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Time-Domain Radiated Fields of a Resistively Loaded Biconical Antenna Based on a Transmission-Line Model

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Abstract

Pulse-radiating biconical antennas have been widely used for nuclear electromagnetic pulse (NEMP) simulation. An early analytical model considered a resistively loaded biconical antenna. Based on a transmission-like model of such an antenna, far fields in the working volume are available in closed form for step function voltage sources. In this note, we extend this analysis to obtain radiated fields in time-domain for practical capacitive voltage sources whose outputs are nearly double-exponential. The expressions developed here are useful in quickly estimating radiated characteristics such as peak field, zero crossing, etc., for a given Marx type of source.

I. Introduction

Resistively loaded dipoles for NEMP simulation purposes was first considered by Baum [1]. The analysis presented in [1] was based on a transmission-line model of the antenna. The resistive loading of the antenna is similar to what was considered earlier in [2 and 3]. Once, the transmission-line like currents on the antenna are known, the radiated fields in the working volumes of such simulators are evaluated by appropriate integrals of the currents. Baum [1] also obtained closed-form expressions, both in frequency and time domains, for the radiated fields. This analysis has led to the design [4], evaluation [5 and 6] and verification [7] of this class of NEMP simulator. Examples of such simulators are ATHAMAS II [8], EMPRESS II [9], NAVES II [10], and EMISS III [11].

Related studies employing integral equation formulation of the antenna currents over a finite number of wires were described by Wilton [12 and 13]. In addition, some external environments outside and away from the working volumes have been reported by Casey [14 and 15]. A technique for obtaining the near fields from known far fields was described by Singaraju and Baum [16]. Kohlberg [17] presented additional theoretical techniques and computational considerations for such antennas.

Our present interest is still in the working volume environment of such simulators and we seek to present closed-form fields, when the antenna is driven by a practical generator e.g., Marx pulser. Such generators are widely employed in EMP simulation and can be characterized by double exponential output waveform. Consequently, we take the step response expressions from [1] and analytically perform the convolution integral and present closed-form radiated field expressions. Using these expressions, one can rapidly estimate the radiated characteristics of such antenna without resorting to excessive numerical computations.

In Section II, we briefly review the step-excited transmission-line model of the antenna and extend in Section III for practical sources. The note is concluded with a list of references.

II. Review of the Step Response

The biconical antenna under consideration is shown in Figure 1, along with rectangular (x, y, z) and spherical (r, θ, ϕ) coordinate systems with their origins at the apex of the bicone. Ψ is the cylindrical distance from the biconical axis. In the context of EMP simulation by a pulse-radiating dipole, one synthesizes the problem by seeking an optimal impedance loading that produces a desired radiated waveform. The case of uniform loading (i.e., constant resistance per unit length) does not optimally shape the radiated waveform after the initial peak. A special form of non-uniform resistance distribution [1, 2 and 3] leads to a set of optimal characteristics in the radiated waveform.

With reference to Figure 1, we note that

$$
\begin{aligned}
V(t) &\equiv \text{voltage source at the apex} = V_0 u(t) \\
u(t) &\equiv \text{unit step function} \\
2h &\equiv \text{height of the bicone} \\
\theta_1 &\equiv \text{semi-angle of the bicone} \\
Z_\infty &\equiv \text{characteristic impedance of the bicone} \\
&= \frac{Z_0}{\pi} \ln\left[\cot(\theta_1/2)\right] \\
Z_0 &\equiv \text{characteristic impedance of free space} \simeq 377\Omega \\
f_g &\equiv \text{geometric factor} = Z_\infty/Z_0 \qquad (1) \\
2a &\equiv \text{diameter of the bicone at its ends} \\
Z'(z) &\equiv \text{loading impedance per unit length} = R'(z) \\
&= \left[\frac{Z_\infty}{h}\frac{1}{\left\{1-\frac{|z|}{h}\right\}}\right] \Omega/m \quad \text{for} \quad (-h \le z \le h)
\end{aligned}
$$

We now summarize the results obtained in [1] for the biconical antenna outlined above. To begin with one can define a retarded time τ_h for convenience,

$$
\tau_h = \frac{ct - r}{h} \qquad (2)
$$

where c is the speed of light in air. The corresponding normalized Laplace transform

