q| \leq 1 \text{ and } A > 0 \text{ as } |s| \to \infty \text{ in RHP} \tag{2.8}$$

As discussed in [14], typically $q = 0$, i.e. $\tilde{Z}_{in}$ is resistive asymptotically in the RHP for typical realistic antennas due to the construction of the antennas where signals are fed into (or out of) them.

In more modern considerations of the analytic properties of finite-size electromagnetic structures comprised of perfect conductors and other simple media, it has been established that such functions are meromorphic in the s plane, justifying the form in (2.6) and (2.7) as being appropriate [8, 9]. Furthermore this can be case in other forms for equivalent networks, with transmission and reception properties included as well [10,14]. There are many appropriate references included in [12].

III. Electrically Small Dipoles

Especially important to our consideration of general antenna properties are those of electrically small dipoles, electric and magnetic and combinations thereof. By electrically small we mean that the radian wavelength $\lambda\!\!\!^{-}$ (in free space as well as other appropriate materials that might be used) is large compared to antenna dimensions. These give the dominant properties of all electrically small antennas as effective radiators [1,2,3,4,13]. For transient antennas the low-frequency properties of the pulse are dominated by the dipole characteristics.

At low frequencies electric dipoles consisting of two separate conductors connected through a port behave in reception as [16]

$$\tilde{V}_{o.c.}(s) = \vec{h}_e \cdot \tilde{\vec{E}}_o(s)$$

$$\tilde{I}_{s.c.}(s) = -s\epsilon_o \vec{A}_e \cdot \tilde{\vec{E}}_o(s)$$

$$\vec{h}_e \equiv \text{equivalent height}$$

$$\vec{A}_e = \frac{C_a}{\epsilon_o}\vec{h}_e \equiv \text{equivalent area}$$

$$C_a \equiv \text{antenna capacitance}$$

$$\tilde{Y}_{in} = sC_a \tag{3.1}$$

In terms of previously defined general reception parameters we have for $s \to 0$

$$\tilde{\vec{h}}_V(\vec{1}_i, s) = \vec{h}_e$$

$$\tilde{\vec{h}}_I(\vec{1}_i, s) = -s\epsilon_o \vec{A}_e = -sC_a \vec{h}_e$$

$$= -sC_a \tilde{\vec{h}}_V(\vec{1}_i, s) \tag{3.2}$$

In transmission at low frequencies the behavior of an electric dipole is dominated by the electric-dipole moment as [2,4]

$$\tilde{\vec{p}}(s) = \tilde{Q}(s)\vec{h}_e = \tilde{V}(s) C_a \vec{h}_e$$

$$\tilde{Q}(s) \equiv \text{antenna charge} \tag{3.3}$$

with the equivalent height applying in both transmission and reception [3]. The radiated far field at low frequencies is

$$\tilde{\vec{E}}_f(\vec{r},s) = -e^{-\gamma r} \frac{\mu_o}{4\pi r} s^2 \; \vec{1}_r \cdot \tilde{\vec{p}}(s)$$

$$= -e^{-\gamma r} \frac{\mu_o}{4\pi r} s^2 \; C_a \; \vec{1}_r \cdot \vec{h}_e \; \tilde{V}(s) = -e^{-\gamma r} \frac{\mu_o s}{4\pi r} \vec{1}_r \cdot \vec{h}_e \; \tilde{I}(s)$$

$$\vec{1}_r = \vec{1} - \vec{1}_r \; \vec{1}_r \tag{3.4}$$

From this we identify for $s \to 0$

$$\tilde{\vec{F}}_V\left(\vec{1}_r, s\right) = -\frac{\mu_o}{4\pi} C_a \; s^2 \; \vec{1}_r \cdot \vec{h}_e$$

$$= -\frac{\gamma^2}{4\pi} \vec{1}_r \cdot \vec{A}_e$$

$$\tilde{\vec{F}}_I\left(\vec{1}_r, s\right) = -s\frac{\mu_o}{4\pi} \vec{1}_r \cdot \vec{h}_e = \frac{1}{s\,C_a} \tilde{\vec{F}}_V\left(\vec{1}_r, s\right) \tag{3.5}$$

This illustrates the time-integral relationship (for low-frequencies here) going from transmission (port voltage) to reception (short circuit current). This applies also going from transmission (port current) to reception (open circuit voltage).

