Sensor and Simulation Notes

Note 353

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Non-Loading Inductive Couplers for Driving Cables

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Abstract

In designing EMP simulators for various systems to be tested, one is often faced with the case of long conductors (cables) to be illuminated. In some cases this can be accomplished by local pulsers which are inductively coupled to the cable. This paper considers the design of such devices with particular attention to the requirement of small equivalent source impedance (compared to the cable (antenna) impedance) so as to not significantly change the complex resonance frequencies. Various other aspects, such as the exciting waveform and distribution of multiple inductive drivers, are also considered.

I. Introduction

EMP simulation comes in various forms [8, 13]. While a "complete simulation" involving the full environmental spatial, amplitude, and temporal distribution gives the best results (involving the fewest assumptions), this is sometimes impractical due to the extreme size of the system to be tested. Such is the case when long conductors (such as power and/or communications cables) are connected to some facility (such as a communications center). In such a case one would like to have some way properly exciting these conductors with appropriate waveforms and associated frequency spectra, even to full environmental amplitudes where practical.

Idealizing this problem somewhat, consider the cases illustrated in Fig. 1.1. Here we have some one or more conductors which might represent some sort of power or communications cable. With the diameter of this conductor bundle (cable) assumed small compared to wavelengths of interest (in the local surrounding medium) one can consider the cable as being excited by a single component of the incident electric field as

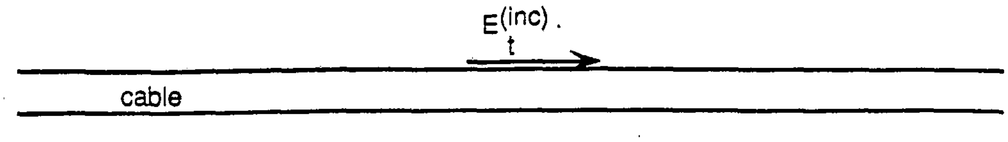
$$E_t^{(inc)}(\zeta,t) = \vec{1}_t(\zeta) \cdot \vec{E}_t^{(inc)}(\zeta,t)$$

$$\zeta \equiv \text{ coordinate (meters) along cable} \qquad (1.1)$$

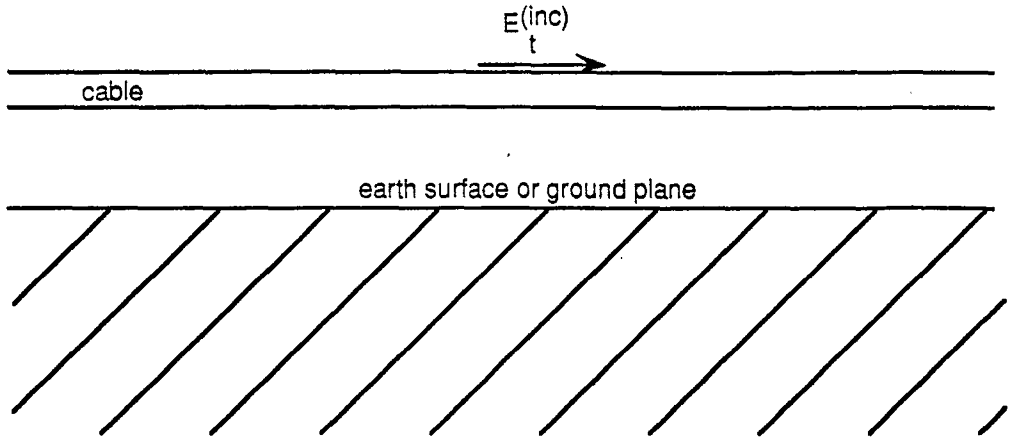
$$\vec{1}_t(\zeta) \equiv \text{ direction parallel to cable}$$

While the examples in Fig. 1.1 have the cable straight, this is not necessary, and $\vec{1}_t(\zeta)$ can assume different orientations along the cable.

