Sensor and Simulation Notes

Note 370

Off-Boresight Field of a Lens IRA

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Abstract

A simple approximation is used to calculate the off-boresight field of a lens impulse radiating antenna (IRA). A sample problem is then calculated to find the "approximately best" radiated field for a given size aperture and given risetime.

I. Introduction

We consider here the radiated field from a lens Impulse Radiating Antenna (IRA), both on-boresight and off-boresight Based on earlier work, we know that lens IRA provides an approximately optimal radiated field for a given size aperture and with a signal of a given risetime for fast pulses. An example of a lens IRA is shown in Figure 1. Other configurations will have smaller fields on boresight, with higher fields off boresight. Another design, a reflector IRA, will provide comparable performance, depending somewhat upon the particular figure of merit. The off-boresight field of a reflector IRA has been calculated in [1], and we use a similar technique here.

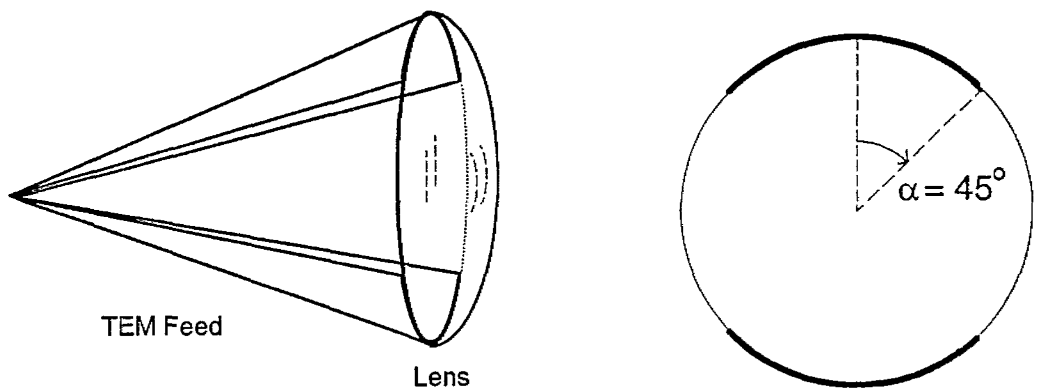

Figure 1. A lens IRA.

The aperture field of the lens IRA is planar, and it is excited by a smooth step function. The impedance of the aperture is chosen to be 189 Ω, which is known to be optimal for this class of antennas. This gives an opening angle of 90 degrees on the top and bottom plates.

We understand that for high-voltage applications, one may not want exactly this shape for the aperture. Instead, one would probably want conductors that curved away from each other, to reduce the fields at the edges. Such a design is somewhat less efficient than the design considered here, but the lens IRA of Figure 1 is somewhat easier to analyze. Thus, we consider this analysis as an approximate "best-case" analysis.

II. Calculations

We begin with the radiated field on boresight. From Appendix A and [2], we have the radiated field on boresight as

$$E(r,t) = \frac{-0.85\,a}{2\pi r c f_g} \frac{dv(t)}{dt} \tag{1}$$

where a is the aperture radius, c is the speed of light in free space, $v(t)$ is the driving voltage, and $f_g = Z_{feed} / Z_o$ is the feed impedance normalized to the impedance of free space.

In order to find the antenna patterns off-boresight, one must calculate the step response, and then convolve with the derivative of the driving voltage. We find the step responses in the H- and E-planes to be [1]

$$\begin{aligned} \vec{E}^{(h)}(r,t) &= \vec{1}_y \left(\frac{-V}{r}\right) \frac{\cot(\theta)}{2\pi} \, \Phi^{(h)}\left(\frac{ct}{\sin(\theta)}\right) \\ \vec{E}^{(e)}(r,\theta,t) &= \pm \vec{1}_\theta \left(\frac{-V}{r}\right) \frac{1}{2\pi \sin(\theta)} \, \Phi^{(e)}\left(\frac{ct}{\sin(\theta)}\right) \end{aligned} \tag{2}$$

where the driving voltage is $v(t) = V\,u(t)$, $u(t)$ is the Heaviside step function, and V is the peak voltage. Furthermore, the normalized potential functions $\Phi^{(h)}(x)$ and $\Phi^{(e)}(y)$ are line integrals of the electric field that sweep over the aperture as

$$\begin{aligned} \Phi^{(h)}(x) &= \frac{-1}{V} \int_{C_1(x)} E_y \, dy \\ \Phi^{(e)}(y) &= \frac{-1}{V} \int_{C_2(y)} E_y \, dx \end{aligned} \tag{3}$$

The location of the contours $C_1(x)$ and $C_2(y)$ are shown in Figure 2. Note that this theory ignores reflections from the lens (4 % of the electric field magnitude for polyethylene, $\varepsilon_r = 2.2$). Note also that we ignore the unfocused portion of the field outside the circle.

