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Uniform Wedge Dielectric Lenses for Bends in Circular Coaxial Transmission Lines

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Abstract

This paper considers the performance of a wedge dielectric lens which can be used at a bend in a circular coaxial waveguide (coaxial cable). The early-time transmission of the TEM mode is calculated as a function of the permittivity and bend angle under the usual equal-time condition. Then, the optimum permittivity is found for a given bend angle. For small bend angles the early-time transmission coefficient can be reasonably good.

1 Introduction

In bending a TEM transmission line, distortions are introduced through the generation of higher-order modes. These modes are important at high frequencies for which wavelengths are less than or comparable to the cross-section dimensions of the transmission line. By introducing an appropriate lens at the bend, one can reduce the distortion. As discussed in [6] one can have a perfect bending lens by appropriate variation of both the permittivity ϵ and the permeability μ of the lens.

If one only varies ϵ, the problem is more difficult due to reflections at the lens boundary, but some approximate lenses can be so found [3, 4]. If one has a uniform plane wave with a single polarization, such as propagates between two infinitely wide conducting sheets [2], then the use of the Brewster angle again makes a perfect transition of the TEM wave.

In this paper, we extend the analysis of the wedge lens to the case that it intersects a coaxial circular-cylinder TEM waveguide (coaxial cable) as in fig. 1.1. Note that in bending the conductors to follow the ray paths, the cross section of the waveguide becomes two elliptic cylinders. As indicated in fig. 1.1, the lens is constructed with a symmetry plane perpendicular to the direction of propagation (in the lens) so that on both sides of the lens the waveguide consists of coaxial circular cylinders with the same medium parameters (ϵ_1, μ_0).

Noting the half angle ψ_ℓ of the lens (dielectric wedge) and the requirement that the ray in the lens be perpendicular to the symmetry plane gives

$$\psi_2 = \psi_\ell. \tag{1.1}$$

For the ray outside the lens there is the usual relation of the angles

$$\frac{\sin(\psi_1)}{\sin(\psi_2)} = \left[\frac{\epsilon_2}{\epsilon_1}\right]^{\frac{1}{2}} \equiv \xi \tag{1.2}$$

involving the ratio of permittivities ξ^2. Furthermore, we have the geometric relation

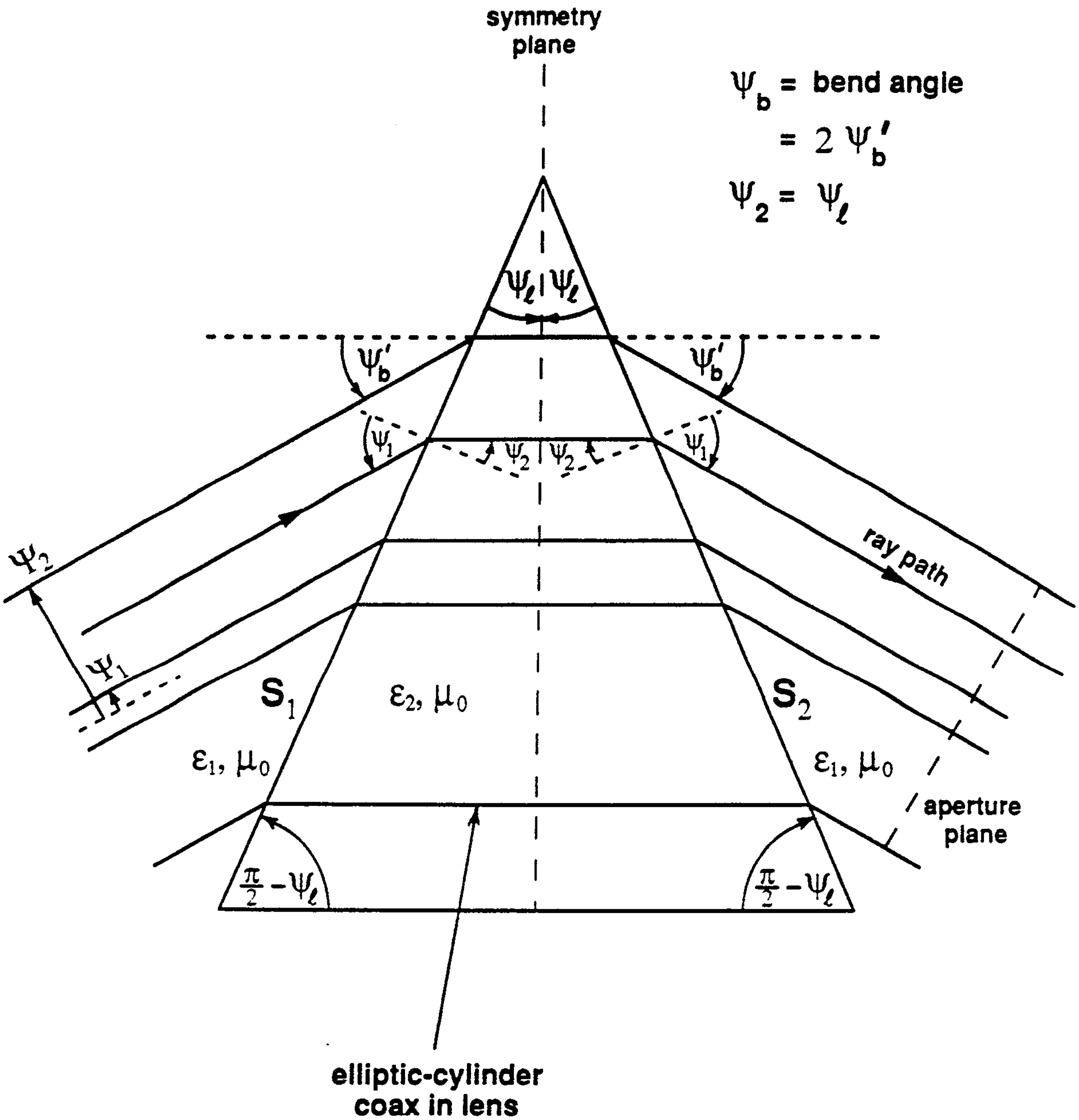

Fig. 1.1: Wedge Lens in Circular Coax

$$
\begin{aligned}
\psi_1 - \psi_2 &= \psi_b' \equiv \frac{\psi_b}{2} \\
\psi_b &\equiv \text{bend angle.}
\end{aligned}
\tag{1.3}
$$

For a given bend angle, we need to determine ξ, ψ_1 and ψ_2 for optimal lens performance.

