Sensor And Simulation Notes

Note 341

27 April 1992

Arrays Of Parallel Conducting Sheets For Two-Dimensional E-Plane Bending Lenses

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Abstract

This paper considers a technique for bending TEM waves by subdividing the space between two infinitely wide parallel perfectly conducting plates by the insertion of additional perfectly conducting sheets. With close spacing these can be bent in various ways to make a transient/broad-band lens.

I. Introduction

A previous paper [1] has shown a special case of a wedge dielectric lens which can bend a transient TEM plane wave between two parallel perfectly conducting sheets without distortion (no reflections, dispersion, etc.). The wave satisfies the Maxwell equations and boundary conditions with radian wavelength $\lambda\!\!\!^{-}$ (or characteristic times) short compared to plate spacing as well as for lower frequencies for which a transmission-line approximation can be used (with the same results).

This paper considers an alternate type of lens in which a set of conducting sheets, parallel with small spacing Δ, is used to make an anisotropic lens which can also be used to bend the propagation of the TEM wave. This is another example of such an anisotropic lens, similar to those discussed in [3].

II. TEM Propagation on Set of Parallel Perfectly Conduction Sheets

As indicated in Fig. 2.1, one can propagate a plane wave between two parallel perfectly conducting sheets (separation Δ) with

$$\vec{E} // \vec{1}_y \ , \ \vec{H} // (-\vec{1}_x)$$
$$(\vec{E} \times \vec{H}) // \vec{1}_z \quad (\textit{propagation direction}) \tag{2.1}$$

the sheets being perpendicular to $\vec{1}_y$. Given a set of such sheets spaced Δ apart (although this can be a spacing Δ_n, i.e. different spacing between different pairs of sheets) it is possible to have such a wave between each adjacent pair of sheets. However, they need not have the same amplitude and phase (in frequency domain) at a given plane of constant z.

For present purposes the medium between the plates is characterized by a permeability μ_ℓ and permittivity ϵ_ℓ, both real and frequency-independent (lossless). The sheets being assumed infinitely wide in the x direction for present analytical purposes, the waves between the two sheets have an admittance per unit width

$$Y_\ell^{(w)} = \frac{1}{\Delta} \sqrt{\frac{\mu_\ell}{\varepsilon_\ell}} \tag{2.2}$$

Furthermore, the waves are assumed to all have the same amplitude but progressively delayed (or advanced) so that there is some effective wavefront as a plane parallel to the x axis, but tilted with respect to the z axis and thereby not parallel to $\vec{E}$. This is quite possible since energy cannot be transported through any of the perfectly conducting sheets. For this purpose, Δ is assumed small compared to radian wavelengths $\lambda\!\!\!^{-}$ of interest in the "lens" medium with propagation speed

$$v_\ell = \frac{1}{\sqrt{\mu_\ell \varepsilon_\ell}} \tag{2.3}$$

So non-zero Δ represents some high-frequency/small-time-change limit on the behavior of this wave ensemble as representing a skewed TEM macroscopic (many Δ) wave with effective wavefront as in Fig. 2.1.

Now consider how to launch this skewed TEM wave on the multiple-parallel-sheet region, or what we can think of as an anisotropic lens. As in Fig. 2.2, consider an incident TEM wave on the left of the lens boundary, between two parallel plates of spacing d in a medium of permeability μ and permittivity ϵ. Let the direction of incidence of this wave make an angle of ψ_1 with respect to the normal to this lens boundary ($\pi/2$ - ψ_1 with respect to the lens-boundary plane). The admittance per unit width is