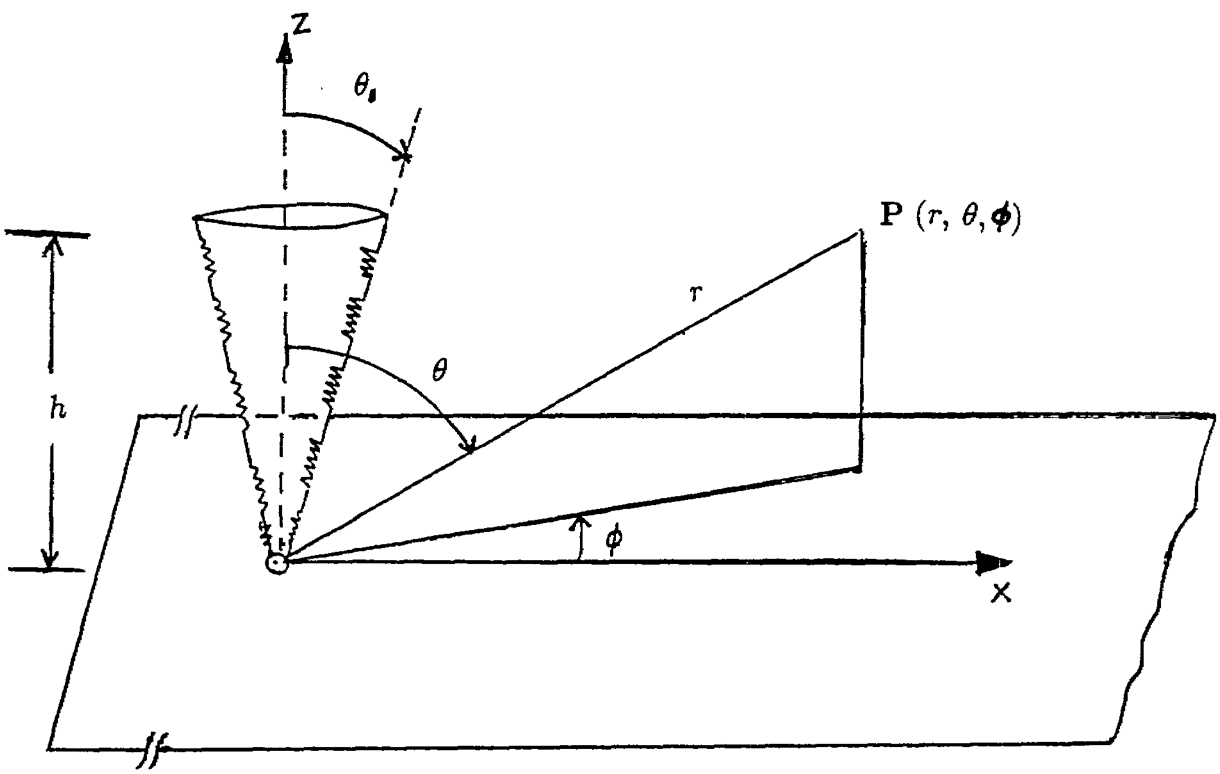

Figure 1. Geometry of a pulse radiating, resistively loaded bi-conical antenna.

variable is

$$\left.\begin{array}{l} s_h = st_h \qquad \text{with} \qquad t_n = h/C \\ s \equiv \text{Laplace transform variable} = \Omega + j\omega \end{array}\right\} \tag{3}$$

ζ is used as the axial coordinate for the transmission-line model of the antenna, and the current distribution for the above case of non-uniform loading is given by [1],

$$\tilde{I}(z) = \frac{V_0}{Z_\infty}\,\frac{t_h}{s_h+\alpha}\left[1-\frac{|z|}{h}\right]e^{-s_h|z|/h} \tag{4}$$

or

$$\tilde{I}(z) = \frac{V_0}{Z_\infty}\,\frac{1}{s+(\alpha c/h)}\left[1-\frac{|z|}{h}\right]e^{-\gamma|z|} \tag{5}$$

where tilde denotes Laplace transformed quantities and

$$\begin{aligned} \alpha &\equiv \text{capacitance parameter} = 1+\frac{C_a}{C_g} \\ C_a &\equiv \text{antenna capacitance} = \mathcal{E}_0 h/f_g \\ \mathcal{E}_0 &\equiv \text{permitivity of free space} = \frac{1}{36\pi\times 10^9} \quad F/m \\ \gamma &= (s/c) \end{aligned} \tag{6}$$