2 Transmission of Wave Through Two Interfaces

As in fig. 2.1, we have the well-known case of plane-wave incidence on a dielectric interface. Consider first an E (or TM) wave defined by the electric field parallel to $\vec{1}_e$, i.e., in the plane of incidence defined by the unit normal $\vec{1}_S$ to the interface S and the direction of incidence (the incident ray). In this case, the incident magnetic field is parallel to S.

For such an E wave (E superscripts), we have the reflection coefficient [2, 5]

$$R_e^{(E)} \equiv \frac{E_{refl}}{E_{inc}} = \frac{H_{refl}}{H_{inc}} \equiv R_h^{(E)} \tag{2.1}$$

and transmission coefficients

$$\begin{aligned} T_h^{(E)} &\equiv \frac{H_{trans}}{H_{inc}} = 1 + R_h^{(E)} \\ T_e^{(E)} &\equiv \frac{E_{trans}}{E_{inc}} = \left[\frac{\epsilon_i}{\epsilon_t}\right]^{\frac{1}{2}} T_h^{(E)}. \end{aligned} \tag{2.2}$$

These are then all evaluated in terms of

$$R_e^{(E)} = R_h^{(E)} = \frac{\cos(\psi_i) - \left[\frac{\epsilon_i}{\epsilon_t}\right]^{\frac{1}{2}} \cos(\psi_t)}{\cos(\psi_i) + \left[\frac{\epsilon_i}{\epsilon_t}\right]^{\frac{1}{2}} \cos(\psi_t)} \tag{2.3}$$

Now apply this result to find the transmission coefficient $T_V^{(E)}$ for the electric field through the lens as the product of such coefficients for both S_1 and S_2, the two lens surfaces in fig. 1.1. Note that the role of the 1 and 2 subscripts reverses at the two interfaces, giving

$$\begin{aligned} T_V^{(E)} &= T_{e,1}^{(E)} T_{e,2}^{(E)} = T_{h,1}^{(E)} T_{h,2}^{(E)} \\ &= \frac{2\cos(\psi_1)}{\cos(\psi_1) + \xi^{-1}\cos(\psi_2)} \quad \frac{2\cos(\psi_2)}{\cos(\psi_2) + \xi\cos(\psi_1)} \\ &= 2\left\{1 + \frac{1}{2}\left[\xi X + \frac{1}{\xi X}\right]\right\}^{-1} \end{aligned} \tag{2.4}$$

