Sensor and Simulation Notes
Note XXXVIII
21 March 1967

Parameters for Some Electrically - Small Electromagnetic Sensors

Capt Carl E. Baum

Air Force Weapons Laboratory

Abstract

Equivalent circuits are considered for electrically-small loops and dipoles, for which equivalent areas and equivalent lengths are defined as appropriate sensitivity parameters. The various sensor parameters (sensitivities and admittances) are related to each other. An equivalent volume for the sensor is considered to relate the energy delivered from the electromagnetic fields to a resistive sensor load. A figure of merit is defined as the ratio of the equivalent volume to an appropriate geometric volume in which the sensor is placed. This figure of merit may prove useful in optimizing the design of electromagnetic sensors for various applications.

Sensor and Simulation Notes
Note XXXVIII
21 March 1967

Parameters for Some Electrically - Small Electromagnetic Sensors

Capt Carl E. Baum

Air Force Weapons Laboratory

Abstract

Equivalent circuits are considered for electrically-small loops and dipoles, for which equivalent areas and equivalent lengths are defined as appropriate sensitivity parameters. The various sensor parameters (sensitivities and admittances) are related to each other. An equivalent volume for the sensor is considered to relate the energy delivered from the electromagnetic fields to a resistive sensor load. A figure of merit is defined as the ratio of the equivalent volume to an appropriate geometric volume in which the sensor is placed. This figure of merit may prove useful in optimizing the design of electromagnetic sensors for various applications.

I. Introduction

In measurements of pulsed electromagnetic fields one often uses electrically-small antennas because of their relatively simple response characteristics. By an electrically-small antenna is meant that the sensor dimensions are small compared to a radian wavelength or a skin depth, as appropriate, in the medium where the fields are to be measured. The sensor consists of good conductors (ideally perfect conductors) in some geometric configuration plus, perhaps, additional media with one or more parameters (permittivity, permeability, and conductivity) differing from those of the external medium. The radian wavelength or the skin depth, as appropriate, in these additional media should also be much larger than an appropriate dimension of the particular medium. One might use such extra media (e.g., if the external medium is conducting) to improve some aspect of the sensor response.[1,2] In some cases these additional media do not necessarily complicate the basic parameters (to be discussed) of the electrically-small antennas, in which case such media are not necessarily excluded. An example of such a case is a loop which is enclosed in an insulator which does not affect its inductance or basic sensitivity to the magnetic field, as long as it is electrically small. A counter example is a dipole totally or partially enclosed in an insulating dielectric; if the external medium is an insulator, the sensitivity to the electric field and the capacitance are well behaved. But, if the external medium is conducting, the sensor capacitance and conductance are not simply related, which in some cases complicates the relation of the sensor response to the electric field. Cases in which the presence of an additional medium complicates the sensor parameters for frequencies of interest are not considered in this note.

The external medium is presumed to have a permittivity, ϵ, a permeability, μ, and a conductivity, σ. All of these are assumed to be scalars and independent of the electromagnetic fields. Where significant, σ is, in addition, taken to be time independent. Various other effects, including those peculiar to the nuclear environment in the source region for the nuclear electromagnetic pulse, are also ignored in this analysis.

In this note we consider some of the basic parameters of electrically-small antennas as used for measuring pulsed electric or magnetic fields. The two classes of such sensors considered are loops and dipoles, the former for measuring pulsed magnetic fields and the latter for measuring pulsed electric fields and/or current densities (conduction plus displacement current densities). Effects such as sensitivity to undesired components of electric and magnetic fields are not considered. Each type of sensor is assumed to be driving a purely resistive load, R, which is independent of

1. Lt Carl E. Baum, Sensor and Simulation Note XIII, Electric Field and Current Density Measurements in Media of Constant Conductivity, Jan. 1965.
2. Capt Carl E. Baum, Sensor and Simulation Note XXX, The Single-Gap Cylindrical Loop in Non-Conducting and Conducting Media, Jan. 1967.

frequency for the frequencies of interest. This is a common approach in using electrically-small antennas to measure pulsed fields.

The admittance of an electrically-small loop is basically due to an inductance. Other admittances, associated with capacitances, conductances, turn resistance, etc., are assumed unimportant compared to the inductance and load resistance for frequencies of interest. These extra admittances could be significant in some cases but are neglected from this analysis. Ideally an electrically-small loop can be designed for which these extra admittances are insignificant and for which the inductance is frequency-independent for frequencies of interest. The admittance of an electrically-small dipole is basically due to a capacitance, and if the conductivity of the external medium is nonzero there is also a conductance in parallel with this capacitance. Other admittances are assumed comparatively unimportant. Again the capacitance and conductance are assumed frequency-independent for frequencies of interest for the electrically-small dipole.

