Sensor and Simulation Notes

Note 332

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Wedge Dielectric Lenses for TEM Waves Between Parallel Plates

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Abstract

This paper considers the design of a uniform isotropic dielectric lens for use in bending the propagation of a TEM wave between two wide parallel plates (ideally infinitely wide). The resulting wedge lens is installed at a corresponding bend in the parallel plates. The bend angle is a function of the relative dielectric constant of the lens. Examples are discussed.

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dielectrics, TEM waves, lens antennas

I. Introduction

For distortionless propagation of transient electromagnetic waves one can use a TEM waveguide (or transmission line) comprised of perfect guiding conductors in a uniform lossless and dispersionless medium (e.g. free space). Here we consider a cylindrical system comprised of two parallel perfectly conducting sheets, infinitely wide, with a uniform TEM plane wave propagating between the sheets in a particular direction.

If one wishes to bend this waveguide to change the direction of propagation of the wave, then distortions are introduced at the bend. Frequencies in the wave with half wavelength smaller than the plate spacing h will appear in various higher order E modes supported by such parallel plates. This can be corrected by insertion of a lens in the bend region which matches the TEM mode into another TEM mode in the lens, and in turn back into the TEM mode of the waveguide after the lens. This paper discusses the design of such a lens made of a uniform isotropic dielectric. There are various other kinds of exact lenses for transitioning TEM waves as discussed in [4], to which the present example can be compared.

II. Plane-Wave Transmission Between Two Dielectric Media: E Waves

As indicated in Figure 2.1 let there be a plane wave in medium 1 incident on a plane surface S which is the boundary between two dielectric media. Let both media have the same permeability μ_0, but different permittivities ϵ_1 and ϵ_2 (real, i.e. no conductivity). With angles as indicated with respect to the surface normal $\vec{1}_S$, matching the phase velocities along the boundary as in the usual derivations [2,3] gives

$$\psi_r = \psi_i$$

$$\sqrt{\epsilon_1}\,\sin(\psi_i) = \sqrt{\epsilon_2}\,\sin(\psi_t) \tag{2.1}$$

Considering an E (or TM) wave incident (and same for reflected and transmitted waves) we have the transmission and reflection of the various fields as

$$R_e \equiv \frac{E_{refl}}{E_{inc}} = \frac{H_{refl}}{H_{inc}} = R_h$$

$$T_h = \frac{H_{trans}}{H_{inc}} = 1 + R_h$$

$$T_e = \frac{E_{trans}}{E_{inc}} = \frac{Z_2}{Z_1}\,T_h = \left[\frac{\epsilon_1}{\epsilon_2}\right]^{\frac{1}{2}} T_h \tag{2.2}$$

$$Z_1 \equiv \sqrt{\frac{\mu_o}{\epsilon_1}}\;,\quad Z_2 \equiv \sqrt{\frac{\mu_o}{\epsilon_2}}$$

