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Field-Containing Solenoidal Inductors

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Abstract

Solenoidal inductors have a large magnetic dipole moment resulting in excessive interfering magnetic fields in certain pulse shaping networks. Improved designs based on the traditional toroidal windings have been analyzed and fabricated in the past for high-voltage operation. In this note, we present improved designs based on the multiple solenoidal windings (an even number), arranged to give rotation / reflection symmetry. These new designs offer the advantage of cancelling the magnetic dipole moments, but adding the inductances of individual solenoids.

Contents

I. Introduction

Field-containing inductors have several applications in pulse-shaping networks and transmission lines. Toroidal windings have been frequently used to contain the magnetic fields caused by the current-flow in the inductor [1]. However, if the terminals of the inductor in a toroid are in close proximity, then their high-voltage and high-frequency performances are seriously restricted. It is possible to overcome these restrictions by improved toroidal designs described in [2, 3]. The improved designs minimize the stray capacitance of the inductor and are also suitable for high-voltage applications since the terminals can be at diametrically opposite points of the toroidal form.

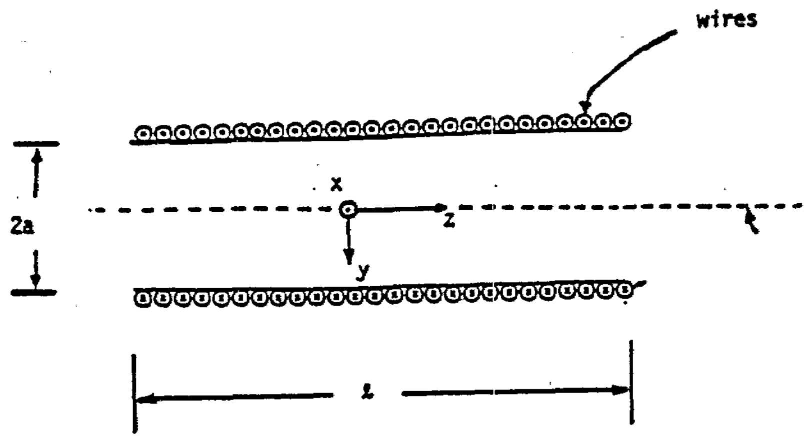
In this note, we reconsider solenoidal inductors with the dual purpose of a) minimizing the magnetic dipole moment and b) adding the individual solenoidal inductances in series. One immediately observes that we have to consider an even number (M) of solenoidal inductors so that a net cancellation of magnetic dipole moments can occur. Consequently, such inductors can be called as bisolenoidal ($M = 2$), quadrasolenoidal ($M = 4$) etc. In general, using symmetry concepts [4], one may consider a multisolenoidal inductor consisting of an even number of individual solenoids interconnected in special ways.

II. Conventional Solenoidal Inductor (M = 1)

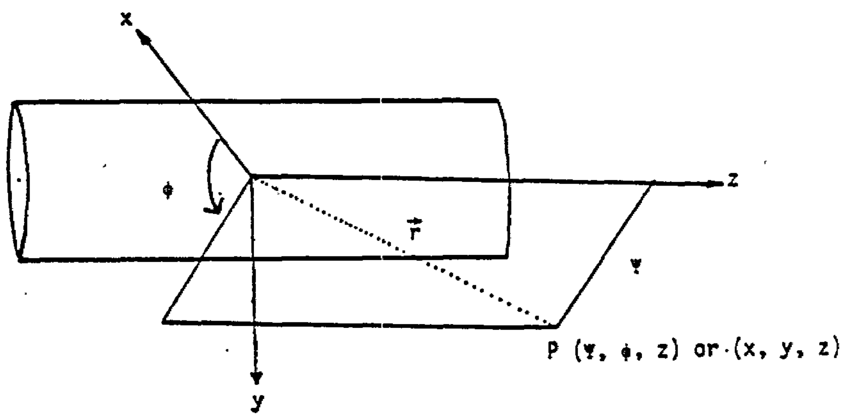
The electrical parameters of a conventional solenoidal inductor are well known [1, 2] as summarized below. Consider a long solenoid of length ℓ and winding radius a as shown in figure 1a. Rectangular (x, y, z) and cylindrical (Ψ, ϕ, z) coordinates are useful as indicated in figure 1b. The general appearance of the magnetic field is shown in figure 1c. Under the assumption of an infinitely long solenoid, the quantities of interest may be easily derived and they are listed below.