Electrically-small loops and dipoles are then each assumed to have simple Thévenin and Norton equivalent circuits. For each type of sensor we define an equivalent area, A_{eq}, and an equivalent length, ℓ_{eq}, (both assumed independent of frequency for frequencies of interest) based on the open circuit voltage and short circuit current from the device, as related to an appropriate electromagnetic field quantity.[3] There are various conventions used in the literature for some of these parameters, often using the terms, "effective area" and "effective height."[4] For a loop the equivalent area used in this note is the same as the effective area often used in other references, but the effective area is sometimes used for something quite different, i.e., the ratio of the power absorbed in an optimum antenna load to the power per unit area in the incident wave. Likewise, for a dipole, the equivalent length used in this note is the same as the effective length or effective height often used in other references. However, the concept of an effective height for a loop (see ref. 4) as sometimes used is quite different from our concept of an equivalent length for a loop. Rorden uses effective areas and effective lengths for loops and dipoles in much the same sense that we use equivalent areas and equivalent lengths, except in the case of a loop where these definitions differ by a factor of the number of loop turns.[5] We think the definitions of equivalent area and equivalent length used in the present note are more natural definitions for a multi-turn loop, directly relating the open circuit voltage and short circuit current to the appropriate field quantities; the number of turns does not enter the equivalent circuit, but rather is included in the inductance, equivalent area, and equivalent length. By using the word, "equivalent", in connection with the appropriate electrical parameters for electrically-small loops and dipoles, we hope to avoid some ambiguity which might arise if we were to use the word, "effective".[6] We hope then to establish a convenient and consistent set of

---

3. Rationalized MKSA units are used for all quantities.
4. S. Schelkunoff and H. Friis, Antennas: Theory and Practice, Chap. 10, 1952.
5. L. H. Rorden, Stanford Research Institute report, A Study of Low-Noise Broadband VLF Receiving Techniques, Sept. 1965.
6. We would like to thank W. E. Blair of the Stanford Research Institute for suggesting to us the use of the word, "equivalent", in this context to avoid ambiguity (private communication).

parameters to describe simple electrically-small loops and dipoles as related to the measurement of electromagnetic field components.

Next we define an equivalent volume, V_{eq}, based on the relation between the electromagentic energy density associated with a step function of the field component being measured, and the total energy delivered to the resistive load. This gives the same result as Rorden's (see ref. 5) which he defines (for sinusoidal fields) as the ratio of the peak stored energy in the antenna to the peak energy in the incident field. This equivalent volume can be related to the geometric volume of the sensor and represents the degree to which the sensor can extract energy from the electromagnetic field. To extract more energy one can make the sensor larger. There may be various reasons for one to limit the size of the sensor, such as a physical restriction on the volume in which the sensor must fit, a restriction on the allowable field distortion near the sensor, and just convenience in handling. Thus, we define a figure of merit, η, as the ratio of the equivalent volume of the sensor to the volume of the smallest geometrical figure of a given type inside which the sensor can be enclosed. Appropriate geometric figures would include spheres and cylinders and various other shapes depending on the application intended for the sensor. This figure-of-merit concept gives one a quantitative standard by which to compare different sensor designs for a given application. Of course there may be a certain arbitrariness in the choice of the geometric figure in which the sensor is inscribed. Nevertheless, the figure of merit should be a useful concept.

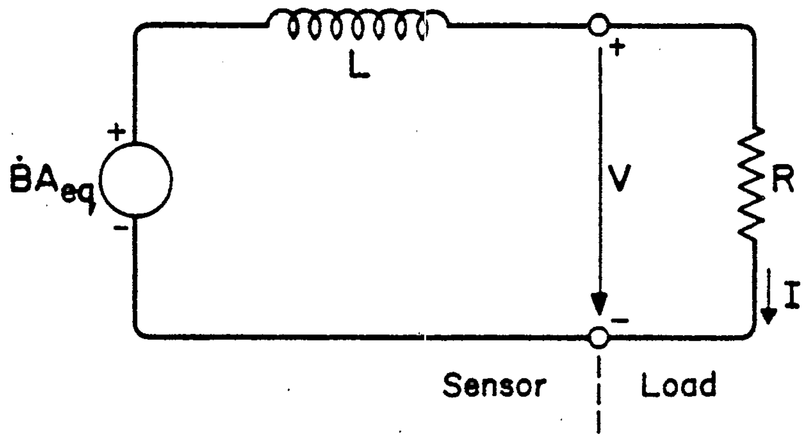
II. Loop Parameters

Consider first the electrically-small loop. The Thévenin and Norton equivalent circuits are given in figures 1A and 1B, respectively. The various parameters used in the equivalent circuits are:

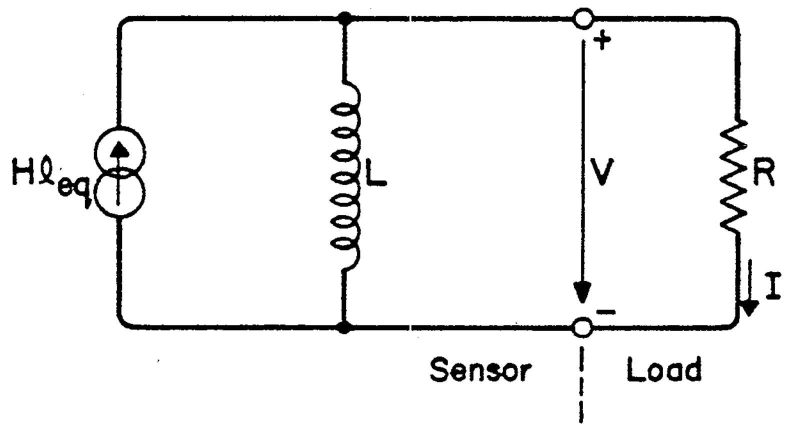
B, H	Component of incident magnetic field in direction of maximum sensitivity of loop
V	Output voltage
I	Output current
A_{eq}	Equivalent area
ℓ_{eq}	Equivalent length
L	Sensor inductance
R	Load resistance

A tilde, $\sim$, over one of these quantities indicates the Laplace transform of the quantity. Replace the Laplace transform variable, s, by $j\omega$ for a frequency domain analysis. Define a time constant and a characteristic frequency for the sensor-load combination by the relation

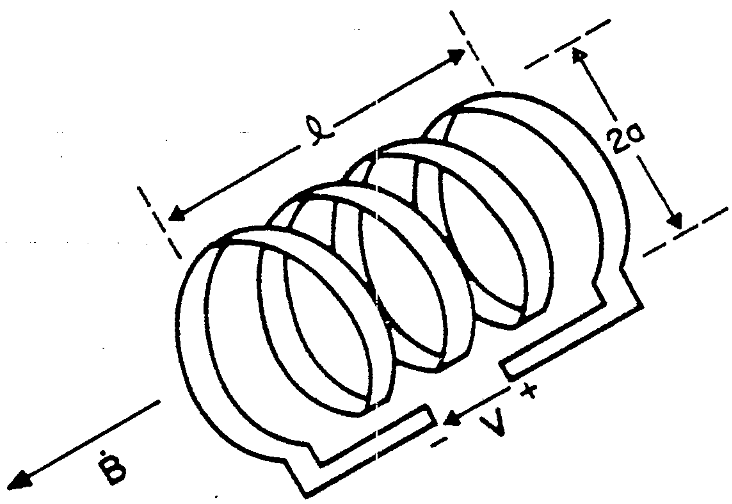
$$t_o = \frac{1}{\omega_o} = \frac{L}{R} \tag{1}$$



A. Thévenin Equivalent Circuit



B. Norton Equivalent Circuit



C. Example: N-Turn Cylindrical Loop

Figure 1. ELECTRICALLY-SMALL LOOP

The Thévenin equivalent circuit has a response given by

$$\tilde{V} = s\tilde{B}A_{eq} \frac{R}{R+sL} = \frac{s\tilde{B}A_{eq}}{1+st_o} \tag{2}$$

For frequencies of interest limited to $\omega \ll \omega_o$, this reduces to

$$\tilde{V} = s\tilde{B}A_{eq} \tag{3}$$

or in the time domain

$$V = \dot{B}A_{eq} \tag{4}$$

This is the characteristic of a $\dot{B}$ loop and is used to define A_{eq} from the open circuit voltage.

The Norton equivalent circuit has a response given by

$$\tilde{I} = \tilde{H}\ell_{eq} \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{sL}} = \tilde{H}\ell_{eq} \frac{st_o}{1+st_o} \tag{5}$$

For frequencies of interest limited to $\omega \gg \omega_o$, this reduces to

$$\tilde{I} = \tilde{H}\ell_{eq} \tag{6}$$

or in the time domain

$$I = H\ell_{eq} \tag{7}$$

A loop designed to give such a response characteristic might be called an H loop (and sometimes referred to as a self-integrating loop). Equation (7) is used to define ℓ_{eq} from the short circuit current.