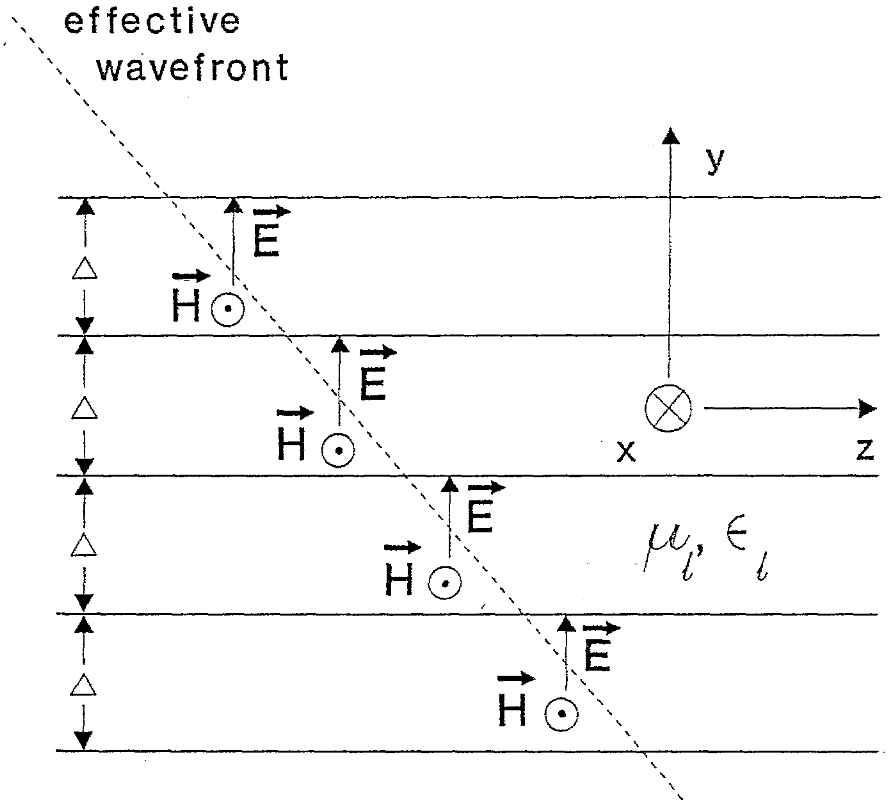

Fig. 2.1 Skewed TEM Wave Propagating on Set of Parallel Perfectly Conducting Sheets