$$\begin{aligned} L_1 &\equiv \text{inductance of a single solenoidal inductor} \\ &= \mu_0 \frac{N^2}{\ell} \pi a^2 = \mu_0 \, N'^2 \, \ell \, \pi a^2 \end{aligned} \tag{1}$$

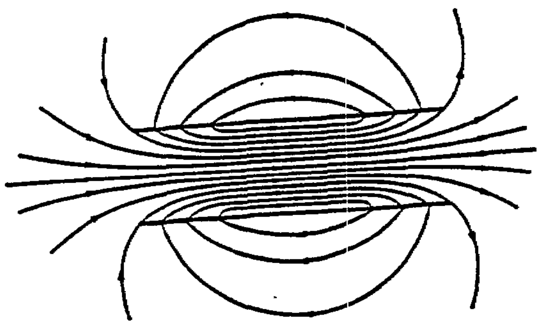
with



(a) Cross sectional view



(b) Rectangular x cylindrical coordinate systems



(c) General appearance of the magnetic field

Figure 1. Solenoidal inductor

$$\mu_0 \equiv \text{permeability of free space} \simeq 4\pi \times 10^{-7}\,\text{H/m}$$

$$N' \equiv \text{number of turns per unit length} = \text{N}/\ell$$

$$N \equiv \text{total number of turns in length } \ell$$

Note that the end effects in a finitely long solenoid makes the inductance L_1 expression approximate. Correction factors are tabulated in [1]. At low frequencies, the quasistatic magnetic field $\vec{H}_s$ from the inductor's current flow is dominated by the magnetic dipole moment $\vec{m}$ given by

$$\vec{m}_1 = \vec{1}_z\, m_z = \vec{1}_z\, N I\, \pi a^2 \tag{2}$$

and the magnetic field itself is given by [5],

$$\vec{H}(\vec{r}) \cong \frac{1}{4\pi r^3}\left[3\,\vec{1}_r\vec{1}_r - \overleftrightarrow{1}\right] \cdot \vec{m}_1 \tag{3}$$

where

$$\vec{1}_\zeta \equiv \text{unit vector in } \zeta \text{ direction}$$

$$\overleftrightarrow{1} \equiv \vec{1}_x\vec{1}_x + \vec{1}_y\vec{1}_y + \vec{1}_z\vec{1}_z \equiv \text{identity dyadic}$$

Substituting (2) in (1)

$$\vec{H}_1(\vec{r}) \cong \frac{1}{4\pi r^3}\left[3\,\vec{1}_r\vec{1}_r - \overleftrightarrow{1}\right] \cdot \vec{1}_z\, N I \pi a^2$$

so that the distant magnetic field is bounded by

$$\left|\vec{H}_1(\vec{r})\right| \leq N a^2 I/(2r^3) \tag{4}$$

This field can far exceed the field due to current flow in the leads of the inductors and can interfere with other parts of circuits in which solenoidal inductors are employed.

Given this review of a single solenoidal inductor, we can now proceed to consider a bisolenoidal inductor ($M = 2$) that has a net cancellation of the magnetic dipole moment while adding the two individual inductors. This design has to be contrasted with some special inductance cancelling winding employed in special "non-inductive" resistors.

III. Bisolenoidal Inductors (M = 2)

We now consider a bisolenoidal inductor, made up of two ($M = 2$) solenoids as illustrated in figure 2. The individual solenoids are wound on cylindrical forms of