Relate the parameters of the two equivalent circuits by

$$V = IR \tag{8}$$

and

$$B = \mu H \tag{9}$$

where μ is the permeability of the external medium. Then combining equations (2) and (5) gives

$$\mu A_{eq} = \ell_{eq} L \tag{10}$$

which gives a rather simple relationship between the equivalent length and equivalent area in terms of the inductance and permeability.

Next consider the effectiveness of an electrically-small loop in extracting energy from a pulsed magnetic field. For convenience let the magnetic field component of interest be of the form

$$B = B_o u(t) \tag{11}$$

where u(t) is a unit step function, rising at t = 0. The corresponding Laplace-transformed magnetic field is

$$\tilde{B} = \frac{B_o}{s} \tag{12}$$

Substituting this into equation (2) gives

$$\tilde{V} = \frac{B_o A_{eq}}{1+st_o} \tag{13}$$

which in the time domain (for t > 0) is

$$V = \frac{B_o A_{eq}}{t_o} e^{-\frac{t}{t_o}} \tag{14}$$

This represents a power into the resistive load (for t > 0) of

$$P = \frac{V^2}{R} = \frac{B_o^2 A_{eq}^2}{Lt_o} e^{-2\frac{t}{t_o}} \tag{15}$$

and a total energy of

$$U = \int_o^\infty P dt = \frac{B_o^2 A_{eq}^2}{Lt_o} \int_o^\infty e^{-2\frac{t}{t_o}} dt = \frac{B_o^2 A_{eq}^2}{2L} \tag{16}$$

Note that this total energy delivered to the resistive load is independent of R, but depends only on the magnetic field and the sensor electrical parameters.

Relate the energy delivered to the resistive load to the energy associated with the magnetic field component of interest. In a given equivalent volume, V_{eq}, the energy associated with B (after t = 0) is

$$U_m = \frac{B_o^2}{2\mu} V_{eq} \tag{17}$$

We equate U_m to U (equation (16)) as the definition of V_{eq}, giving

$$V_{eq} = \frac{\mu A_{eq}^2}{L} \tag{18}$$

Using equation (10) this result can be expanded into several forms as

$$V_{eq} = \frac{\mu A_{eq}^2}{L} = A_{eq}\ell_{eq} = \frac{L\ell_{eq}^2}{\mu} \tag{19}$$

Actually the definition of the equivalent volume is somewhat arbitrary, depending on the pulse shape chosen for the incident magnetic field component. However, the definition used here is a convenient one with a rather simple form for the result. Compare this equivalent volume to the geometric volume of a loop by considering the example of an N-turn cylindrical loop as illustrated in figure 1C. The equivalent area is

$$A_{eq} = N\pi a^2 \tag{20}$$

and the inductance is roughly (for $\ell >> 2a$)

$$L \simeq \frac{\mu N^2 \pi a^2}{\ell} \tag{21}$$

which, from equation (10), gives an equivalent length (for $\ell >> 2a$) as

$$\ell_{eq} \simeq \frac{\ell}{N} \tag{22}$$

Using any of the expressions from equation (19) then gives an equivalent volume as

$$V_{eq} \simeq \pi a^2 \ell \tag{23}$$

Note, however, that $\pi a^2 \ell$ is also the geometric volume of the cylinder approximating the loop shape. So, the equivalent volume would seem to be related to the geometry of the sensor. For other loop shapes the equivalent and geometrical volumes are not necessarily so simply related. Another characteristic of this example of a cylindrical loop is that the number of turns, N, does not appear in the equivalent volume in equation (23).

Consider the relationship of the equivalent volume to the sensitivity and frequency response characteristics of the loop. Rewrite equation (19), substituting from equation (1), as

$$V_{eq} = \frac{\mu}{R}\left(A_{eq}^2 \omega_o\right) = \frac{R}{\mu}\left(\frac{\ell_{eq}^2}{\omega_o}\right) \tag{24}$$

For a fixed R, the equivalent volume then combines sensitivity and bandwidth together. For the case of $\dot{B}$ loop, ω_o is the upper frequency response and A_{eq} is the sensitivity; the equivalent volume is proportional to sensitivity squared times upper frequency response. For the case of an H loop (or self-integrating loop), ω_o is the lower frequency response (which one would like to make small) and ℓ_{eq} is the sensitivity; the equivalent volume is now proportional to sensitivity squared divided by lower frequency response. The equivalent volume then combines sensitivity and bandwidth in a form sensitivity squared times bandwidth. For a given equivalent volume one can increase sensitivity, but at the expense of bandwidth, and vice versa. As an example again consider the N-turn cylindrical loop in figure 1C, maintaining R and the dimensions constant and verying N; if one doubles N, A_{eq} is doubled but L is quadrupled, thereby quartering ω_o.