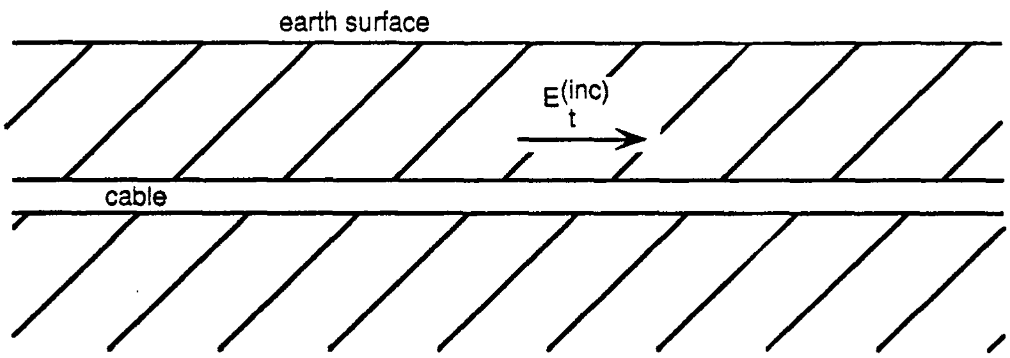
This paper is in part based on some notes I generated in 1983 in The Hague, Netherlands for Peter Sevat at the TNO Physics and Electronics Laboratory concerning low-insertion-impedance inductive line drivers. Subsequently a set of these were built by Physics International for this laboratory. The present paper is then intended to document these concepts. While this exposition is based on one-dimensional (line) structures, there is another concept (PARTES) relating to two-dimensional (surface) conducting structures driven by discrete sources [10].



A. Cable in uniform medium (e.g., free space)



B. Cable above a conducting medium (e.g., lossy earth or a ground plane)



C. Cable buried near surface of conducting medium

Fig. 1.1. One-Dimensional Conductor (Cable) Excited by Parallel Component of Incident Electric Field

II. Integral-Equation Representation of Cable Current in Terms of an Appropriate Incident Electric Field

In order to compare discrete versus distributed excitation, consider first a general way to describe both. One can write a general linear relation between the incident electric field (field with no system (cable) present) and the surface current density on the system as an integral equation of the form

$$\left\langle \tilde{\overleftrightarrow{Z}}_t\left(\vec{r}_s, \vec{r}\,'_s; s\right); \tilde{\vec{J}}_s\left(\vec{r}\,'_s; s\right)\right\rangle = \overleftrightarrow{1}_t\left(\vec{r}_s\right) \cdot \tilde{\vec{E}}^{(inc)}_{\left(\vec{r}_s, s\right)}$$

$\overleftrightarrow{1}_t\left(\vec{r}_s\right) \equiv$ tangential dyad on S at coordinate $\vec{r}_s$

$S \equiv$ system surface

$\langle \;,\; \rangle \equiv$ symmetric product

$\equiv$ integration with respect to common coordinates $\left(\vec{r}_s\,' \text{ here}\right)$ over S (2.1)

$s \equiv \Omega + j\omega \equiv$ Laplace - transform variable or complex frequency

$\sim \;\equiv$ Laplace - transform (two sided) with respect to time t

Here $\tilde{\overleftrightarrow{Z}}_t$ is a dyadic kernel related to the Green's function for the system currents, including the effect of the surrounding medium (such as soil, water, etc., as in Fig. 1.1). Furthermore, it has the properties of a passive impedance operator.

With the assumption of a basically one-dimensional structure as in Fig. 1.1, the integral equation (2.1) can be reduced to the general one-dimensional form

$$\left\langle \tilde{Z}_t(\zeta, \zeta'; s); \tilde{I}\;(\zeta', s)\right\rangle = \tilde{E}_t^{(inc)}(\zeta,s) \qquad (2.2)$$

with integration over the common coordinate ζ'. Note now that this is strictly correct for the case of a single thin conductor for which a single current is appropriate, noting the assumption of wavelengths large compared to the conductor cross-section dimensions. There are various traditional forms (such as the Pocklington form) that such one-dimensional integral equations take, with perhaps various degrees of approximation involved. Here it is the form, and not the details, that concern us.

If the structure of interest is a bundle of distinct conductors (multiconductor cable) then the problem is more complicated. While $E_t^{(inc)}$ can still represent the source, strictly one may wish to consider the differences between the incident electric fields on the different conductors. More importantly, there are a number of differential modes of propagation supported by the conductor set. These interact with the terminating impedances at discontinuities (e.g. ends) and thereby can significantly alter the impedance properties of the system as driven locally by $E_t^{(inc)}$. In a special case of interest where there is an outer cable shield, there is an approximate separation of outside from inside characteristics, allowing one to first solve for the shield current before considering the internal response [14].

