Sensor and Simulation Note

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Limited-Angle-of-Incidence and Limited-Time Electric Sensors

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Abstract

This paper considers two classes of antennas which under the right conditions can be considered as accurate time-domain sensors of an incident electric field. One class gives an output proportional to the incident time-domain field, while the second class gives the time integral. As these are electrically large but not infinitely large, there is in general a limited time based on when the truncation sends a signal to the output terminal pair. Furthermore there are limitations on the allowable angles of incidence of a plane wave for ideal performance. They do not measure a single component of the incident field, independent of angle of incidence, in the manner of the classical electrically small electric-dipole sensor.

I. Introduction

By an electromagnetic sensor we do not mean just any antenna. As discussed in [8, 20, 21] a sensor is a special kind of antenna which has been optimized in certain respects. Limiting ourselves to fields (instead of voltage and current which involve spatial integrals), we consider the usual passive linear antennas but require that their sensitivity be accurately calculable for frequencies/times of interest (calibratable by a ruler, a few percent accuracy being easily attainable). In addition one would like such a sensitivity parameter to be simple across a wide frequency band. This could be a constant times a vector component of the field, its time derivative, its time integral, or some other simple mathematical operation. Which form one chooses depends on various considerations such as signal-to-noise ratio, geometry of the measurement, etc. From the chosen mathematical form of the transfer function, other forms are generated by simple mathematical operations. There is no a priori reason to prefer in general one form such as the waveform of the field component as compared to say its time derivative. If one thinks in terms of the important frequencies of interest in the waveform, then the field-component waveform, or even its time integral may be more useful. However, for high-frequencies (as say in the numerical Fourier transform) or for good resolution around the fast rise of the waveform the time-derivative of the field-component waveform may be more useful. For constructing the Fourier transform over a wideband of frequencies, all of the above forms (and others, e.g. second derivative) may be useful in composite sense.

An important concept in such field measurements is that of a point measurement. By this is meant that at some specified point $\vec{r}_o$ in space one measures a field in the sense of some voltage (or current) at a terminal pair as

$$\tilde{V}(s) = \tilde{T}(s)\ \vec{1}_m \cdot \begin{cases} \tilde{\vec{E}}^{(inc)}(\vec{r}_o, s) \\ or \\ \tilde{\vec{H}}^{(inc)}(\vec{r}_o, s) \end{cases}$$

$\vec{1}_m \equiv$ measurement direction (constant unit vector)

$\tilde{T}(s) \equiv$ sensor transfer function

$\sim \equiv$ Laplace transform (two sided) (1.1)

$s \equiv \Omega + j\omega \equiv$ Laplace-transform variable or complex frequency

$\tilde{\vec{E}}^{(inc)}(\vec{r}, s)\ ,\ \tilde{\vec{H}}^{(inc)}(\vec{r}, s) \equiv$ incident electric and magnetic fields in the absense of the sensor

Note that the incident fields of course satisfy the Maxwell equations, but are not assumed to be of any particular form such as a single plane wave.

These conditions are somewhat restrictive in that the dot-product relationship leads to basically electric- and magnetic-dipole sensors. There is also a special form of point measurement combining the electric and magnetic fields [10] via two dipoles (electric and magnetic) which may even be made to be present in the same antenna. This class of sensors is basically electrically small so that only the appropriate dipole moment (in the multipole expansion) is significant. By use of symmetries and multiple sampling positions around the sensor structure, some of the unwanted multipoles can be suppressed leading to a higher frequency response in the dipole mode for a given antenna size and associated sensitivity. This aspect is discussed in some detail in [9] and briefly reviewed in [15]. While these are often used in the mode of the time-derivative of the field-component waveform (because of the relatively large signals from the passive sensor) they also can function in the mode of the field-component waveform itself.

Suppose now that one relaxes the constraint of (1.1). This allows the possibility of antennas which are not characterized by a particular electric or magnetic dipole oriented in some particular direction such as $\vec{1}_m$, at least for some frequencies (particularly high frequencies) and associated times of interest. This raises the possibility of electrically large antennas. Well, if the response is not to be dipolar, what other kind of response might be useful. Suppose that the incident field is constrained to be a single plane wave as

$$\tilde{\vec{E}}^{(inc)}(\vec{r},s) = E_o \tilde{f}(s)\ \vec{1}_p\ e^{-\gamma \vec{1}_1 \cdot \vec{r}}$$

$$\tilde{\vec{H}}^{(inc)}(\vec{r},s) = \frac{E_o}{Z_o} \tilde{f}(s)\ \vec{1}_1 \times \vec{1}_p\, e^{-\gamma \vec{1}_1 \cdot \vec{r}}$$

$$\tilde{\vec{E}}^{(inc)}(\vec{r},t) = E_o\ \vec{1}_p\ f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right)$$

$$\tilde{\vec{H}}^{(inc)}(\vec{r},t) = \frac{E_o}{Z_o}\ \vec{1}_1 \times \vec{1}_p\, f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right)$$

$$\vec{1}_p \equiv \text{polarization unit vector}$$

$$\vec{1}_1 \equiv \text{direction-of-incidence unit vector}$$

$$\vec{1}_p \cdot \vec{1}_1 = 0$$

$$\overleftrightarrow{1}_1 \equiv \overleftrightarrow{1} - \vec{1}_1 \vec{1}_1 \equiv \text{transverse identity}$$

$$Z_o = \left[\frac{\mu_o}{e_o}\right]^{\frac{1}{2}} \equiv \text{wave impedance of free space} \tag{1.2}$$

$$c = [\mu_o \in_o]^{-\frac{1}{2}} \equiv \text{speed of light}$$

$$\gamma \equiv \frac{s}{c} \equiv \text{propagation constant}$$

Here the polarization is taken as time invariant, but this is often not important since an arbitrary polarization can be decomposed into two orthogonal polarizations. The antenna of interest can also be designed with appropriate symmetry planes [17] such that it is only sensitive to one of these polarizations. Direction of incidence is a different matter which requires careful consideration. One can observe that general incident fields as in (1.1) can be considered as an integral over a distribution of plane waves [19]. However, we shall now restrict the direction of incidence not to vary over 4π steradians, but over some more limited range consistent with the types of antennas under consideration, such that over this limited range some response similar to (1.1) results.

II. Voltage Proportional to Incident Electric Field

For our first class of these limited-angle-of-incidence sensors, consider the case that the sensor conductors, etc. are positioned in space such that there is no scattering of the incident wave. Noting that this implies that no energy is delivered to a load (such as the input to a coaxial cable) let us consider one of the two possibilities, i.e. the open circuit voltage (the second being the short-circuit current). Later, the load is reinserted, for which case one needs the antenna impedance.