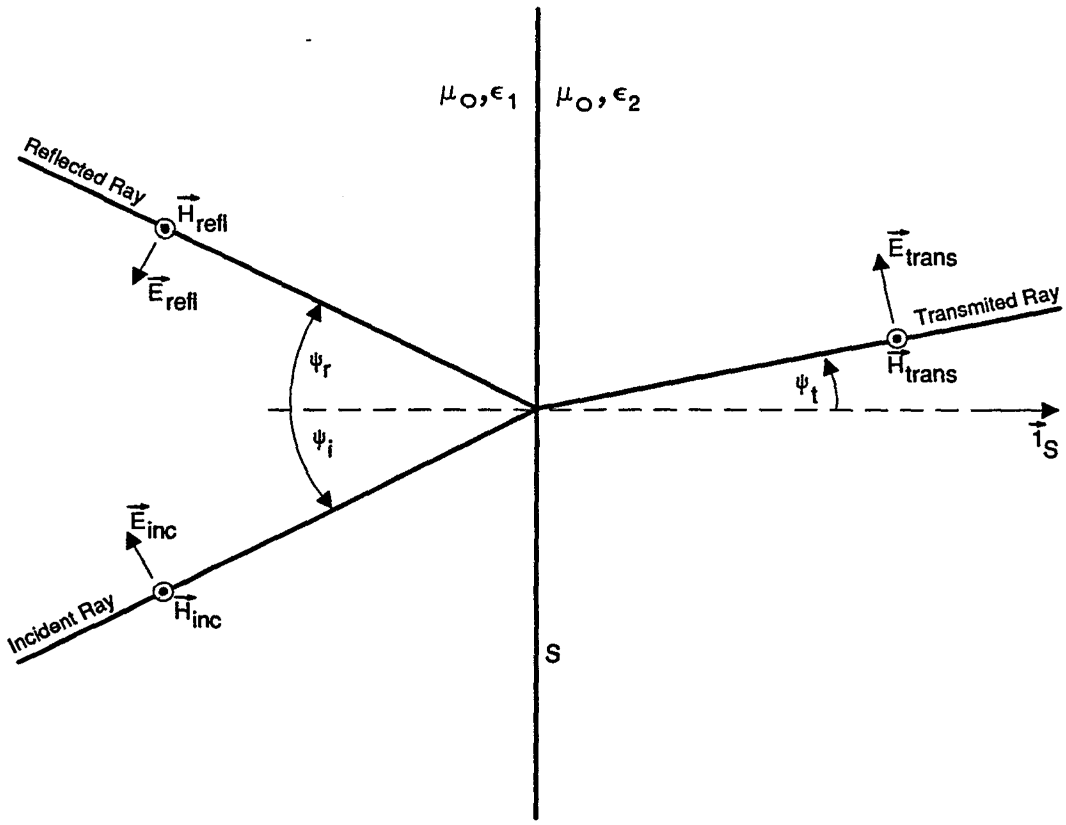

Figure 2.1. Transmission and Reflection of E Wave at Interface Between Two Dielectric Media

The fields here are vector components with orientations as in Figure 2.1. As in [2,3] we have

$$R_e = R_h = \frac{\left[\frac{\epsilon_2}{\epsilon_1}\right]^{\frac{1}{2}} \cos(\psi_i) - \left[1 - \frac{\epsilon_1}{\epsilon_2} \sin^2(\psi_i)\right]^{\frac{1}{2}}}{\left[\frac{\epsilon_2}{\epsilon_1}\right]^{\frac{1}{2}} \cos(\psi_i) + \left[1 - \frac{\epsilon_1}{\epsilon_2} \sin^2(\psi_i)\right]^{\frac{1}{2}}} \tag{2.3}$$

This has the special important case under which the reflection is zero giving the Brewster angle (subscript B) as

$$\cos(\psi_{iB}) = \left[\frac{\epsilon_1}{\epsilon_1 + \epsilon_2}\right]^{\frac{1}{2}} = \sin(\psi_{tB})$$

$$\sin(\psi_{iB}) = \left[\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}\right]^{\frac{1}{2}} = \cos(\psi_{tB})$$

$$\cot(\psi_{iB}) = \left[\frac{\epsilon_1}{\epsilon_2}\right]^{\frac{1}{2}} = \tan(\psi_{tB}) \tag{2.4}$$

$$\psi_{iB} + \psi_{tB} = \frac{\pi}{2}$$

The relative magnitudes of incidence and transmission Brewster angles depend inversely on the square roots of the respective permittivities on the two sides of S.

As discussed in [4 (Appendix J)], this case of no reflection allows one to introduce perfectly conducting sheets perpendicular to the electric field without distorting the wave. One can then have two such parallel sheets in medium 1 parallel to $\vec{H}_{inc}$ and the incident ray direction with the wave propagating only between these two sheets. On passing through S these sheets bend to be parallel to the transmitted ray direction, the spacing being now different. This forms a two-parallel-plate transmission line (infinitely wide) with a bend by an angle $\psi_i - \psi_t$. It propagates a uniform TEM wave with no distortion (or introduction of other modes) due to the bend for all frequencies.

III. Wedge Lens

Now consider a symmetrical three-medium problem as in Figure 3.1. Here a lens with permeability μ_0 and permittivity ϵ_ℓ has a cross section as an isosceles triangle ABC. This lens is placed as indicated in a parallel-plate waveguide (in principle infinitely wide) at the place of a desired bend. The upper plate bends at B, while the lower plate bends twice at A and C. Note that this lens can also be a trapezoid if the upper plate is lowered to pass through the lens, bending twice, once at each face (surfaces AB and BC).

Apply the results of Section II on face AB where the incident wave first encounters the lens. Then we have, with respect to the surface normal, incidence angle ψ_1 and transmission angle ψ_2 with the Brewster condition satisfied by

$$\tan(\psi_1) = \sqrt{\epsilon_r} = \cot(\psi_2)$$

$$\psi_1 + \psi_2 = \frac{\pi}{2} \tag{3.1}$$

$$\epsilon_r \equiv \frac{\epsilon_\ell}{\epsilon} \equiv \text{relative dielectric constant of lens}$$

One may choose the permittivity of the parallel-plate region as ϵ_0 (free space) as desired. Note that in the lens the ray bends to become parallel to AC. This establishes another smaller isosceles triangle DBE similar to ABC with angles ψ_1 at D and E as well as A and C (using 3.1). Then at B the angle is $2\psi_2$. One can repeat the Brewster condition on face BC or simply appeal to symmetry and reciprocity.