In time domain, in terms of finite energy in the energy source (pulser), we can think of a late-time electric-dipole moment $\vec{p}(\infty)$ representing the resulting electric-dipole moment from a capacitive source (into a capacitive load C_a) or the change from the discharge of the precharged condition of such a capacitive antenna. Then in a low-frequency sense [2, 4]

$$\tilde{\vec{p}}(s) = \frac{1}{s} \vec{p}(\infty) = \frac{Q(\infty)}{s} \vec{h}_e = \frac{C_a V(\infty)}{s} \vec{h}_e = \frac{\epsilon_o \, V(\infty)}{s} \vec{A}_e$$

$$\tilde{\vec{E}}_f(s) = -e^{-\gamma r} \frac{\mu_o}{4\pi r} s \; \vec{1}_r \cdot \vec{p}(\infty) \tag{3.6}$$

Considering the non-zero energy

$$U_e = \frac{1}{2} C_a \; V^2(\infty) = \frac{1}{2} Q(\infty) \; V(\infty) \tag{3.7}$$

required to charge up the late-time electric-dipole moment, then (3.6) represents the best one can do at low frequencies. As discussed in [2, 4] this implies that the radiated time-domain waveform must have at least one zero crossing (sign reversal for a not-identically-zero component) to preserve zero integral over time (or "area" of the waveform).

A related question concerns the response of such an electric dipole in reception of an incident step-function plane wave [1, 16]. The received energy into a resistive load is proportional to an equivalent volume

$$V_e = \frac{\epsilon_o}{C_a} \vec{A}_e \cdot \vec{A}_e = \vec{A}_e \cdot \vec{h}_e = \frac{C_a}{\epsilon_o} \vec{h}_e \cdot \vec{h}_e \tag{3.8}$$

Given some geometrical volume with a given shape (spherical or whatever) one can consider various electric-dipole designs to maximize V_e. Note that this equivalent volume assumes that frequencies of interest are dominantly low, i.e. that for a resistive load R, the characteristic time $R\,C_a$ is much larger than transit times across the antenna. For some cases (low-frequency dominated) V_e gives a design optimization parameter for receiving and by reciprocity for radiation.

The dual low-frequency antenna is a magnetic dipole consisting of a loop (perfectly conducting) connected to a port. In reception this behaves as [16]

$$\tilde{V}_{o.c.}(s) = s\,\mu_o\,\vec{A}_h \cdot \tilde{\vec{H}}_o(s)$$

$$\tilde{I}_{s.c.}(s) = -\vec{\ell}_h \cdot \tilde{\vec{H}}_o(s)$$

$$\vec{\ell}_h \equiv \text{equivalent length}$$

$$\vec{A}_h = \frac{L_a}{\mu_o}\,\vec{\ell}_h \equiv \text{equivalent area}$$

$$L_a \equiv \text{antenna inductance}$$

$$\tilde{Z}_{in} = sL_a$$

$$\tilde{\vec{H}}_o(s) = \frac{1}{Z_o}\,\vec{1}_i \times \tilde{\vec{E}}_o(s) \tag{3.9}$$

In terms of previously defined reception parameters we have for $s \to 0$

$$\vec{h}_V(\vec{1}_i,s) = \gamma\,\vec{A}_h \times \vec{1}_i$$

$$\vec{h}_I(\vec{1}_i,s) = -\frac{1}{Z_o}\,\vec{\ell}_h \times \vec{1}_i = -\frac{1}{cL_a}\,\vec{A}_h \times \vec{1}_i$$

$$= -\frac{1}{sL_a}\,\vec{h}_V(\vec{1}_i,s) \tag{3.10}$$

In transmission at low frequencies the behavior of a magnetic dipole is dominated by the magnetic-dipole moment as [4]

$$\tilde{\vec{m}}(s) = \tilde{I}(s)\,\vec{A}_h \tag{3.11}$$

The equivalent area also applies in reception (3.9). The radiated far field at low frequencies is

$$\tilde{\vec{E}}_f(\vec{r},s) = \frac{e^{-\gamma r}}{4\pi r} \frac{\mu_o}{c} s^2 \vec{1}_r \times \tilde{\vec{m}}(s)$$

$$= \frac{e^{-\gamma r}}{4\pi r} Z_o \gamma^2 \vec{1}_r \times \vec{A}_h \tilde{I}(s) \tag{3.12}$$

From this we identify for $s \to 0$

$$\tilde{\vec{F}}_V(\vec{1}_r,s) = \frac{\mu_o}{4\pi} \frac{\gamma}{L_a} \vec{1}_r \times \vec{A}_h = \frac{\gamma}{4\pi} \vec{1}_r \times \vec{\ell}_h$$

$$\tilde{\vec{F}}_I(\vec{1}_r,s) = \frac{Z_o}{4\pi} \gamma^2 \vec{1}_r \times \vec{A}_h = sL_a \tilde{\vec{F}}_V(\vec{1}_r,s) \tag{3.13}$$

Again we have the time-integral relationship at low frequencies going from transmission (port voltage) to reception (short circuit current), as well as going from transmission (port current) to reception (open circuit voltage).

