NOTE 40

CONDUCTING SHIELDS FOR ELECTRICALLY-SMALL CYLINDRICAL LOOPS

by

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Abstract

The response characteristics of an electrically-small cylindrical loop with two external conducting shields is calculated. The shields have the effect of reducing the electric field sensitivity of the loop. The shield parameters may be chosen so that the shields do not appreciably affect the response of the loop to $\dot{B}$ for frequencies of interest. If the external medium is conducting the electric field sensitivity of the loop can also be reduced by placing insulating media around the loop in its immediate vicinity. For this note the electric field sensitivity is defined as the response of the loop to a particular term in an electromagnetic field expansion.

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ABSTRACT

The response characteristics of an electrically-small cylindrical loop with two external conducting shields is calculated. The shields have the effect of reducing the electric field sensitivity of the loop. The shield parameters may be chosen so that the shields do not appreciably affect the response of the loop to $\dot{B}$ for frequencies of interest. If the external medium is conducting the electric field sensitivity of the loop can also be reduced by placing insulating media around the loop in its immediate vicinity. For this note the electric field sensitivity is defined as the response of the loop to a particular term in an electromagnetic field expansion.

I. Introduction

A $\dot{B}$ loop is a common sensor for measuring pulsed magnetic fields. Typically, if the loop can be considered electrically small, i.e., if wavelengths or skin depths of interest (as appropriate) are much larger than the sensor dimensions, then the response characteristics of the loop simplify somewhat. The loop can then be characterized by an equivalent area, giving the sensitivity of the loop to $\dot{B}$, and an inductance.

Actually the characterization of the loop as sensitive only to an incident $\dot{B}$ is an approximation. For simplicity we often consider the response of such sensors to an incident plane wave. Depending on the coordinate system used for the analysis (e.g., cylindrical or spherical coordinates centered on the sensor) the plane wave can be expanded as an infinite series of some characteristic functions. Corresponding to each term in such a series there is a current pattern on the sensor. For an electrically-small sensor one of these current patterns is generally dominant and of primary interest. For an appropriately oriented $\dot{B}$ loop this dominant term is associated with the magnetic field in the incident wave near the sensor.

Suppose, however, that the incident wave is not a simple plane wave. Then in the expansion of the incident wave the relative size of the various terms can be quite different from the case of a plane wave. Specifically, the term(s) associated with the magnetic field near the sensor may be considerably reduced to the point that other terms may give comparatively significant currents on the sensor, even though the sensor may be electrically small. Then one may be required to consider even lower frequencies such that the currents on the sensor associated with the magnetic field terms are dominant. One can also associate one or more terms in the electromagnetic field expansion with the electric field near the sensor. The response of a loop to such terms is then sometimes called the electric field sensitivity of the loop, although this is a somewhat simplified concept.

An interesting question is how to minimize the influence of unwanted terms in the expansion of the incident electromagnetic fields. One approach to this problem consists of completely enclosing the sensor in one or more conducting shields. In conjunction with these shields some of the electromagnetic parameters of the media near the sensor are altered (in certain cases) to further improve the response characteristics. One can think of the conducting shields as allowing the magnetic field to penetrate to the sensor for frequencies of interest for a measurement, while the shields short out the electric field before it reaches the sensor.

In order to illustrate some of the effects of such shields and to obtain some quantitative estimate of the response characteristics of such shielded loops, we consider a simplified problem. We consider the response of three concentric conducting shells to an incident plane wave, propagating perpendicular to the axis of the shells. The innermost shell represents a cylindrical loop (of some unspecified number of turns and sensitive to the z component of the magnetic field); the outer two shells are the shields. The cylinders are assumed of infinite length for the calculations so that the results apply for the length much greater than the diameter. In the case of finite length cylinders we might extend the shields slightly and cap the ends with the same type of conducting sheet(s) so as to minimize any coupling from the loop to the electric field through the otherwise open ends.

Expanding the incident wave in cylindrical coordinates we solve the above problem. This solution relates the loop response to the incident magnetic field in terms of loop and shield parameters. By making the surface conductance of the inner cylinder (the loop) infinite we calculate the short circuit surface current density associated with the term corresponding to the electric field near the sensor. The results of this calculation show the effect of the shields in reducing the currents associated with this term in the field expansion. The electromagnetic parameters of the media near the sensor also play a significant role in these currents. The currents associated with the incident electric field are considered for both the case in which the external medium is nonconducting and in which it is highly conducting.

II. Boundary Value Problem

Consider then the response of the multiple cylindrical conducting sheets to an incident plane wave as illustrated in figure 1A. There are 3 cylindrical conducting sheets with parameters identified by subscripts 1 through 3 in order of increasing radius. The innermost conducting sheet represents a cylindrical loop; the remainder of the conducting sheets are shields. The surface conductance is G_s and the surface current density is J_s. There are also 4 separate media defined by the cylindrical conducting sheets, again with parameters subscripted in order of increasing radius except for the external medium which has unsubscripted parameters. These media are assumed to be characterized by a permittivity, ϵ, a permeability, μ, and a conductivity, σ, which are scalar parameters independent of both time and position. Associated with each of the media we have two other parameters of the form