Using the above current distribution, the radiated electric field in frequency and time domains were found to be [1],

$$\tilde{E}_{f_\theta}(r,\theta,s) = \left[\frac{V_0}{r}\,\frac{h}{2\pi c f_g}\,e^{-\gamma_0 r}\,\tilde{\xi}'(\theta)\right]V/(m-Hz) \tag{7a}$$

$$\tilde{\xi}'(\theta) = \frac{\sin(\theta)}{2(s_h+\alpha)}\left\{\begin{array}{l} \dfrac{1}{1-\cos(\theta)}\left[\dfrac{e^{-s_h(1-\cos(\theta))}-1}{s_h(1-\cos(\theta))}+1\right] \\ +\ \dfrac{1}{1+(\cos(\theta))}\left[\dfrac{e^{-s_h(1+\cos(\theta))}}{s_h(1+\cos(\theta))}+1\right] \end{array}\right\} \tag{7b}$$

$$E_{f_\theta}(r,\theta,t) = \frac{V_0}{r}\,\frac{1}{2\pi f_g}\quad \xi'(\theta) \tag{8a}$$

$$\tilde{\xi}'(\theta) = \frac{\sin(\theta)}{2} \left\{ \begin{aligned} & \left[\frac{e^{-\alpha\tau_h}}{1-\cos(\theta)} - \frac{1-e^{-\alpha\tau_h}}{\alpha(1-\cos(\theta))^2} \right] u(\tau_h) \\ & + \frac{1}{\alpha} \frac{1-e^{-\alpha[\tau_h-(1-\cos(\theta))]}}{(1-\cos(\theta))^2} u(\tau_h - [1-\cos(\theta)]) \\ & + \left[\frac{e^{-\alpha\tau_h}}{1+\cos(\theta)} - \frac{1-e^{-\alpha\tau_h}}{\alpha(1+\cos(\theta))^2} \right] u(\tau_h) \\ & + \frac{1}{\alpha} \frac{1-e^{-\alpha[\tau_h-(1+\cos(\theta))]}}{(1+\cos(\theta))^2} u(\tau_h - [1+\cos(\theta)]) \end{aligned} \right\} \tag{8b}$$

We thus conclude summarizing the results in terms of the radiated electric field (frequency and time domains) for the case of step-function excited biconical antenna that is resistively loaded in a special non-uniform way.

III. Closed-Form Radiated Fields for Double-Exponential Pulser Excitation

In the past, researchers have used the results of Section II for the case of a practical pulser excitation by employing numerical convolution procedures. The convolution procedure may be illustrated using the following notations in terms of linear system theory.

The "input" is considered to be the pulser voltage $V(t)$ and the "output" is taken to be the radiated electric field $E_{f_\theta}(r,\, \theta,\, t)$. We already know that for $V(t) = V_0\, u(t)$, the output $E_{f_\theta}(r,\, \theta,\, t)$ is given by (8). This implies for a double-exponential pulse excitation of the form,

$$V_p(t) = V_0(-e^{-at} + e^{-bt})u(t) \tag{9}$$

the "output" or the radiated θ-component of the electric field is given by a convolution integral

$$\begin{aligned} \mathcal{E}_{f_\theta}(r,\, \theta,\, t) &\equiv \text{far field with a double exponential excitation} \\ &= \frac{1}{V_0}\int_0^t \left[\frac{d}{dt}\, V_p(t-t')\right] E_{f_\theta}(r,\, \theta,\, t')dt' \end{aligned} \tag{10}$$

where $E_{f_\theta}(r,\, \theta,\, t)$ is the step response given by (8). We also observe that, for a fast-rising and slow-decaying double exponential $a \gg b > 0$, and the various risetimes of the pulser are given by [18]

$$\begin{aligned} t_r(\text{e-fold rise}) &\simeq (1/a) \\ t_{10-90} &\simeq \ln(9)/a \simeq 2.2/a \\ t_{mr} &\equiv \text{maximum rate of rise} = \frac{V_{\text{max}}}{\left.\frac{\partial V}{\partial t}\right|_{\text{max}}} \\ &= \frac{1}{a-b} \simeq \frac{1}{a} \end{aligned} \tag{11}$$

Substituting (8) and (9) in (10), the integral in (10) can be analytically performed resulting in the following closed form expression

$$\mathcal{E}_{f_\theta}(r,\theta,t) = \frac{V_0}{r}\,\frac{1}{2\pi f_g}\,\sin(\theta)$$
$$\left[\begin{array}{l} \left\{A_1 e^{-\alpha(t-t_r)/\tau} - A_2 e^{-b(t-t_r)} + A_3 e^{-a(t-t_r)}\right\} u(t-t_r) \\ -\left\{B_1 e^{-\alpha(t-t_+)/\tau} - B_2 e^{-b(t-t_+)} + B_3 e^{-a(t-t_+)}\right\} u(t-t_+) \\ -\left\{C_1 e^{-\alpha(t-t_-)/\tau} - C_2 e^{-b(t-t_-)} + C_3 e^{-a(t-t_-)}\right\} u(t-t_-) \end{array}\right] \quad (12)$$