In time domain a finite energy source (pulser) can drive a late-time magnetic-dipole moment $\vec{m}(\infty)$ representing the resulting magnetic-dipole moment from an inductive source (opening switch into an inductive load L_a), or the change from the interruption of the current already flowing in such an inductive antenna. Then in a low-frequency sense [4]

$$\tilde{\vec{m}}(s) = \frac{1}{s} \vec{m}(\infty) = \frac{I(\infty)}{s} \vec{A}_h = \frac{L_a}{\mu_o} I(\infty) \vec{\ell}_h$$

$$\tilde{\vec{E}}_f(s) = e^{-\gamma r} \frac{\mu_o \gamma}{4\pi r} \vec{1}_r \times \vec{m}(\infty) \tag{3.14}$$

Considering the non-zero energy

$$U_m = \frac{1}{2} L_a I^2(\infty) \tag{3.15}$$

required to produce the late-time magnetic-dipole moment, then (3.14) is the best that one can do at low frequencies. Again as in [4] the radiated time-domain waveform must have at least one zero crossing (for a not-identically zero component) to preserve zero integral over time.

In receiving a step-function incident plane wave the received energy is proportional to an equivalent volume [1, 16]

$$V_h = \frac{\mu_o}{L_a} \vec{A}_h \cdot \vec{A}_h = \vec{A}_h \cdot \vec{\ell}_h = \frac{L_a}{\mu_o} \vec{\ell}_h \cdot \vec{\ell}_h \tag{3.16}$$

For a given geometrical volume and shape one can optimize the design of a magnetic-dipole antenna to maximize V_h. Again frequencies of interest should be sufficiently low so that L_a/R (for resistive load R) is large compared to transit times across the antenna for this to apply.

IV. Antenna Reciprocity

The concept of antenna reciprocity can be considered in various ways [11, 17]. For present purposes consider two antennas designated "1" and "2" as in fig. 4.1. These are considered as a 2-port network characterized by

$$\left(\tilde{Z}_{n,m}(s)\right) = \begin{pmatrix} \tilde{Z}_{1,1}(s) & \tilde{Z}_{1,2}(s) \\ \tilde{Z}_{2,1}(s) & \tilde{Z}_{2,2}(s) \end{pmatrix} \equiv \text{impedance matrix}$$

$$\left(\tilde{Y}_{n,m}(s)\right) = \begin{pmatrix} \tilde{Y}_{1,1}(s) & \tilde{Y}_{1,2}(s) \\ \tilde{Y}_{2,1}(s) & \tilde{Y}_{2,2}(s) \end{pmatrix} \equiv \text{admittance matrix} \qquad (4.1)$$

which relate the two port voltages and the two port currents. With reciprocal media everywhere (permeability, permitivity, and conductivity matrices all symmetric) we have the well-known reciprocity results

$$\tilde{Z}_{2,1}(s) = \tilde{Z}_{1,2}(s)$$

$$\tilde{Y}_{2,1}(s) = \tilde{Y}_{1,2}(s) \qquad (4.2)$$

For completeness note that the diagonal terms are just the input impedances or admittances for the respective antennas (if they are sufficiently far apart).

In terms of antenna-port parameters (current positive into port) substituted in (4.2) we have

$$\frac{\tilde{V}^{(2)}_{o.c.}(s)}{\tilde{I}^{(1)}(s)} = \frac{\tilde{V}^{(1)}_{o.c.}(s)}{\tilde{I}^{(2)}(s)}$$

$$\frac{\tilde{I}^{(2)}_{s.c.}(s)}{\tilde{V}^{(1)}(s)} = \frac{\tilde{I}^{(1)}_{s.c.}(s)}{\tilde{V}^{(2)}(s)} \qquad (4.3)$$

which we can write as

$$\tilde{V}^{(1)}_{o.c.}(s)\ \tilde{I}^{(1)}(s) = \tilde{V}^{(2)}_{o.c.}(s)\ \tilde{I}^{(2)}(s)$$

$$\tilde{V}^{(1)}(s)\ \tilde{I}^{(1)}_{s.c.}(s) = \tilde{V}^{(2)}(s)\ \tilde{I}^{(2)}_{s.c.}(s) \qquad (4.4)$$

Note that parameters neither open nor short circuit are port parameters used in transmission.