$$k = \sqrt{-j\omega\mu(\sigma+j\omega\epsilon)} \tag{1}$$

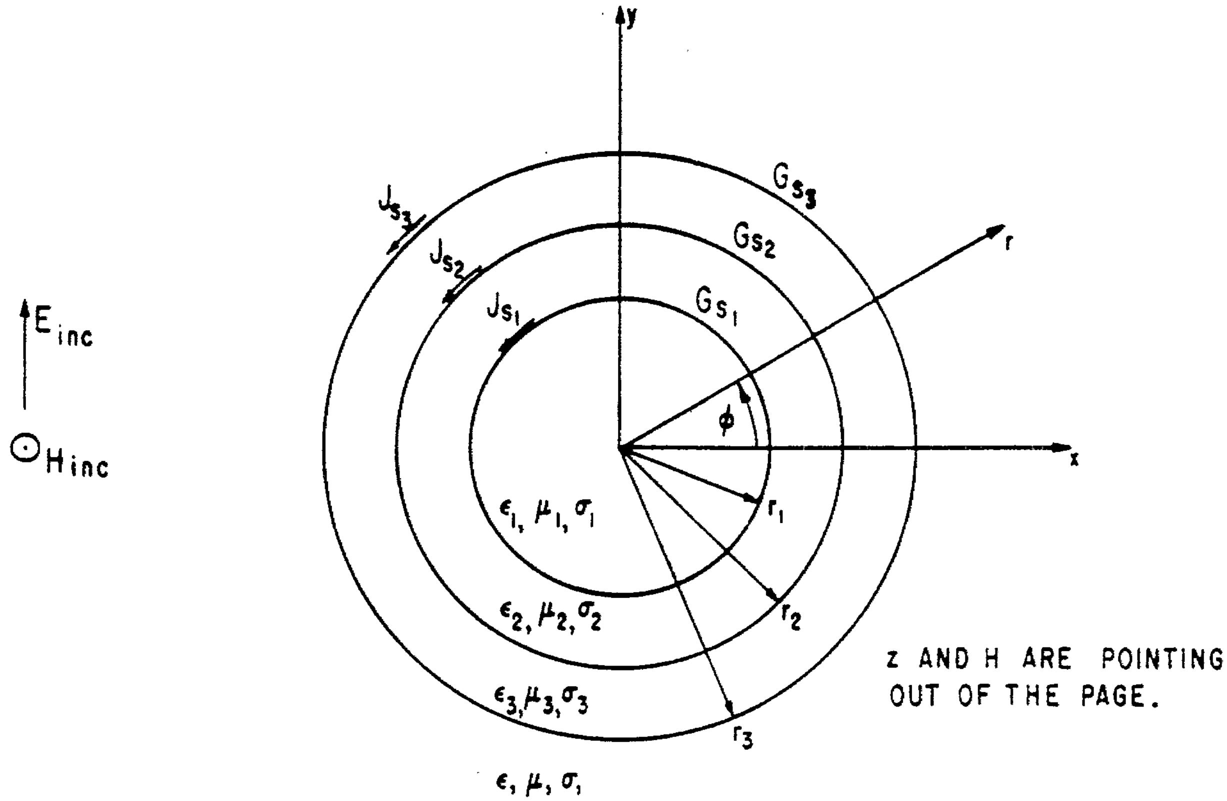
for the propagation constant and

$$Z = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} \tag{2}$$

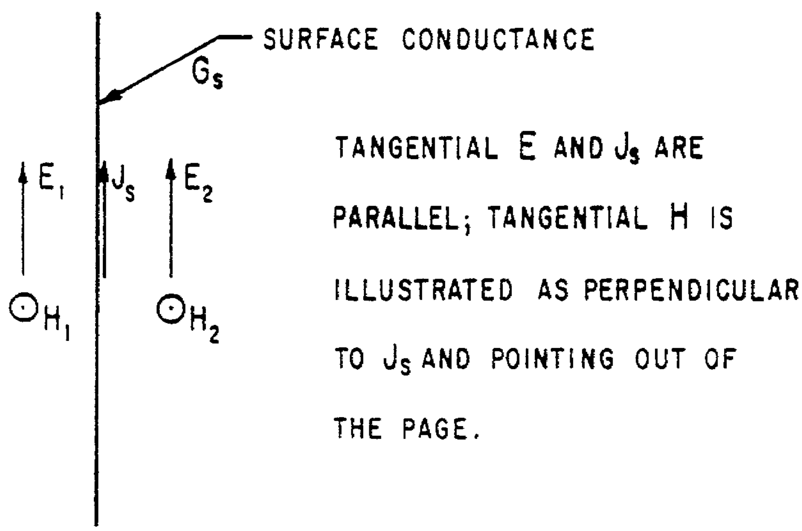
for the wave impedance. There is also a convenient relationship using these two parameters of the form

$$\frac{Z_\ell}{Z_m} = \frac{\mu_\ell}{\mu_m}\frac{k_m}{k_\ell} \tag{3}$$

where ℓ and m are subscripts applying to two different media.



A. MULTIPLE SHIELD GEOMETRY



B. CONDUCTING SHEET BOUNDARY CONDITIONS

FIGURE 1 BOUNDARY VALUE PROBLEM

Since we are assuming an incident wave with only a z component of the magnetic field (which is also z independent) the field expansions are of the form[1]

$$H_z = H_{z_o} \sum_{n=0}^{\infty} a_n C_n^{(\ell)}(kr) \begin{Bmatrix} \cos(n\phi) \\ \sin(n\phi) \end{Bmatrix} \tag{4}$$

$$E_r = -jZH_{z_o} \sum_{n=0}^{\infty} a_n \frac{C_n^{(\ell)}(kr)}{kr} n \begin{Bmatrix} -\sin(n\phi) \\ \cos(n\phi) \end{Bmatrix} \tag{5}$$

and

$$E_\phi = jZH_{z_o} \sum_{n=0}^{\infty} a_n C_n^{(\ell)\prime}(kr) \begin{Bmatrix} \cos(n\phi) \\ \sin(n\phi) \end{Bmatrix} \tag{6}$$

where $C_n^{(\ell)}(kr)$ denotes one of the cylindrical Bessel functions and where a prime over a Bessel function denotes the derivative with respect to the argument. The braces around the trigonometric functions indicate a linear combination of the two functions, the same linear combination being used for all three components. A time dependence of the form $e^{j\omega t}$ is assumed but is suppressed from all the expressions.