As indicated in fig. 2.1 let us consider that the incident wave is polarized in the z direction, i.e.

$$\vec{1}_p = \vec{1}_z \tag{2.1}$$

Then in cylindrical coordinates (Ψ, ϕ, z) we have

$$\begin{aligned} x &= \Psi \cos(\phi) \quad , \quad y = \Psi \sin(\phi) \\ \vec{1}_1 &= \vec{1}_x \cos(\phi_1) + \vec{1}_y \sin(\phi_1) \\ \vec{1}_1 \times \vec{1}_p &= \vec{1}_x \sin(\phi_1) - \vec{1}_y \cos(\phi_1) \end{aligned} \tag{2.2}$$

so that $0 \leq \phi_1 < 2\pi$ characterizes the direction of incidence. The orientation of the magnetic field is then only constrained to be perpendicular to $\vec{1}_z$. With the sensor structure assumed to lie only on two planes, $z = \pm b$, the incident field is everywhere perpendicular to these planes and no scattering occurs. Note that the sensor structure on each of these planes is idealized as infinitesimally thin (or at least much thinner than h), and can consist of conducting sheets (idealized as perfectly conducting), or even resistive sheets. (Permeable sheets are in general avoided.)

Let us define the open-circuit voltage as

$$V_{o.c.}(t) \equiv - \int_{-h}^{h} \vec{E}^{(inc)}(\vec{r},t)\Big|_{(x,y)=(0,0)} \cdot \vec{1}_z \, dz = -2b \; E_o \, f(t) \tag{2.3}$$

where, for convenience, the integral is taken along the z axis. The two points (x, y, z) = 0, 0, $\pm$ b) then can be used to define a terminal pair where eventually a load impedance will be added connecting these two points. Note now that the open-circuit voltage is independent of the limited direction of incidence.

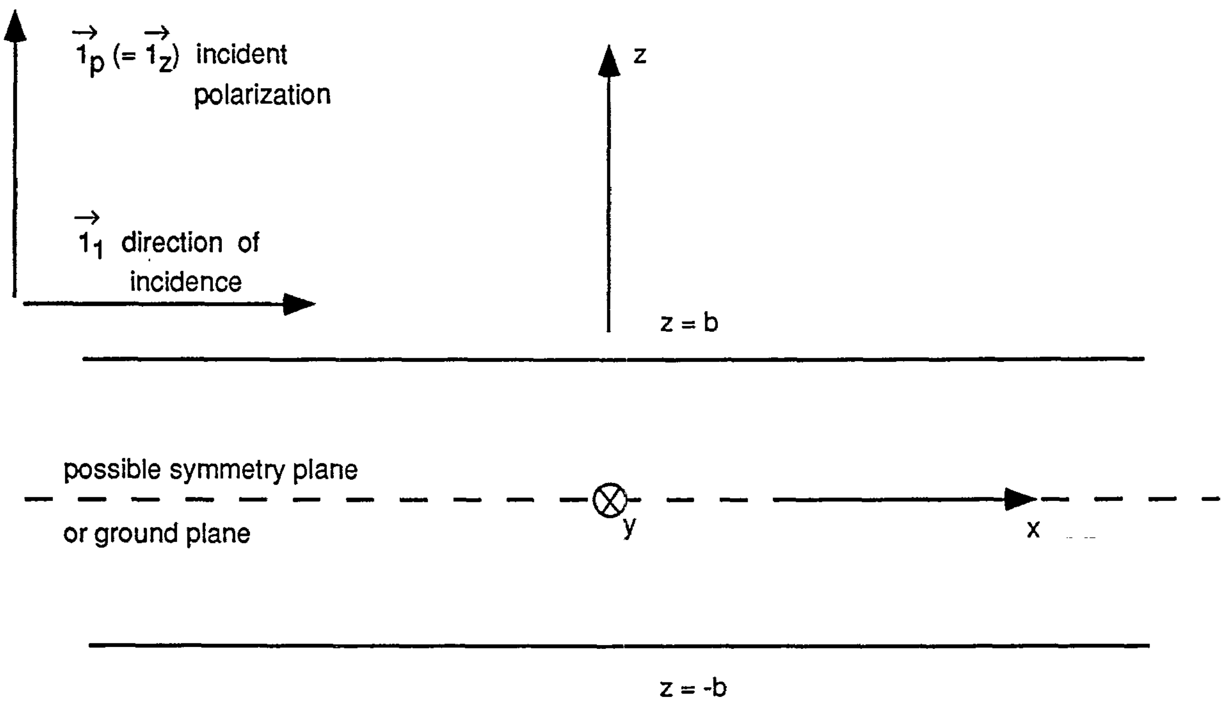

Fig. 2.1 Sensor Structure on Planes of Constant z

At this point note that the concept is similar to that developed in [3,4] which gives the design principles of the PPD (parallel-plate dipole) sensor. As indicated there and in (2.3) the sensor bandwidth in this open-circuit bandwidth can be quite high. The size of the sensor structure is not a limitation but b can be, depending on the details of how the signal is extracted at the terminal. One kind of high-resistance output is discussed in [4] which allows one to match the resistor into the field configuration, thereby avoiding some of the problems with stray capacitance, but that is not the approach considered in the present paper.

Suppose now that a wave has the direction of incidence $\vec{1}_1$ limited as in (2.2), but with polarization

$$\vec{1}_p = \vec{1}_1 \times \vec{1}_z \quad , \quad \vec{1}_p \cdot \vec{1}_z = 0 \tag{2.4}$$

By making the z = 0 plane a symmetry plane such an incident wave is symmetric as compared with the antisymmetric case given by (2.1) [14, 17]. By making the two parts of the sensor structure (on $z = \pm b$) have the same projection on the z = 0 plane (a plane of reflection symmetry), then the scattered field due to this cross-polarized incident field in (2.4) is also a symmetric wave which gives exactly zero in the integral along the z axis in (2.3). So now let us impose such symmetry on the sensor. Note that if the z = 0 plane is a ground plane (ideally perfectly conducting) only the antisymmetric field distribution (for z > 0) is allowed to exist.

The sensor structure is still rather general as in fig. 2.1 consisting of a structure on two parallel planes, symmetric with respect to the z = 0 plane. This leaves a lot of flexibility concerning the shape etc. of these plates which can be exploited for other desirable characteristics. In particular, suppose that the load impedance is a finite resistance R_L (say 50 Ω or 100 Ω) such as is characteristic of one or more transmission lines (coaxial cables, etc.). One may wish the antenna impedance $\tilde{Z}_a(s)$ to have some nice properties so that the voltage delivered to the load is simply related to $V_{o.c.}$. Considering the case of circular disks as in [3,4], the source impedance is approximately characterized by Bessel functions due to the cylindrical propagation of the wave outward from the load. More convenient would be a transmission-line geometry with a characteristic impedance Z_c independent of position along the line. If infinitely long, this would be an antenna impedance of $Z_c/2$ due to the waves propagating in both directions away from the load (or Z_c if the transmission line extended in only one direction away from the load). For a finite length, still in time domain there is a certain clear time given by the round-trip time to one or both truncations, during which time the transmission-line behaves as though it were infinitely long.