where

$$X \equiv \frac{\cos(\psi_1)}{\cos(\psi_2)}. \tag{2.5}$$

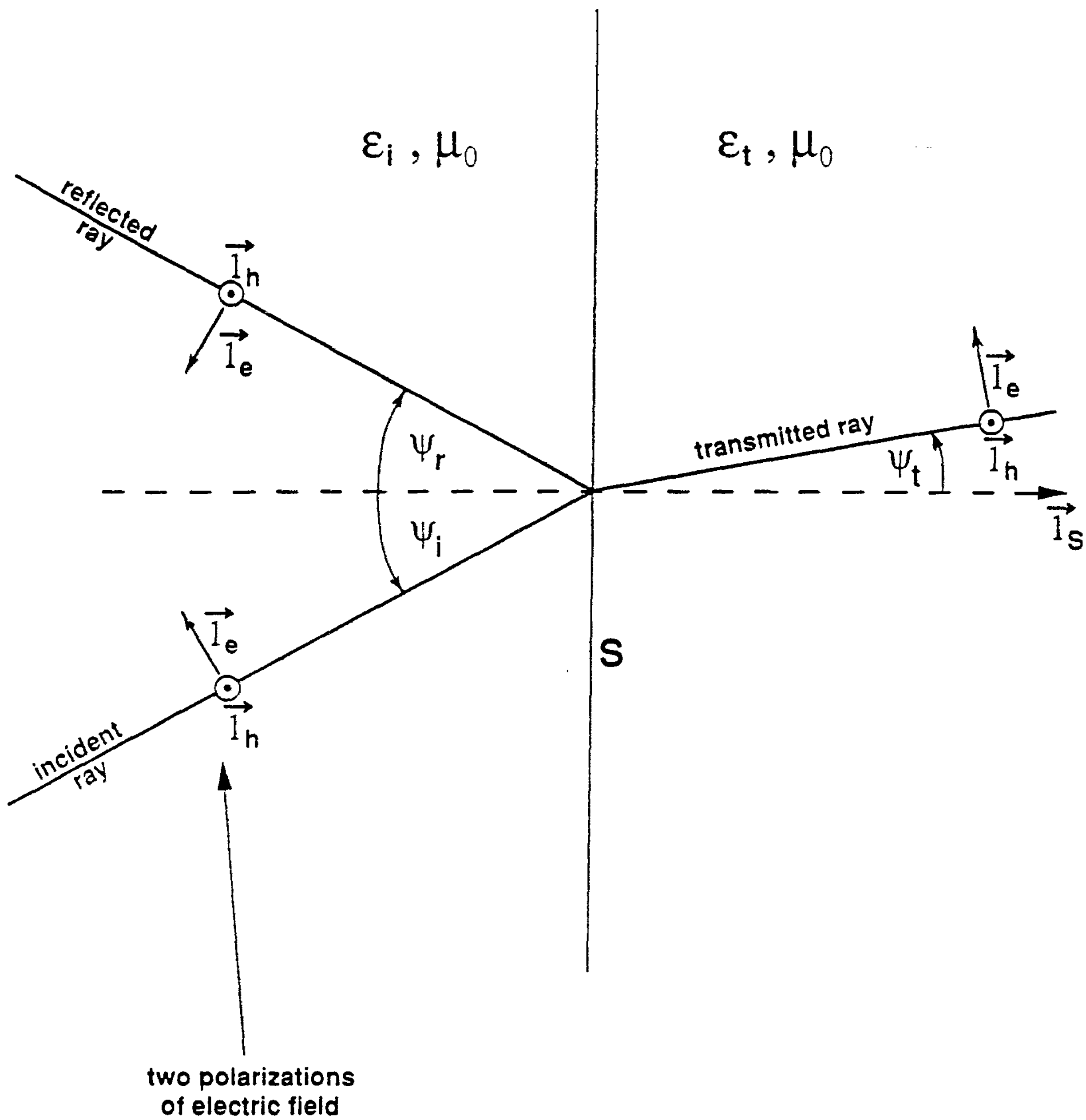

Fig. 2.1: Transmission and Reflection of Plane Wave at Interface Between Two Dielectric Media

Second, an H (or TE) wave has the electric field polarized parallel to $\vec{1}_h$ in fig. 2.1, i.e., parallel to S. Now the magnetic field has a component parallel to $\vec{1}_S$.

For such an H wave (H superscripts), we have the reflection coefficient [5]

$$R_e^{(H)} = \frac{E_{refl}}{E_{inc}} = \frac{H_{refl}}{H_{inc}} = R_h^{(H)} \tag{2.6}$$

and transmission coefficients

$$\begin{aligned} T_e^{(H)} &\equiv \frac{E_{trans}}{E_{inc}} = 1 + R_e^{(E)} \\ T_h^{(H)} &\equiv \frac{H_{trans}}{H_{inc}} = \left[\frac{\epsilon_t}{\epsilon_i}\right]^{\frac{1}{2}} T_h^{(E)}. \end{aligned} \tag{2.7}$$

These are all evaluated in terms of

$$R_e^{(H)} = R_h^{(H)} = \frac{\cos(\psi_i) - \left[\frac{\epsilon_t}{\epsilon_i}\right]^{\frac{1}{2}} \cos(\psi_t)}{\cos(\psi_i) + \left[\frac{\epsilon_t}{\epsilon_i}\right]^{\frac{1}{2}} \cos(\psi_t)}. \tag{2.8}$$

Now, find the transmission coefficient $T_V^{(H)}$ for the electric field through the lens as a product of such coefficients for both S_1 and S_2, giving

$$\begin{aligned} T_V^{(H)} &= T_{e,1}^{(H)} T_{e,2}^{(H)} = T_{h,1}^{(H)} T_{h,2}^{(H)} \\ &= \frac{2\cos(\psi_1)}{\cos(\psi_1) + \xi\cos(\psi_2)} \quad \frac{2\cos(\psi_2)}{\cos(\psi_2) + \xi^{-1}\cos(\psi_2)} \\ &= 2\left\{1 + \frac{1}{2}\left[\frac{X}{\xi} + \frac{\xi}{X}\right]\right\}^{-1}. \end{aligned} \tag{2.9}$$

Note that both these waves satisfy a relation like (1.2) as

$$\sqrt{\epsilon_i}\ \sin(\psi_i) = \sqrt{\epsilon_t}\ \sin(\psi_t). \tag{2.10}$$

For later use, we can solve for X in terms of ξ and ψ_b'. Write from (1.2) and (1.3)

$$\xi^{-1} = \frac{\sin(\psi_2)}{\sin(\psi_1)} = \frac{\sin(\psi_1 - \psi_b')}{\sin(\psi_1)}$$

$$\begin{aligned}
&= \cos(\psi_b') - \cot(\psi_1)\sin(\psi_b') \\
\cot(\psi_1) &= \frac{\cos(\psi_b') - \xi^{-1}}{\sin(\psi_b')} \equiv C_1 \qquad (2.11)\\
\sin(\psi_1) &= \left[1 + C_1^2\right]^{-\frac{1}{2}},\ \cos(\psi_1) = C_1\left[1 + C_1^2\right]^{-\frac{1}{2}}.
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
\xi &= \frac{\sin(\psi_1)}{\sin(\psi_2)} = \frac{\sin(\psi_2 + \psi_b')}{\sin(\psi_2)} \\
&= \cos(\psi_b') + \cot(\psi_2)\sin(\psi_b') \\
\cot(\psi_2) &= \frac{\xi - \cos(\psi_b')}{\sin(\psi_b')} \equiv C_2 \qquad (2.12)\\
\sin(\psi_2) &= \left[1 + C_2^2\right]^{-\frac{1}{2}},\ \cos(\psi_2) = C_2\left[1 + C_2^2\right]^{-\frac{1}{2}}.
\end{aligned}$$

Then, we have

$$\begin{aligned}
X &= \frac{\cos(\psi_1)}{\cos(\psi_2)} = \frac{C_1}{C_2}\left\{\frac{1 + C_2^2}{1 + C_1^2}\right\}^{\frac{1}{2}} = \left\{\frac{1 + C_2^{-2}}{1 + C_1^{-2}}\right\}^{\frac{1}{2}} \\
&= \left|\frac{\cos(\psi_b') - \xi^{-1}}{\xi - \cos(\psi_b')}\right| \left\{\frac{\xi^2 - 2\cos(\psi_b')\,\xi + 1}{\xi^2 - 2\cos(\psi_b')\,\xi^{-1} + 1}\right\}^{\frac{1}{2}} \qquad (2.13)\\
&= \left|\frac{\xi\cos(\psi_b') - 1}{\xi - \cos(\psi_b')}\right|.
\end{aligned}$$

Noting that $\cot(\psi_1)$, $\cot(\psi_2)$ and $\sin(\psi_b')$ are all positive, then C_1 and C_2 are positive, and hence the numerator and denominator above are separately positive, giving

$$X = \frac{\xi\cos(\psi_b') - 1}{\xi - \cos(\psi_b')}. \qquad (2.14)$$

3 TEM Mode After Lens at Early Times

Transmission through this lens is not perfect. At low frequencies one can view the lens as some sort of loading impedances (series and/or parallel) on the coaxial transmission line of characteristic impedance

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_1}}\, f_g \quad , \; f_g = \frac{1}{2\pi}\ell n\left(\frac{\Psi_2}{\Psi_1}\right). \tag{3.1}$$

Local changes in the inductance and capacitance associated with the lens have negligible effect at sufficiently low frequencies.