III. Driving the Cable at Discrete Positions

Now, as indicated in Fig. 3.1, let us consider some localized driver which is designed to insert some series voltage source in the cable, with in general some series source impedance to give the Thevenin equivalent circuit. Note that this is an equivalent source, and the cable need not be broken providing it takes the form of a transformer (inductive coupling). This is especially important in the case of a multiconductor cable, such as a shielded cable. The polarity of the voltage source is chosen according to the scattered tangential electric field (negative of the incident tangential electric field) so that the current flows in the proper direction on the cable.

Consider first what should the equivalent voltage source V_s be. In some sense it should approximate the distributed source $E_t^{(inc)}$. As a first approximation one might set

$$V_s(t) \equiv \int_{\zeta^{(1)}}^{\zeta^{(2)}} E_t^{(inc)}(\zeta,t)\, d\zeta \tag{3.1}$$

Now, what should the limits $\zeta^{(1)}$ and $\zeta^{(2)}$ be? They might correspond to the ends of the cable (including the terminations, such as the vertical risers in the case of a power line). However, if $\zeta^{(2)} - \zeta^{(1)}$ is greater than a radian wavelength λ at some frequency of interest, such a lumped source does not approximate the true distributed source.

As will be considered later, some source with inductive coupler has some equivalent series source impedance $\tilde{Z}_s(s)$ (Thevenin equivalent) inserted in the cable. As indicated in Fig. 3.1, one can think of this equivalent source as being at some position ζ_1 on the cable, driving between positions marked 1′ and 1″. As such it is driving some impedance we might call $\tilde{Z}_1(s)$, thinking of this as some antenna input impedance. This is computed as

$$\tilde{I}(\zeta_1,s) = \left[\tilde{Z}_1(s) + \tilde{Z}_s(s)\right]^{-1} \tilde{V}_s(s) \tag{3.2}$$

which leads directly to the idea that one should have

$$\left|\tilde{Z}_s(s)\right| << \left|\tilde{Z}_1(s)\right| \tag{3.3}$$

for a good simulation. Now $\tilde{Z}_1$ will have to be considered when putting restrictions on $\tilde{Z}_s$. Depending on the medium in which the cable lies, other adjacent structures, length, etc., $\tilde{Z}_1$ can vary considerably. Note that for certain (complex) values of s, namely natural frequencies s_α, we have

$$\tilde{Z}_1(s_\alpha) = 0 \tag{3.4}$$

so one much replace (3.3) for s near s_α by a requirement that the change of the natural frequency Δs_α be small where

$$\tilde{Z}_1(s_\alpha + \Delta s_\alpha) + \tilde{Z}_s(s_\alpha + \Delta s_\alpha) = 0 \tag{3.5}$$

One may also have another kind of loading (capacitive or conductive) associated with the presence of the physical source in that it is in general somewhat larger than the cable cross-section dimensions. This is a parallel kind of loading with admittance $\tilde{Y}$ which should be small enough not to perturb the cable current.