As indicated in fig. 2.2 we have the special case of a transmission line of total length ℓ with the terminal pair in the center. This has two additional symmetry planes (x = 0 and y = 0). The antenna impedance can be estimated for radian wavelength $\lambda >> b, a$ and $\ell >> b, a$ as

$$\tilde{Z}_a(s) = \frac{Z_c}{2} \coth\left(\frac{\gamma \ell}{2}\right) \tag{2.5}$$

For the clear time of ℓ / c as discussed above this is just $Z_c / 2$. The voltage delivered to the load is just

$$\begin{aligned} \tilde{V}(s) &= R_L \left[R_L + \tilde{Z}_a(s)\right]^{-1} \tilde{V}_{o.c.}(s) \\ &= \left[1 + \frac{Z_c}{2 R_L} \coth\left(\frac{\gamma \ell}{2}\right)\right]^{-1} \tilde{V}_{o.c.}(s) \end{aligned} \tag{2.6}$$

For the special case that the load is matched in an early-time sense to the two transmission lines in parallel we have

$$\begin{aligned} Z_c &= 2 R_L \\ \tilde{V}(s) &= \left[1 + \coth\left(\frac{\gamma \ell}{2}\right)\right]^{-1} \tilde{V}_{o.c.}(s) \\ &= \frac{1}{2}\left[1 - e^{-\gamma \ell}\right] \tilde{V}_{o.c.}(s) \\ V(t) &= \frac{1}{2}\left[V_{o.c.}(t) - V_{o.c.}\left(t - \frac{\ell}{c}\right)\right] \end{aligned} \tag{2.7}$$

so that what would otherwise be an infinite series of such time-domain terms truncates at two terms. Rearranging (2.7) as

$$V_{o.c.}(t) = 2 V(t) + V_{o.c.}\left(t - \frac{\ell}{c}\right) \tag{2.8}$$

then consider a signal which starts at t = 0. $V_{o.c.}(t)$ can then be reconstructed from the measured $V(t)$ as

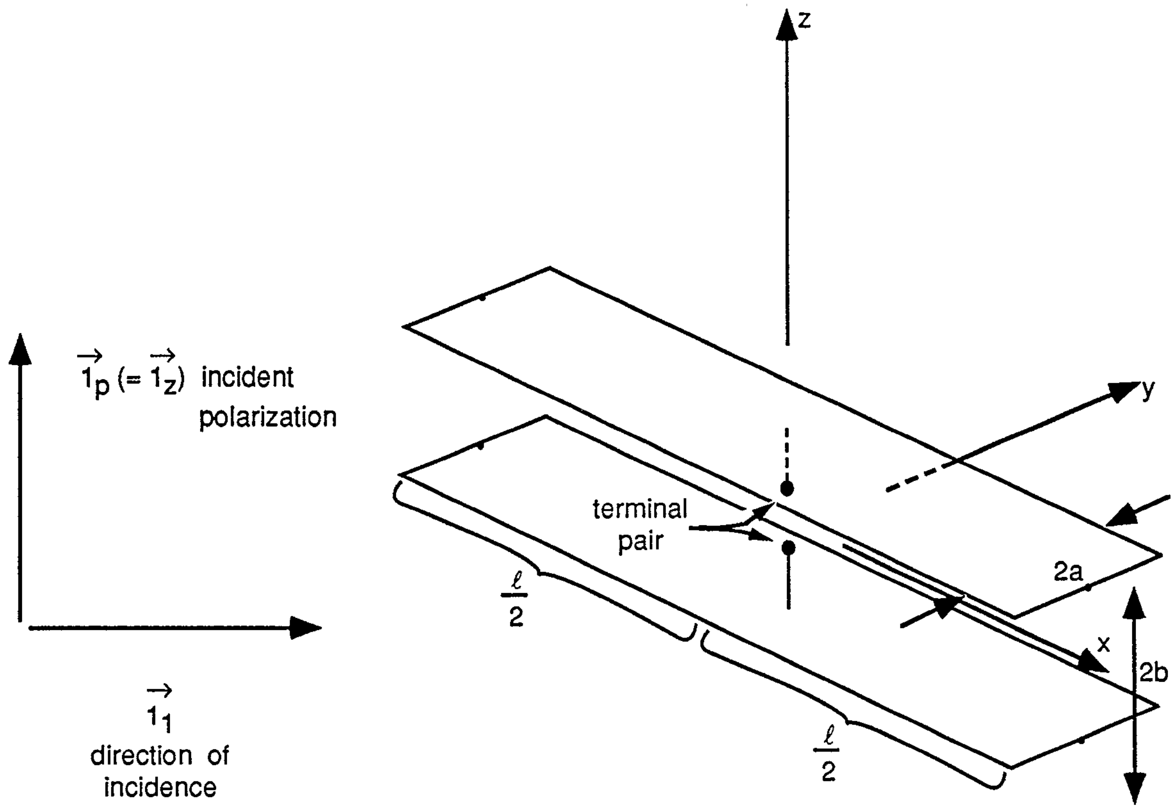

Fig. 2.2 Transmission-Line Sensor

$$V_{o.c.}(t) = \begin{cases} 2\,V(t) \;\; \text{for} \;\; 0 \le t < \dfrac{\ell}{c} \\ 2\,V(t) + V_{o.c.}\left(t - \dfrac{\ell}{c}\right) \;\; \text{for} \;\; n\dfrac{\ell}{c} \le t < (n+1)\dfrac{\ell}{c} \\ \qquad n = 1, 2, 3, \cdots \end{cases} \tag{2.9}$$

After the first time interval of length ℓ/c one goes to successive intervals using the reconstructed $V_{o.c.}$ from the previous interval. One should be cautious, however, in this reconstruction for later time intervals since there is some high-frequency loss for the waves propagating on the not-perfectly-conducting transmission-line structure, and some high-frequency limitation in the reflections at the ends of line ($x = \pm\,\ell/2$).

For comparison consider the case of an approximate integrator, such as given by a simple series RC circuit (Appendix A) as

$$\frac{\tilde{V}_{out}(s)}{\tilde{V}_{in}(s)} = \left[1 + s\,t_i\right]^{-1} \quad , \quad t_i \equiv RC \tag{2.10}$$

where the output voltage V_{out} is measured across the capacitor. Note that the transfer function goes to zero at low frequencies. Rearranging gives

$$\begin{gathered} \frac{1}{s}\,\tilde{V}_{in}(s) = \left[\frac{1}{s} + t_i\right]\tilde{V}_{out}(s) \\ \int_{-\infty}^{t} V_{in}(t')dt' = t_i\,V_{out}(t) + \int_{-\infty}^{t} V_{out}(t')dt' \end{gathered} \tag{2.11}$$

showing how to correct V_{out} to obtain the time integral of V_{in}. The summation from previous time intervals in (2.9) is replaced by an integral over previous times in (2.11). This type of integration correction is sometimes used in conjunction with differentiating field sensors such as discussed in Section I.