At early times, when the wave first reaches the aperture plane (figs. 1.1 and 3.1) there is some reduction in the fields according to the transmission coefficients calculated in section 2. These fields (all the transmitted rays) on the coax cross section all arrive on the aperture plane (perpendicular to the direction of propagation) of the coax at the same time due to the equal-time property of the lens.

Assuming a step-rising TEM wave incident on the lens with voltage V_0, the initial amplitude ($at\ t = t_{a+}$) of the TEM mode is [1, 3]

$$\begin{aligned} V(t_{a+}) &= V_0 T_V \\ T_V &= \frac{1}{V_0}\frac{\int_{S_a} \vec{E}(\Psi,\phi;t_{a+})\cdot\vec{e_0}(\Psi)dS}{\int_{S_a}\vec{e_0}(\Psi)\cdot\vec{e_0}(\Psi)dS} \end{aligned} \tag{3.2}$$

where S_a denotes the region for $\Psi_2 \leq \Psi \leq \Psi_1$ (cylindrical coordinates) in fig. 3.1. The TEM mode function (for electric field) is [3]

$$\begin{aligned} u(\Psi) &= \frac{\ell n\left(\frac{\Psi}{\Psi_2}\right)}{\ell n\left(\frac{\Psi_1}{\Psi_2}\right)} = \begin{cases} 0 & \text{for } \Psi = \Psi_2 \\ 1 & \text{for } \Psi = \Psi_1 \end{cases} \\ \vec{e_0}(\Psi) &= -\vec{1}_\Psi \frac{du}{d\Psi} = \left\{\Psi \ell n\left(\frac{\Psi_2}{\Psi_1}\right)\right\}^{-1}. \end{aligned} \tag{3.3}$$

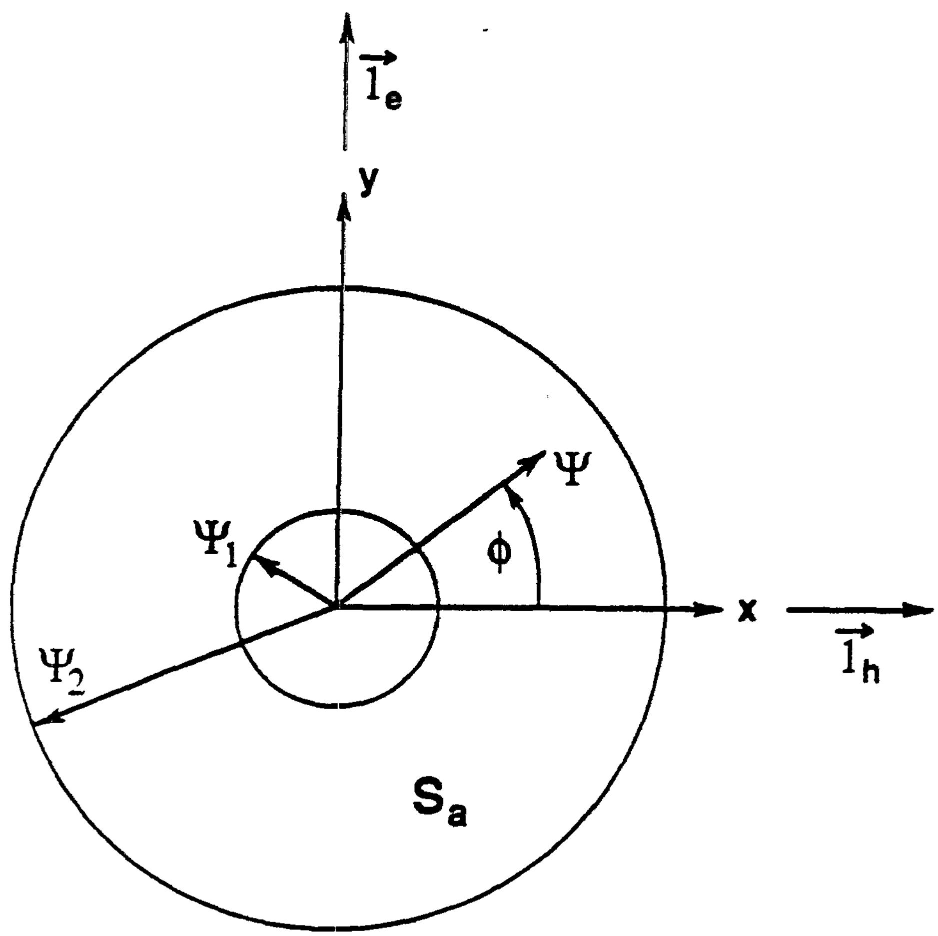

Fig. 3.1: Aperture Plane

Now, the initial electric field on the aperture plane consists of E-wave and H-wave parts. The transmission coefficients through the lens are expressed in terms of the cartesian coordinates in fig. 3.1 with the E wave polarized in the y direction and the H wave polarized in the x direction. With $V_0\vec{e}_0$ representing the incident electric field in the first coax (with a change of coordinates to those of the first coax), then the electric field first reaching the aperture plane can be written as

$$\vec{E}\left(\Psi,\phi;t_{a+}\right) = V_0\left[T_V^{(E)}\vec{1}_y\vec{1}_y + T_V^{(H)}\vec{1}_x\vec{1}_x\right]\cdot\vec{e}_0(\Psi). \tag{3.4}$$