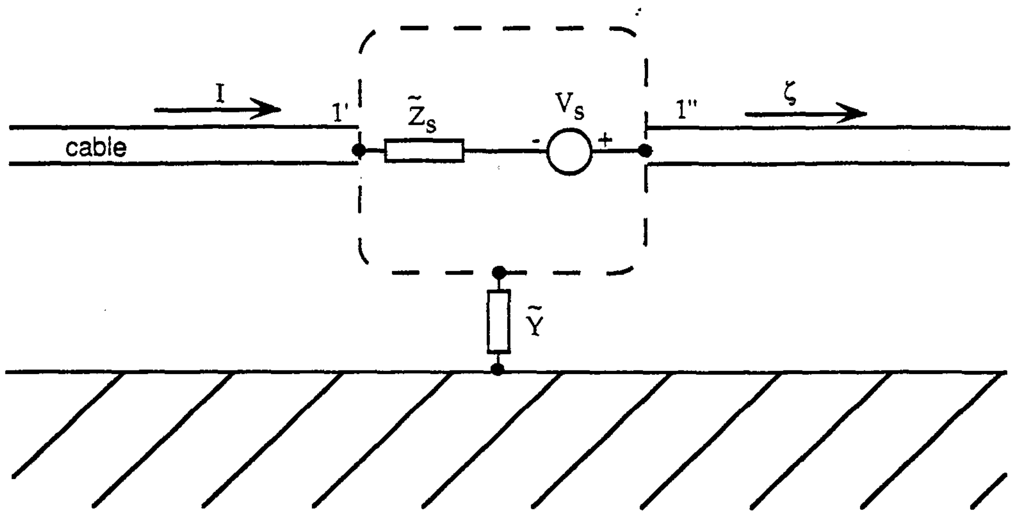


Fig. 3.1. Insertion of a Local Equivalent Source

As discussed earlier, a single source has limitations when simulating the effect of a distributed source (incident field), particularly at high frequencies. So one can limit the range of ζ in (3.1) to be simulated by a single source. So let there be N sources located at ζ_n for $n = 1, 2, \cdots, N$. Then define the individual sources by

$$V_{s_n}(t) = \int_{\zeta_{n-\frac{1}{2}}}^{\zeta_{n+\frac{1}{2}}} E_t^{(inc)}(\zeta, t)\, d\zeta$$

$$\zeta_{n+\frac{1}{2}} \equiv \text{position between } \zeta_n \text{ and } \zeta_{n+1} \tag{3.6}$$

where the end points of the integration are chosen for convenience, such as to give uniform amplitudes to the V_{s_n}, perhaps allowing for time delays (phase shifts) from one source to the next.

Now the cable driven at N positions is represented by an N X N impedance matrix and (3.2) is replaced by

$$\left(\tilde{I}_n(s)\right) = \left[\left(\tilde{Z}_{n,m}^{(c)}(s)\right) + \left(\tilde{Z}_{s_{n,m}}^{(c)}(s)\right)\right]^{-1} \cdot \left(\tilde{V}_{s_n}(s)\right)$$

$$\tilde{I}_n(s) \equiv \tilde{I}(\zeta_n, s) \equiv \text{ current at } \zeta_n$$

$$\tilde{V}_{s_n}(s) \equiv \text{equivalent voltage source at } \zeta_n$$

$$\left(\tilde{Z}_{n,m}^{(c)}(s)\right) \equiv \text{ impedance matrix for cable} \tag{3.7}$$

$$\left(\tilde{Z}_{s_{n,m}}(s)\right) \equiv \tilde{Z}_s(s)\ \left(1_{n,m}\right)$$

$$\tilde{Z}_s(s) \equiv \text{source impedance assumed same for each of the N sources} \tag{3.7}$$

$$1_{\text{n,m}} = \begin{cases} 1 & \text{for } n = m \\ 0 & \text{for } n \neq m \end{cases}$$

Of course, one can have more general forms than the above, but this simplifies the consideration. This is the same kind of equation as in [12], and as such the same norm concepts can be applied. This gives the requirement (like (3.3)) that

$$\left|\tilde{Z}_s(s)\right| << \text{ smallest eigenvalue magnitude of } \left(\tilde{Z}_{n,m}(s)\right) \tag{3.8}$$

The natural frequencies are now given by

$$\det\left(\left(\tilde{Z}_{n,m}(s_\alpha)\right)\right) = 0 \tag{3.9}$$

and the perturbation as in (3.5) is given by

$$\det\left(\left(\tilde{Z}_{n,m}\left(s_\alpha + \Delta s_\alpha\right)\right) + \tilde{Z}_s\left(s_\alpha + \Delta s_\alpha\right)\left(1_{n,m}\right)\right) = 0 \tag{3.10}$$

Note that with N sources the source impedances are effectively in series connection at low frequencies, for which case the individual source impedances need to be reduced by 1/N from that for the previously discussed single source.

IV. Inductive Line Driver with Low Insertion Impedance

Consider first a somewhat idealized source as indicated in Fig. 4.1. Here an ideal transformer of turns ratio N_p:1 is used to connect a pulser (or even CW source) of voltage $\tilde{V}_p(s)$ (open circuit) and impedance $\tilde{Z}_p(s)$ to the effective load of the cable $\tilde{Z}_1(s)$ as discussed in the previous section. Of course this could also be one of N pulsers connected to a matrix load.