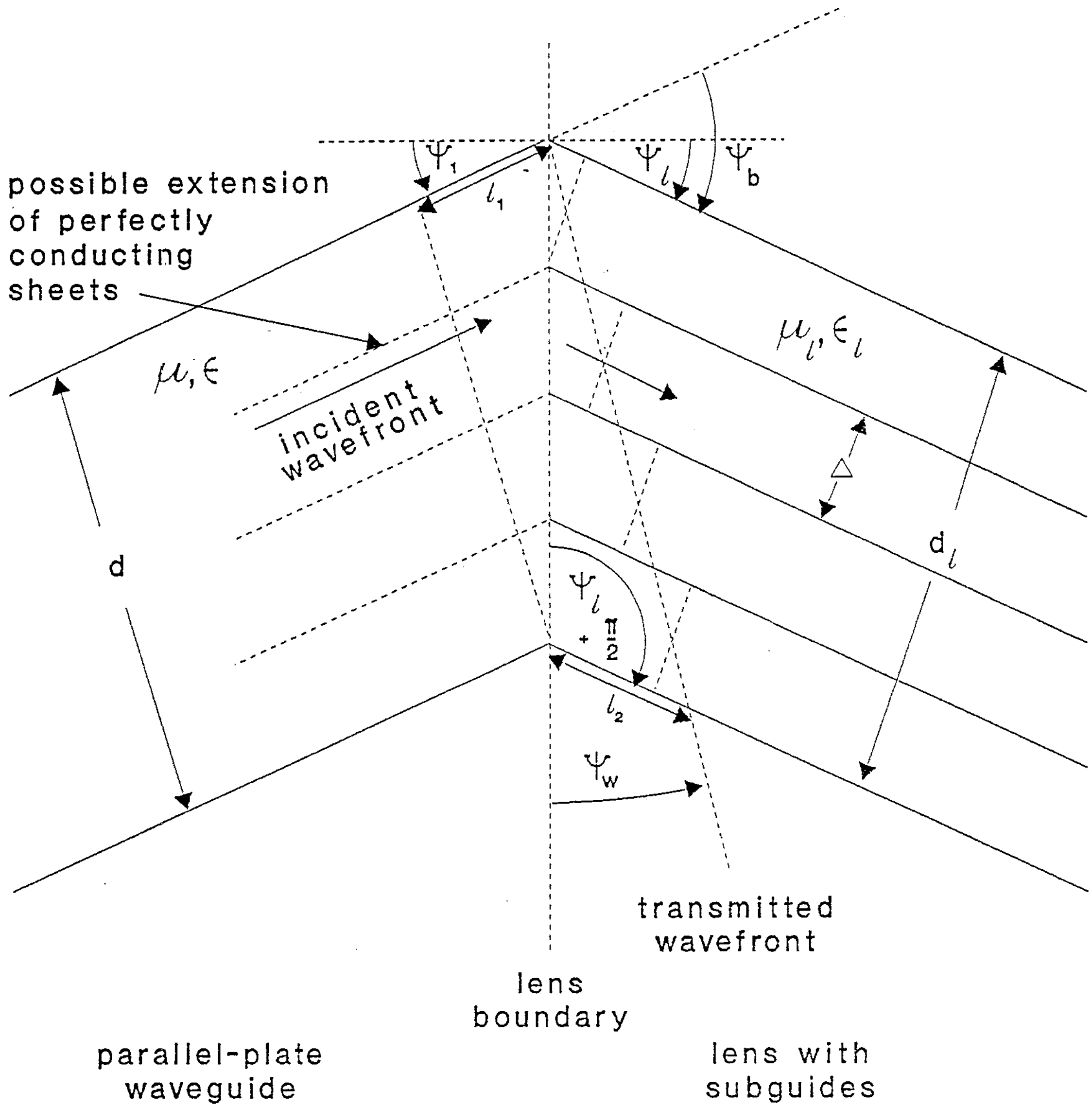

Fig. 2.2 Matched Transmission of TEM Wave at Bend from Parallel-Plate Region to Lens Region

$$Y^{(w)} = \frac{1}{d}\sqrt{\frac{\mu}{\varepsilon}} \tag{2.4}$$

The wave in this region propagates with speed

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \tag{2.5}$$

To the right of the lens boundary we have the lens region with a set of perfectly conducting sheets as discussed before. These divide the region between two sheets of spacing d_ℓ into N regions of thickness Δ with

$$d\ell = N\Delta \tag{2.6}$$

and overall admittance per unit width

$$Y_\ell^{(w)} = \frac{1}{N} y_\ell^{(w)} = \frac{1}{N\Delta}\sqrt{\frac{\mu_\ell}{\varepsilon_\ell}} = \frac{1}{d_\ell}\sqrt{\frac{\mu_\ell}{\varepsilon_\ell}} \tag{2.7}$$

Matching impedance on both sides of the lens boundary gives

$$\frac{1}{d}\sqrt{\frac{\mu}{\varepsilon}} = \frac{1}{d_\ell}\sqrt{\frac{\mu_\ell}{\varepsilon_\ell}} \tag{2.8}$$

Matching distances along the lens boundary gives

$$\frac{d}{\cos(\psi_1)} = \frac{d_\ell}{\cos(\psi_\ell)} \tag{2.9}$$

where ψ_ℓ as in Fig 2.2 gives the direction of the transmitted wave with respect to the normal to the lens boundary. Combining the above gives

$$\frac{1}{\cos(\psi_1)}\sqrt{\frac{\mu}{\varepsilon}} = \frac{1}{\cos(\psi_\ell)}\sqrt{\frac{\mu_\ell}{\varepsilon_\ell}} \tag{2.10}$$

relating the angles on both sides of the lens boundary. The total bend angle is

$$\psi_b = \psi_1 + \psi_\ell \tag{2.11}$$

Specifying ψ_b and the constitutive parameters then determines ψ_1 and ψ_ℓ.