Another detail concerns the optimization of the connection to the load at the terminal pair. As illustrated in fig. 2.3 consider the case of a single strip parallel to a (perfectly conducting) ground plane ($z = 0$). Then considering the incident field as propagating along the ground plane one can replace $2b$ by b in (2.3) and regard Z_c as the characteristic impedance of the strip over the ground plane. For a load R_L as, say, a 50 Ω coax, a Z_c of 100 Ω is appropriate giving a ratio of $b/a \simeq 1.235$ where a is the strip half

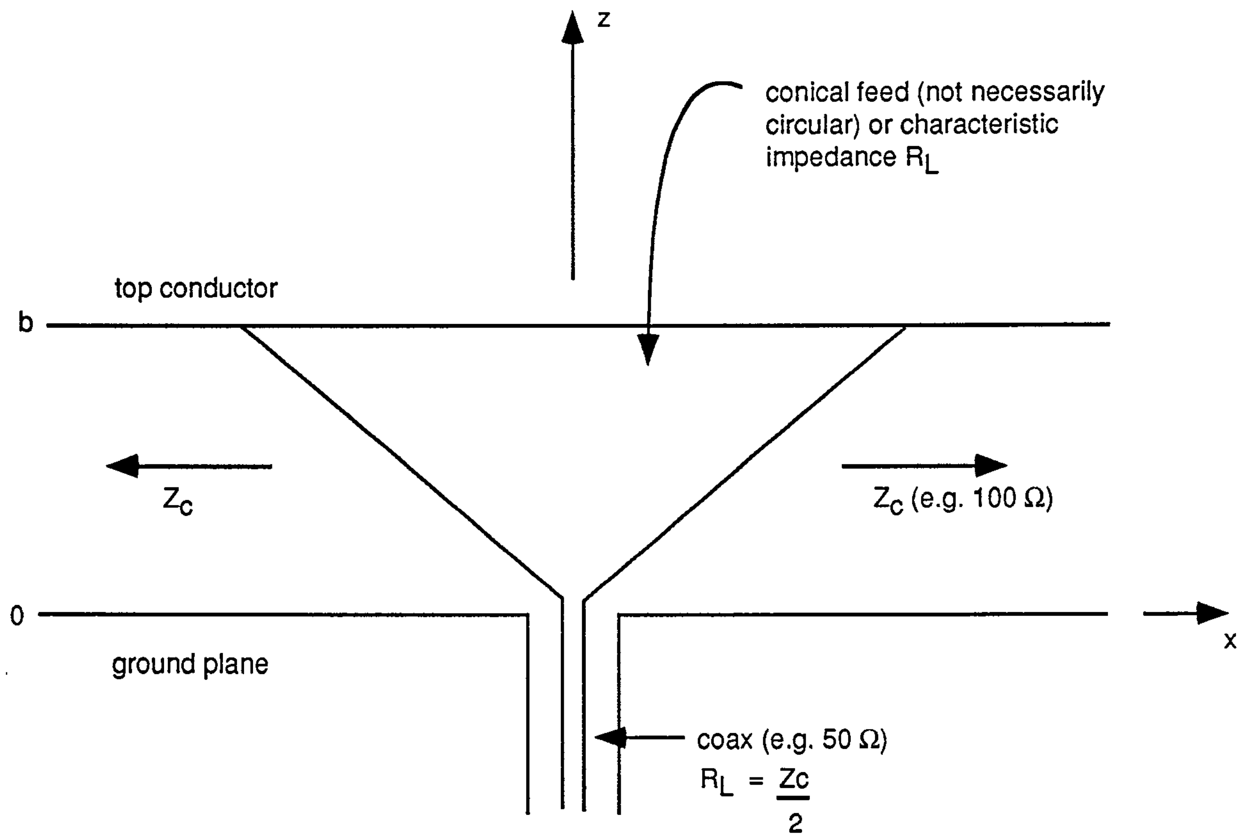

Fig. 2.3 Feed Details for Terminal Pair in Case With Ground Plane

width [5]. With the coax diameter assumed small compared to b one can transition from the strip on $z = b$ to the coax center conductor via a conducting cone of characteristic impedance R_L (with respect to the ground plane). This can be a circular cone with a half-angle of ~47° or a flat-plate cone with a half angle of ≃78° for the case of $R_L = 50\ \Omega$ [1, 2]. Of course we are assuming that $\ell >> b$ so that the local distortion of the field near $\vec{r} = \vec{0}$ negligibly affects the open circuit voltage (for $\lambda >> b$).

Considering the orientation of the sensor with respect to the direction of incidence, some choices of ϕ_1 may be better than others. Suppose that the incident wave comes from some small source a distance d away from the origin. The far field behaves as r'^{-1} for distances r' from the source. Other higher order terms go as r'^{-2} and r'^{-3}. Referring to fig. 2.2, if $\vec{1}_1 = \vec{1}_x$ (i.e., $\phi_1 = 0$) then there is maximum variation of r'^{-1} over the sensor. Another way to look at this is that there is a small component of the electric field in the x direction on $z = b$. If, on the other hand, we select $\vec{1}_1 = \vec{1}_y$ (i.e. $\phi_1 = \pi/2$) this problem is minimized since $a << \ell$ and the variation of r'^{-1} over the 2a strip width is much smaller. An added benefit of choosing this particular direction of incidence is indicated in fig. 2.4 where $\vec{1}_1$ is kept parallel to the $x = 0$ plane, but is allowed to arrive from a source positioned above the $z = 0$ plane. Then there is an image wave from below the ground plane and the open circuit voltage is proportional to $2b\, E_0 \sin(\theta)$ including the ground-plane reflection. The high-frequency limitation is based on the a and b dimensions which are assumed small. For this raised direction of incidence with $\phi_1 = \pi/2$ there is still no component of the incident electric field in the x direction along the major dimension of the sensor conductor on $z = b$.

The sensor conductor on $z = b$ can even be curved in an S-shaped curve or whatever provided the desired transmission-line impedance properties are retained as in (2.5), and the plane-wave properties are retained as in (2.1) and (2.2). However, the more refined considerations involving r'^{-1} variation of the fields, and the possibility of elevated incidence suggests the straight conductor on $z = b$ with $\phi_1 = \pi/2$ as a better choice. Note that while the coaxial output is indicated as connected in the center of the strip, it can also be connected at one end with a change to $Z_c \coth(\gamma\ell)$ in (2.5) and following equations. While the ideal considerations here indicate a thin sheet for the sensor conductor(s) on $z = \pm b$, one can also use wire(s) (circular cylindrical conductors) for this purpose if the equivalent height is substituted for b [18].