From this (3.2) gives

$$T_V = \frac{\displaystyle\int_{S_a}\vec{e}_0(\Psi)\cdot\left[T_V^{(E)}\vec{1}_y\vec{1}_y + T_V^{(H)}\vec{1}_x\vec{1}_x\right]\cdot\vec{e}_0(\Psi)dS}{\displaystyle\int_{S_a}\vec{e}_0(\Psi)\cdot\vec{e}_0(\Psi)dS}. \tag{3.5}$$

Substituting from (3.3) and using cylindrical coordinates gives

$$\begin{aligned} T_V &= \frac{\displaystyle\int_{\Psi_1}^{\Psi_2}\int_0^{2\pi}\left\{\Psi\ell n\left(\tfrac{\Psi_2}{\Psi_1}\right)\right\}^{-2}\left[T_V^{(E)}\sin^2(\phi) + T_V^{(H)}\cos^2(\phi)\right]\Psi d\phi d\Psi}{\displaystyle\int_{\Psi_1}^{\Psi_2}\int_0^{2\pi}\left\{\Psi\ell n\left(\tfrac{\Psi_2}{\Psi_1}\right)\right\}^{-2}\Psi d\phi d\Psi} \\ &= \frac{\pi\left[T_V^{(E)} + T_V^{(H)}\right]\displaystyle\int_{\Psi_1}^{\Psi_2}\left\{\Psi\ell n\left(\tfrac{\Psi_2}{\Psi_1}\right)\right\}^{-2}\Psi d\Psi}{2\pi\displaystyle\int_{\Psi_1}^{\Psi_2}\left\{\Psi\ell n\left(\tfrac{\Psi_2}{\Psi_1}\right)\right\}^{2}\Psi d\Psi} \\ &= \frac{1}{2}\left[T_V^{(E)} + T_V^{(H)}\right]. \end{aligned} \tag{3.6}$$

The simplicity of this result is due to the rotation symmetry of the coax. The early-time TEM-mode coefficient is just the average of the E-wave and H-wave transmission coefficients.

4 Properties of the Transmission Coefficients

The transmission is a function of ψ_b' (the half-bend angle) and ξ, the other angles ψ_1 and ψ_2 being determined by these. From (2.4) and (2.14) we have

$$\begin{aligned} T_V^{(E)} &= 2\left\{1+\frac{1}{2}\left[\xi X+\frac{1}{\xi X}\right]\right\}^{-1} \\ &= \frac{4\xi X}{\left[\xi X+1\right]^2} \\ &= \frac{4\xi\left[\xi-\cos\left(\psi_b'\right)\right]\left[\xi\cos\left(\psi_b'\right)-1\right]}{\cos^2\left(\psi_b'\right)\left[\xi^2-1\right]^2}. \end{aligned} \tag{4.1}$$

Similarly, we have from (2.9) and (2.14)

$$\begin{aligned} T_V^{(H)} &= 2\left\{1+\frac{1}{2}\left[\frac{X}{\xi}+\frac{\xi}{X}\right]\right\}^{-1} \\ &= \frac{4\xi X}{\left[X+\xi\right]^2} \\ &= \frac{4\xi\left[\xi-\cos\left(\psi_b'\right)\right]\left[\xi\cos\left(\psi_b'\right)-1\right]}{\left[\xi^2-1\right]^2} \\ &= \cos^2\left(\psi_b'\right)T_V^{(E)}. \end{aligned} \tag{4.2}$$

Thus, we have the very interesting result that, for a given ψ_b', these two transmission coefficients have the same dependence on ξ except for a constant multiplier. Furthermore, the result from section 3 becomes

$$\begin{aligned} T_V &= \frac{1+\cos^2\left(\psi_b'\right)}{2}T_V^{(E)} \\ &= \frac{1+\sec^2\left(\psi_b'\right)}{2}T_V^{(H)} \\ &= \frac{2\xi\left[\xi-\cos\left(\psi_b'\right)\right]\left[\xi\cos\left(\psi_b'\right)-1\right]\left[1+\sec^2\left(\psi_b'\right)\right]}{\left[\xi^2-1\right]^2}. \end{aligned} \tag{4.3}$$

So, for a fixed half-bend-angle all three of these transmission coefficients have the same dependence on ξ (except for a constant multiplier).

The range of ξ is limited by physical realizability of the lens. Referring to fig. 1.1 smaller ϵ_2/ϵ_1 corresponds to a thicker lens (larger $\psi_\ell = \psi_2$) to achieve the required delay for a given ψ_b' (for $\epsilon_2/\epsilon_1 \geq 1$). However ψ_ℓ is limited by the orientation of the coax, i.e.

$$\psi_2 = \psi_\ell \leq \frac{\pi}{2} - \psi_b'. \tag{4.4}$$

Setting these equal gives ξ_0, a lower limit on ξ, from (1.2) and (1.3) as

$$\begin{aligned} \psi_1 &= \psi_2 + \psi_b' = \frac{\pi}{2} \quad , \quad \psi_2 = \frac{\pi}{2} - \psi_b' \\ \xi_0 &= \frac{\sin(\psi_1)}{\sin(\psi_2)} = \sec(\psi_b'). \end{aligned} \tag{4.5}$$

For this special case, we have

$$\begin{aligned} X &= \frac{\cos(\psi_1)}{\cos(\psi_2)} = 0 \\ T_V^{(E)} &= T_V^{(H)} = T_V = 0. \end{aligned} \tag{4.6}$$