Considering an incident wavefront just reaching the lens boundary in Fig. 2.2, there is still a distance for the farthest position to still reach the lens boundary of

$$\ell_1 = d\tan(\psi_1) \tag{2.12}$$

As the incident wavefront reaches all of the lens boundary, the farthest penetration ℓ_2 of the wavefront into the lens is

$$\frac{\ell_2}{v_\ell} = \frac{\ell_1}{v_1}$$

$$\ell_2 = \sqrt{\frac{\mu\varepsilon}{\mu_\ell\varepsilon_\ell}}\,\ell_1 = \sqrt{\frac{\mu\varepsilon}{\mu_\ell\varepsilon_\ell}}\,d\,\tan(\psi_1) \tag{2.13}$$

This gives an angle ψ_w for the wavefront propagating into the lens as (law of sines)

$$\frac{\sin(\psi_w)}{\ell_2} = \left[\frac{\cos(\psi_1)}{d}\right]\sin\left(\frac{\pi}{2} - \psi_\ell - \psi_w\right)$$

$$= \frac{\cos(\psi_1)\cos(\psi_\ell + \psi_w)}{d}$$

$$\sin(\psi_w) = \frac{\ell_2}{d}\cos(\psi_1)\cos(\psi_2 + \psi_w) \tag{2.14}$$

$$= \sqrt{\frac{\mu\varepsilon}{\mu_\ell\varepsilon_\ell}}\,\sin(\psi_1)\cos(\psi_\ell + \psi_w)$$

Note that this wavefront in the lens is defined in a limiting sense as $\Delta \to 0$ or $N \to \infty$.

One can note that this type of bend can be referred to as an E-plane bend in the usual waveguide convention. The bend angle is in a plane containing the electric field polarization both before and after the bend.

As a simple special case let the media be the same, i.e.

$$\varepsilon_\ell = \varepsilon \quad , \mu_\ell = \mu \quad , \quad v_\ell = v \tag{2.15}$$

Then we have

$$d_\ell = d, \ \ell_2 = \ell_1$$

$$\psi_1 = \psi_\ell \quad , \quad \psi_b = 2\psi_1 = 2\psi_\ell \tag{2.16}$$

which gives a symmetrical geometry with respect to the lens boundary. Furthermore, (2.14) reduces to

$$\begin{aligned}\sin(\psi_w) &= \sin(\psi_1)\cos(\psi_1+\psi_w)\\ &= \sin(\psi_1)[\cos(\psi_1)\cos(\psi_w) - \sin(\psi_1)\ \sin\ (\psi_w)]\\ \cot(\psi_w) &= \frac{1}{\cos(\psi_1)}\left[\frac{1}{\sin(\psi_1)} + \sin(\psi_1)\right]\\ \psi_w &= \arctan\left(\frac{\sin(\psi_1)\cos(\psi_1)}{1+\sin^2(\psi_1)}\right)\end{aligned} \tag{2.17}$$

which for small ψ_1 gives

$$\begin{aligned}\frac{\sin(\psi_w)}{\ell_2} &= \left[\frac{\cos(\psi_1)}{d}\right]\sin\left(\frac{\pi}{2} - \psi_\ell - \psi_w\right)\\ &= \frac{\cos(\psi_1)\cos(\psi_\ell+\psi_w)}{d}\end{aligned} \tag{2.14}$$

Another way to see how this lens boundary works is to imagine the extension of the perfectly conduction sheets over into the incident-wave region as indicated in Fig. 2.2. The incident wave enters into the region between adjacent sheets with no distortion and admittance per unit width $NY^{(w)}$ (from (2.4)). On reaching the lens boundary the impedance is matched (per (2.8)) into a similar region on the lens side of the boundary. This impedance matching is, of course, a transmission-line concept which does not apply for wavelengths short compared to Δ. But this is precisely the point of introducing the perfectly conducting sheets in the lens region. By making Δ sufficiently small one can make the effective bandwidth for transporting the TEM wave into the lens as high as desired. Wavelengths can be small compared to both d and d_ℓ with distortionless transmission from the TEM wave into the skewed TEM wave in the lens.