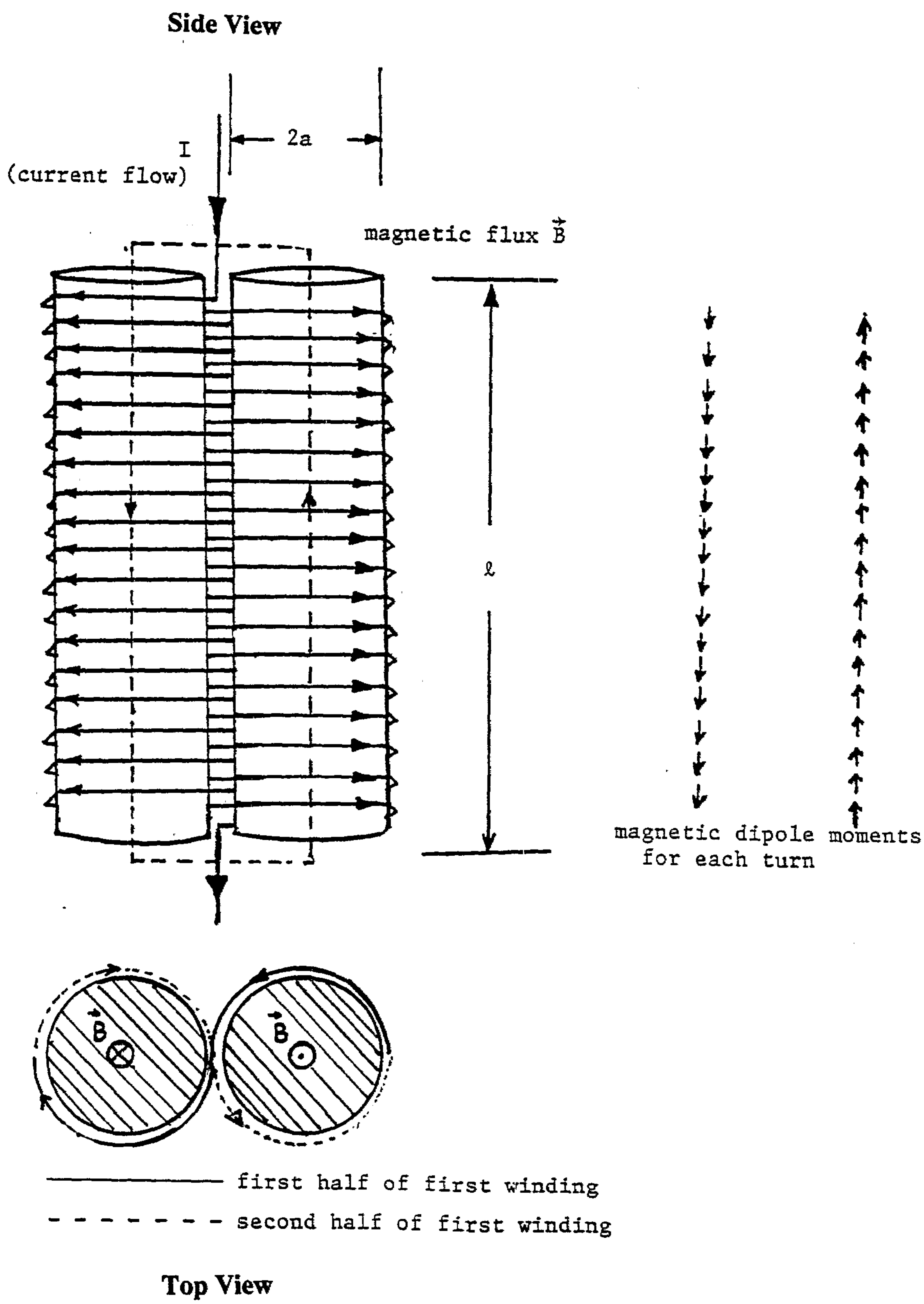


Figure 2. Bisolenoidal inductor S_2

length ℓ and winding radius a. The winding is special going successively over and under each form. The current flowing in the inductor is I and let there be N turns on each side. It is noted that the cylindrical form itself can be magnetic material e.g., ferrite core with $\mu = \mu_0 \mu_r$. However, without any loss of generality, we will assume that the core is non-magnetic with a permeability μ_0 equal to that of free space. Under the solenoidal approximation of $(\ell \gg a)$ which in practice can be $(\ell \geq 5a)$, the magnetic flux in each turn is

$$\phi = B \times A = \mu_0 \frac{N}{\ell} I A \qquad \text{in each turn} \tag{5}$$

total flux ϕ_t is then given by

$$\phi_t = 2N \phi = 2 \mu_0 \frac{N^2 I}{\ell} \pi a^2 \tag{6}$$

so that the net inductance of the bisolenoidal inductor is given by

$$L_2 = 2 \mu_0 N^2 \frac{\pi a^2}{\ell} \tag{7}$$

It is observed that $L_2 = 2L_1$ derived earlier. In other words, the individual inductors of the two solenoids add in series. One could also derive L_2 from energy considerations

$$\begin{aligned} \text{energy } U_m \simeq \frac{1}{2} L_2 I^2 &= (\text{volume}) \times \frac{1}{2} \mu H^2 \\ &= (2\,\mathrm{A}\ell) \times \frac{1}{2} \mu_0 \times \left(\frac{NI}{\ell}\right)^2 \end{aligned} \tag{8}$$

or

$$\frac{1}{2} L_2 I^2 = \mu_0 A N^2 \frac{I^2}{\ell} \tag{9}$$

Substituting $A = \pi a^2$, we find that

$$L_2 = 2\mu_0 N^2 \frac{\pi a^2}{\ell} = 2L_1 \tag{10}$$

which is same as the result in (7). Using the expression for L_2, one can design for the required inductance by choosing ℓ, a, N, etc.