Now with this ideal transformer the pulser has the equivalent parameters (as a Therenin equivalent series source in the cable) as

$$\tilde{V}_s(s) = \frac{\tilde{V}_p(s)}{N_p}$$

$$\tilde{Z}_s(s) = \frac{\tilde{Z}_p(s)}{N_p^2} \tag{4.1}$$

This points out one function of the inductive coupler, namely to reduce the source impedance by a factor N_p^{-2} to achieve the low source impedance required. However, the source voltage is decreased by a factor $1/N_p$, or for a given $\tilde{V}_s$ the pulser voltage $\tilde{V}_p$ required is correspondingly increased.

Referring to Fig. 4.2 we have some idea of what a practical inductive line driver might look like. There is a core of permeability μ (or μ_o for an air core) of dimensions as indicated. On this is wound a coil made of coaxial (or even triaxial etc.) cable with characteristic impedance Z_c, approximately a constant resistance such as 50 Ω. This winding can use various of the techniques discussed in [3, 4, 6, 11] to suppress resonances on the winding and improve bandwidth. In the example. the winding is differential with grounds to case at both ends (A) and midpoint (B). The signal enters at the center of each half winding where one may terminate the coaxial signal in $\tilde{Z}_T^{(1)}/2$ which may be selected as Z_c (to avoid resonances). Alternately one can terminate the cables in $\tilde{Z}_T^{(2)}/2$ back at the pulser. These terminations are similar to those which can be used with electromagnetic sensors [1, 2]. For the present analysis we will use the first form, noting that a similar result applies with the second form.

Figure 4.3 gives an equivalent circuit for a realistic inductive line driver, noting the coaxial cables are not included in this simplified equivalent circuit, but can be included if desired. So this equivalent circuit applies to sufficiently low frequencies that cable effects (such as inductance and capacitance) can be neglected compared to the effects of the other impedances. As previously noted the cable effects at high frequencies are a delay from pulser to winding gaps, with resonances suppressed by suitable choice of $\tilde{Z}_T^{(1)}$ (or $\tilde{Z}_T^{(2)}$ if desired).

This type of transformer is characterized by an impedance matrix

$$\left(\tilde{Z}_{n,m}^{(t)}(s)\right) = s\left(L_{n,m}\right)$$

$$\left(L_{n,m}\right) = \begin{pmatrix} L_{1,1} & L_{n,m} \\ L_{2,1} & L_{2,2} \end{pmatrix} = \begin{pmatrix} L & M \\ M & L_o \end{pmatrix} \tag{4.2}$$