The response characteristics of an electrically-small loop can then be improved by increasing the equivalent volume which one can do just by making the loop larger. Suppose, however, that the size of the sensor is constrained in some way by necessity, practicality, convenience, etc. Then a useful question is how to best design the sensor to fit in a specified geometric volume. One way to quantitatively consider this question is to try to maximize the equivalent volume. A useful number to consider is then the ratio of the equivalent volume to the specified geometric volume; this ratio is one measure of the efficiency of a given sensor design for a given application and we define the ratio as a figure of merit, η.

There are various choices for the geometric volume on which to base a figure of merit. If for the N-turn cylindrical loop of figure 1C, one took the geometric volume of the sensor, $\pi a^2 \ell$, then η would be about one. However, it would seem appropriate to specify a geometric volume first and then evaluate various sensor designs for their figures of merit. One such geometric volume would be a sphere of radius, r_o, and corresponding volume, $\frac{4}{3}\pi r_o^3$, for which case we define η_s as the figure of merit. A sphere has the maximum volume for a fixed maximum linear dimension. As such the sphere may be an appropriate, but arbitrary, shape for figures of merit for conveniently handleable sensors. Fitting the N-turn cylindrical loop into a sphere with the smallest possible r_o still makes V_{eq} (as in equation (23)) somewhat less than the volume of the sphere so that η_s is less than one for this type of loop. One might even try to find a length-to-diameter ratio for an N-turn cylindrical loop (using a more accurate form for the inductance than equation (21)) which gives the largest η_s. Another geometric volume one might consider is a circular cylinder of given radius, r_o, length, z_o, and volume, $\pi r_o^2 z_o$; this geometric shape might be appropriate for considering sensors which must fit into a sounding-rocket body, a circular hole in the ground, etc. However, a circular cylinder does not have the same symmetry as a sphere, so that there are two cases of interest: the magnetic field of interest parallel to the cylinder axis for which we use η_{cp} as the figure of merit, and the magnetic field of interest normal (perpendicular) to the cylinder axis for which we use η_{cn} as the figure of merit. There are undoubtedly many other geometric shapes on which one can base a figure of merit; the sphere and circular cylinder may have wide application. In some cases we may be interested in measuring fields near a conducting plane with sensors utilizing the conducting plane as a symmetry plane. It may then be appropriate to use geometric shapes like hemispheres and hemicylinders for the figure of merit and we can use the same notation for the figure of merit as for spheres and cylinders.

III. Dipole Parameters

Consider second the electrically-small dipole. The Thévenin and Norton equivalent circuits are given in figures 2A and 2B, respectively. The various parameters used in the equivalent circuits are:

E	Component of incident electric field in direction of maximum sensitivity of dipole
J_t	Component of total current density (conduction plus displacement) in direction of maximum sensitivity of dipole
V	Output voltage
I	Output current
ℓ_{eq}	Equivalent length
A_{eq}	Equivalent area
C	Sensor capacitance
G	Sensor conductance
R	Load resistance

The component of interest of the total current density is given by

$$J_t = \left(\sigma + \varepsilon\frac{\partial}{\partial t}\right) E \qquad (25)$$

The sensor conductance and capacitance are assumed related as

$$\frac{G}{\sigma} = \frac{C}{\varepsilon} \qquad (26)$$

If G and σ are nonzero (and significant), the above relation is necessary for ℓ_{eq} and A_{eq} to be frequency independent, and thereby giving simple equivalent circuits. One way to achieve this relation between G and C is to have the dipole consist of good (ideally perfect) conductors in the external medium with no other media included. Then G and C both come from the same solution of Laplace's equation, since σ and ε are both assumed independent of the electric field, and since boundary layer or plasma sheath problems are assumed negligible. Again define a time constant and a characteristic frequency for the sensor-load combination as