While we have been discussing this type of an antenna as a receiving sensor, reciprocity gives its performance as a transmitter [11]. With its open-circuit voltage characterized as a constant times the incident field in (2.3), then driving a current into the antenna terminal pair produces a far field proportional to the time derivative of this current. Of course this result applies to far fields near the $z = 0$ plane as well as to fields on the $x = 0$ plane with a factor of $\sin(\theta_1)$, since these are the conditions under

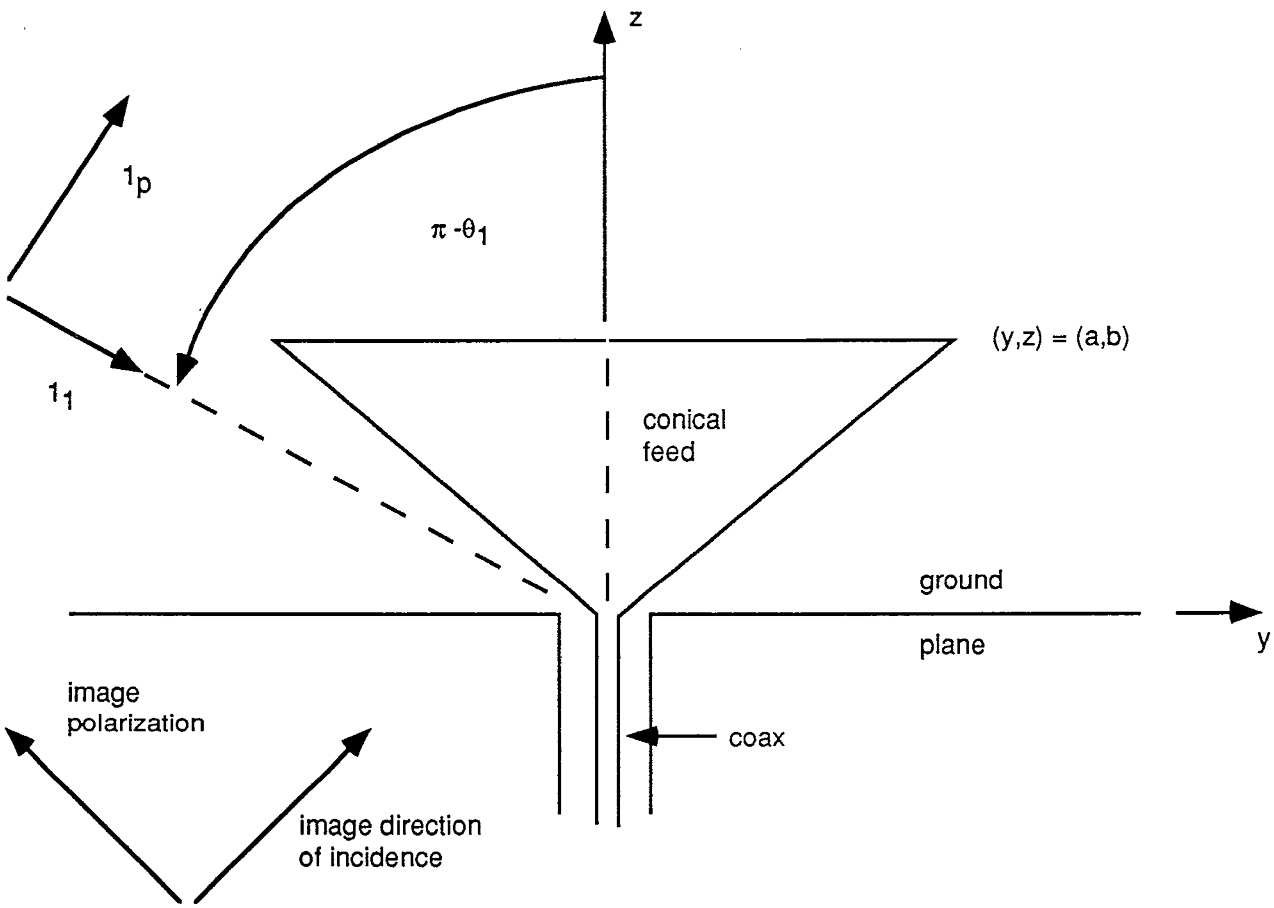

Fig. 2.4 Elevated Source in x = 0 plane Incident on Transmission-Line Sensor

which the open-circuit voltage has been considered. Another way to look at this antenna in transmission is to consider the directions to which the currents on $z = b$ will not radiate (including the image on $z = -b$), leaving only the currents in the z direction in the terminal region to radiate.

III. Voltage Proportional to Time Integral of Incident Electric Field

For a sensor with voltage proportional to the time integral (out to some clear time) of the incident electric field, let us consider the case of conical structures (perfectly conducting) such as illustrated in fig. 3.1. In approaching this problem it is convenient to look at such antennas in transmission and then apply reciprocity to determine their response in reception. Consider two perfectly conducting cones of arbitrary cross section with common apex at $\vec{r} = \vec{0}$ where the terminal pair is also taken. The cones are of finite length, but behave as though infinite until some clear time t_c determined by when the signal from the truncation reaches the origin provided the direction of incidence is from outside the cones.

Considering the cones in transmission, let them be infinitely long and introduce a transient signal $V_s(t)$ at the terminal pair. The outgoing spherical wave is then a TEM wave [6, 7, 12, 13, 16, 22] of the form

$$V(r,\theta,\phi;t) = V_s\left(t-\frac{r}{c}\right)u(\theta,\phi)$$

$$\nabla^2_{\theta,\phi}\, u(\theta,\phi) = \frac{1}{\sin(\theta)}\frac{\partial}{\partial\theta}\left[\sin(\theta)\, u(\theta,\phi)\right]+\frac{1}{\sin^2(\theta)}\frac{\partial^2}{\partial\phi^2}u(\theta,\phi)=0 \tag{3.1}$$

where u can be thought of as part of a complex potential

$$\mathrm{w}(\theta,\phi) = u(\theta,\phi) + j\ \mathrm{v}(\theta,\phi) = \mathrm{w}\left(2e^{j\phi}\tan\left(\frac{\theta}{2}\right)\right)$$

$$\nabla^2_{\theta,\phi}\,\mathrm{w}(\theta,\phi) = 0 \tag{3.2}$$

This leads to the well-known concept of the stereographic transformation, which in an equivalent cylindrical coordinate system (Ψ', ϕ, z) is

$$\Psi' = 2\, z_o \tan\left(\frac{\theta}{2}\right)$$

$$\phi = \phi \tag{3.3}$$

$$z' = r$$

which is solved for various geometries by conformal transformation.