The directions of individual magnetic dipole moments ($\vec{m}_1$ and $\vec{m}_2$) are along the $\pm z$ axes and the net dipole moment is ideally zero. Magnetic field lines from closed loops as indicated, satisfying $(\nabla \cdot \vec{B} = 0)$.

It can also be observed that the bisolenoidal inductor possesses rotation-reflection symmetry and belongs to the S_2 symmetry group [4, 6]. Rotation-reflection symmetry S_M is defined by

$$S_M = \{(S_M)_\ell \ |\ell = 1, 2 \ldots M\} \tag{11a}$$

In group theory notation, we also observe that

$$\begin{aligned}
&(S_M)_1 = (C_M)_1(R_z) = (R_z)(C_M)_1 \\
&(S_M)_\ell = (S_M)_1^\ell \\
&(S_M)_2 = (C_M)_2 \\
&(S_M)_\ell = (C_M)_{\ell/2} \qquad \text{for } \ell \text{ even} \\
&(S_2)_1 = (I) \quad \text{or} \quad S_2 = I
\end{aligned} \tag{11b}$$

where C_M, R_z and I are respectively rotation, reflection and inversion symmetry groups. In the present case M even is of interest and $(S_M)_1$ represents a rotation by $(2\pi/M)$ and reflection in a plane perpendicular to $\vec{1}_z$. After the rotation and reflection, the object replicates itself. For the simple case of S_2, one considers the cross-sectional plane $(x - y)$, rotates by π and then a reflection yields the original inductor. It can also be noted that special case of S_2 is equivalent to inversion symmetry I for which $\vec{r} \rightarrow -\vec{r}$.

In the pulse generator for a prototype impulse radiating antenna (IRA) [7], an S_2 or bisolenoidal inductor has been fabricated, tested and used. It is employed as an isolation inductor to prevent waves getting launched back towards the pulser, after the closure of the output switch located at the focal point of the IRA. This S_2 had the following parameters

$$a = 5\text{mm} \quad , \quad \ell = 45\text{mm} \quad , \quad N = 10 \text{ per side}$$

$$L_2 = 2\mu_0 N^2 \pi a^2 / \ell \simeq 440 \text{ nH}$$

The calculated value of about 440 nH is in fair agreement with measurements. This also confirms that, under the solenoidal approximation, the inductance of the two individual solenoids add in series, i.e., $L_2 \simeq 2L_1$.

IV. Quadrasolenoidal Inductor (M = 4)

Quadrasolenoidal inductor consists of four solenoids arranged and wound in a special way as shown in figure 3. The winding scheme is shown in the figure by illustrating two successive halves of the first winding.

It is observed that the forms on which the wire is wound can be hollow. We do require that the length of the form ℓ be large compared to its radius ($\ell/a \geq 5$) for solenoidal approximations to be valid for individual solenoidal inductor. For this case of $M = 4$ (S_4), the centers of the forms are evenly spaced on a circle whose radius is slightly larger than that of the individual solenoids. To a first order, the over all inductance will be 4 times individual inductance as given by

$$\begin{aligned} L_4 &\simeq 4L_1 \simeq 2L_2 \\ &\simeq 4\,\mu_0\,N^2\,\frac{\pi a^2}{\ell} \end{aligned} \tag{12}$$

where

$$a \equiv \text{radius of each form}$$

$$N \equiv \text{number of turns on each form}$$

$$\ell \equiv \text{length of each of the four forms}$$

It is also easy to observe that the net magnetic dipole moment of an S_4 inductor vanishes. In figure 3, we observe that the magnetic flux from adjacent pairs cancel out.

V. Multisolenoidal Inductors (M even)

One could easily extend the concept of S_2 and S_4 described earlier to a general case of S_M, as long as M is even. One needs an even number of solenoids so that the net dipole moment is zero. An odd number of solenoids would leave the dipole moment of one of the solenoids uncancelled, and violates the rotation/reflection symmetry in that M applications of $(S_M)_1$ does not give the identity (but rather a reflection (R_z)).