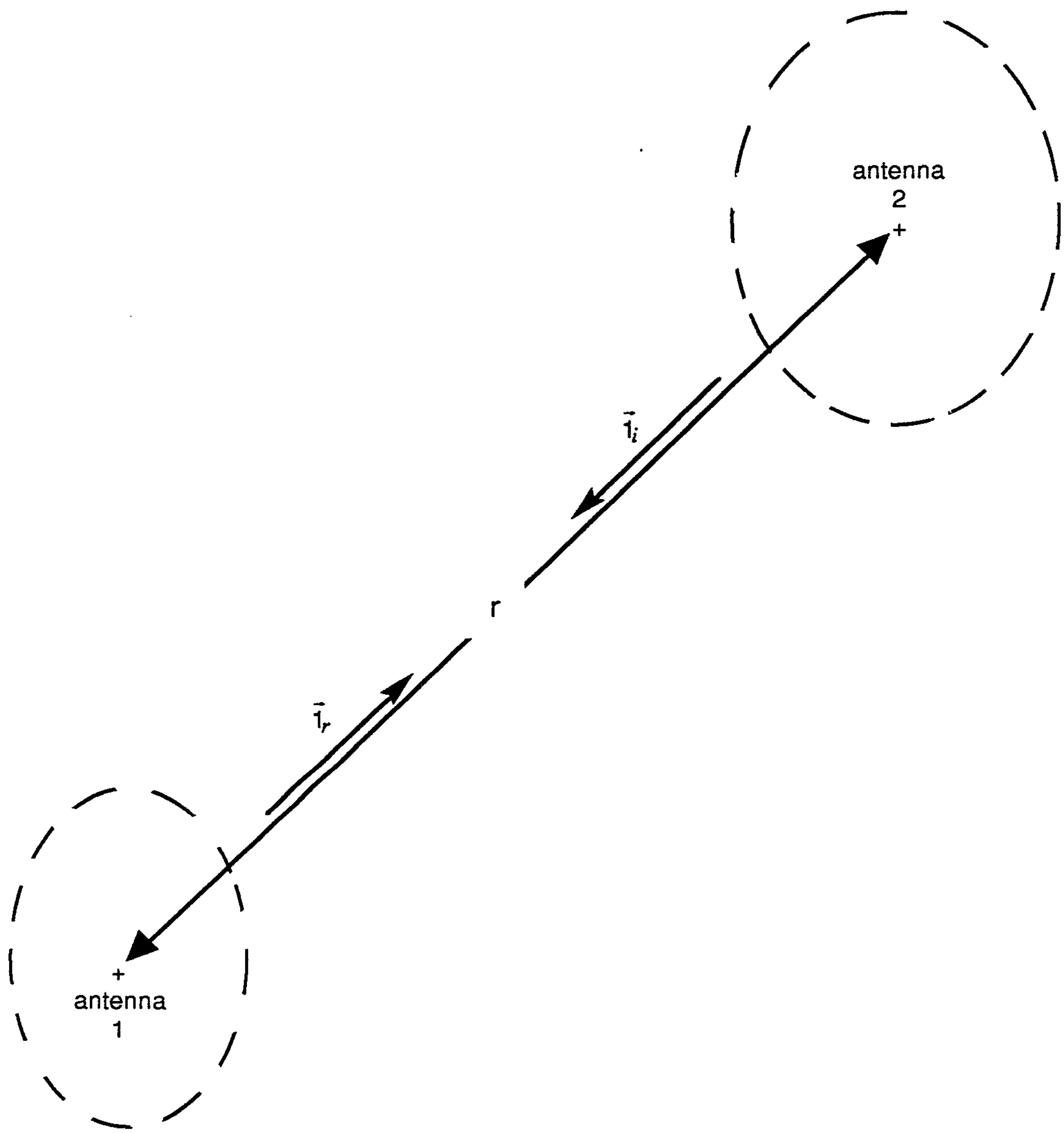

Figure 4.1. Mutual Transmission Between Two Antennas

Using the conventions in fig. 4.1 we have

$\vec{1}_r \equiv$ transmission direction from "1"

$\equiv$ reception direction for "2"

$\vec{1}_i \equiv$ transmission direction from "2"

$\equiv$ reception direction for "1"

$\vec{1}_i = -\vec{1}_r$

$$\overleftrightarrow{1}_r = \overleftrightarrow{1}_i = \overleftrightarrow{1} - \vec{1}_r\,\vec{1}_r = \overleftrightarrow{1} - \vec{1}_i\,\vec{1}_i \equiv \text{transverse dyad} \tag{4.5}$$