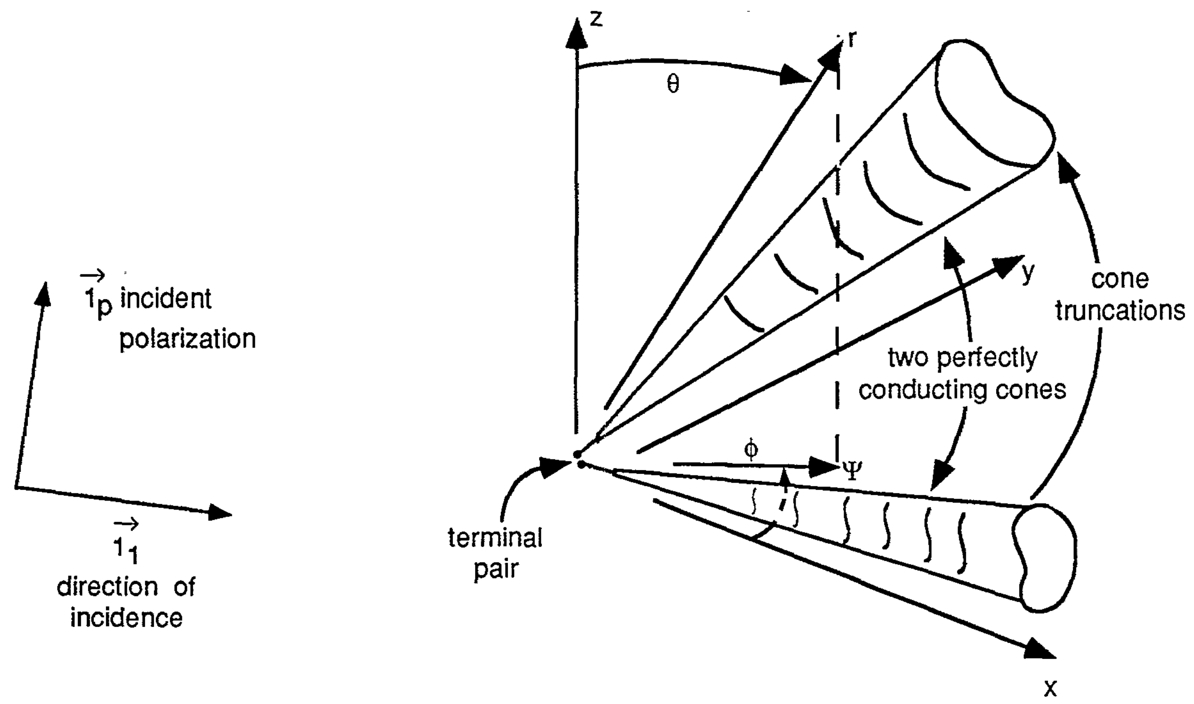

Fig. 3.1 General Conical Sensor

With

$$\Delta u \equiv \text{difference of } u \text{ on the two conical conductors (positive)}$$
$$\Delta v \equiv \text{change of v in going around either of the conical conductors (positive)} \tag{3.4}$$

we have the characteristic impedance for the TEM mode as

$$Z_c = \frac{\Delta u}{\Delta v} Z_o \tag{3.5}$$

The electric field is given by

$$\vec{E}(\vec{r},t) = -\frac{1}{r} V_s\left(t-\frac{r}{c}\right) \frac{1}{\Delta u} \nabla_{\theta,\phi}\, u(\theta,\phi)$$
$$\nabla_{\theta,\phi} = \vec{1}_\theta \frac{\partial}{\partial\theta} + \vec{1}_\phi \frac{1}{\sin(\theta)} \frac{\partial}{\partial\phi} \tag{3.6}$$

In [11] the transmission and receiving characteristics of an antenna are related. In complex-frequency form the transmission is characterized by

$$\tilde{\vec{E}}_f(\vec{r},s) = \frac{e^{-\gamma r}}{r} \tilde{\vec{F}}_V(\vec{1}_r,s)\, \tilde{V}(s) = \frac{e^{-\gamma r}}{r} \tilde{\vec{F}}_I(\vec{1}_r,s)\, \tilde{I}(s) = \frac{e^{-\gamma r}}{r} \tilde{\vec{F}}_w(\vec{1}_1,s)\, \tilde{V}_s(s)$$
$$\vec{1}_r \cdot \tilde{\vec{F}}_V(\vec{1}_r,s) = 0 \quad , \quad \tilde{Z}_{in}(s) = Z_c = \frac{\tilde{V}(s)}{\tilde{I}(s)}$$
$$\tilde{\vec{F}}_1(\vec{1}_r,s) = Z_c\, \tilde{\vec{F}}_V(\vec{1}_r,s) = \left[Z_c + R_L\right] \tilde{\vec{F}}_w(\vec{1}_r,s) \tag{3.7}$$
$$\tilde{V}_s(s) = \left[1+\frac{R_L}{Z_c}\right] \tilde{V}(s) = \text{source voltage with source impedance also } R_L$$

Note that the current convention is here taken into the port (instead of into the load). We can identify the far field in the general case with the exact r^{-1} dependence in (3.6) giving

$$\tilde{\vec{F}}_V(\vec{1}_r,s) = -\frac{1}{\Delta u} \nabla_{\theta,\phi}\, u(\theta,\phi) \equiv \vec{F}_V(\vec{1}_r) \tag{3.8}$$

In reception we have

$$\tilde{V}_{o.c.}(s) \equiv \tilde{\vec{h}}_V(\vec{1}_1,s) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s) \equiv \text{open-circuit voltage}$$

$$\tilde{I}_{s.c.}(s) \equiv \tilde{\vec{h}}_I(\vec{1}_1,s) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s) \equiv \text{short-circuit voltage}$$

$$\tilde{V}_L(s) \equiv \tilde{\vec{h}}_w(\vec{1}_1,s) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s) \equiv \text{voltage into impedance } R_L \text{ loading antenna port} \tag{3.9}$$

$$-Z_c\, \tilde{\vec{h}}_I(\vec{1}_1,s) = \tilde{\vec{h}}_V(\vec{1}_1,s) = \left[1+\frac{Z_c}{R_L}\right] \tilde{\vec{h}}_w(\vec{1}_1,s)$$

where the incident wave is a plane wave as in (1.2). Replacing $\vec{1}_r$ by $-\vec{1}_1$ the receive and transmit parameters are related by

$$\vec{F}_V(-\vec{1}_1) = -\frac{s\mu_o}{4\pi} \overleftrightarrow{1}_1 \cdot \tilde{\vec{h}}_I(\vec{1}_1,s)$$

$$\vec{F}_I(-\vec{1}_1) = \frac{s\mu_o}{4\pi} \overleftrightarrow{1}_1 \cdot \tilde{\vec{h}}_V(\vec{1}_1,s) \tag{3.10}$$

$$\vec{F}_w(-\vec{1}_1) = \frac{s}{R_L}\frac{\mu_o}{4\pi} \overleftrightarrow{1}_1 \cdot \tilde{\vec{h}}_w(\vec{1}_1,s)$$

Thus the reception parameters can be written (noting that the incident field is perpendicular to $\vec{1}_1$) as

$$\tilde{V}_{o.c.}(s) = \frac{4\pi}{\mu_o s} \vec{F}_I(-\vec{1}_1) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s)$$

$$\tilde{I}_{s.c.}(s) = -\frac{4\pi}{\mu_o s} \vec{F}_V(-\vec{1}_1) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s) \tag{3.11}$$

$$\tilde{V}_L(s) = \frac{4\pi R_L}{\mu_o s} \vec{F}_w(-\vec{1}_1) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s)$$

Considering first the open circuit voltage we have

$$\tilde{V}_{o.c.}(s) = \frac{1}{s}\vec{v}_{o.c.}(\vec{1}_1) \cdot \vec{\tilde{E}}^{(inc)}(\vec{0},s) \quad , \quad V_{o.c.}(t) = \vec{v}_{o.c.}(\vec{1}_1) \cdot \int_{-\infty}^{t} \vec{E}^{(inc)}(\vec{0},t')\,dt' \tag{3.12}$$

which in time domain is valid for truncated cones out to some clear time t_c after first arrival of the signal at the cone apex. The sensitivity parameter is

$$\vec{v}_{o.c.}(\vec{1}_1) = \frac{4\pi}{\mu_o}\vec{F}_I(-\vec{1}_1) = 4\pi\frac{Z_c}{\mu_o}\vec{F}_V(-\vec{1}_1) = -4\pi\frac{Z_c}{\mu_o}\frac{1}{\Delta u}\nabla_{\theta,\phi}\, u(\theta,\phi) \tag{3.13}$$

The symbolism is that of a velocity (m/s) which gives the units of this parameter. This can be related to other parameters as

$$V_L(t) = \vec{v}_L(\vec{1}_1) \cdot \int_{-\infty}^{t} \vec{E}^{(inc)}(\vec{0},t')\,dt' \quad , \quad \vec{v}_L(\vec{1}_1) = \left[1 + \frac{Z_c}{R_L}\right]^{-1} \vec{v}_{o.c.}(\vec{1}_1)$$

$$I_{s.c.}(t) = \vec{v}_{s.c.}(\vec{1}_1) \cdot \int_{-\infty}^{t} \vec{E}^{(inc)}(\vec{0},t')\,dt' \quad , \quad \vec{v}_{s.c.}(\vec{1}_1) = -Z_c^{-1}\vec{v}_{o.c.}(\vec{1}_1) \tag{3.14}$$

In short circuit the parameter has an extra factor of admittance in the units.