$$\det\left(L_{n,m}\right) = LL_o - M^2 \geq 0$$

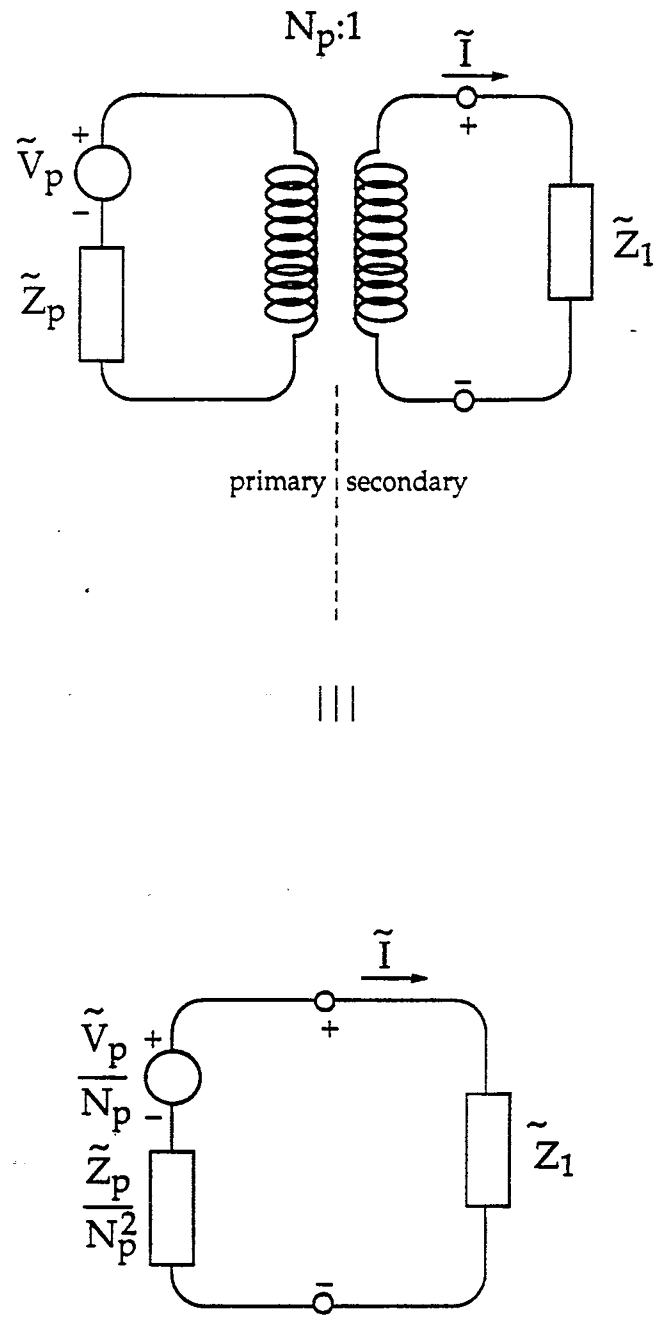


Figure 4.1. Equivalent Circuit of Pulser with Ideal Transformer.