III. Lateral Displacement Lens

As an application of the concept discussed above, consider the case of two lens boundaries as in Fig. 3.1. Here, symmetry is employed to assure that the waves in the lens leave to the second parallel-plate waveguide (spacing d, like the first) in a way to give a wave propagating parallel to the plates. Note the two-fold rotation axis (C_2 symmetry) around which the structure can be rotated by π to leave the geometry unchanged. The parameters are the same as in the previous section. Note that the length L of each subguide (spacing Δ) in the lens is the same. How much displacement $L \sin(\psi_b)$ is achieved depends on L and $\psi_b = \psi_1 + \psi_\ell$ as discussed previously.

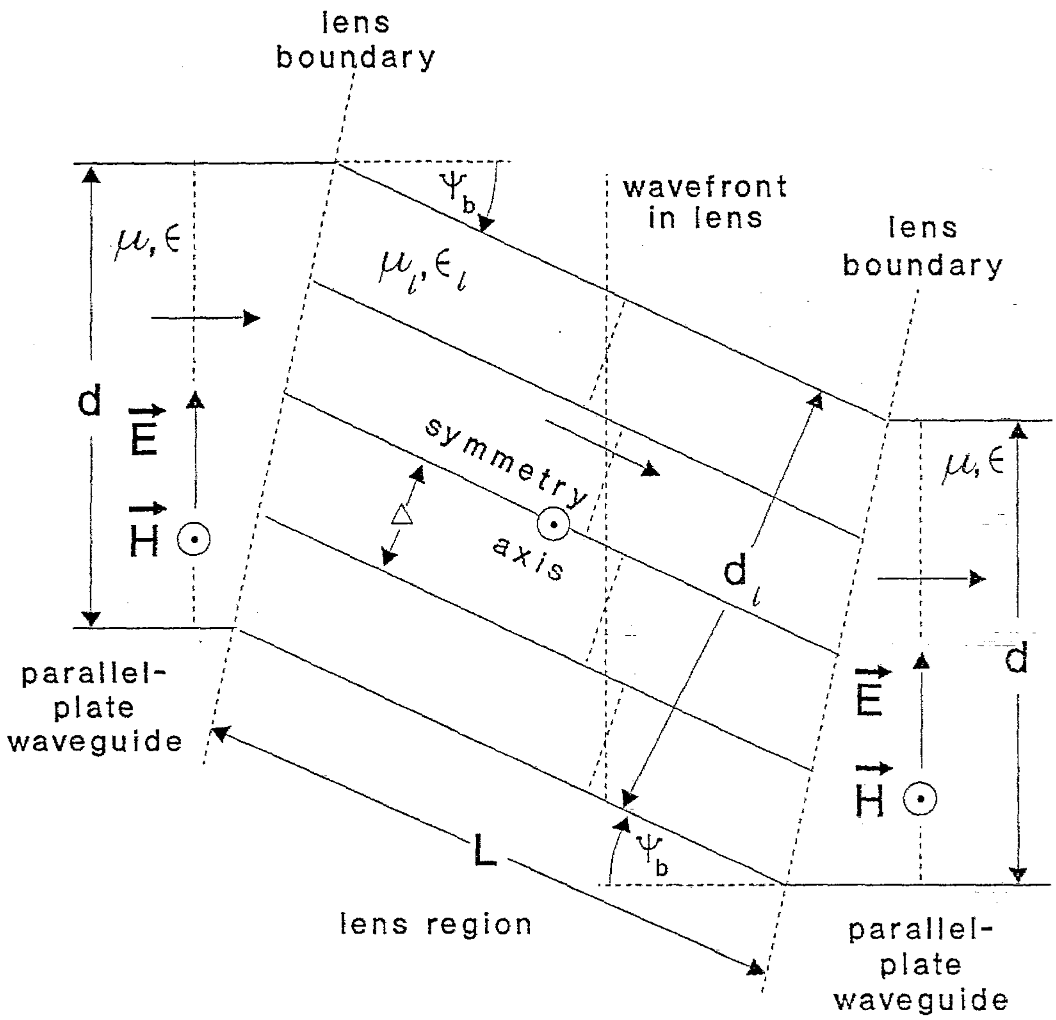

Fig. 3.1 Lateral Displacement Lens

IV. Bending Lens

Another example is the bending lens (E-plane bend) illustrated in Fig. 4.1. Now take the wavefront in the lens as in Figs. 2.1 and 2.2 as a symmetry plane and bend the conducting sheets to form equal angles on both sides of one such wavefront. Note that the spacing Δ (and hence admittance per unit width) is maintained between adjacent plates, giving no reflection at the symmetry plane (in the transmission-line approximation appropriate for $\lambda\!\!\!^{-} \ll \Delta$).