From Section II we relate the open-circuit voltages and port currents via

$$\tilde{V}^{(1)}_{o.c.}(s) = \tilde{\vec{h}}^{(1)}_V\left(\vec{1}_i,s\right) \cdot \left\{\frac{e^{-\gamma r}}{r}\,\tilde{\vec{F}}^{(2)}_I\left(\vec{1}_i,s\right)\,\tilde{I}^{(2)}(s)\right\}$$

$$\tilde{V}^{(2)}_{o.c.}(s) = \tilde{\vec{h}}^{(2)}_V\left(\vec{1}_r,s\right) \cdot \left\{\frac{e^{-\gamma r}}{r}\,\tilde{\vec{F}}^{(1)}_I\left(\vec{1}_r,s\right)\,\tilde{I}^{(1)}(s)\right\} \tag{4.6}$$

which with the reciprocity relations give

$$\tilde{\vec{h}}^{(1)}_V\left(\vec{1}_i,s\right) \cdot \tilde{\vec{F}}^{(2)}_I\left(\vec{1}_i,s\right) = \tilde{\vec{h}}^{(2)}_V\left(\vec{1}_r,s\right) \cdot \tilde{\vec{F}}^{(1)}_I\left(\vec{1}_r,s\right) \tag{4.7}$$

Similarly the short-circuit currents and port voltages are related via

$$\tilde{I}^{(1)}_{s.c.}(s) = \tilde{\vec{h}}^{(1)}_I\left(\vec{1}_i,s\right) \cdot \left\{\frac{e^{-\gamma r}}{r}\,\tilde{\vec{F}}^{(2)}_V\left(\vec{1}_i,s\right)\,\tilde{V}^{(2)}(s)\right\}$$

$$\tilde{I}^{(2)}_{s.c.}(s) = \tilde{\vec{h}}^{(2)}_I\left(\vec{1}_r,s\right) \cdot \left\{\frac{e^{-\gamma r}}{r}\,\tilde{\vec{F}}^{(1)}_V\left(\vec{1}_r,s\right)\,\tilde{V}^{(1)}(s)\right\} \tag{4.8}$$

which now give

$$\tilde{\vec{h}}^{(1)}_I\left(\vec{1}_i,s\right) \cdot \tilde{\vec{F}}^{(2)}_V\left(\vec{1}_i,s\right) = \tilde{\vec{h}}^{(2)}_I\left(\vec{1}_r,s\right) \cdot \tilde{\vec{F}}^{(1)}_V\left(\vec{1}_r,s\right) \tag{4.9}$$

Note that (4.7) and (4.9) are equivalent by interchanging V and I subscripts and including the input admittances of the two antennas required by this interchange via the formulas in Section II. This can also be expressed in terms of wave parameters with the various impedances included in the expression.

V. Antenna Self Reciprocity

As is well known antennas have closely related properties in transmission and reception. As discussed in Section III elementary dipoles have particularly simple responses in transmission and reception. In the case of the electric dipole the equivalent height characterizes both transmission and reception as is easily seen in special cases [3]. A similar simplicity can be seen for the equivalent area in the case of a magnetic dipole (loop). A most important result concerns similar results for general antennas [11, 17].

A simple approach to this problem is to select one of the antennas, say number 2, as an elementary electric dipole for which (from (3.2) and (3.5))

$$\tilde{\vec{h}}_V^{(2)}\left(\vec{1}_r,s\right) = \tilde{\vec{h}}_e = -\frac{1}{sC_a}\,\tilde{\vec{h}}_I\left(\vec{1}_r,s\right)$$

$$\tilde{\vec{F}}_I^{(2)}\left(\vec{1}_i,s\right) = -s\frac{\mu_o}{4\pi}\,\vec{1}_i\cdot\tilde{\vec{h}}_e = \frac{1}{sC_a}\,\tilde{\vec{F}}_V^{(2)}\left(\vec{1}_i,s\right) \tag{5.1}$$

Substituting in (4.7) gives

$$-s\frac{\mu_o}{4\pi}\,\tilde{\vec{h}}_e\cdot\vec{1}_i\cdot\tilde{\vec{h}}_V^{(1)}\left(\vec{1}_i,s\right) = \tilde{\vec{h}}_e\cdot\tilde{\vec{F}}_I^{(1)}\left(\vec{1}_r,s\right) \tag{5.2}$$

Noting that $\tilde{\vec{h}}_e$ can be chosen with arbitrary orientation and replacing $\vec{1}_i$ by $\vec{1}_r$, we have for an arbitrary reciprocal antenna (superscript "1" no longer needed)

$$\tilde{\vec{F}}_I\left(\vec{1}_r,s\right) = -s\frac{\mu_o}{4\pi}\,\vec{1}_r\cdot\tilde{\vec{h}}_V\left(-\vec{1}_r,s\right) \tag{5.3}$$

Stated in words the radiation from a port current is proportional to the (negative) time derivative of the open-circuit voltage, and the angular variation (pattern) is the same with a reversal of direction.