The incident wave is taken as

$$\vec{H}_{inc} = H_{z_o} \vec{e}_z e^{-jkx} = H_{z_o} \vec{e}_z e^{-jkr\cos(\phi)} \tag{7}$$

and

$$\vec{E}_{inc} = ZH_{z_o} \vec{e}_y e^{-jkx} = ZH_{z_o} \vec{e}_y e^{-jkr\cos(\phi)} \tag{8}$$

where $\vec{e}_y$ and $\vec{e}_z$ are unit vectors in the y and z directions, respectively. Expanding the incident magnetic field in cylindrical coordinates gives[2]

$$H_{z_{inc}} = H_{z_o} \left[J_o(kr) + 2 \sum_{n=1}^{\infty} (-j)^n J_n(kr) \cos(n\phi) \right] \tag{9}$$

Associated with this there is an azimuthal electric field of the form

1. J. A. Stratton, Electromagnetic Theory (Chap. VI), 1941.

2. See AMS 55, Handbook of Mathematical Functions, National Bureau of Standards, 1964, for the expansions of cos[krcos(ϕ)] and sin[krcos(ϕ)].

$$E_{\phi_{inc}} = jZH_{z_o}\left[J_o'(kr) + 2\sum_{n=1}^{\infty}(-j)^n J_n'(kr)\cos(n\phi)\right] \tag{10}$$

There is also an associated radial electric field but this is not needed in the problem solution and is not listed with the field expansions in the various media. Expanding the terms in equations (9) and (10) in powers of kr one can note that for small $|kr|$ the magnetic field is associated with the first (or n=0) term in equation (9) while the electric field is associated with the second (or n=1) term in equation (10) (together with the corresponding term in $E_{r_{inc}}$). As one example of varying the ratio of electric and magnetic fields we might think of adding another plane wave travelling in the -x direction with the electric field (at the origin) in the same direction as that in equation (8) but with the magnetic field in the opposite direction to that in equation (7). In the cylindrical coordinate expansion this would increase the magnitude of the n=1 term in relation to the magnitude of the n=0 term. Thus, we consider the effect of the conducting shields in varying the currents on the sensor associated with the first two terms in the field expansions. Ideally we would like to reduce the currents associated with the n=1 term without significantly affecting those associated with the n=0 term. We might then say that the shields decrease the electric field sensitivity of the sensor.

In addition to the incident wave there are several other field expansions needed. In the external medium there is a reflected wave of the form

$$H_{z_{refl}} = H_{z_o}\left[c_o H_o^{(2)}(kr) + 2\sum_{n=1}^{\infty}(-j)^n c_n H_n^{(2)}(kr)\cos(n\phi)\right] \tag{11}$$

and

$$E_{\phi_{refl}} = jZH_{z_o}\left[c_o H_o^{(2)'}(kr) + 2\sum_{n=1}^{\infty}(-j)^n c_n H_n^{(2)'}(kr)\cos(n\phi)\right] \tag{12}$$

The regions inside the cylindrical sheets also have field expansions which are of the form for region 1

$$H_{z_1} = H_{z_o}\left[a_{1_o} J_o(k_1 r)+2\sum_{n=1}^{\infty}(-j)^n a_{1_n} J_n(k_1 r)\cos(n\phi)\right] \tag{13}$$

and

$$E_{\phi_1} = jZ_1 H_{z_o}\left[a_{1_o} J_o'(k_1 r)+2\sum_{n=1}^{\infty}(-j)^n a_{1_n} J_n'(k_1 r)\cos(n\phi)\right] \tag{14}$$

for region 2

$$H_{z_2} = H_{z_o}\left\{\left[a_{2_o}J_o(k_2r)+b_{2_o}Y_o(k_2r)\right] +2\sum_{n=1}^{\infty}(-j)^n\left[a_{2_n}J_n(k_2r)+b_{2_n}Y_n(k_2r)\right]\cos(n\phi)\right\} \quad (15)$$

and

$$E_{\phi_2} = jZ_2H_{z_o}\left\{\left[a_{2_o}J'_o(k_2r)+b_{2_o}Y'_o(k_2r)\right] +2\sum_{n=1}^{\infty}(-j)^n\left[a_{2_n}J'_n(k_2r)+b_{2_n}Y'_n(k_2r)\right]\cos(n\phi)\right\} \quad (16)$$

and for region 3

$$H_{z_3} = H_{z_o}\left\{\left[a_{3_o}J_o(k_3r)+b_{3_o}Y_o(k_3r)\right] +2\sum_{n=1}^{\infty}(-j)^n\left[a_{3_n}J_n(k_3r)+b_{3_n}Y_n(k_3r)\right]\cos(n\phi)\right\} \quad (17)$$

and

$$E_{\phi_3} = jZ_3H_{z_o}\left\{\left[a_{3_o}J'_o(k_3r)+b_{3_o}Y'_o(k_3r)\right] +2\sum_{n=1}^{\infty}(-j)^n\left[a_{3_n}J'_n(k_3r)+b_{3_n}Y'_n(k_3r)\right]\cos(n\phi)\right\} \quad (18)$$

In order to calculate the various coefficients in the field expansions, we need to relate the field components at the conducting cylindrical sheets. The manner of doing this is illustrated in figure 1B. The conducting sheet is assumed to be much thinner than other dimensions of interest and also to be much thinner than a skin depth for frequencies of interest. It is then approximated as having zero thickness. The conducting sheet is then characterized by a surface conductance, G_s, which relates the surface current density to the tangential components of the electric field (parallel to the surface current density) on both sides of the sheet as

$$E_1 = E_2 = \frac{J_s}{G_s} \quad (19)$$

The tangential components of the magnetic field which are perpendicular to the surface current density are discontinuous across the sheet by the relation

$$H_1 - H_2 = J_s \quad (20)$$

Note that in this problem the tangential electric field is only in the ϕ direction while the tangential magnetic field is in the z direction so that H_z and J_s are perpendicular.