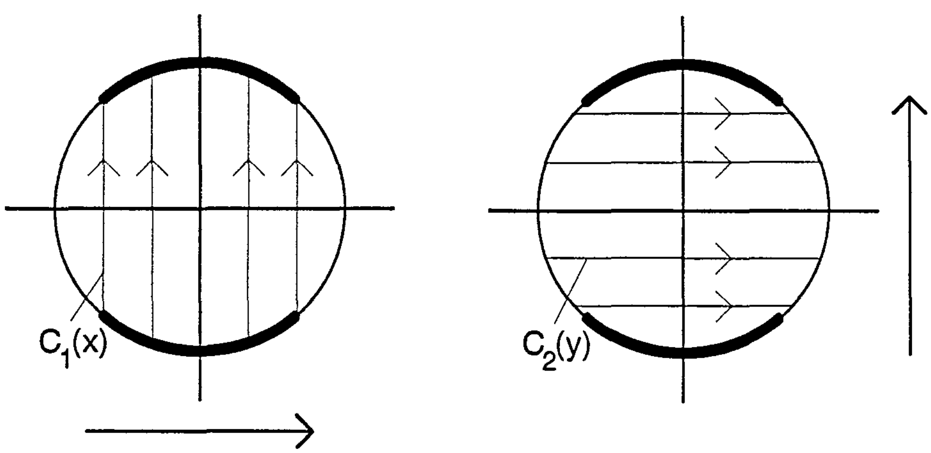

Figure 2. Location of $C_1(x)$, left and $C_2(y)$, right.

To calculate the normalized potential function $\Phi^{(h)}(x)$, we must make some approximations. For most of the aperture, where the contour $C_1(x)$ cuts through the two conductors, the normalized potential is just unity. At the edge of the aperture , where $x/a = \pm 1$, the normalized potential is zero. Thus the potential function $\Phi^{(h)}(x)$ is not known rigorously only in the small region where $1/\sqrt{2} < |x|/a < 1$. It is somewhat complicated to carry out the integral exactly, but since there is such a small region where the function is unknown, we simply assume a piecewise linear approximation to the function. Thus, we have

$$\Phi^{(h)}(x) = \begin{cases} 1 & |x|/a \le 1/\sqrt{2} \\ (1-|x|/a)/(1-1/\sqrt{2}) & 1/\sqrt{2} \le |x|/a \le 1 \\ 0 & |x|/a > 1 \end{cases} \tag{4}$$

We have plotted this function in Figure 3. The normalized potential $\Phi^{(e)}(y)$ has a similar behavior, due to symmetry conditions (which apply only when $f_g = 0.5$, as in our case). Thus, one obtains $\Phi^{(e)}(y)$, by simply replacing x with y in th above equation.