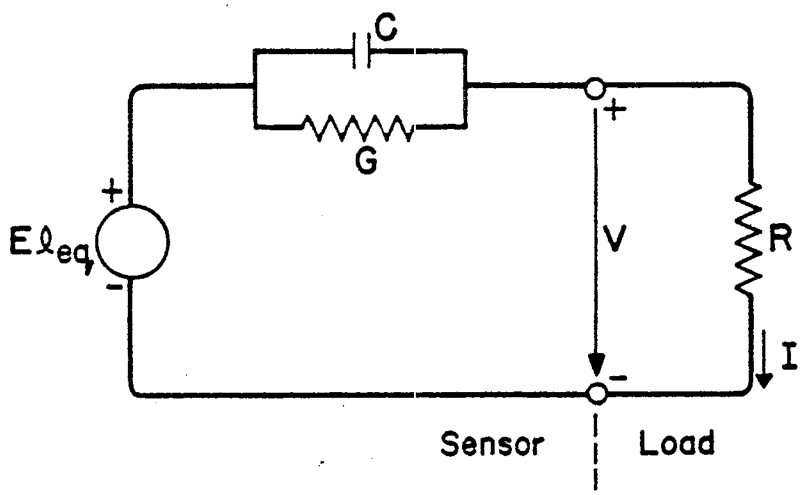
$$t_o = \frac{1}{\omega_o} = RC \qquad (27)$$

The Thévenin equivalent circuit has a response given by

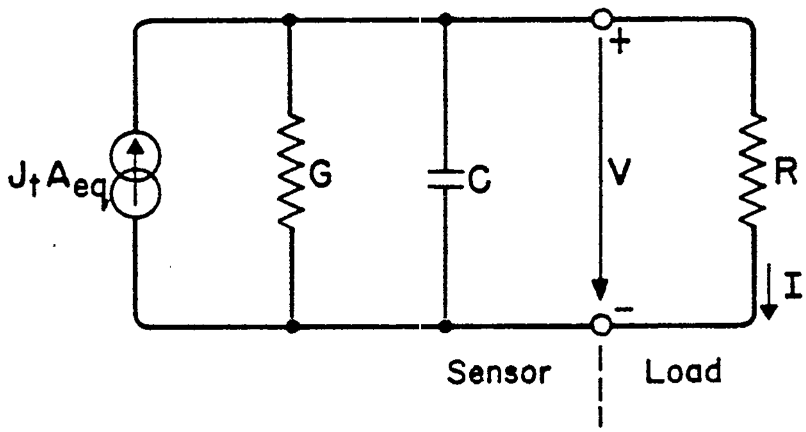
$$\tilde{V} = \tilde{E}\ell_{eq} \frac{R}{R + \frac{1}{G+sC}} = \tilde{E}\ell_{eq} \frac{GR+st_o}{1+GR+st_o} \qquad (28)$$

For $R \gg 1/G$, and/or frequencies of interest limited to $\omega \gg \omega_o$, this reduces to

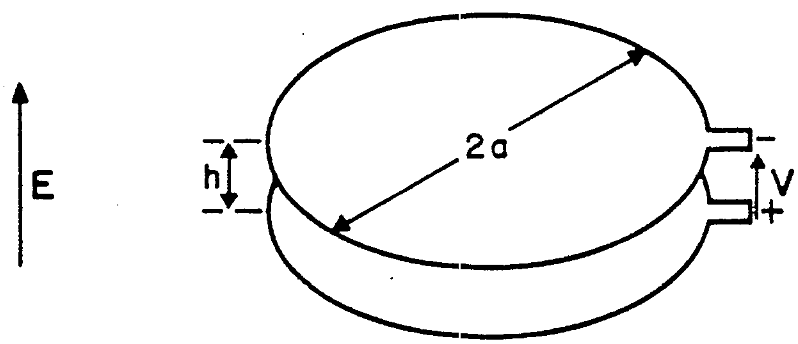
$$\tilde{V} = \tilde{E}\ell_{eq} \qquad (29)$$



A. Thévenin Equivalent Circuit



B. Norton Equivalent Circuit



C. Example: Parallel-Plate Dipole

Figure 2. ELECTRICALLY-SMALL DIPOLE

or in the time domain

$$V = E\ell_{eq} \tag{30}$$

A dipole designed to give this response might be called an E dipole. Equation (30) is used to define ℓ_{eq} from the open circuit voltage.

The Norton equivalent circuit has a response given by

$$\tilde{I} = \tilde{J}_t A_{eq} \frac{\frac{1}{R}}{\frac{1}{R} + G + sC} = \frac{\tilde{J}_t A_{eq}}{1+GR+st_o} \tag{31}$$

For $R<<1/G$, and frequencies of interest limited to $\omega<<\omega_o$, this reduces to

$$\tilde{I} = \tilde{J}_t A_{eq} \tag{32}$$

or in the time domain

$$I = J_t A_{eq} \tag{33}$$

A dipole designed to give this response might be called a total-current-density dipole. Equation (33) is used to define A_{eq} from the short circuit current.