Now we have an E-wave lens with no reflection at either face passing a plane wave from one parallel-plate region to another. Note that in passing through one face of the lens the ray is bent through an angle $\psi_1 - \psi_2$, and likewise at the second face. The angle of bend ψ_b of the parallel-plate transmission line (TEM waveguide) is then

$$\begin{aligned}\psi_b &= 2[\psi_1 - \psi_2] = 4\psi_1 - \pi \\ &= 4\arctan\left(\sqrt{\epsilon_r}\right) - \pi\end{aligned} \tag{3.2}$$

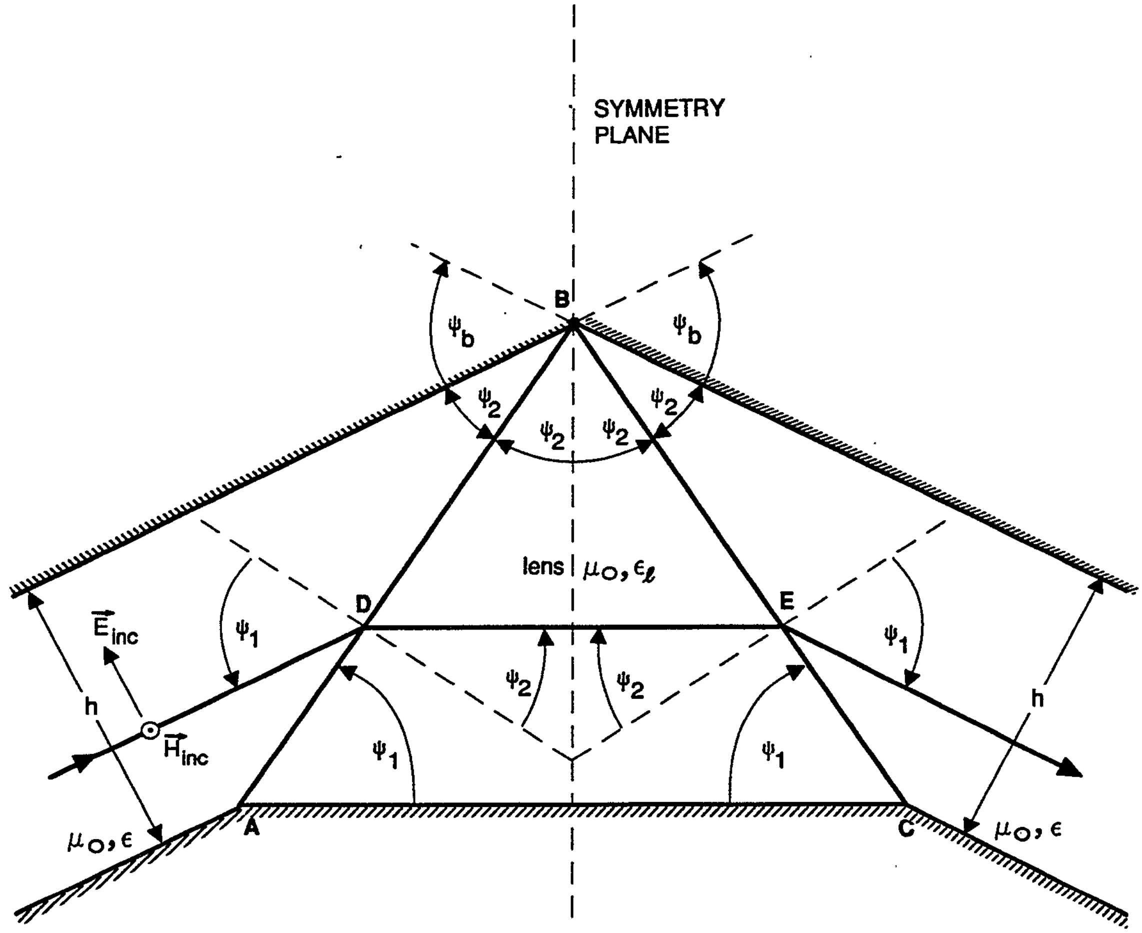

Figure 3.1. Wedge Lens in Parallel-Plate Geometry

Given a desired ψ_b we have

$$\epsilon_r = \tan^2\left(\frac{\psi_b + \pi}{4}\right)$$

$$\psi_1 = \frac{\pi + \psi_b}{4} \quad , \quad \psi_2 = \frac{\pi - \psi_b}{4} \tag{3.3}$$

thereby specifying the desired lens parameters.