where

$$\begin{aligned} (r,\theta) &\equiv \text{observer location in the far zone} \\ V_p(t) &= V_0(-e^{-at} + e^{-bt})u(t) \equiv \text{excitation function} \\ \alpha &= 1 + (C_a/C_g) \equiv \text{capacitance factor} \\ C_a &\equiv \text{antenna capacitance} \\ C_g &\equiv \text{generator capacitance} \\ \tau &= h/c \\ h &\equiv \text{height of the antenna above the ground plane} \\ c &\equiv \text{speed of light in air} \simeq 3\times 10^8\ m/s \\ t_r &= r/c \\ t_+ &= t_r + [1+\cos(\theta)]\tau \\ t_- &= t_r + [1-\cos(\theta)]\tau \end{aligned} \quad (13)$$

$$\left.\begin{aligned} A_1 &= \frac{\tau(b-a)[1+\cos^2(\theta)+\alpha\ \sin^2(\theta)]}{\sin^4(\theta)(\alpha-a\tau)(\alpha-b\tau)} \\ A_2 &= \frac{1+\cos^2(\theta)+b\tau\ \sin^2(\theta)}{\sin^4(\theta)(\alpha-b\tau)} \\ A_3 &= \frac{1+\cos^2(\theta)+a\tau\ \sin^2(\theta)}{\sin^4(\theta)(\alpha-a\tau)} \end{aligned}\right\} \quad (14)$$

$$\left.\begin{aligned} B_1 &= \frac{\tau(b-a)}{2[1+\cos(\theta)]^2(\alpha-a\tau)(\alpha-b\tau)} \\ B_2 &= \frac{1}{2[1+\cos(\theta)]^2(\alpha-b\tau)} \\ B_3 &= \frac{1}{2[1+\cos(\theta)]^2(\alpha-a\tau)} \end{aligned}\right\} \quad (15)$$

$$\left.\begin{aligned} C_1 &= \frac{\tau(b-a)}{2[1-\cos(\theta)]^2[\alpha - b\tau][\alpha - a\tau]} \\ C_2 &= \frac{1}{2[1-\cos(\theta)]^2(\alpha - b\tau)} \\ C_3 &= \frac{1}{2[1-\cos(\theta)]^2(\alpha - a\tau)} \end{aligned}\right\} \tag{16}$$

The general shape of the far field $\mathcal{E}_{f_\theta}(r, \theta, T)$ may be depicted as in Figure 2.

In figure 2, we observe that

a) the area under the curve must vanish

b) the low-frequency radiation is proportional to $\vec{p}(\infty)$ and $\vec{m}(\infty)$ which are the late-time electric and magnetic dipole moments respectively; note also that this particular antenna has no magnetic dipole moment.

The constraints on the radiated field $\vec{E}_f$ may be summarized as follows [19 and 20].

$$\int_{-\infty}^{\infty} \vec{E}_f(r, t)dt = 0 \tag{17a}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{t} \vec{E}_f(r, t')dt'\,dt = -\frac{\mu_0}{4\pi r}\left[\vec{p}(\infty) - \frac{\vec{1}_z}{c} \times \vec{m}(\infty)\right] \tag{17b}$$

The second integral (17b) also vanishes in the present case because the late-time dipole moments go to zero. This also implies that the far field has at least two zero crossings in the time domain.

c) there are two times where one has slope discontinuities t_+ and t_- given by

$$T_+ = (t_+ - t_r) = [1 + \cos(\theta)](h/c)$$

$$T_- = (t_- - t_r) = [1 - \cos(\theta)](h/c)$$

d) t_+ and t_- become equal to $t_r + (h/c)$ along the ground plane where $\theta = (\pi/2)$

e) all of the early-time characteristics including the first zero crossing may be derived by considering the first term in (12).