Relate the parameters of the two equivalent circuits using equations (8) and (25). Then combining equations (28) and (31) gives

$$(GR+st_o)\ \ell_{eq} = (\sigma+s\varepsilon)\ A_{eq}R \tag{34}$$

or

$$(G+sC)\ \ell_{eq} = (\sigma+s\varepsilon)\ A_{eq} \tag{35}$$

If one desires that both ℓ_{eq} and A_{eq} be independent of frequency for frequencies of interest, then equation (35) requires that

$$\varepsilon A_{eq} = \ell_{eq}\ C \tag{36}$$

and

$$\sigma A_{eq} = \ell_{eq}\ G \tag{37}$$

These last two equations can be combined to give the restriction of equation (26) relating G and C through σ and ε, showing that this restriction is necessary for ℓ_{eq} and A_{eq} to be both independent of frequency. Note that the simple relations between the equivalent area and equivalent length for an electrically-small dipole (in equations (36) and (37)) are of the same form as that for an electrically-small loop (in equation (10)).

Next consider the effectiveness of an electrically-small dipole in extracting energy from a pulsed electric field. For this calculation (leading to an equivalent volume) we assume that the conductivity, σ, is zero; otherwise the total energy delivered to the dipole load could be infinite. For convenience let the electric field component of interest be of the form

$$E = E_o u(t) \tag{38}$$

The corresponding Laplace-transformed electric field is

$$\tilde{E} = \frac{E_o}{s} \tag{39}$$

Substituting this into equation (28) gives

$$\tilde{V} = \frac{E_o \ell_{eq} t_o}{1+st_o} \tag{40}$$

which in the time domain (for $t > 0$) is

$$V = E_o \ell_{eq} e^{-\frac{t}{t_o}} \tag{41}$$

This represents a power into the resistive load (for $t > 0$) of

$$P = \frac{V^2}{R} = \frac{E_o^2 \ell_{eq}^2}{R} e^{-2\frac{t}{t_o}} \tag{42}$$

and a total energy of

$$U = \int_0^\infty P dt = \frac{E_o^2 \ell_{eq}^2}{R} \int_0^\infty e^{-2\frac{t}{t_o}} dt = \frac{E_o^2 \ell_{eq}^2 C}{2} \tag{43}$$

As with the loop, the total energy delivered to the resistive load is independent of R, but depends only on the electric field and the dipole electrical parameters.

In a given equivalent volume, V_{eq}, the energy associated with E (after $t = 0$) is

$$U_e = \frac{\varepsilon E_o^2}{2} V_{eq} \tag{44}$$

We equate U_e and U to define V_{eq} as

$$V_{eq} = \frac{C \ell_{eq}^2}{\varepsilon} \tag{45}$$

Using equation (36) we have

$$V_{eq} = \frac{\varepsilon A_{eq}^2}{C} = A_{eq} \ell_{eq} = \frac{C \ell_{eq}^2}{\varepsilon} \tag{46}$$

Note the similarity in the results for the equivalent volumes for the electrically-small dipole and the electrically-small loop (equation (19)).

Consider the example of a parallel-plate dipole as illustrated in figure 2C for comparing the equivalent and geometrical volumes. The equivalent length is

$$\ell_{eq} = h \tag{47}$$

the capacitance is roughly (for a>>h)

$$C \approx \frac{\epsilon\pi a^2}{h} \tag{48}$$

and the equivalent area is roughly (for a>>h)

$$A_{eq} \approx \pi a^2 \tag{49}$$

The equivalent volume (from equation (46)) is then

$$V_{eq} \approx \pi a^2 h \tag{50}$$

which is also the geometric volume of the cylinder approximating the dipole shape. As with the loop, the dipole equivalent volume seems to be related to the geometry of the sensor.

Rewrite equation (46) for the equivalent volume in the form (substituting from equation (27))

$$V_{eq} = \epsilon R(A_{eq}^2 \omega_o) = \frac{1}{\epsilon R}\left(\frac{\ell_{eq}^2}{\omega_o}\right) \tag{51}$$

For a fixed R the equivalent volume then is proportional to sensitivity squared times bandwidth for both the total current density dipole and the E dipole. This is the same result as in the case of the loop. Note that these results for the dipole apply only to the assumed case of $\sigma = 0$. For $\sigma \neq 0$ the bandwidth of the two types of dipole change somewhat.