Similarly substituting in (4.9) gives

$$-\frac{s^2C_a\mu_o}{4\pi}\,\tilde{\vec{h}}_e\cdot\vec{1}_i\cdot\tilde{\vec{h}}_I^{(1)}\left(\vec{1}_i,s\right) = -sC_a\,\tilde{\vec{h}}_e\cdot\tilde{\vec{F}}_V^{(1)}\left(\vec{1}_r,s\right) \tag{5.4}$$

Again varying the $\tilde{\vec{h}}_e$ orientation over 4π steradians and replacing $\vec{1}_i$ by $-\vec{1}_r$ we have for general reciprocal antennas

$$\tilde{\vec{F}}_V\left(\vec{1}_r,s\right) = s\frac{\mu_o}{4\pi}\,\vec{1}_r\cdot\tilde{\vec{h}}_I\left(-\vec{1}_r,s\right) \tag{5.5}$$

Stated in words the radiation from a port voltage is proportional to the time derivative of the short-circuit current, and the angular variation (pattern) is the same with a reversal of direction.

In terms of wave parameters we have, substituting from (2.1) and (2.4),

$$\left[\tilde{Z}_{in}(s) + \tilde{Z}_s(s)\right]\tilde{\vec{F}}_w\left(\vec{1}_r,s\right) = -s\frac{\mu_o}{4\pi}\,\frac{\tilde{Z}_{in}(s)+\tilde{Z}_L(s)}{\tilde{Z}_L(s)}\,\vec{1}_r\cdot\tilde{\vec{h}}_w\left(-\vec{1}_r,s\right) \tag{5.6}$$

While this is more complicated in form than (5.3) and (5.5) it can be simplified in interesting special cases. First let the source and load impedance be the same giving

$$\tilde{Z}_s(s) \equiv \tilde{Z}_L(s)$$

$$\tilde{\vec{F}}_w(\vec{1}_r, s) = -\frac{s}{\tilde{Z}_L(s)} \frac{\mu_o}{4\pi} \vec{1}_r \cdot \tilde{\vec{h}}_w(-\vec{1}_r, s) \tag{5.7}$$

Next let the load impedance be a simple resistance giving

$$\tilde{Z}_L(s) \equiv R$$

$$\tilde{\vec{F}}_w(\vec{1}_r, s) = -s \frac{\mu_o}{4\pi R} \vec{1}_r \cdot \tilde{\vec{h}}_w(-\vec{1}_r, s) \tag{5.8}$$

In this form the (negative) time derivative relationship reappears.

An interesting application of this result (5.8) concerns an antenna fed by some length of transmission line of characteristic impedance Z_c (frequency independent). Choosing

$$Z_c = R \tag{5.9}$$

Then the transmission line is terminated at the load/source end in its characteristic impedance (no reflections there). Then, if the transmission line has transit time t_o, let there be a pulse from the source $V_s(t)$ beginning at $t = 0$. For a time $2t_o$ (at least) the transmitted voltage onto the transmission (at the input) is just

$$V_t(t) = \frac{V_s(t)}{2} \quad \text{for } 0 \le t < 2t_o \tag{5.10}$$

In terms of the radiating properties of the antenna we can then define

$$\tilde{\vec{F}}_t(\vec{1}_r, s) = 2\tilde{\vec{F}}_w(\vec{1}_r, s) \tag{5.11}$$

so that the same radiated fields result. Noting that $\vec{h}_w$ corresponds to the voltage into the load R we can write

$$\tilde{\vec{h}}_t(-\vec{1}_r, s) = \tilde{\vec{h}}_w(-\vec{1}_r, s) \tag{5.12}$$

giving

$$\tilde{\vec{F}}_t(\vec{1}_r, s) = -\frac{s\mu_o}{4\pi R} \vec{1}_r \cdot \tilde{\vec{h}}_t(-\vec{1}_r, s) \tag{5.13}$$

Note, however, that this applies, for general antennas, over times limited as in (5.10).

VI. Combined Dipoles

As discussed in [4, 5, 6] there is an important case of dipoles which might be called a combined dipole. This is characterized by

$$\vec{p}(t) = p(t)\,\vec{1}_p \quad , \quad \vec{m}(t) = m(t)\,\vec{1}_m$$

$$\vec{1}_p \cdot \vec{1}_m = 0 \quad , \quad m(t) = c\,p(t)$$

$$\vec{1}_1 = \vec{1}_p \times \vec{1}_m = \text{principal radiation direction (center of beam)} \tag{6.1}$$

Considerations to date have been for time invariant polarization, but $\vec{1}_p$ (and $\vec{1}_m$) can in principle vary as a function of time. The complete dipole fields can be written out [4] (applying as long as r and λ are large compared to antenna dimensions). In the $\vec{1}_1$ (forward) direction all 3 dipole terms are balanced in the TEM sense with impedance Z_o. In the $-\vec{1}_1$ (backward) direction there is a radiation null with both $e^{-\gamma r}/r$ and $e^{-\gamma r}/r^2$ terms being zero. The third order term $\left(e^{-\gamma r}/r^3\right)$ term is nonzero in the $-\vec{1}_1$ direction, but still has

$$\vec{1}_1 \times \vec{E} = Z_o\,\vec{H} \tag{6.2}$$

i.e. TEM in the $+\vec{1}_1$ direction, just like in the forward direction from the antenna.