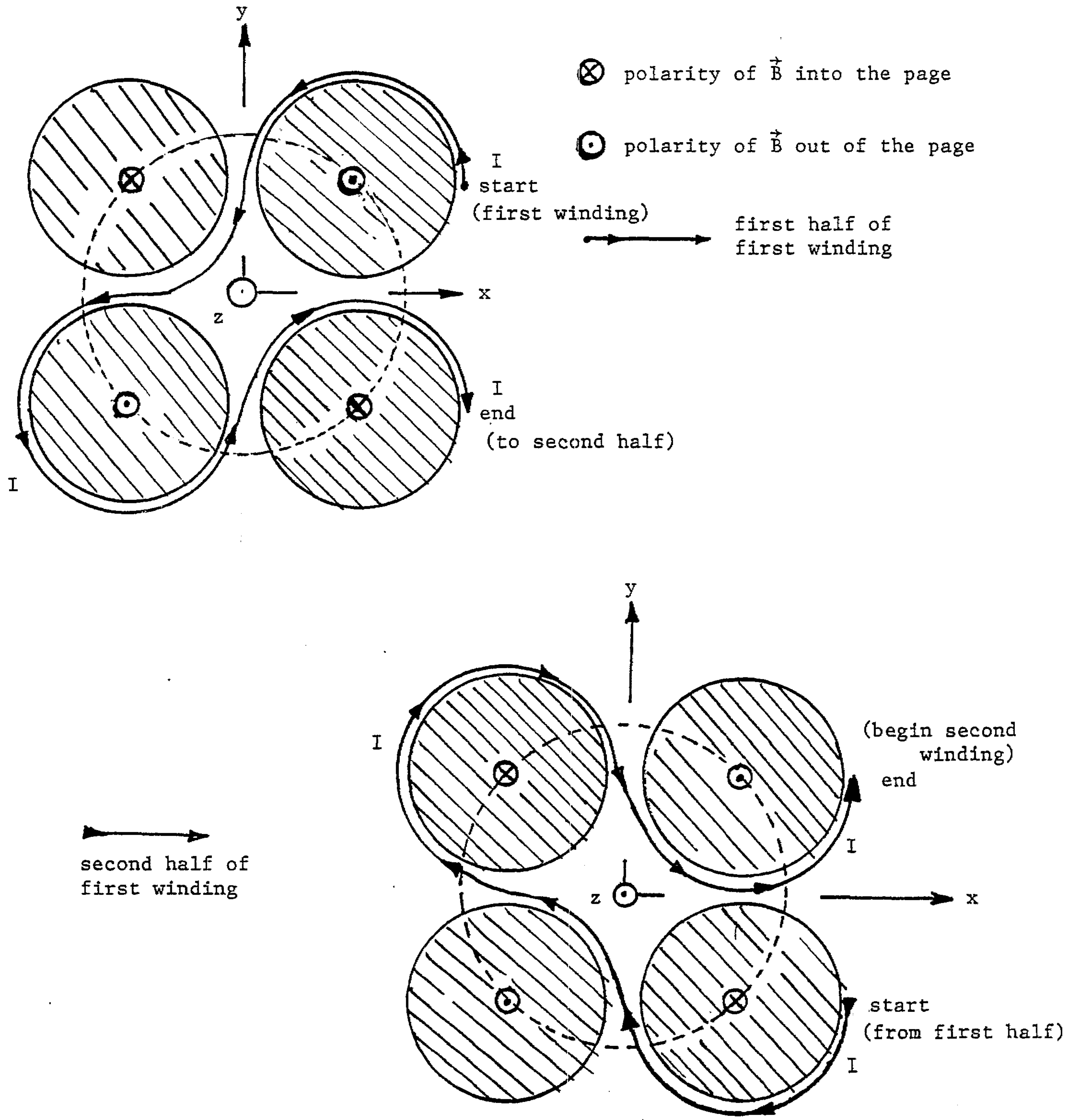


Figure 3. Quadrasolenoidal inductor S_4

As indicated earlier S_M has rotation-reflection symmetry. The rotation is by an angle $(2\pi/M)$ and reflection in a plane perpendicular to $\vec{1}_z$ with reference to the coordinate system indicated in S_4 of figure 3 for example. Again, to a first order the inductance L_M of an S_M inductor may be approximated by

$$\begin{aligned} L_M &\simeq ML_1 \\ &\simeq M\left\{\frac{\mu_0\, N^2\, \pi a^2}{\ell}\right\} \end{aligned} \tag{13}$$

with $\pi a^2 = A$ being the cross sectional area, and ℓ the length of each of the M (even) forms with N windings perform.

VI. Summary

Conventional single solenoidal coils have excessive dipole moments leading to large interfering magnetic fields. The use of toroidal forms is one way of containing the fields within the inductor. In addition, we have found that any even number of solenoids may be employed in conjunction with special ways of winding to yield a net zero dipole moment. The magnetic flux from adjacent solenoid pairs can be made equal and opposite. While the field is thus contained, the inductance of each of the solenoids is added in series.

References

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