Now apply the boundary conditions of tangential E continuous and tangential H discontinuous by an amount, J_s, at the conducting sheets. This gives at $r = r_1$

$$a_{1_n}Z_1J'_n(k_1r_1) = a_{2_n}Z_2J'_n(k_2r_1)+b_{2_n}Z_2Y'_n(k_2r_1) \quad (21)$$

and

$$a_{1_n}\left[J_n(k_1r_1)-jG_{s_1}Z_1J_n'(k_1r_1)\right] = a_{2_n}J_n(k_2r_1)+b_{2_n}Y_n(k_2r_1) \tag{22}$$

at $r = r_2$

$$a_{2_n}Z_2J_n'(k_2r_2)+b_{2_n}Z_2Y_n'(k_2r_2) = a_{3_n}Z_3J_n'(k_3r_2)+b_{3_n}Z_3Y_n'(k_3r_2) \tag{23}$$

and

$$a_{2_n}\left[J_n(k_2r_2)-jG_{s_2}Z_2J_n'(k_2r_2)\right] +b_{2_n}\left[Y_n(k_2r_2)-jG_{s_2}Z_2Y_n'(k_2r_2)\right]$$
$$= a_{3_n}J_n(k_3r_2)+b_{3_n}Y_n(k_3r_2) \tag{24}$$

and at $r = r_3$

$$a_{3_n}Z_3J_n'(k_3r_3)+b_{3_n}Z_3Y_n'(k_3r_3) = ZJ_n'(kr_3)+c_nZH_n^{(2)\,'}(kr_3) \tag{25}$$

and

$$a_{3_n}\left[J_n(k_3r_3)-jG_{s_3}Z_3J_n'(k_3r_3)\right] +b_{3_n}\left[Y_n(k_3r_3)-jG_{s_3}Z_3Y_n'(k_3r_3)\right]$$
$$= J_n(kr_3) + c_nH_n^{(2)}(kr_3) \tag{26}$$

Now arrange these six equations in a more convenient form by combining them by pairs and substituting for some of the wave impedances from equation (3). A Wronskian relationship is used to simplify some of the combinations of Bessel functions.[3] Solve for a_{2_n} and b_{2_n} in terms of a_{1_n} from equations (21) and (22) giving

$$a_{2_n} = -a_{1_n}\frac{\pi k_2r_1}{2}\left\{\frac{\mu_1k_2}{\mu_2k_1}J_n'(k_1r_1)Y_n(k_2r_1)-Y_n'(k_2r_1)\left[J_n(k_1r_1)-jG_{s_1}Z_1J_n'(k_1r_1)\right]\right\} \tag{27}$$

and

$$b_{2_n} = a_{1_n}\frac{\pi k_2r_1}{2}\left\{\frac{\mu_1k_2}{\mu_2k_1}J_n'(k_1r_1)J_n(k_2r_1)-J_n'(k_2r_1)\left[J_n(k_1r_1)-jG_{s_1}Z_1J_n'(k_1r_1)\right]\right\} \tag{28}$$

3. See reference 2 for the various Bessel function relationships.

Solve for a_{3_n} and b_{3_n} in terms of a_{2_n} and b_{2_n} from equations (23) and (24) giving

$$a_{3_n} = -a_{2_n} \frac{\pi k_3 r_2}{2} \left\{ \frac{\mu_2 k_3}{\mu_3 k_2} J_n'(k_2r_2) Y_n(k_3r_2) - Y_n'(k_3r_2) \left[J_n(k_2r_2) - jG_{s_2} Z_2 J_n'(k_2r_2) \right] \right\}$$

$$-b_{2_n} \frac{\pi k_3 r_2}{2} \left\{ \frac{\mu_2 k_3}{\mu_3 k_2} Y_n'(k_2r_2) Y_n(k_3r_2) - Y_n'(k_3r_2) \left[Y_n(k_2r_2) - jG_{s_2} Z_2 Y_n'(k_2r_2) \right] \right\} \quad (29)$$

and

$$b_{3_n} = a_{2_n} \frac{\pi k_3 r_2}{2} \left\{ \frac{\mu_2 k_3}{\mu_3 k_2} J_n'(k_2r_2) J_n(k_3r_2) - J_n'(k_3r_2) \left[J_n(k_2r_2) - jG_{s_2} Z_2 J_n'(k_2r_2) \right] \right\}$$

$$+b_{2_n} \frac{\pi k_3 r_2}{2} \left\{ \frac{\mu_2 k_3}{\mu_3 k_2} Y_n'(k_2r_2) J_n(k_3r_2) - J_n'(k_3r_2) \left[Y_n(k_2r_2) - jG_{s_2} Z_2 Y_n'(k_2r_2) \right] \right\} \quad (30)$$

Relate a_{3_n} and b_{3_n} from equations (25) and (26) giving

$$1 = -a_{3_n} \frac{j\pi k r_3}{2} \left\{ \frac{\mu_3 k}{\mu k_3} J_n'(k_3r_3) H_n^{(2)}(kr_3) - H_n^{(2)\prime}(kr_3) \left[J_n(k_3r_3) - jG_{s_3} Z_3 J_n'(k_3r_3) \right] \right\}$$

$$-b_{3_n} \frac{j\pi k r_3}{2} \left\{ \frac{\mu_3 k}{\mu k_3} Y_n'(k_3r_3) H_n^{(2)}(kr_3) - H_n^{(2)\prime}(kr_3) \left[Y_n(k_3r_3) - jG_{s_3} Z_3 Y_n'(k_3r_3) \right] \right\} \quad (31)$$

Equations (27) through (31) can be combined to solve for a_{1_n}, but before doing this let us make some more simplifications.

Since we are primarily concerned with the case of electrically-small sensors in these calculations we can expand the Bessel functions for small arguments as

$$\left.\begin{array}{ll} J_o(z) \simeq 1 & J_o'(z) = -J_1(z) \simeq -\frac{z}{2} \\ J_1(z) \simeq \frac{z}{2} & J_1'(z) \simeq \frac{1}{2} \\ Y_o(z) \simeq \frac{2}{\pi}\ln(z) & Y_o'(z) = -Y_1(z) \simeq \frac{2}{\pi z} \\ Y_1(z) \simeq -\frac{2}{\pi z} & Y_1'(z) \simeq \frac{2}{\pi z^2} \end{array}\right] \quad (32)$$

Note that we are primarily interested in a_{1_o} and a_{1_1} so that equation (32) covers only the cases of $n = 0$ and $n = 1$ for the Bessel functions and their derivatives.