For present purposes we have the radiated far field as

$$\begin{aligned}
\tilde{\vec{E}}_f(\vec{r},s) &= -e^{-\gamma r}\frac{\mu_o}{4\pi r}\,s^2\left\{\vec{1}_r \cdot \tilde{\vec{p}}(s) - \frac{1}{c}\,\vec{1}_r \times \tilde{\vec{m}}(s)\right\} \\
&= -e^{-\gamma r}\frac{\mu_o}{4\pi r}\,s^2\left\{\vec{1}_r \cdot \vec{1}_p - \vec{1}_r \times \vec{1}_m\right\}\tilde{p}(s) \\
&= -e^{-\gamma r}\frac{\mu_o}{4\pi r}\,s^2\left\{\vec{1}_r \cdot \vec{1}_p - \vec{1}_r \times \left[\vec{1}_1 \times \vec{1}_p\right]\right\}\tilde{p}(s)
\end{aligned} \tag{6.3}$$

The angular variation includes polarization. If we square this to obtain the power angular variation (noting the TEM character of the far field) we have

$$\left|\vec{1}_r \cdot \vec{1}_p - \vec{1}_r \times \vec{1}_m\right|^2 = 2\left[1 + \vec{1}_1 \cdot \vec{1}_r\right] - \left(\vec{1}_p \cdot \vec{1}_r\right)^2 - \left(\vec{1}_m \cdot \vec{1}_r\right)^2 \tag{6.4}$$

using standard vector/dyadic identities [15]. Defining coordinates via

$$\vec{1}_x = \vec{1}_p \quad , \quad \vec{1}_y = \vec{1}_m \quad , \quad \vec{1}_z = \vec{1}_1$$

$$x = \Psi \cos(\phi) \quad , \quad y = \Psi \sin(\phi)$$

$$z = r \cos(\theta) \quad , \quad \Psi = r \sin(\theta) \tag{6.5}$$

the pattern function (4.4) is 4 in the +2 direction, 0 in the -2 direction, and 1 in the Ψ direction $(\theta = \pi/2)$ independent of ϕ. This pattern is rotationally symmetric about the z axis.

In principle there are many possible designs for such a combined dipole or $\vec{p} \times \vec{m}$ antenna [4, 5]. For energy efficiency one may wish to have two sources separately driving the $\vec{p}$ (capacitive load) and $\vec{m}$ (inductive load). However, one may also wish to drive both (in proper balance) from a single source. Let us consider this latter case.

As discussed for transmission (MEDIUS) [5] or as a sensor (or receiver, designated BTW [6]), this type of antenna is a length of specially designed TEM transmission line, terminated in its characteristic impedance (hence BTW (balanced transmission-line wave)). As indicated in fig. 6.1 this consists of a transmission line of characteristic impedance Z_c, resistively terminated in this impedance, i.e.

$$R = Z_c \tag{6.6}$$

To a good approximation

$$\tilde{Z}_{in}(s) \simeq R \tag{6.7}$$

This is found from transmission-line theory for $\lambda >> 2h$. For higher frequencies this still holds if the input section (length ℓ_1) is a conical transmission line of this same impedance, the TEM mode dominating the results.

As discussed in [5, 6] the dipole moments for each incremental length of transmission line are balanced as in (6.1) with vector orientation as indicated in fig. 6.1. For the magnetic-dipole moment one has the area simply as

$$\vec{A}_h = A_h \vec{1}_m$$

$$A_h = [\ell_1 + 2\ell_2 + \ell_3]\, h \tag{6.8}$$

Of course we need

$$\lambda >> \ell = \ell_1 + \ell_2 + \ell_3 \tag{6.9}$$

for the dipole behavior to dominate, one can integrate the charge per unit length times local spacing to get the electric dipole moment, or just note that when driven the port parameters are related by