So now consider the angular dependence of $\vec{v}_{o.c.}(\vec{1}_1)$. This if found from the TEM modal distribution in (3.13) where it is in terms of θ, ϕ which is related to the transmission direction $\vec{1}_r (= -\vec{1}_1)$. Rewriting

$$Z_c = f_g\, Z_o = f_g\sqrt{\frac{\mu_o}{\epsilon_o}}$$

$$f_g = \frac{\Delta u}{\Delta v} = \text{geometric impedance factor} \tag{3.15}$$

$$\vec{v}_{o.c.}(\vec{1}_1) = -c\left[\frac{4\pi f_g}{\Delta u}\nabla_{\theta,\phi}\, u(\theta,\phi)\right] = -c\left[\frac{4\pi}{\Delta v}\nabla_{\theta,\phi}\, u(\theta,\phi)\right]$$

To what extent can we make $\vec{v}_{o.c.}$ independent of angle, giving a response similar to (1.1)?

Appealing to symmetry let us consider a body of revolution, i.e. let the perfectly conducting cones be circular and coaxial with respect to the z axis. This removes the ϕ dependence. A special case of interest is a monocone with a ground plane (the z = 0 plane) as illustrated in fig. 3.2. This case has been considered in transmission in [1]. With the cone truncated at slant length ℓ the clear time is

$$t_c = \frac{\ell}{c}\left[1-\cos(\theta-\theta_o)\right]$$

$$\theta_o \equiv \text{half angle of circular cone} \tag{3.16}$$

If one wishes one can remove some of the cone structure on the side away from the incident wave ("lee" side) without reducing the clear time.

With the ϕ dependence removed we have

$$f_g = \frac{1}{2\pi}\,\ell n\left[\cot\left(\frac{\theta_o}{2}\right)\right] \tag{3.17}$$

which for 50 Ω gives $\theta_o \simeq 47°$. The normalized potential distribution is

$$\frac{u(\theta)}{\Delta u} = \frac{\ell n\left[\cot\left(\frac{\theta}{2}\right)\right]}{\ell n\left[\cot\left(\frac{\theta_o}{2}\right)\right]} = \begin{cases} 1 \text{ for } \theta = \theta_o \\ 0 \text{ for } \theta = \frac{\pi}{2} \end{cases} \tag{3.18}$$

From this we find

$$-\frac{1}{\Delta u}\nabla_{\theta,\phi}\,u(\theta) = \left[\sin(\theta)\,\ell n\left[\cot\left(\frac{\theta_o}{2}\right)\right]\right]^{-1}\vec{1}_\theta$$

$$= \left[2\pi\, f_g\ \sin(\theta)\right]^{-1}\vec{1}_\theta \tag{3.19}$$

$$\vec{v}_{o.c.}(\vec{1}_1) = \frac{2\,c}{\sin(\theta)}\ \vec{1}_\theta$$

which is conveniently independent of θ_o.

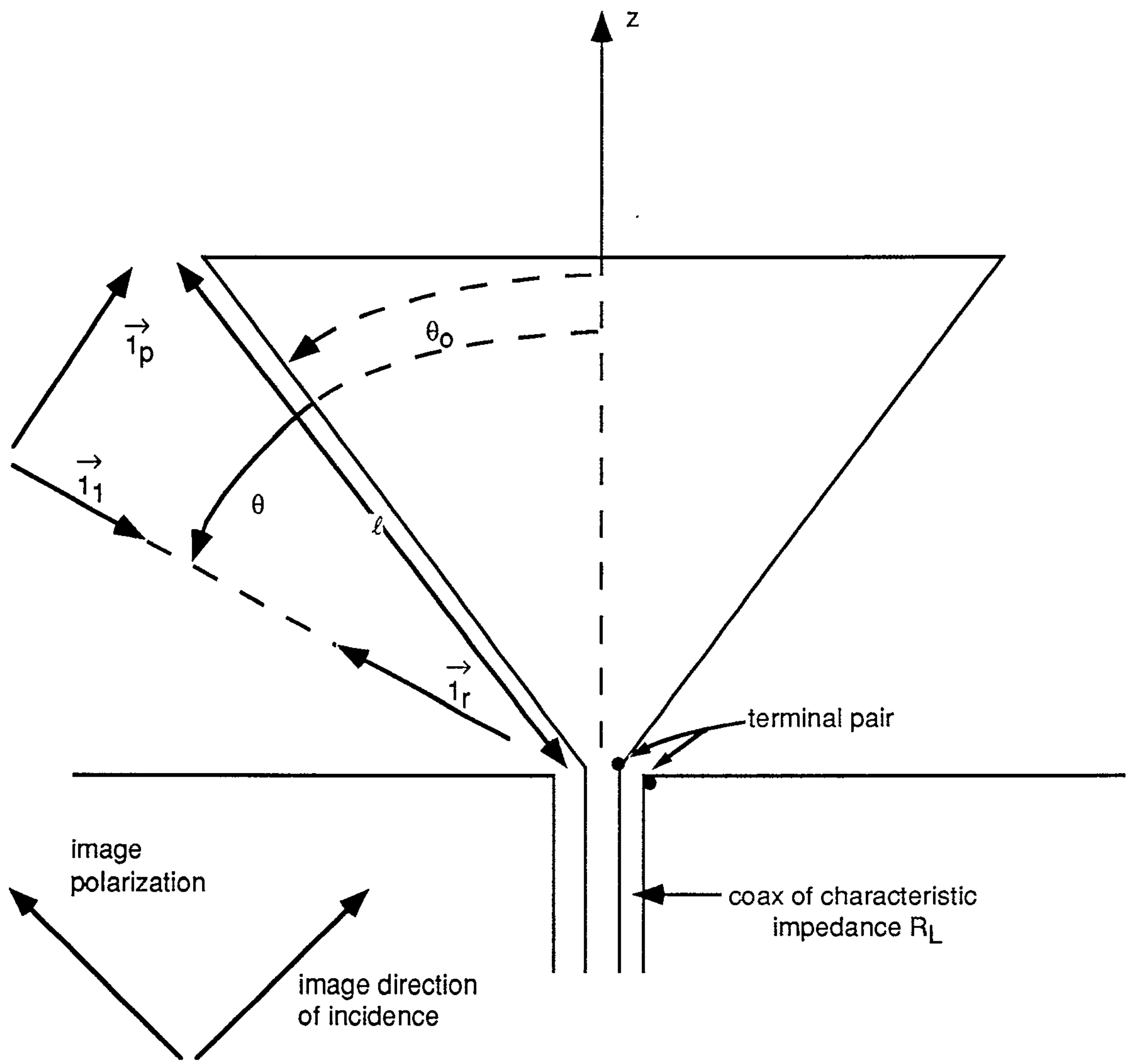

Fig. 3.2 Circular Conical Sensor with Ground Plane

Suppose now that we let the incident wave be characterized by

$$\vec{1}_p = -\vec{1}_\theta \tag{3.20}$$

i.e. by "vertical" polarization. Then we have

$$V_{o.c}(t) = -\frac{2c}{\sin(\theta)} E_o \int_{-\infty}^{t} f(t')\, dt' \tag{3.21}$$