For large ξ with fixed ψ_b' we have

$$\begin{aligned} T_V^{(H)} &= \frac{4\cos(\psi_b')}{\xi} + O\left(\xi^{-2}\right) \text{ as } \xi \to \infty \\ T_V^{(E)} &= \frac{4}{\cos(\psi_b')\,\xi} + O\left(\xi^{-2}\right) \text{ as } \xi \to \infty \\ T_V &= \frac{2}{\xi}\left[\cos(\psi_b') + \sec(\psi_b')\right] + O\left(\xi^{-2}\right) \text{ as } \xi \to \infty. \end{aligned} \tag{4.7}$$

The transmission coefficients can now be maximized for a given ψ_b' by considering any one of the three, since they have the same dependence on ξ except for a constant multiplier. One can set the derivative with respect to ξ to zero to find a particular ξ, say ξ_1, which maximizes these transmission coefficients. For the E wave this is the Brewster-angle condition [2] for which

$$
\begin{aligned}
\cos(\psi_{1B}) &= \left[1+\xi^2\right]^{-\frac{1}{2}} = \sin(\psi_{2B}) \\
\sin(\psi_{1B}) &= \xi\left[1+\xi^2\right]^{-\frac{1}{2}} = \cos(\psi_{2B}) \\
\tan(\psi_{1B}) &= \xi = \cot(\psi_{2B}) \\
\psi_{1B}+\psi_{2B} &= \frac{\pi}{2} \\
X &= \frac{\cos(\psi_{1B})}{\cos(\psi_{2B})} = \xi_1^{-1} \\
T_{V_1}^{(E)} &\equiv \left. T_V^{(E)}\right|_{\xi=\xi_1} = 1.
\end{aligned}
\tag{4.8}
$$

The half-bend angle is

$$
\begin{aligned}
\psi'_{b1} &\equiv \left.\psi_b\right|_{\xi=\xi_1} \\
\sin(\psi'_{b1}) &= \sin(\psi_{1B}-\psi_{2B}) \\
&= \sin(\psi_{1B})\cos(\psi_{2B}) - \cos(\psi_{1B})\sin(\psi_{2B}) \\
&= \frac{\xi_1^2-1}{\xi_1^2+1} \\
\cos(\psi'_{b1}) &= \cos(\psi_{1B}-\psi_{2B}) \\
&= \cos(\psi_{1B})\cos(\psi_{2B}) + \sin(\psi_{1B})\sin(\psi_{2B}) \\
&= \frac{2\xi_1}{\xi_1^2+1}.
\end{aligned}
\tag{4.9}
$$

That the Brewster condition maximizes $T_V^{(E)}$ can be obtained by noting that power conservation (passive lens) gives a bound of 1.0 on power transmission.

It now follows that at the same Brewster condition the other transmission coefficients maximize and we have

$$
T_{V_1}^{(H)} = \cos^2(\psi'_{b1}) = \frac{4\xi_1^2}{[\xi_1^2+1]^2}
$$

$$T_{V_1} = \frac{1+\cos^2(\psi'_{b1})}{2} = \frac{1}{2} + \frac{2\xi_1^2}{[\xi_1^2+1]^2} \tag{4.10}$$

$$= \begin{cases} 1 \text{ for } \psi'_b = 0 \ , \ \psi_b = 0 \\ \frac{1}{2} \text{ for } \psi'_b = \frac{\pi}{2} \ , \ \psi_b = \pi . \end{cases}$$

Figure 4.1 illustrates the dependence of T_V on ξ for various choices of ψ'_b. As ξ is varied the transmission passes through a maximum given by T_{V_1} as indicated. For a specified ψ'_b and ξ the values of ψ_1 and ψ_2 are obtained from (2.11) and (2.12). Selecting an example value of ξ corresponding to $\epsilon_2/\epsilon_1 = 2.26$ (transformer oil (or polyethylene) and air) these angles are plotted as a function of ψ'_b in fig. 4.2. For this same example T_V is also given in fig. 4.3. Note for small ψ'_b that T_V is only a weak function of ψ'_b, but that above a certain angle (about 30° in this case) T_V decreases rapidly. For this permittivity ratio of 2.26, one might conveniently select $\psi'_b = 22.5°$ (or full bend angle $\psi_b = 45°$), for which $T_V = 0.926$, $\psi_1 = 55.8°$, and $\psi_2 = 33.3°$.