Item (e) above leads to

$$\mathcal{E}_{f_\theta}(r, \theta, t) = \frac{V_0}{r}\frac{1}{2\pi f_g}\sin(\theta)\left[A_1 e^{-\alpha c t/h} - A_2 e^{-bt} + A_3 e^{-at}\right]u(t)$$

$$\text{for} \quad t < t < t_- \tag{18}$$

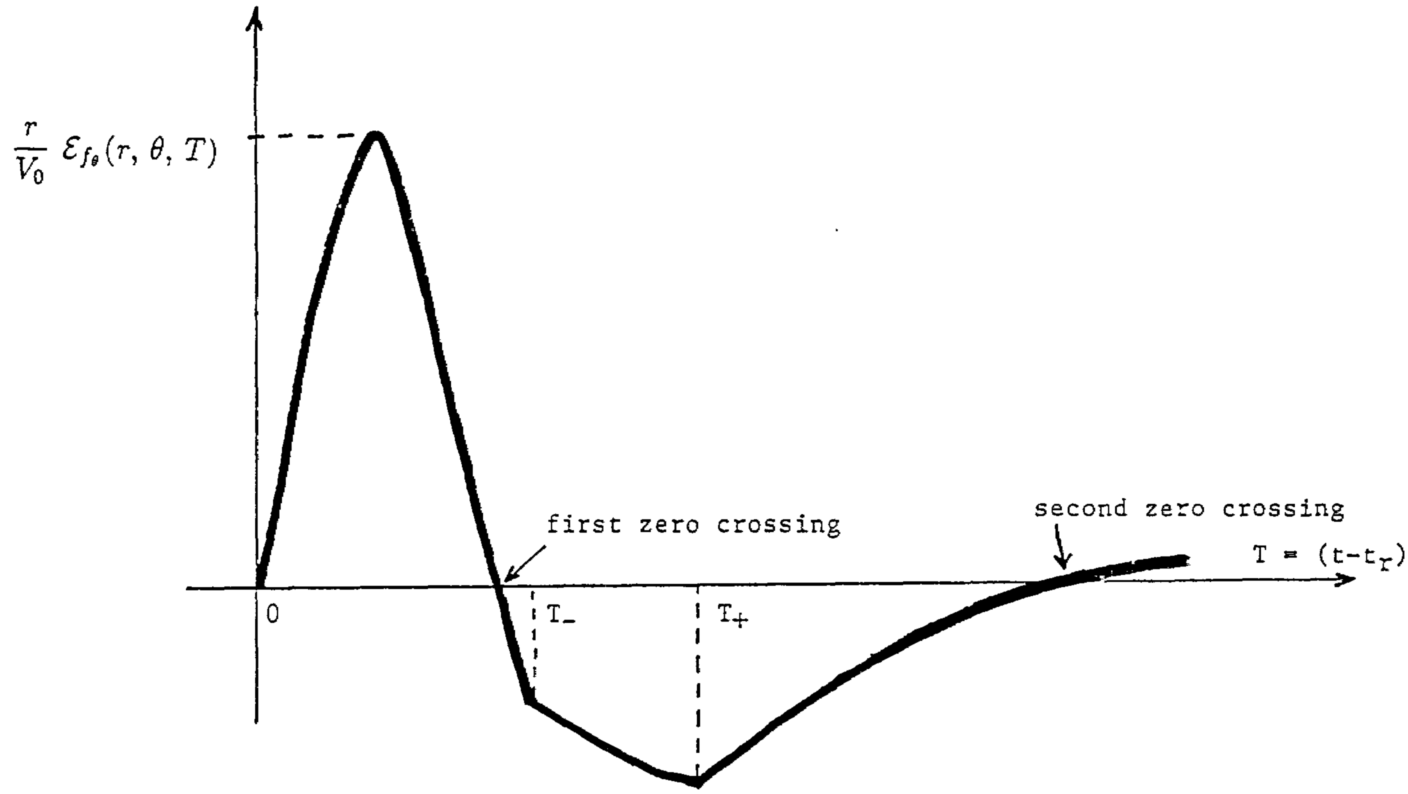

Figure 2. General shape of the radiated waveform

where we have set $t_r = 0$ without any loss of generality. If the risetime, peak amplitude, time at which peak occurs, zero crossing time are of interest, they can be derived from a consideration of (18) only. We establish the accuracy of (12) by evaluating the far field $\mathcal{E}_{f_\theta}$ from numerically evaluating the convolution integral of (10) and comparing it with an evaluation of the analytical closed form expression of (12) for the following set of assumed parameters.