Consider first the case of $n = 0$. Equations (27) through (31) simplify to

$$a_{2_o} \simeq a_{1_o}\left[1 + \frac{j\omega\mu_1 r_1 G_{s_1}}{2}\right] \tag{33}$$

$$b_{2_o} \simeq a_{1_o}\frac{\pi}{4}(k_2 r_1)^2\left[1 - \frac{\mu_1}{\mu_2} + \frac{j\omega\mu_1 r_1 G_{s_1}}{2}\right] \tag{34}$$

$$a_{3_o} \simeq a_{2_o}\left[1 + \frac{j\omega\mu_2 r_2 G_{s_2}}{2}\right] + b_{2_o}\frac{2}{\pi(k_2 r_2)^2}\left[-j\omega\mu_2 r_2 G_{s_2}\right] \tag{35}$$

$$b_{3_o} \simeq a_{2_o}\frac{\pi}{4}(k_3 r_2)^2\left[1 - \frac{\mu_2}{\mu_3} + \frac{j\omega\mu_2 r_2 G_{s_2}}{2}\right] + b_{2_o}\left(\frac{k_3}{k_2}\right)^2\left[\frac{\mu_2}{\mu_3} - \frac{j\omega\mu_2 r_2 G_{s_2}}{2}\right] \tag{36}$$

and

$$1 \simeq a_{3_o}\left[1 + \frac{j\omega\mu_3 r_3 G_{s_3}}{2}\right] + b_{3_o}\frac{2}{\pi(k_3 r_3)^2}\left[-j\omega\mu_3 r_3 G_{s_3}\right] \tag{37}$$

Combining these equations we have

$$a_{1_o} \simeq \left\{\left\{\left[1 + \frac{j\omega\mu_1 r_1 G_{s_1}}{2}\right]\left[1 + \frac{j\omega\mu_2 r_2 G_{s_2}}{2}\right] - \frac{j\omega\mu_2 r_1^2 G_{s_2}}{2 r_2}\left[1 - \frac{\mu_1}{\mu_2} + \frac{j\omega\mu_1 r_1 G_{s_1}}{2}\right]\right\}\left[1 + \frac{j\omega\mu_3 r_3 G_{s_3}}{2}\right] - \frac{j\omega\mu_3 r_3 G_{s_3}}{2}\left(\left(\frac{r_2}{r_3}\right)^2\left[1 + \frac{j\omega\mu_1 r_1 G_{s_1}}{2}\right]\left[1 - \frac{\mu_2}{\mu_3} + \frac{j\omega\mu_2 r_2 G_{s_2}}{2}\right] + \left(\frac{r_1}{r_3}\right)^2\left[\frac{\mu_2}{\mu_3} - \frac{j\omega\mu_2 r_2 G_{s_2}}{2}\right]\left[1 - \frac{\mu_1}{\mu_2} + \frac{j\omega\mu_1 r_1 G_{s_1}}{2}\right]\right)\right\}^{-1} \tag{38}$$

Next consider the case of $n = 1$. Equations (27) through (31) simplify to

$$a_{2_1} \simeq a_{1_1}\left[\frac{1}{2}\frac{\mu_1 k_2}{\mu_2 k_1} + \frac{1}{2}\frac{k_1}{k_2} - j\frac{G_{s_1} Z_1}{2 k_2 r_1}\right] \tag{39}$$

$$b_{2_1} \simeq a_{1_1}\frac{\pi}{4}(k_2 r_1)^2\left[\frac{1}{2}\frac{\mu_1 k_2}{\mu_2 k_1} - \frac{1}{2}\frac{k_1}{k_2} + \frac{j G_{s_1} Z_1}{2 k_2 r_1}\right] \tag{40}$$

$$a_{3_1} \simeq a_{2_1}\left[\frac{1}{2}\,\frac{\mu_2 k_3}{\mu_3 k_2} + \frac{1}{2}\,\frac{k_2}{k_3} - \frac{jG_{s_2}Z_2}{2k_3 r_2}\right] + b_{2_1}\,\frac{4}{\pi(k_2 r_2)^2}\left[\frac{1}{2}\,\frac{\mu_2 k_3}{\mu_3 k_2} - \frac{1}{2}\,\frac{k_2}{k_3} - \frac{jG_{s_2}Z_2}{2k_3 r_2}\right] \tag{41}$$

$$b_{3_1} \simeq a_{2_1}\,\frac{\pi}{4}\,(k_3 r_2)^2\left[\frac{1}{2}\,\frac{\mu_2 k_3}{\mu_3 k_2} - \frac{1}{2}\,\frac{k_2}{k_3} + \frac{jG_{s_2}Z_2}{2k_3 r_2}\right] + b_{2_1}\left(\frac{k_3}{k_2}\right)^2\left[\frac{1}{2}\,\frac{\mu_2 k_3}{\mu_3 k_2} + \frac{1}{2}\,\frac{k_2}{k_3} + \frac{jG_{s_2}Z_2}{2k_3 r_2}\right] \tag{42}$$

and

$$1 \simeq a_{3_1}\left[\frac{1}{2}\,\frac{\mu_3 k}{\mu k_3} + \frac{1}{2}\,\frac{k_3}{k} - \frac{jG_{s_3}Z_3}{2k r_3}\right] + b_{3_1}\,\frac{4}{\pi(k_3 r_3)^2}\left[\frac{1}{2}\,\frac{\mu_3 k}{\mu k_3} - \frac{1}{2}\,\frac{k_3}{k} - \frac{jG_{s_3}Z_3}{2k r_3}\right] \tag{43}$$