The lens dimensions in Figure 3.1 depend on the plate spacing h. Note that the angle between the left top plate and surface AB is just $\pi/2 - \psi_2 - \psi_b/2$ which is just ψ_2 as indicated. The slant height (length of AB) is then just (using (2.4))

$$h_{slant} = \frac{h}{\sin(\psi_2)} = [\epsilon_r + 1]^{\frac{1}{2}} h \tag{3.4}$$

and the lens height (along the symmetry plane) is then

$$\begin{aligned} h_{lens} &= \cos(\psi_2)\ h_{slant} = \cot(\psi_2)\ h \\ &= \tan(\psi_1)\ h = \sqrt{\epsilon_r}\ h \end{aligned} \tag{3.5}$$

Note that this last result is consistent with increasing the plate spacing in the lens to make the transmission-line characteristic impedance in the lens match that outside (before and after) the lens. The lens thickness (AC) at the base is just

$$h_{base} = 2\sin(\psi_2)\ h_{slant} = 2h \tag{3.6}$$

independent of ϵ_r.

For small bend angles we have

$$\begin{aligned} &\epsilon_r \to 1 \ , \ \psi_b \to 0 \\ &\psi_1 \to \frac{\pi}{4} \ , \ \psi_2 \to \frac{\pi}{4} \\ &h_{slant} \to \sqrt{2}\, h \ , \ h_{lens} \to h \end{aligned} \tag{3.7}$$

Note that the geometry does not tend to a thin lens, but rather has the angle at B (i.e. 2 ψ_2) tending to a right angle. The largest bend angle has

$$\epsilon_r \rightarrow \infty \quad , \quad \psi_b \rightarrow \pi$$

$$\psi_1 \rightarrow \frac{\pi}{2} \quad , \quad \psi_2 \rightarrow 0 \tag{3.8}$$

$$h_{slant} \rightarrow \infty \quad , \quad h_{lens} \rightarrow \infty$$

In this case the geometry tends to a thin lens with grazing incidence angle. This establishes a range for our lens parameters as

$$\begin{gathered} 1 \leq \epsilon_r < \infty \quad , \quad 0 \leq \psi_b < \pi \\ \frac{\pi}{4} \leq \psi_1 < \frac{\pi}{2} \quad , \quad 0 < \psi_2 \leq \frac{\pi}{4} \end{gathered} \tag{3.9}$$

As an example, suppose one wished to bend the waveguide by $\pi/2$. This gives

$$\psi_b = \frac{\pi}{2} \ (=90°)$$

$$\psi_1 = \frac{3\pi}{8} \ (=67.5°) \ , \ \psi_2 = \frac{\pi}{8} \ (=22.5°)$$

$$\begin{aligned} \epsilon_r &= \tan^2\left(\frac{3\pi}{8}\right) = \frac{1 - \cos\left(\frac{3\pi}{4}\right)}{1 + \cos\left(\frac{3\pi}{4}\right)} \\ &= \frac{1 + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \\ &= \left[\sqrt{2}+1\right]^2 = 3 + 2\sqrt{2} \simeq 5.83 \end{aligned} \tag{3.10}$$

This is in a range appropriate to certain kinds of glass. Note that two such lenses can be used to make a 180° bend which reverses the direction of propagation.

As a second example consider a lens for bending the waveguide by $\pi/4$. This gives

$$\psi_b = \frac{\pi}{4} \ (=45°)$$

$$\psi_1 = \frac{5\pi}{16} \ (=56.25°) \ , \ \psi_2 = \frac{3\pi}{16} \ (=33.75°)$$

$$\epsilon_r = \tan^2\left(\frac{5\pi}{16}\right) = \frac{1 - \cos\left(\frac{5\pi}{8}\right)}{1 + \cos\left(\frac{5\pi}{8}\right)} = \frac{1 + \sin\left(\frac{\pi}{8}\right)}{1 - \sin\left(\frac{\pi}{8}\right)} \tag{3.11}$$

$$\sin\left(\frac{\pi}{8}\right) = \left[\frac{1 - \cos\left(\frac{\pi}{4}\right)}{2}\right]^{\frac{1}{2}} = \left[\frac{\sqrt{2} - 1}{2\sqrt{2}}\right]^{\frac{1}{2}}$$

$$\epsilon_r \simeq 2.24$$

This case has a relative dielectric constant that approximates that of polyethylene or transformer oil.

IV. Concluding Remarks

This type of bending lens is quite simple in shape, construction, and function. It is applicable in the case that the parallel-plate waveguide has low impedance, i.e. that the plate spacing h is small compared to the width (say w). Then most of the energy in the TEM wave is in regions where the electric field is polarized as an E wave with respect to the lens surface. For various h/w ratios one can estimate the fractional amount of such energy by use of the exact TEM solutions and associated potential and field plots [1]. While for $w \to \infty$ the lens is exact in the sense of [4], matching perfectly the various TEM waves, in a practical application the finite width introduces imperfections.

References

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