Note the dependence as $1/\sin(\theta)$ which is opposite to that of a dipole in (1.1) (i.e. proportional to $\sin(\theta)$ from a dot product). So one needs to know θ accurately to use this as the sensor sensitivity. Another way to view this case is to consider the resultant field on the ground plane in the absence of the cone. At $(x, y, z) = (0, 0, 0+)$ the electric field is just

$$\vec{E}(t) = 2(\vec{1}_p \cdot \vec{1}_z)\, E_o\, \vec{1}_z\, f(t) \tag{3.22}$$

In terms of this field with the polarization as in (3.20) we have

$$\begin{aligned} \vec{E}(t) &= 2\sin(\theta)\, E_o\, \vec{1}_z\, f(t) \\ V_{o.c.}(t) &= -\frac{c}{\sin^2(\theta)}\, \vec{1}_z \cdot \int_{-\infty}^{t} \vec{E}(t')\, dt' \end{aligned} \tag{3.23}$$

In this latter form we can let $\theta \to \pi/2$ for incidence along the ground plane with the image field combined with the incident field as the usual resultant field. This gives

$$V_{o.c.}(t) = -c\, \vec{1}_z \cdot \int_{-\infty}^{t} \vec{E}(t')\, dt' \quad \text{for} \quad \theta = \frac{\pi}{2} \tag{3.24}$$

which is quite simple in form. These results are readily adapted to various loads R_L via (3.14). For the case of $R_L = Z_c$ the voltage is simply reduced by half.

These results are also directly applicable to coaxial biconical antennas, even if the two angles defining the cones, say θ_o and θ_o', are not related by $\theta_o + \theta_o' = \pi$. A way to interpret these simple results is to note that in transmission, for coaxial circular cones, the field is proportional to the driving current, independent of θ_0 and θ_0'. The reciprocity theorem relates open-circuit voltage to driving current. For all these cases (3.21) applies.

Besides the clear-time limitation (3.16), there are other accuracy issues. As discussed in Section II, the incident field is usually a spherical wave instead of a plane wave. If the small source of the spherical wave is at a distance d away, and r' is the spherical radius from the source, then the variation of r' over the antenna structure (out to distances that at a particular time $< t_C$ can influence the signal at the terminal pair) can introduce some error. So d should be much larger than ct_C for an accurate result out to $t = t_C$. The dimensions of the terminal-pair region (e.g. coax radius, etc.) also need to be small compared to $\bar{\lambda}$ for high frequencies of interest. Furthermore, the direction of incidence (or equivalently θ) must be accurately known.

IV. Concluding Remarks

So as discussed in this paper there are these two classes of interest electrically large antennas. One gives an output proportional to the incident field in time domain. The second gives an output proportional to the time integral of the incident field. Due to the finite size of such antennas this response is generally limited to some maximum time. There are also restrictions on the angle of incidence for this behavior to apply. Provided these limitations and accuracy considerations are observed then these antennas can be regarded as electromagnetic sensors and used as primary standards, being "calibratable by a ruler" [20, 21].

Appendix A: Approximate Integrator

An oft used technique is the approximate integration of the time-derivative waveform via an RC integrator, i.e. one characterized by a transfer function

$$\tilde{T}_{RC}(s) = [1 + st_i]^{-1}$$

$$t_i \equiv \text{integration time constant} \tag{A.1}$$

$$= RC$$

which goes to one at low frequencies, but to $1/(st_i)$ at high frequencies (like an ideal integrator which behaves as constant/s). Assuming that the sensor from (1.1) behaves as

$$\tilde{V}(s) = s\,A\,\overrightarrow{1}_m \cdot \begin{cases} \tilde{\overrightarrow{D}}^{(inc)}(\overrightarrow{r}_o, s) = \frac{1}{\epsilon}\tilde{\overrightarrow{E}}^{(inc)}(\overrightarrow{r}_o, s) \\ \text{or} \\ \tilde{\overrightarrow{B}}^{(inc)}(\overrightarrow{r}_o, s) = \frac{1}{\mu}\tilde{\overrightarrow{H}}^{(inc)}(\overrightarrow{r}_o, s) \end{cases}$$

$$\tilde{T}(s) = s\,A \begin{cases} \epsilon^{-1} \\ \text{or} \\ \mu^{-1} \end{cases} \quad \text{(ideal differentiator)} \tag{A.2}$$

$$A \equiv \text{equivalent area of sensor}$$

then the voltage out of approximate integrator is

$$\tilde{V}_{RC}(s) = \tilde{T}_{RC}(s)\,\tilde{V}(s) = s\,A\,\tilde{T}_{RC}(s)\,\overrightarrow{1}_m \cdot \begin{cases} \tilde{\overrightarrow{E}}^{(inc)}(\overrightarrow{r}_o, s) \\ \\ \tilde{\overrightarrow{H}}^{(inc)}(\overrightarrow{r}_o, s) \end{cases} \tag{A.3}$$

Define the corrected voltage as (with zero initial conditions)

$$\tilde{V}_{cor}(s) \equiv \frac{1}{st_i}\tilde{V}(s) \quad , \quad V_{cor}(t) = \frac{1}{t_i}\int_{-\infty}^{t} V(t')\,dt' \tag{A.4}$$

i.e. as the true time integral. Solving (A.3) and (A.4) for $\tilde{V}(s)$ gives

$$\tilde{V}(s) = [1+st_i]\ \tilde{V}_{RC}(s) = st_i\ \tilde{V}_{cor}(s) \tag{A.5}$$

from which we find (with zero initial conditions)

$$\tilde{V}_{cor}(s) = \left[1+\frac{1}{st_i}\right] \tilde{V}_{RC}(s)$$

$$V_{cor}(t) = V_{RC}(t) + \frac{1}{t_i} \int_{-\infty}^{t} V_{RC}(t')\, dt' \tag{A.6}$$

Thus the approximate integrator has a simple correction procedure which can be numerically integrated and/or used to estimate times for which the integrator output has a specified accuracy. Of course, one need not limit oneself to integrators, but similar considerations can be applied to differentiators (for say first or second time derivatives of the field-component waveform).

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