Combining these equations we have

$$\begin{aligned}
a_{1_1} = \Bigg\{ &\Bigg\{\left[\frac{1}{2}\,\frac{\mu_1 k_2}{\mu_2 k_1} + \frac{1}{2}\,\frac{k_1}{k_2} - \frac{jG_{s_1}Z_1}{2k_2 r_1}\right]\left[\frac{1}{2}\,\frac{\mu_2 k_3}{\mu_3 k_2} + \frac{1}{2}\,\frac{k_2}{k_3} - \frac{jG_{s_2}Z_2}{2k_3 r_2}\right] \\
&+ \left(\frac{r_1}{r_2}\right)^2\left[\frac{1}{2}\,\frac{\mu_1 k_2}{\mu_2 k_1} - \frac{1}{2}\,\frac{k_1}{k_2} + \frac{jG_{s_1}Z_1}{2k_2 r_1}\right]\left[\frac{1}{2}\,\frac{\mu_2 k_3}{\mu_3 k_2} - \frac{1}{2}\,\frac{k_2}{k_3} - \frac{jG_{s_2}Z_2}{2k_3 r_2}\right]\Bigg\}\left[\frac{1}{2}\,\frac{\mu_3 k}{\mu k_3} + \frac{1}{2}\,\frac{k_3}{k} - \frac{jG_{s_3}Z_3}{2k r_3}\right] \\
&+ \Bigg\{\left(\frac{r_2}{r_3}\right)^2\left[\frac{1}{2}\,\frac{\mu_1 k_2}{\mu_2 k_1} + \frac{1}{2}\,\frac{k_1}{k_2} - \frac{jG_{s_1}Z_1}{2k_2 r_1}\right]\left[\frac{1}{2}\,\frac{\mu_2 k_3}{\mu_3 k_2} - \frac{1}{2}\,\frac{k_2}{k_3} + \frac{jG_{s_2}Z_2}{2k_3 r_2}\right] \\
&+ \left(\frac{r_1}{r_3}\right)^2\left[\frac{1}{2}\,\frac{\mu_1 k_2}{\mu_2 k_1} - \frac{1}{2}\,\frac{k_1}{k_2} + \frac{jG_{s_1}Z_1}{2k_2 r_1}\right]\left[\frac{1}{2}\,\frac{\mu_2 k_3}{\mu_3 k_2} + \frac{1}{2}\,\frac{k_2}{k_3} + \frac{jG_{s_2}Z_2}{2k_3 r_2}\right]\Bigg\}\left[\frac{1}{2}\,\frac{\mu_3 k}{\mu k_3} - \frac{1}{2}\,\frac{k_3}{k} - \frac{jG_{s_3}Z_3}{2k r_3}\right]\Bigg\}^{-1}
\end{aligned} \tag{44}$$

The expressions for a_{1_0} and a_{1_1} are still rather complicated. We thus go on to consider some special cases to simplify the results and more readily estimate the effects of such conducting shields.

III. Effect of Conducting Shields on Loop Response

The presence of the conducting shields affects the response characteristics of the loop in at least two ways. The frequency response of the loop (for measuring $\dot{B}$) is lowered somewhat, and the currents on the loop structure associated with the electric field are significantly reduced. These two effects are included in the two coefficients given in equations (38) and (44).

A. Effect of Conducting Shields on Response to Magnetic Field

We now look at some special cases for a_{1_o}. For convenience we define characteristic frequencies of the form

$$\omega_{h_m} \equiv \frac{2}{\mu_m r_m G_{s_m}} \tag{45}$$

where m is 1, 2, or 3. The cylindrical loop has the characteristic frequency, ω_{h_1}, which one can think of as the load resistance connected to the loop, divided by the loop inductance (ignoring the presence of the shields). The other two characteristic frequencies, ω_{h_2} and ω_{h_3}, are associated with the shield parameters. The coefficient, a_{1_o}, represents both the penetration of a unit magnetic field (sinusoidal) inside the cylindrical loop and the response of the loop to $\dot{B}$, normalized by dividing by the limiting form of this response for low frequencies.

Consider the special case that the permeabilities of all four media are the same. (Typically the permeabilities would be μ_o.) Equation (38) then becomes

$$a_{1_o} \simeq \left\{ \left\{ \left[1 + \frac{j\omega}{\omega_{h_1}}\right]\left[1 + \frac{j\omega}{\omega_{h_2}}\right] - \left(\frac{r_1}{r_2}\right)^2 \frac{j\omega}{\omega_{h_1}}\frac{j\omega}{\omega_{h_2}} \right\}\left[1 + \frac{j\omega}{\omega_{h_3}}\right] \right.$$

$$\left. - \frac{j\omega}{\omega_{h_3}}\left\{ \left(\frac{r_2}{r_3}\right)^2 \left[1 + \frac{j\omega}{\omega_{h_1}}\right]\frac{j\omega}{\omega_{h_2}} + \left(\frac{r_1}{r_3}\right)^2 \left[1 - \frac{j\omega}{\omega_{h_2}}\right]\frac{j\omega}{\omega_{h_1}} \right\} \right\}^{-1} \tag{46}$$

Suppose we remove the two shields (by making G_{s_2} and G_{s_3} both zero). Then a_{1_o} has the very simple form

$$a_{1_o} \simeq \left[1 + \frac{j\omega}{\omega_{h_1}}\right]^{-1} \tag{47}$$

This is the normalized response characteristic of an electrically-small $\dot{B}$ loop. Setting only G_{s_3} to zero to leave one shield plus the sensor gives

$$a_{1_o} \simeq \left\{ \left[1 + \frac{j\omega}{\omega_{h_1}}\right]\left[1 + \frac{j\omega}{\omega_{h_2}}\right] - \left(\frac{r_1}{r_2}\right)^2 \frac{j\omega}{\omega_{h_1}}\frac{j\omega}{\omega_{h_2}} \right\}^{-1} \tag{48}$$

Note the progressive complication of the mathematical form of the response in going from zero to two shields. A three shield case would undoubtedly be significantly more complicated.