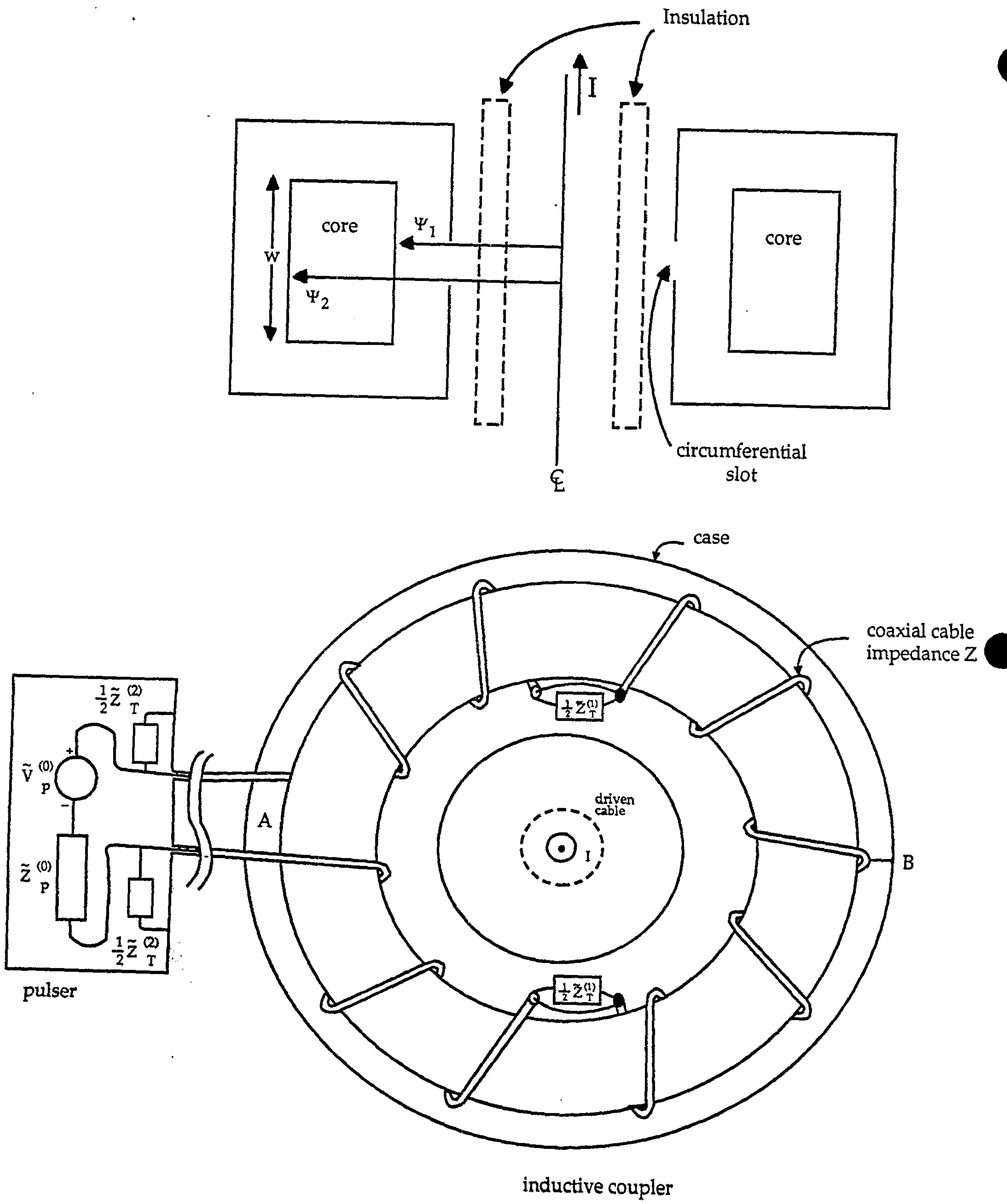


Figure 4.2. Special Transformer: Example of Differential Coaxial Winding

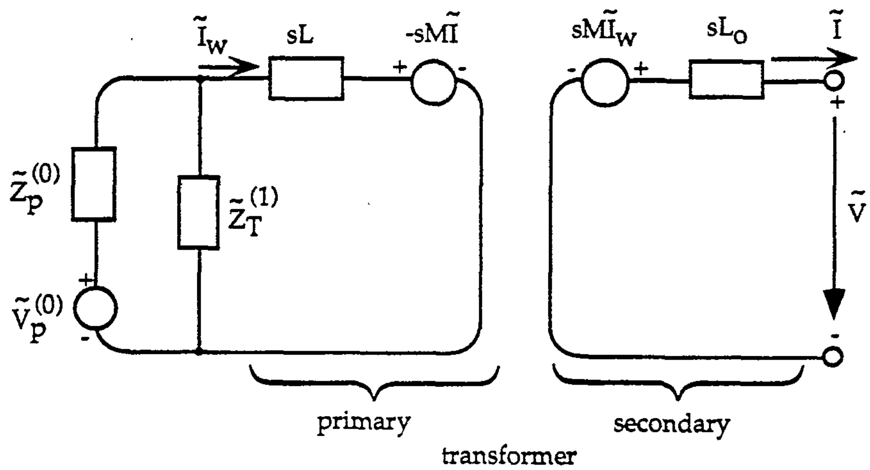


Figure 4.3. Equivalent Circuit of Pulser with Inductive Coupler (Simplified)

where the 1 indices refer to primary and 2 indices to secondary. A strictly positive determinant is associated with the (realistic) condition of non-unity coupling through the transformer (non ideal).

Considering the secondary first, let the primary winding be open circuited ($I_W = 0$) and note that the transformer core presents an inductance to the cable (through the core of permeability μ with dimensions as in Figure 4.2) of

$$L_o = \frac{\mu}{2\pi} \, w \, \ell n\left(\frac{\Psi_2}{\Psi_1}\right) \tag{4.3}$$

Looking at the primary, let the secondary be open circuited ($I = 0$), giving an inductance

$$L \simeq N_p^2 \, L_o \tag{4.4}$$

A more accurate estimate is slightly larger owing to the finite number of turns N_p and the resulting nonuniform distribution of the current around the core. The mutual inductance is found by driving a current from either side (by reciprocity) and looking at the open-circuit voltage on the other side, giving

$$M = N_p L_o \tag{4.5}$$

Following the procedure of driving the secondary, the flux in the core is quite uniform giving (4.5) as an accurate estimate for the voltage on the primary. Note now that the determinant relation in (4.2) is satisfied.

With this we can now find the equivalent source in the secondary cable as in Figure 3.1. The equivalent voltage source is found by open circuiting the secondary and computing

$$\begin{aligned}
\frac{\tilde{V}_s(s)}{\tilde{V}_p^{(o)}(s)} = \frac{sM\tilde{I}_w(s)}{\tilde{V}_p^{(0)}(s)} &= \frac{M}{L} \, \frac{\left[\tilde{Z}_T^{(1)^{-1}}(s) + [sL]^{-1}\right]^{-1}}{\left[\tilde{Z}_T^{(1)^{-1}}(s) + [sL]^{-1}\right]^{-1} + \tilde{Z}_p^{(0)}} \\
&= \frac{M}{L}\left\{1 + \tilde{Z}_p^{(0)}(s)\left[\tilde{Z}_T^{(1)^{-1}}(s) + [sL]^{-1}\right]\right\}^{-1} \\
&\simeq N_p^{-1}\left\{1 + \tilde{Z}_p^{(0)}(s)\left[\tilde{Z}_T^{(1)^{-1}}(s) + [sL]^{-1}\right]\right\}^{-1}
\end{aligned} \tag{4.6}$$