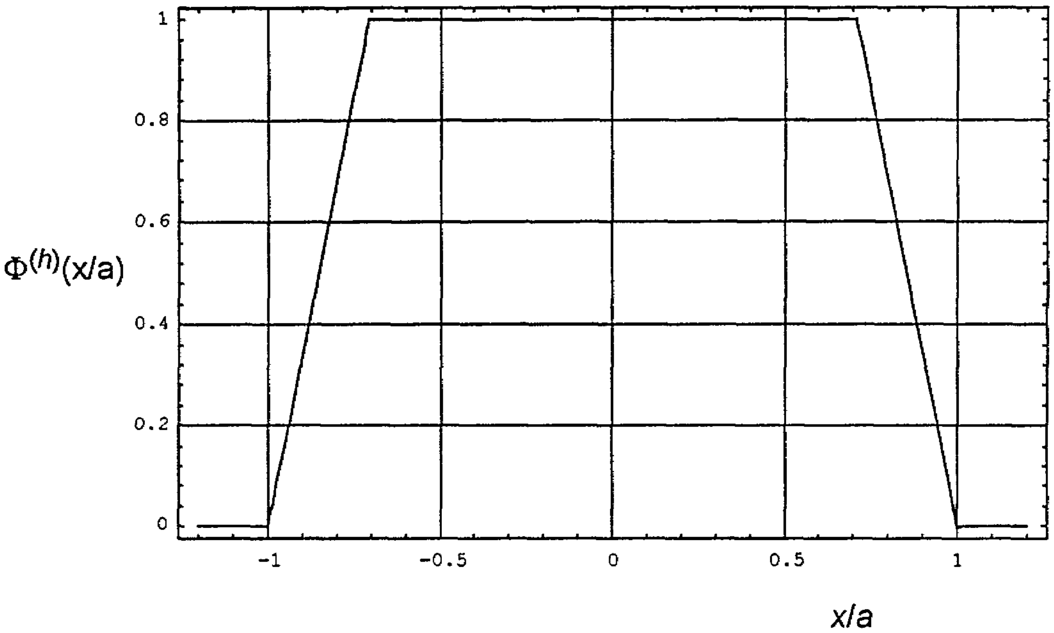

Figure 3. Plot of the normalized potential function $\Phi^{(h)}(x)$. Note that the normalized potential function $\Phi^{(e)}(y)$ has the same shape due to symmetry.

We now have the step response off-boresight. This must be convolved with the derivative of the driving voltage to obtain the actual radiated field. Thus,

$$E(r,\theta,\phi,t) = \frac{1}{V}\frac{dv(t)}{dt} \circ E^{step}(r,\theta,\phi,t) \tag{5}$$

where the " ∘ " symbol indicates a convolution. The excitation voltage is described most simply in terms of its derivative, which is just a Gaussian curve. Thus,

$$\frac{dv(t)}{dt} = \frac{V}{t_d} e^{-\pi(t/t_d)^2} \qquad t_{FWHM} = 0.940\, t_d$$
$$v(t) = \int_{-\infty}^{t} \frac{dv(t')}{dt'} dt' \qquad t_{10-90} = 1.023\, t_d \tag{6}$$

where we have defined the risetime in terms of the so-called derivative risetime

$$t_d = \frac{\max(v(t))}{\max(dv(t)/dt)} \tag{7}$$

Note that for a Gaussian curve this is just two percent different from the 10-90% risetime. Thus, for this shape waveform it does not matter very much which risetime definition one uses. We have plotted the voltage and its derivative in Figure 4.

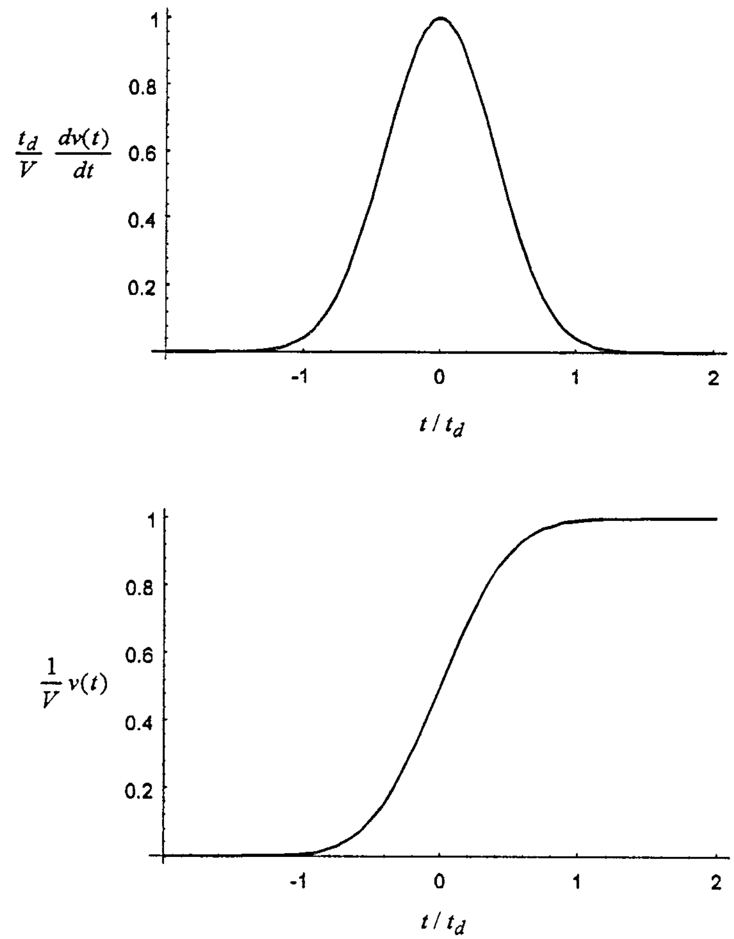

Figure 4. Derivative of driving voltage (top), and its integral (bottom).