One approximate form for equation (46) is to neglect terms of order ω^2 and higher in $(a_{1_o})^{-1}$. This gives

$$a_{1_o} \simeq \left[1 + \frac{j\omega}{\omega_{h_1}} + \frac{j\omega}{\omega_{h_2}} + \frac{j\omega}{\omega_{h_3}}\right]^{-1} \tag{49}$$

Considering the loop as a $\dot{B}$ loop we are roughly interested in frequencies below ω_{h_1}, and we assume that at $\omega = \omega_{h_1}$ the loop is electrically small. Then from equation (49) one can see that the response of the loop below ω_{h_1} will not be significantly affected if both ω_{h_2} and ω_{h_3} are made somewhat larger than ω_{h_1}. Thus, the shields can be made to have negligible effect on the response of the loop to the magnetic field.

B. Effect of Conducting Shields on Response to Electric Field

In comparing the response of the loop to the electric field to its response to the magnetic field it is convenient to consider the short circuit surface current densities. Typically, the cylindrical loop has one or more turns made of good conductors and a resistive load is introduced at some position on the loop structure. Thus, the loop structure does not behave just like a conducting cylindrical sheet. However, if we consider the short circuit current by making the load resistance zero, the loop structure can be approximated as a continuous, perfectly conducting sheet (for conducting azimuthal (ϕ) currents). The surface current density on the loop associated with the $n = 0$ term is

$$G_{s_1} jZ_1 H_{z_o} a_{1_o} J_o'(k_1 r_1) \simeq -jG_{s_1} Z_1 H_{z_o} a_{1_o} \frac{k_1 r_1}{2} = -\frac{j\omega}{\omega_{h_1}} H_{z_o} a_{1_o} \tag{50}$$

and the surface current density associated with the $n = 1$ term is

$$2G_{s_1} Z_1 H_{z_o} a_{1_1} J_1'(k_1 r_1)\cos(\phi) \simeq G_{s_1} Z_1 H_{z_o} a_{1_1} \cos(\phi) \tag{51}$$

To obtain the short circuit current densities associated with these terms one takes the limit of arbitrarily large G_{s_1}.

In the limit of large G_{s_1}, and for frequencies much less than ω_{h_2} and ω_{h_3}, one has from equation (46) that

$$a_{1_o} \approx \left(\frac{j\omega}{\omega_{h_1}}\right)^{-1} \tag{52}$$

Combining this with equation (50) shows that the short circuit surface current density associated with the magnetic field is $-H_{z_o}$ for frequencies of interest. Thus, we define a new expression as

$$a' = \lim_{G_{s_1}\to\infty} G_{s_1} Z_1 a_{1_1} \tag{53}$$

When multiplied by $-\cos(\phi)$ this represents the ratio of the short circuit current density associated with the electric field to that associated with the magnetic field. This parameter, a', then gives the relative contribution of the term associated with the electric field, in a plane wave, to the sensor signal. The effectiveness of the shields and other associated media in reducing the electric field sensitivity of the loop is reflected in a decrease in $|a'|$.

From equations (44) and (53) a' can now be calculated giving

$$\begin{aligned} a' \approx j2k_2r_1 \Bigg\{ & \left\{\left[\frac{1}{2}\frac{\mu_2k_3}{\mu_3k_2} + \frac{1}{2}\frac{k_2}{k_3} - \frac{jG_{s_2}Z_2}{2k_3r_2}\right] - \left(\frac{r_1}{r_2}\right)^2\left[\frac{1}{2}\frac{\mu_2k_3}{\mu_3k_2} - \frac{1}{2}\frac{k_2}{k_3} - \frac{jG_{s_2}Z_2}{2k_3r_2}\right]\right\} \\ & \cdot \left[\frac{1}{2}\frac{\mu_3k}{\mu k_3} + \frac{1}{2}\frac{k_3}{k} - \frac{jG_{s_3}Z_3}{2kr_3}\right] \\ & + \left\{\left(\frac{r_2}{r_3}\right)^2\left[\frac{1}{2}\frac{\mu_2k_3}{\mu_3k_2} - \frac{1}{2}\frac{k_2}{k_3} + \frac{jG_{s_2}Z_2}{2k_3r_2}\right] - \left(\frac{r_1}{r_3}\right)^2\left[\frac{1}{2}\frac{\mu_2k_3}{\mu_3k_2} + \frac{1}{2}\frac{k_2}{k_3} + \frac{jG_{s_2}Z_2}{2k_3r_2}\right]\right\} \\ & \cdot \left[\frac{1}{2}\frac{\mu_3k}{\mu k_3} - \frac{1}{2}\frac{k_3}{k} - \frac{jG_{s_3}Z_3}{2kr_3}\right] \Bigg\}^{-1} \end{aligned} \tag{54}$$

This is a somewhat simpler expression than that for a_{1_1}. As a first case let G_{s_2} and G_{s_3} both be zero and let ε, μ, and σ apply to media 2 and 3, the same as for the external medium. This is just the case of no shields or other distinct media external to the cylindrical loop structure, giving

$$a' \approx j2kr_1 \tag{55}$$

The results of other cases can be compared to this one in order to estimate any improvements gained by adding conducting shields and other media around the loop. Note that we are only considering electrically-small sensors and shields so that $|kr_1| \ll 1$, and thus also $|a'| \ll 1$. This means that for such an electrically-small loop the electric field sensitivity is insignificant compared to the magnetic field sensitivity if the electric and magnetic fields are related as in a simple plane wave. However, there can be situations in which the ratio of electric to magnetic fields can have a much larger magnitude. In such situations one may need a smaller $|a'|$.