The equivalent source impedance is found by setting $\tilde{V}_p^{(0)}$ to zero and driving the secondary giving

$$\tilde{I}_w(s) = s\ M\ \tilde{I}(s)\left\{\left[\tilde{Z}_T^{(1)^{-1}}(s) + \tilde{Z}_p^{(0)^{-1}}(s)\right]^{-1} + sL\right\}^{-1}$$

$$\tilde{I}(s) = -\left[\tilde{V}(s) + s\ M\ \tilde{I}_w(s)\right]\left[sL_o\right]^{-1}$$

$$= -\left\{\tilde{V}(s) + [sM]^2\ \tilde{I}(s)\left[\left[\tilde{Z}_T^{(1)^{-1}}(s) + \tilde{Z}_p^{(0)^{-1}}(s)\right]^{-1} + sL\right]^{-1}\right\}\left[sL_o\right]^{-1} \tag{4.7}$$

$$\tilde{Z}_s(s) = -\frac{\tilde{V}(s)}{\tilde{I}(s)} = sL_o - [sM]^2\left\{\left[\tilde{Z}_T^{(1)^{-1}}(s) + \tilde{Z}_p^{(0)^{-1}}(s)\right]^{-1} + sL\right\}^{-1}$$

Setting

$$\tilde{Z}_T^{(1)}(s) = 2Z_c \quad \text{(resistive)} \tag{4.8}$$

to terminate the coaxial cables at high frequencies, and setting

$$\tilde{Z}_T^{(0)}(s) \equiv \frac{1}{sC_p}$$

$$C_p \equiv \text{ pulser capacitance} \tag{4.9}$$

gives a pulser example (such as for a Marx generator). At low frequencies we then have

$$\tilde{Z}_s(s) = sL_o \text{ as } s \to 0 \tag{4.10}$$

which can be small (as desired). At high frequencies we have

$$\tilde{Z}_s(s) = \left[1 - \frac{M^2}{L\ L_o}\right] sL_o \text{ as } s \to \infty \tag{4.11}$$

Note that the coefficient can be quite small (particularly for large N_p) as the transformer coupling is made to approach unity. For intermediate frequencies this impedance depends on the size of the various parameters, but can be kept reasonably low.

For such a capacitive pulser one can select a voltage source

$$V_p^{(0)}(t) = V_o u(t) \quad , \quad V_p^{(0)}(t) = \frac{V_o}{s} \tag{4.12}$$

which is limited by a finite rise time for a real pulser. For low frequencies the equivalent source voltage for the cable is

$$\tilde{V}_s(s) = \frac{M}{L}\ V_o s\ C_p\ L \to s\ \frac{V_o C_p L}{N_p} \text{ as } s \to 0 \tag{4.13}$$

For high frequencies this is

$$\tilde{V}_s(s) = \frac{M}{L}\frac{V_o}{s} \rightarrow \frac{V_o}{sN_p} \quad \text{as} \quad s \rightarrow \infty \tag{4.14}$$

which corresponds to a step rise in time domain. For intermediate frequencies the relative sizes of Z_C, C_g, and L need to be adjusted to control the waveform, such as by making it non resonant.

V. Concluding Remarks

One of the fundamental problems of simulator design is simulator/test-object interaction [7, 9, 13] which limits simulation accuracy, including extrapolation to (partly) correct for environmental deficiencies. By choice of sufficiently large N_p one can make $\tilde{Z}_s$ small enough that it does not significantly load the driven cable (a form of simulator/test-object interaction). Of course, this reduces the coupling efficiency, lowering the pulser voltage by $1/N_p$ to give $\tilde{V}_s$. Note also that this inductive line driver is like an inductive current sensor operated in reverse direction [5].

By use of multiple inductive couplers (electrically isolated from each other) one can distribute the source over the cable to better simulate the incident wave. By triggering the pulsers at different times one can vary the direction of incidence of the incident wave being simulated.

One can also vary the design of the pulser by addition of various impedance elements. Thereby one has some control over the shape of $V_s(t)$.

While this paper has considered inductive couplers, capacitive couplers are also possible. There are certain advantages to inductive couplers, including efficiency due the transformer action (multiple turns and core). However, one may wish to consider an array of couplers containing both kinds.

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