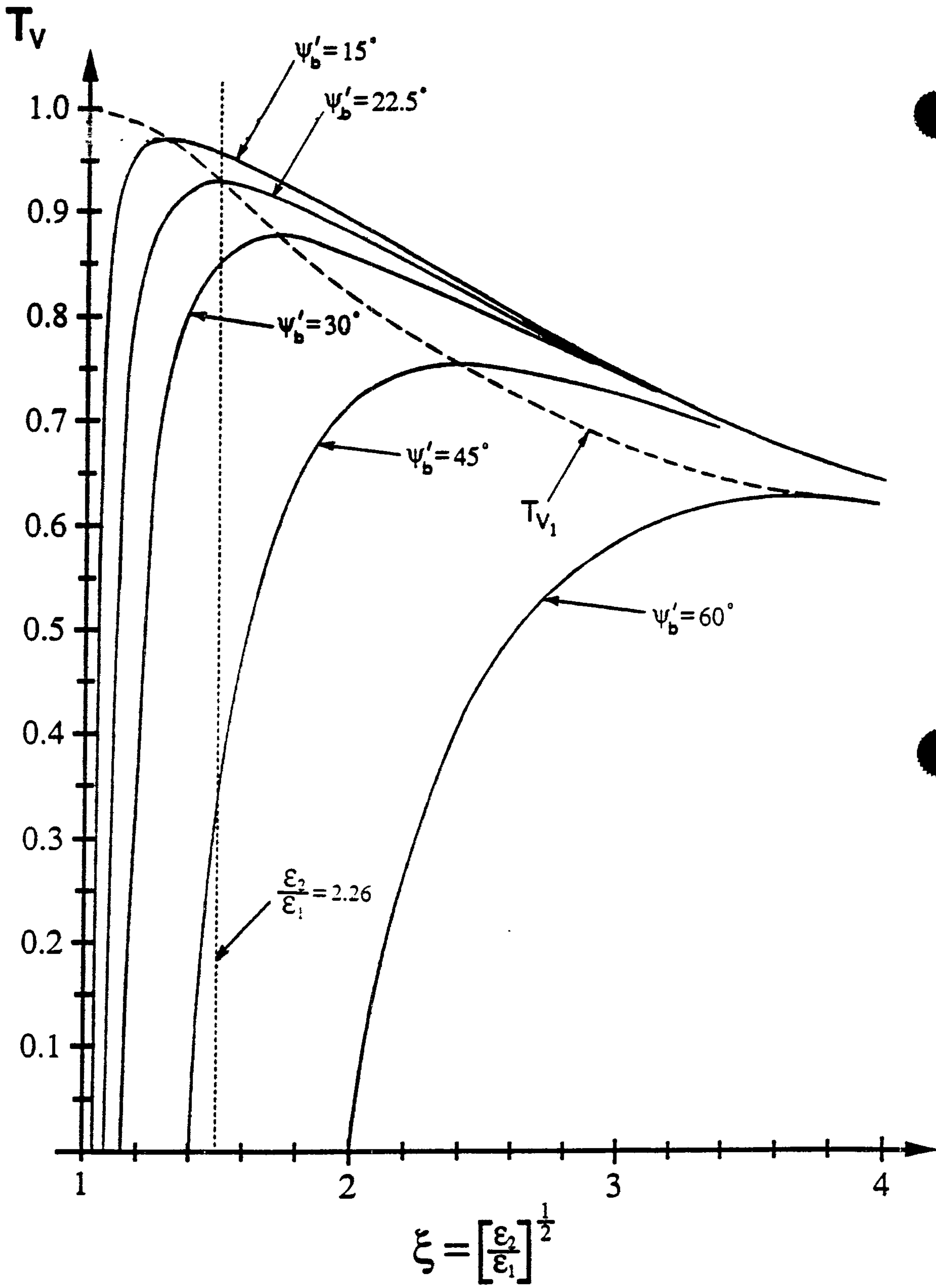

Fig. 4.1: Initial Transmission of TEM Mode

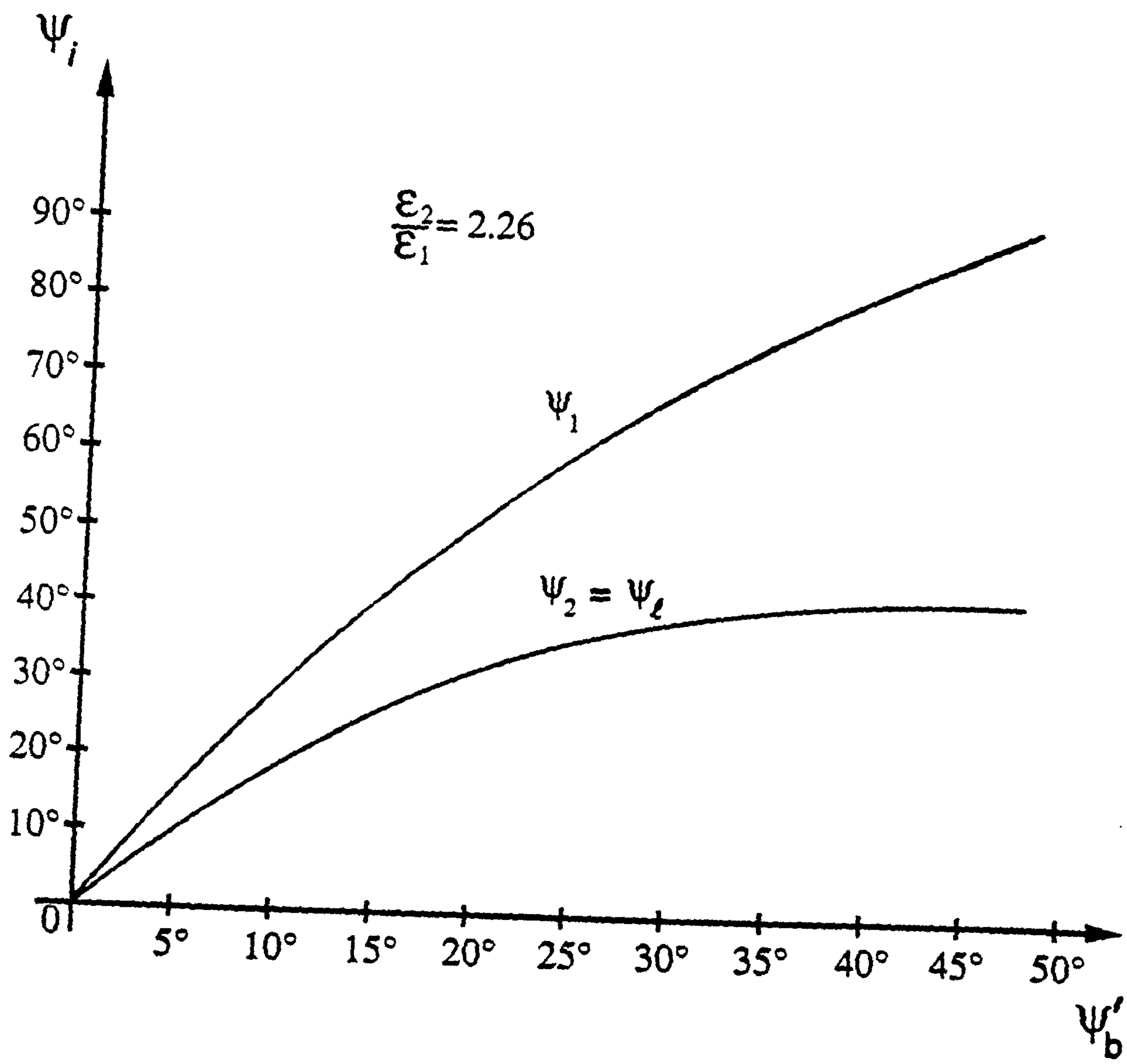

Fig. 4.2: Lens Angles for Particular Permittivity Ratio

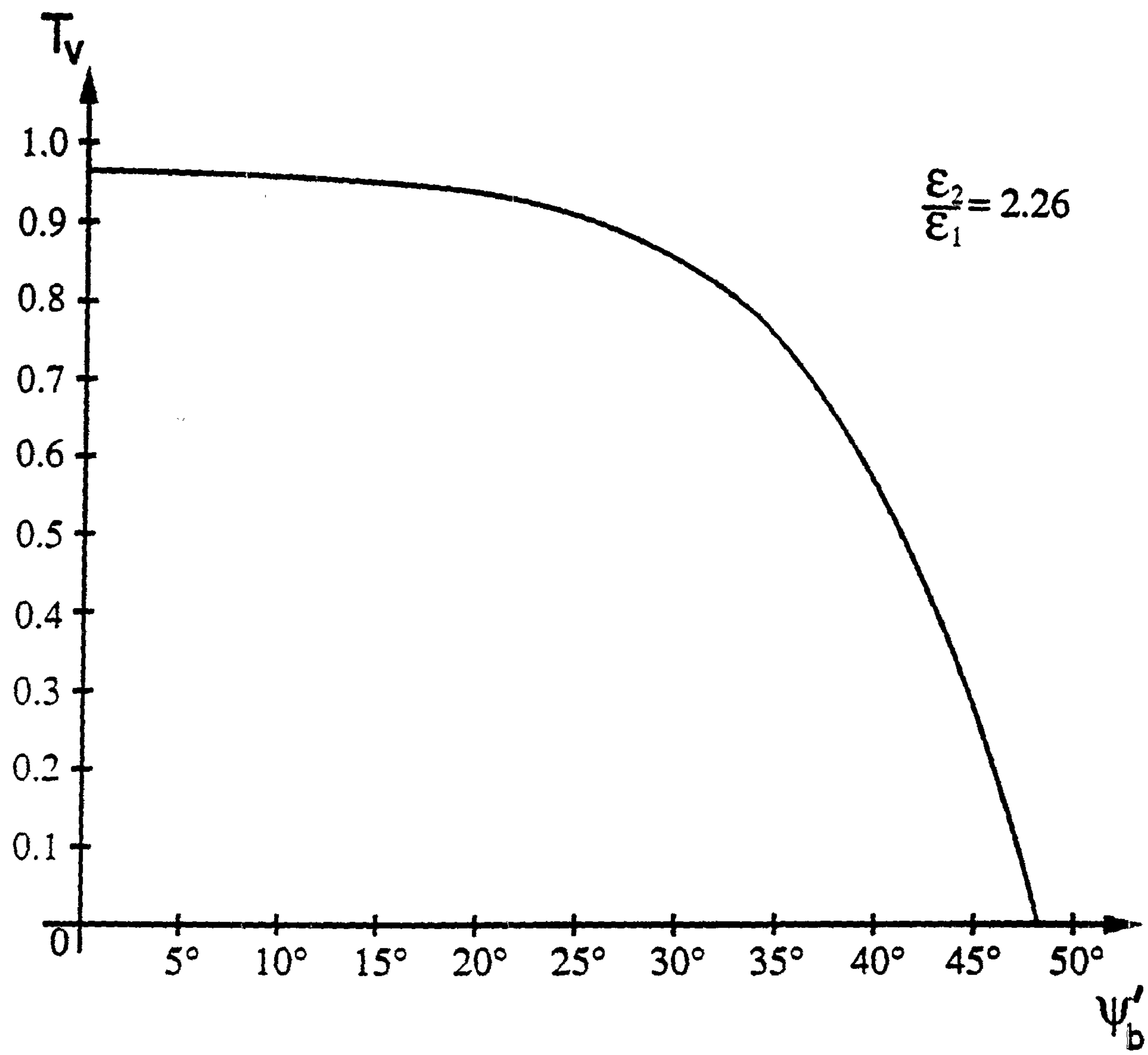

Fig. 4.3: Initial TEM Transmission for Particular Permittivity Ratio

5 Concluding Remarks

For such a dielectric bending lens, we now have the trade-off between bend angle and TEM-mode transmission. As indicated in fig. 4.1, as $\psi_b \, (= 2\psi_b')$ is increased, the maximum transmission T_{V_1} decreases. Instead of, for example, $\psi_b' = 45°$ (or $\psi_b = 90°$) for a single lens, one can choose two lenses, each with $\psi_b' = 22.5°$ (or $\psi_b = 45°$) to achieve the same effective right-angle bend. Then the maximum achievable T_V (for the combination, i.e. $\left[T_V^{(E)^2} + T_V^{(H)^2}\right]/2$ when considering the E-wave and H-waves along the ray paths through the two lenses) is increased and the permittivity ratio is reduced to what may be a more practical value, near to that of transformer oil and polyethylene. In the limit of a large number of lenses one may think of this as a nonuniform dielectric lens which continuously bends the rays through the total bend angle.

This paper has considered the case of a circular coax passing through a wedge dielectric lens. However, the procedure for evaluating the E-wave and H-wave contributions in section 3 is not limited to the circular coaxial cross section. It applies to any cylindrical TEM transmission line, such as a parallel plate transmission line, a wire parallel to a ground plane, etc. However, the integrals over the aperture S_a are not necessarily as simple as in the present case.

Note that the impedance matching is before and after the lens, but not in it. One could try to improve on this by changing the position of some of the conductors as they pass through the lens so as to increase the transmission-line impedance in the lens without interfering with the ray paths passing through the lens from the first coaxial (or other) transmission line to the second.

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