Input parameters

$$h = 28 \text{ m}$$

$$\theta_1 = 40.4^\circ \qquad \Rightarrow \qquad f_g = \frac{1}{\pi} \simeq 0.318$$

$$Z_\infty \simeq 120\,\Omega \quad \text{(full bicone)}$$

$$Z_\infty \text{ (half)} \simeq 60\,\Omega \quad \text{(monocone)}$$

$$\alpha = 1 + \frac{C_a}{C_g} \simeq 1 \qquad (C_a \ll C_g)$$

$$V_p(t) = V_0 \ (-e^{-at} + e^{-bt}) u(t)$$

$$V_0 = 1 \text{ Volt}$$

$$a = 5 \times 10^8/\text{sec} \qquad\qquad (a \gg b)$$

$$b = 4 \times 10^6/\text{sec}$$

$$t_{10-90} \simeq \ln(9)/a = 4.4 \text{ ns}$$

$$(r,\, \theta) = (300 \text{ m},\, 90^\circ),\ (301.5 \text{ m},\, 84.29^\circ)$$

$$\text{and } (305.94 \text{ m},\, 78.69^\circ)$$

The above input parameters and the observer locations are also illustrated in Figure 3. The electric fields $\mathcal{E}_{f_\theta}(r,\, \theta,\, t)$ at the three observer locations computed from a numerical convolution process in (10) are shown in Figure 4. The electric fields at the same locations computed from the closed form expression of (12) are shown in Figure 5. Several observations are in order

a) The results from (10) and (12) are almost indistinguishable establishing the accuracy of the closed form result of (12)

b) The risetimes can be seen in the curves on left

c) The quantities plotted are $\mathcal{E}_{f_\theta}(r,\, \theta,\, t)$ in V/m for 1 volt double exponential excitation of the antenna. For a MV pulser, multiply the result by a factor

Antenna	*Pulser*	*Observers*
$h = 28$ m	$V_p(t) = V_0(-e^{-at} - e^{-bt})u(t)$	① $(r_2, \theta_2) = (300\text{m}, 90°)$
$\theta_1 = 40.4°$	$V_0 = 1$ Volt	② $(r_3, \theta_3) = (301.5\text{m}, 84.29°)$
$f_g = (1/\pi) \simeq 0.318$	$a = 5 \times 10^8/s$	③ $(r_4, \theta_4) = (305.94\text{m}, 78.69°)$
$Z_\infty = 120\,\Omega$ full	$b = 4 \times 10^6/s$	
$C_a \ll C_g$	$a \gg b$	
$\alpha = 1 + (C_a/C_g) \simeq 1$	$t_{10-90} \simeq 4.4$ ns	