The response characteristics of an electrically-small dipole can then be improved by increasing the equivalent volume, which can be done by making the dipole larger. As for the loop we define a figure of merit, η, for the dipole as the ratio of the equivalent volume to a specified geometric volume into which the dipole should fit. Such geometric volumes might include spheres and circular cylinders (and, in some cases, hemispheres or hemicylinders) and various other appropriate shapes. For a given type of geometric volume one might then try to maximize the dipole figure of merit.

Actually for some cases of interest G and C need not be related as in equation (26). For the case of $\sigma = 0$ and $G = 0$, then there can even be additional insulating dielectric media in the immediate vicinity of the sensor; such media may affect A_{eq} and/or ℓ_{eq} but these two parameters will still be independent of frequency (consistent with the electrically-small restriction). If one appropriately restricts the frequency range of interest he can make the effect of the conductance or the capacitance negligible (compared to the effect of the other). As an example consider the total current density dipole for $\omega \ll \omega_o$ and for $GR \ll 1$; the capacitance is relatively unimportant and G may even be reduced through the addition of insulators, giving a frequency response as in equation (33) without making A_{eq} frequency dependent.[7] Thus, there may be advantage in some cases in not restricting the relation of G and C, particularly in cases where one of the two is of dominant concern.

IV. Summary

An electrically-small loop or dipole (under some restrictions) has an equivalent circuit with an equivalent area or equivalent length as a sensitivity to the appropriate electromagnetic quantity. The loop admittance is due to an inductance; the dipole admittance is due to a capacitance, possible in parallel with a conductance. One can use either a Thévenin or Norton form of the equivalent circuit; the Thévenin equivalent is more convenient if the desired response is based on the open circuit voltage, while the Norton equivalent is more convenient if the desired response is based on the short circuit current.

We define an equivalent volume for the sensor based on the amount of energy delivered from a step function field to a resistive load. This equivalent volume is related to the geometric volume of the sensor; it combines the electrical parameters of the sensor in a form which, for a frequency-independent resistive load, is proportional to sensitivity squared times bandwidth. For convenience we include the following table of some of the parameters for electrically-small antennas.

	loop	dipole
Open circuit voltage	$\dot{B}A_{eq}$	$E\ell_{eq}$
Short circuit current	$H\ell_{eq}$	$J_t A_{eq}$
Time constant, $t_o = \frac{1}{\omega_o}$	$\frac{L}{R}$	RC
Equivalent volume, V_{eq}	$\frac{\mu}{L} A_{eq}^2$	$\frac{\varepsilon}{C} A_{eq}^2$
	$A_{eq}\ell_{eq}$	$A_{eq}\ell_{eq}$
	$\frac{L}{\mu} \ell_{eq}^2$	$\frac{C}{\varepsilon} \ell_{eq}^2$
	$\frac{\mu}{R} (A_{eq}^2 \omega_o)$	$\varepsilon R (A_{eq}^2 \omega_o)$
	$\frac{R}{\mu} \left(\frac{\ell_{eq}^2}{\omega_o}\right)$	$\frac{1}{\varepsilon R} \left(\frac{\ell_{eq}^2}{\omega_o}\right)$

Table I. Parameters for Electrically-small Loops and Dipoles

7. Capt Carl E. Baum, Sensor and Simulation Note XXXIII, Two Types of Vertical Current Density Sensors, Feb. 1967.

A significant feature of the equivalent volume, as indicated in the last forms of V_{eq} in Table I, is that for fixed V_{eq} and fixed R it expresses a trade off between sensitivity and bandwidth. Sensitivity can be increased, but at the expense of bandwidth, and vice versa. Note, however, that the equivalent volume is somewhat arbitrary, in that we have defined it in terms of the energy delivered to a resistive load by a step function incident field. If the pulse shape of the incident field is changed a different equivalent volume can be defined. Using the step function incident field gives V_{eq} a simple form which is conveniently independent of R. In the case of the dipole we have assumed $\sigma = 0$ for the equivalent volume calculation, otherwise the energy delivered to R, as well as the bandwidth, can be quite different.

As part of an efficient sensor design, one might then desire to maximize the equivalent volume. Since the equivalent volume of the sensor is related to its geometric volume, one can then increase the geometric volume of the sensor in order to increase the equivalent volume. For one reason or another, however, one may wish to limit the physical size of the sensor. We find it then convenient to define a figure of merit, η, as the ratio of the equivalent volume to the volume of a chosen geometric figure, inside of which the sensor is placed. The figure of merit is then one quantitative measure of the efficiency with which the sensor utilizes or "fills" a particular chosen geometric volume. The figure-of-merit concept may then be a useful tool in electromagnetic sensor design.