The various angles are treated in Section II. As a simple case one can let the permeability and permittivity in the lens be the same as that outside. This gives the simpler results in (2.15) through (2.18). Referring to Fig. 4.1 the total angle of bend is

$$\psi_B = 2\,[\psi_1 - \psi_w] \tag{4.1}$$

This can be combined with (2.14) to give ψ_B in terms of ψ_1 or ψ_w separately. For the special case of same permeability and permittivity (2.17) gives

$$\psi_B = 2\left[\psi_1 - arctan\left(\frac{\sin(\psi_1)\cos(\psi_1)}{1 + \sin^2(\psi_1)}\right)\right] \tag{4.2}$$

which for small angles with (2.18) reduces to

$$\psi_B = 4\psi_1^3 + 0(\psi_1^5) \tag{4.3}$$

So in this case, ψ_1 needs to be large for a significant bend. An extreme limiting case has

$$\psi_1 = \psi_\ell = \frac{\pi}{2} \quad , \quad \psi_w = 0 \quad , \quad \psi_B = \pi \tag{4.4}$$

Note also that at the symmetry plane there is ideally no net current crossing on the conducting sheets except the outermost (spacing d_ℓ). On the internal sheets the surface current density (or magnetic field) is the same on both sides since this symmetry plane is a common phase surface for all subguides. Stated another way the wave arrives at the symmetry plane at the same time in all subguides. Of course, we are considering $\Delta \ll \lambda\!\!\!^{-}$ so that we are neglecting small times (order Δ/c).

One of the potential problems with a set of finite-length parallel plate regions is the possibility of resonances with same number of half wavelengths along the length of the subguide. Some of these (odd number of half wavelengths) can be suppressed by insertion of resistors (or short resistive sheets) in the conducting sheets at the symmetry plane. These resistors do not couple to the desired mode of propagation in which the currents through these resistors are zero, the currents on each side of the sheets being equal and opposite crossing the symmetry plane.