We finally have all the pieces in place to carry out the calculation. We use $a = 0.25$ m, and $t_d = 150$ ps. We plot the radiated fields normalized to V / r as function of time in the H-plane for various angles off-boresight in Figure 5. We do the same for the E-plane in Figure 6.

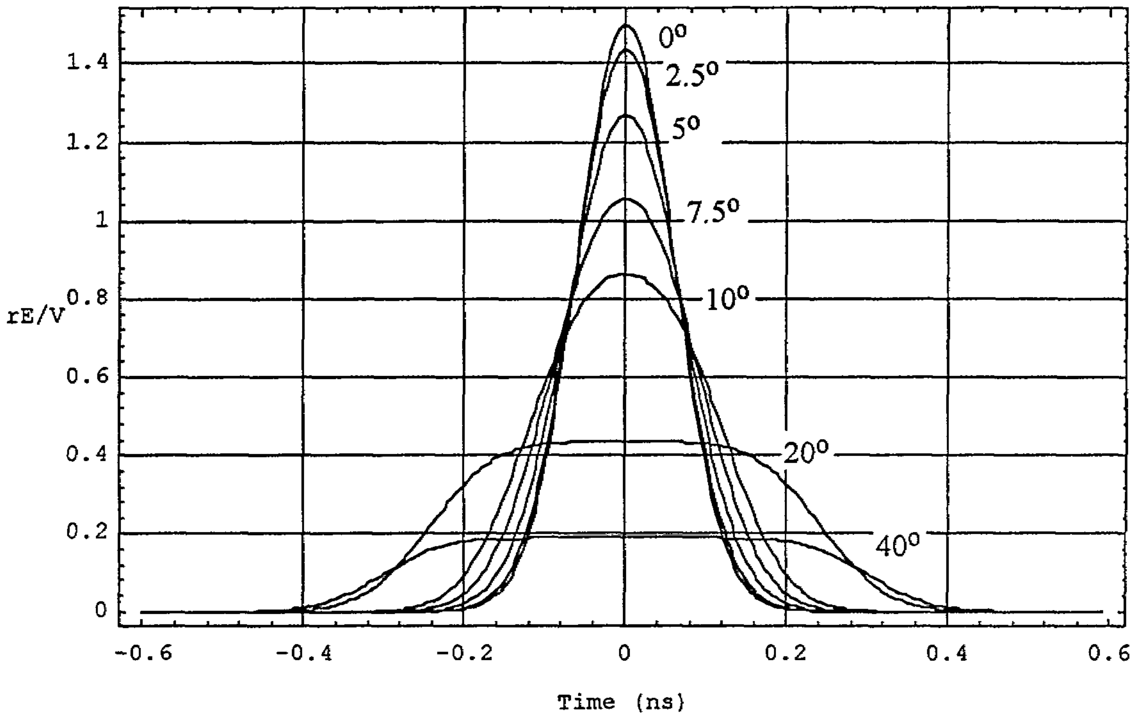

Figure 5. Plots of the normalized electric field in the H-plane for various angles θ off boresight.