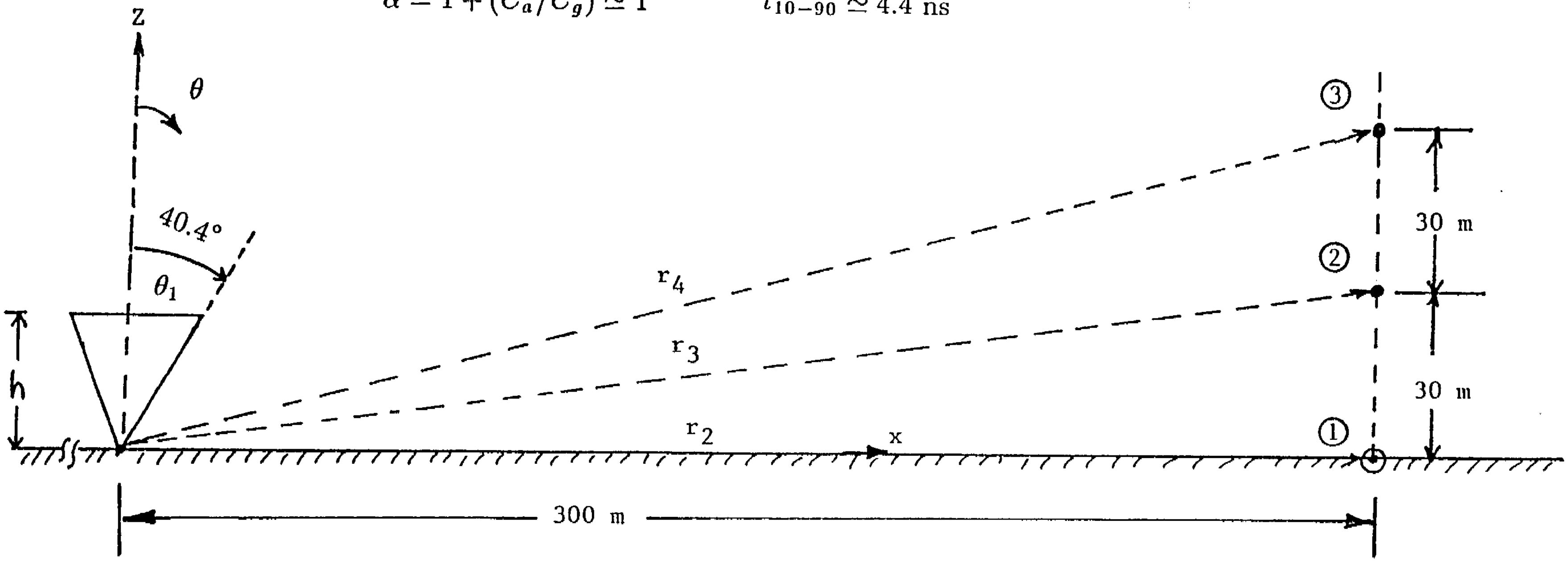

Figure 3. Geometry of an illustrative example problem

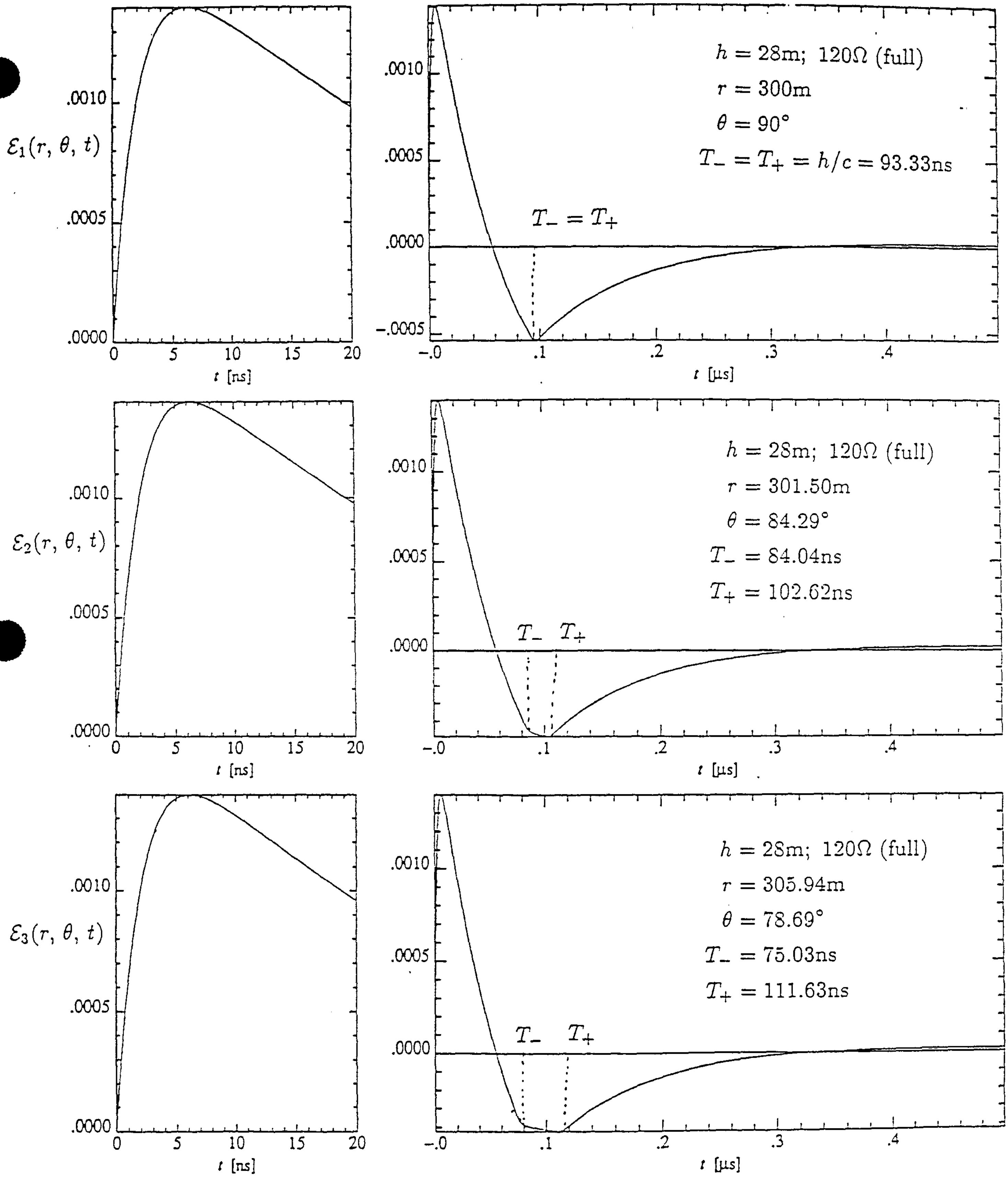

Figure 4. Far fields from a numerical convolution procedure [see (10)]