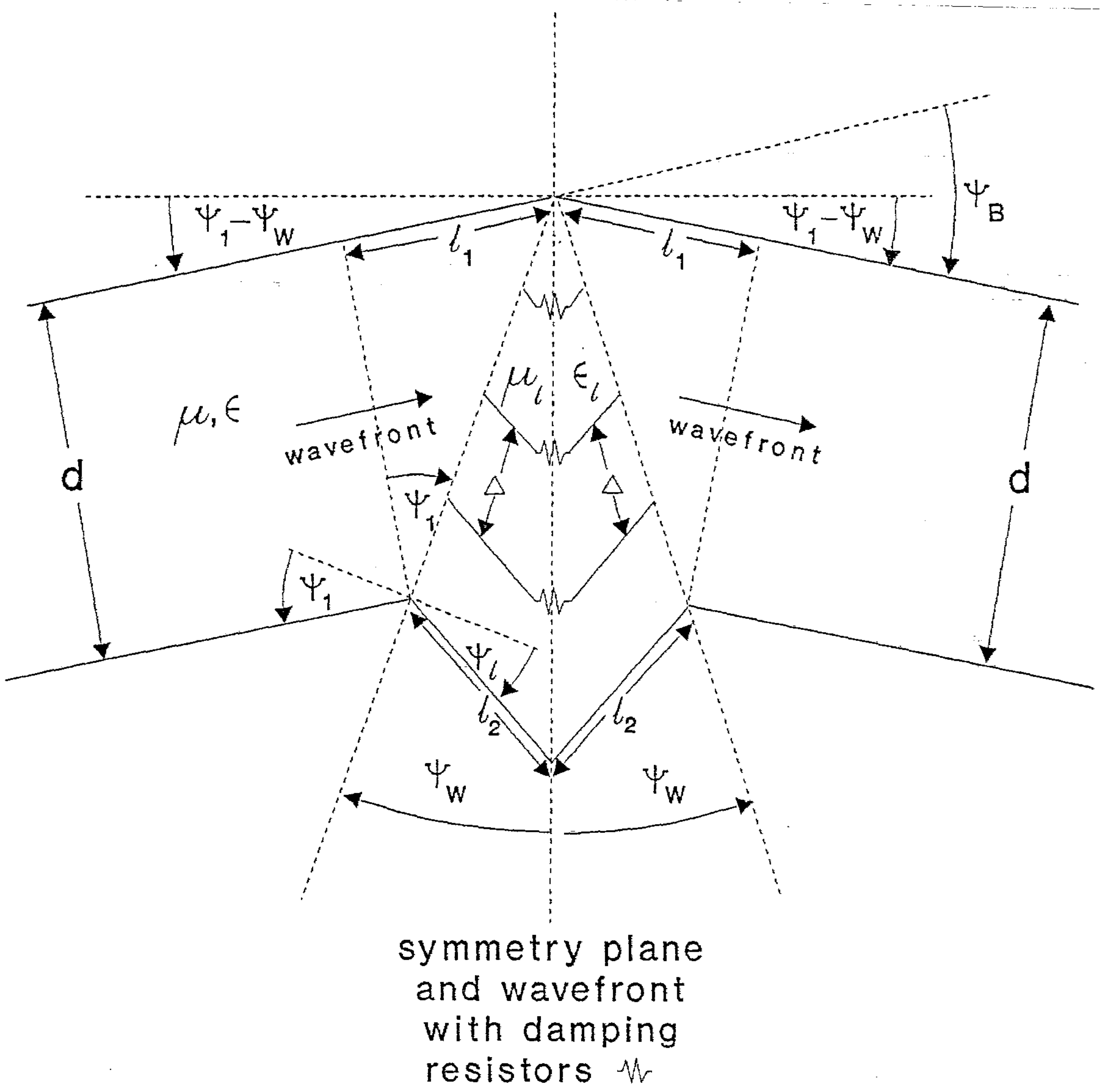

Fig. 4.1 Bending Lens

V. Concluding Remarks

The lens represents yet another way to control TEM wave propagation in a way that allows $\bar{\lambda} \ll$ parallel plate spacing in the original waveguide. Of course, as a two-dimensional structure which cannot be practically of infinite width, there are fringe fields at the edges of the plates which are not accommodated by the two-dimensional lens structure. Depending on the ratio of plate spacing to width, the lens intercepts some fraction of the energy in the incident TEM wave, roughly that between the plates. For a large intercepted fraction (near 1) one may expect good lens performance.

While this kind of lens can have various (ideally lossless and dispersionless) materials between the conducting sheets, one application may lie in evacuated transmission systems. At very high electric fields one sometimes uses intense magnetic fields to retard electron motion (from field emission) parallel to the electric field. Such magnetic insulation is appropriate in some cases that dielectric insulation is not sufficient.

In this paper the added conducting sheets are discussed for cases that they are flat. This technique also allows for curved sheets such as in [2], except that here we have TEM propagation instead of the $H_{1,0}$ mode characteristic of a rectangular waveguide. The basic concept still carries through for plate spacing $\ll \bar{\lambda}$, impedance matching into and out of each subguide, constant impedance along each subguide, and equal transit times between TEM wavefronts on each side of the lens.

References

1. C. E. Baum, Wedge Dielectric Lenses for TEM Waves Between Parallel Plates, Sensor and Simulation Note 332, Sept. 1991.

2. C. E. Baum, Matching Modulated Electron Beam to Waveguide, Circuit and Electromagnetic System Design Note 39, April 1990.

3. C. E. Baum, And A. P. Stone, Transient Lens Synthesis: Differential Geometry in Electromagnetic Theory, Hemisphere Publishing Corp., 1991.