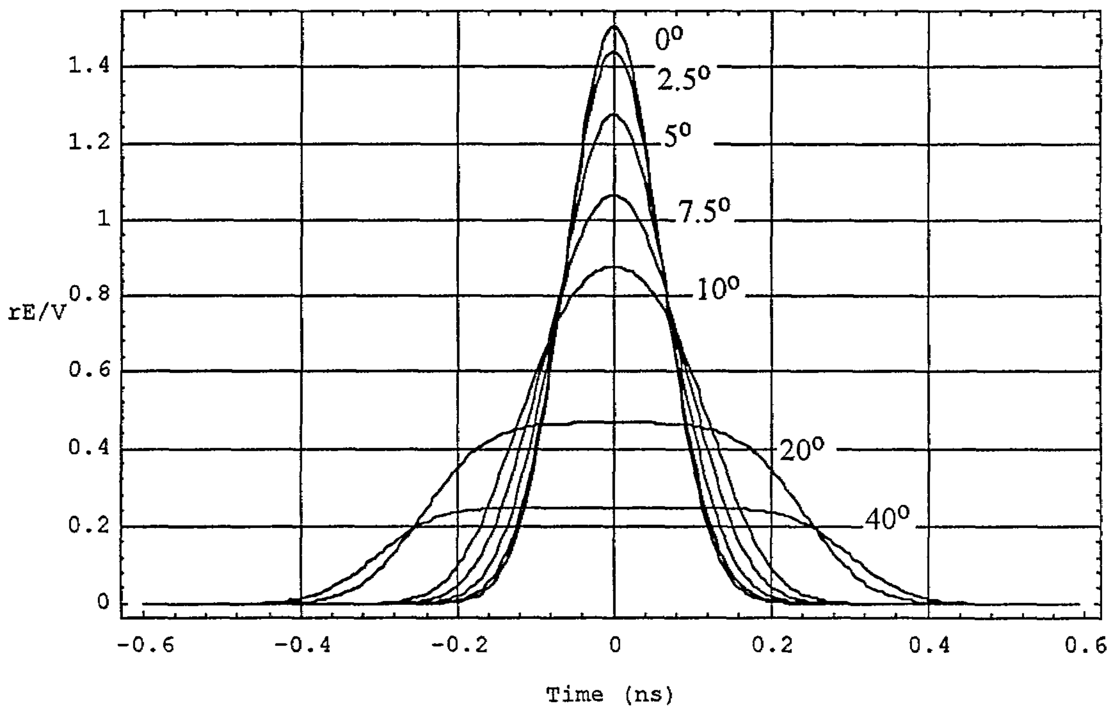

Figure 6. Plots of the normalized electric field in the E-plane for various angles θ off boresight.

III. Analysis

In this sample problem we have a rather narrow beam, in spite of a having a small aperture. The half-power beam width (where the peak electric field is down by 0.707 from its peak) is about ±7.5 degrees. One can spread out the beam by defocusing the aperture or by using a smaller aperture. But peak field on boresight will be sacrificed in doing so.

IV. Concluding Remarks

We have provided a simple linear approximation that allows the calculation of off-boresight fields for a lens IRA. A sample problem was calculated that can give a feel for the beamwidths that are generated by a focused aperture.

There are fundamental limitations on the maximum radiated far field from an aperture of a given size. It is premature to state categorically that we have found that limit in this calculation, but we believe we are close to it.

Appendix A

The radiated field on boresight for a lens IRA as provided in eqn. (1) was estimated in Ref. [1], but was never rigorously proven. We fill in the details here.

From [3], we know the radiated field on boresight due to a step-function driving voltage is

$$E(r,t) = \frac{E_o}{r} \frac{a^2}{2c} \delta_a(t) \tag{A.1}$$

where E_o is the field at the center of the antenna aperture. Recall that for this configuration, the fields outside the aperture make no net contribution to the total radiated field on boresight. From [4], we have an alternative expression in terms of the aperture height, h_a, as

$$E(r,t) = \frac{-V}{r} \frac{h_a}{2\pi c f_g} \delta_a(t) \tag{A.2}$$

where $h_a = p'/2Q'$, p' is the line dipole moment of the aperture, and Q' is the line charge of the aperture. Comparing the above two equations, we have

$$h_a = -\pi a^2 \frac{f_g E_o}{V} \tag{A.3}$$

We know that h_a is somewhat smaller than the radius, so we express it in terms of the radius as

$$\begin{aligned} h_a &= K_p a \\ K_p &= -\pi a \frac{f_g E_o}{V} \end{aligned} \tag{A.4}$$

where we expect the proportionality constant K_p to be equal to 0.85 as in (1). From [5] we have

$$\begin{aligned} E_o &= \frac{-V}{a K(m) (1 + m^{1/2})} \\ f_g &= \frac{K(m)}{K(1-m)} \end{aligned} \tag{A.5}$$

where $K(m)$ is the complete elliptic integral of the first kind, and m is determined from the impedance. Combining the above two equations, we find

$$K_p = \frac{\pi}{K(1-m)(1+m^{1/2})} \tag{A.6}$$

For our geometry, $f_g = 0.5$, so $m = 0.0294373$, and $K_p = 0.847213$, thus validating our earlier estimate of 0.85 in [1].

References

[1] E. G. Farr and C. E. Baum, The Radiation Pattern of Reflector Impulse Radiating Antennas: Early-Time Response, Sensor and Simulation Note 358, June 1993.

[2] E. G. Farr and C. Jerald Buchenauer, Experimental Validation of IRA Models, Sensor and Simulation Note 364, January 1994.

[3] E. G. Farr and C. E. Baum, "Radiation from Self-Reciprocal Apertures," Chapter 6 in *Electromagnetic Symmetry* (C. E. Baum and H. N. Kritikos, eds.), Taylor and Francis, to be published in 1995.

[4] C. E. Baum, Aperture Efficiencies for IRAs, Sensor and Simulation Note 328, June 1991.

[5] T. K. Liu, Impedances and Field Distributions of Curved Parallel-Plate Transmission-Line Simulators, Sensor and Simulation Note 170, February 1973.

Acknowledgment

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