$$\frac{\tilde{V}(s)}{\tilde{I}(s)} = R \tag{6.10}$$

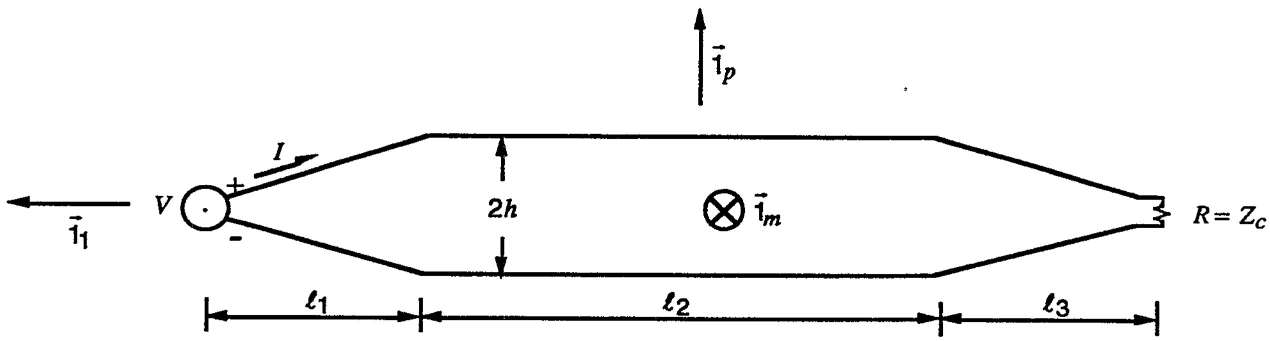

Figure 6.1. Combined Dipole Antenna

The dipole moments are

$$\tilde{\vec{m}}(s) = \vec{A}_h \; \tilde{I}(s)$$

$$\tilde{\vec{p}}(s) = \vec{h}_e \; \tilde{Q}(s) = C_a \; \vec{h}_e \; \tilde{V}(s) \tag{6.11}$$

Note this height now represents behavior in transmission, not open-circuit reception (due to the R termination). Now we have

$$C_a \; \vec{h}_e = \frac{1}{cR} \left[\ell_1 + 2\ell_2 + \ell_3 \right] h \; \vec{1}_p = \frac{A_h}{cR} \; \vec{1}_p = \frac{1}{cR} \; \vec{A}_h \times \vec{1}_1 \tag{6.12}$$

so that (6.11) satisfies (6.1).

Now consider the parameters defined in Section II together with their relation as in Section V. In transmission we have (in the electrically small regime)

$$\tilde{\vec{\vec{F}}}_V\left(\vec{1}_r, s\right) = -s^2 \; \frac{\mu_o}{4\pi} \; \frac{A_h}{cR} \left\{ \vec{1}_r \cdot \vec{1}_p - \vec{1}_r \times \vec{1}_m \right\}$$

$$\tilde{\vec{\vec{F}}}_I\left(\vec{1}_r, s\right) = R \, \tilde{\vec{\vec{F}}}_V\left(\vec{1}_r, s\right)$$

$$\tilde{\vec{\vec{F}}}_w\left(\vec{1}_r, s\right) = \frac{1}{2} \; \tilde{\vec{\vec{F}}}_V\left(\vec{1}_r, s\right) \qquad \left(\text{with } \tilde{Z}_s(s) = R\right)$$

$$\tilde{\vec{\vec{F}}}_t\left(\vec{1}_r, s\right) = \tilde{\vec{\vec{F}}}_V\left(\vec{1}_r, s\right) \tag{6.13}$$

In reception these become (noting that the incident field is already transverse to $\vec{1}_r$)

$$\vec{1}_r \cdot \tilde{\vec{\vec{h}}}_V\left(-\vec{1}_r, s\right) = \frac{s}{c} \; A_h \left\{ \vec{1}_r \cdot \vec{1}_p - \vec{1}_r \times \vec{1}_m \right\}$$

$$\tilde{\vec{\vec{h}}}_I\left(-\vec{1}_r, s\right) = -\frac{1}{R} \; \tilde{\vec{\vec{h}}}_V\left(-\vec{1}_r, s\right)$$

$$\tilde{\vec{\vec{h}}}_w\left(-\vec{1}_r, s\right) = \frac{1}{2} \; \tilde{\vec{\vec{h}}}_V\left(-\vec{1}_r, s\right) \qquad \left(\text{with } \tilde{Z}_L(s) = R\right)$$

$$= \tilde{\vec{\vec{h}}}_t\left(-\vec{1}_r, s\right) \tag{6.14}$$

As discussed in Section V a section of transmission line can be used to define transmitted voltage at the antenna input (subscript-t parameters). With this chosen to have characteristic impedance R it is matched (approximately for all frequencies) to the transmission-line antenna with termination R. The restriction of $2t_o$ for pulses (based on time before reflections occur in transmission) can be listed for this special kind of antenna due to the termination conditions.

VII. Concluding Remarks

As we have seen, the transmission and reception properties of antennas are closely related. One can express these in various forms. Note, however, the restriction to distances and frequencies for which the far-field approximation is valid.

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