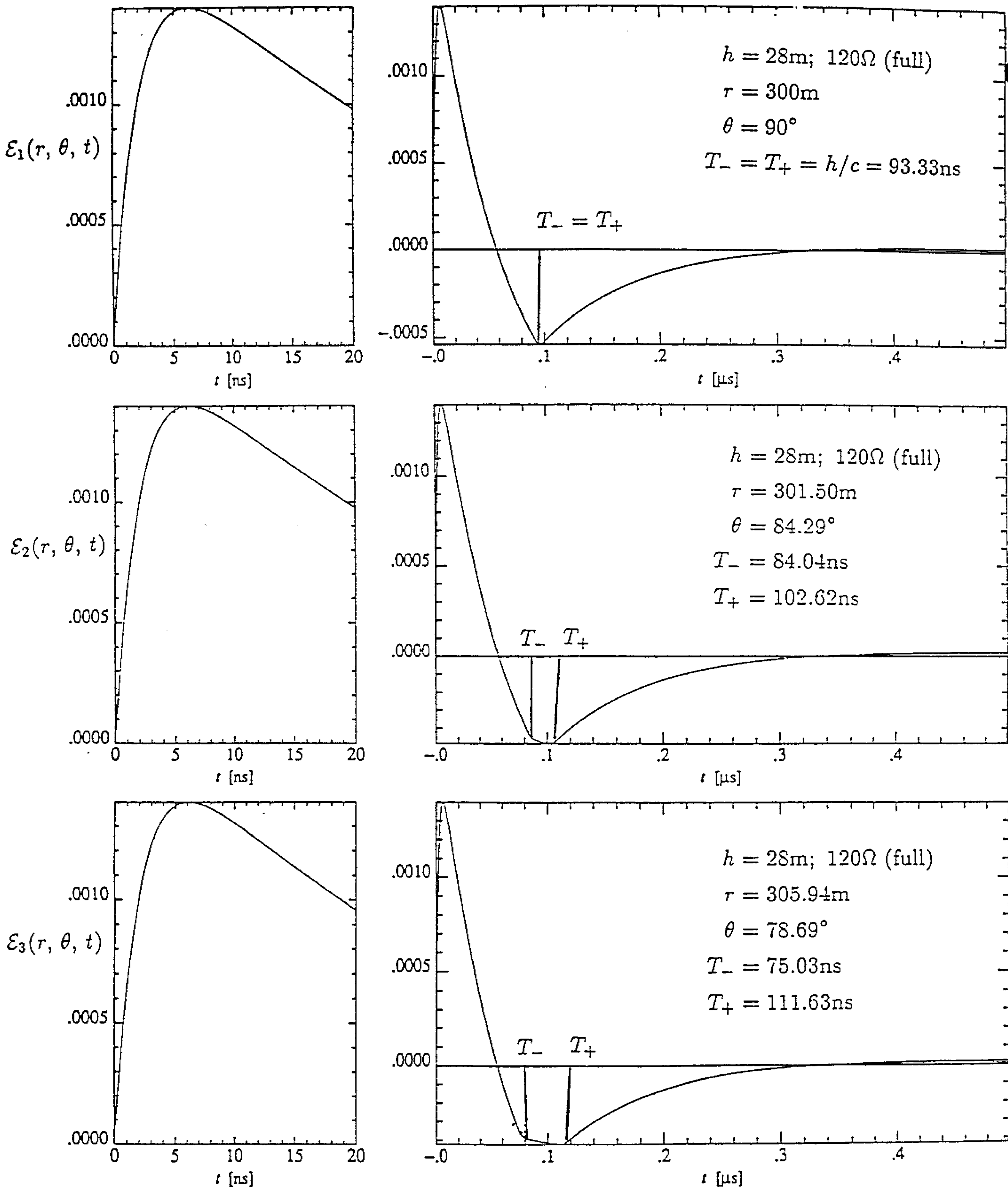

Figure 5. Same fields as in Figure 4, calculated from closed form expression (12)

of 10^6.

d) The two instants of time with slope discontinuities $t_\pm = [1 \pm \cos(\theta)](h/c)$ are clearly seen in the two lower figures. When $\theta = 90°$, t_+ and t_- become equal as seen in the top portion of Figures 4 and 5.

IV. Summary

In this note, we have extended the past analysis of a pulse radiating biconical antenna that is resistively loaded. The analysis was based on a transmission-line model of the antenna and obtained the far fields for a step function generator. In this note, a more realistic double-exponential generator function is used and the analytical convolution has been performed. The far fields are now available in closed form for this excitation function consisting of a fast rise and slow decay.

The expressions of far field will be directly useful in future designs of such pulse-radiating antennas.

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