As a simplification of a' consider the special case that ε, μ, and σ apply to media 2 and 3 as well as to the external medium. Then equation (54) reduces to

$$a' \approx j2kr_1 \left\{ \left\{ \left[1 - \frac{jG_{s_2} Z}{2kr_2} \right] + \left(\frac{r_1}{r_2} \right)^2 \frac{jG_{s_2} Z}{2kr_2} \right\} \left[1 - \frac{jG_{s_3} Z}{2kr_3} \right] - \frac{jG_{s_3} Z}{2kr_3} \left\{ \left(\frac{r_2}{r_3} \right)^2 \frac{jG_{s_2} Z}{2kr_2} - \left(\frac{r_1}{r_3} \right)^2 \left[1 + \frac{jG_{s_2} Z}{2kr_2} \right] \right\} \right\}^{-1} \tag{56}$$

Now let $\sigma \lll \omega\varepsilon$. We define characteristic frequencies of the form

$$\omega_{e_m} \equiv \frac{G_{s_m}}{2\varepsilon_m r_m} \tag{57}$$

where m = 1, 2, or 3. Equation (56) then becomes

$$a' \approx j2kr_1 \left\{ \left\{ \left[1 + \left(\frac{j\omega}{\omega_{e_2}} \right)^{-1} \right] - \left(\frac{r_1}{r_2} \right)^2 \left(\frac{j\omega}{\omega_{e_2}} \right)^{-1} \right\} \left[1 + \left(\frac{j\omega}{\omega_{e_3}} \right)^{-1} \right] - \left(\frac{j\omega}{\omega_{e_3}} \right)^{-1} \left\{ \left(\frac{r_2}{r_3} \right)^2 \left(\frac{j\omega}{\omega_{e_2}} \right)^{-1} + \left(\frac{r_1}{r_3} \right)^2 \left[1 - \left(\frac{j\omega}{\omega_{e_2}} \right)^{-1} \right] \right\} \right\}^{-1} \tag{58}$$

Setting G_{s_3} to zero gives the results for a single shield with the sensor as

$$a' \simeq j2kr_1 \left\{ 1 + \left(\frac{j\omega}{\omega_{e_2}}\right)^{-1} \left[1 - \left(\frac{r_1}{r_2}\right)^2\right] \right\}^{-1} \tag{59}$$

An approximate form for equation (58) is obtained by assuming that the frequencies of interest are much less than both ω_{e_2} and ω_{e_3} giving

$$a' \simeq j2k_1 r \frac{j\omega}{\omega_{e_2}} \frac{j\omega}{\omega_{e_3}} \left[1 - \left(\frac{r_1}{r_2}\right)^2 - \left(\frac{r_2}{r_3}\right)^2 + \left(\frac{r_1}{r_3}\right)^2\right]^{-1} \tag{60}$$

For the case of a single shield as in equation (59) and for frequencies much less than ω_{e_2}, then the expression simplifies to

$$a' \simeq j2kr_1 \frac{j\omega}{\omega_{e_2}} \left[1 - \left(\frac{r_1}{r_2}\right)^2\right]^{-1} \tag{61}$$

In equations (60) and (61) one can see that for sufficiently low frequencies a' decreases more rapidly with frequency as more shields are added, showing the effectiveness of such shields in reducing the electric field sensitivity of the loop. One should be careful in using approximate forms such as in equations (60) and (61)(and others to follow) that the various parameters are in a range (i.e., large enough or small enough) to make the expression valid.

Now let $\sigma \gg \omega\varepsilon$ with the same restriction that ε, μ, and σ be the same in media 2 and 3 as well as in the external medium. Equation (56) then becomes

$$a' \simeq j2kr_1 \left\{ \left\{ \left[1 + \frac{G_{s_2}}{2\sigma r_2}\right] - \left(\frac{r_1}{r_2}\right)^2 \frac{G_{s_2}}{2\sigma r_2} \right\} \left[1 + \frac{G_{s_3}}{2\sigma r_3}\right] - \frac{G_{s_3}}{2\sigma r_3} \left\{ \left(\frac{r_2}{r_3}\right)^2 \frac{G_{s_2}}{2\sigma r_2} + \left(\frac{r_1}{r_3}\right)^2 \left[1 - \frac{G_{s_2}}{2\sigma r_2}\right] \right\} \right\}^{-1} \tag{62}$$

For $G_{s_2}/2\sigma r_2$ and $G_{s_3}/2\sigma r_3$, both much larger than one equation (62) becomes

$$a' \simeq j2kr_1 \frac{2\sigma r_2}{G_{s_2}} \frac{2\sigma r_3}{G_{s_3}} \left[1 - \left(\frac{r_1}{r_2}\right)^2 - \left(\frac{r_2}{r_3}\right)^2 + \left(\frac{r_1}{r_3}\right)^2\right]^{-1} \tag{63}$$

Again the conducting shields reduce the electric field sensitivity. Note the similarity in the forms for a' for the highly conducting case (equations (62) and (63)) and the negligibly conducting case (equations (58) and (60)). Setting G_{s_3} to zero in equation (62) gives the results for a single shield as

$$a' \approx j2kr_1 \left\{ 1 + \frac{G_{s_2}}{2\sigma r_2} \left[1 - \left(\frac{r_1}{r_2} \right)^2 \right] \right\}^{-1} \tag{64}$$

For $G_{s_2}/2\sigma r_2 >> 1$ this reduces to

$$a' \approx j2kr_1 \frac{2\sigma r_2}{G_{s_2}} \left[1 - \left(\frac{r_1}{r_2} \right)^2 \right]^{-1} \tag{65}$$

Equations (63) and (65) are again only limiting forms which require that certain parameters be in a certain range and also that the various r_m's be distinct from each other.

Consider another special case for a' in which the permeabilities are the same in media 2 and 3 and the external medium. However, let the permittivity and conductivity of medium 2 be the same as those of medium 3, but different from those of the external medium. Equation (54) then becomes

$$\begin{aligned} a' \approx j2k_2 r_1 \Bigg\{ & \left\{ \left[1 - \frac{jG_{s_2} Z_2}{2k_2 r_2} \right] + \left(\frac{r_1}{r_2} \right)^2 \frac{jG_{s_2} Z_2}{2k_2 r_2} \right\} \left[\frac{1}{2} \frac{k}{k_2} + \frac{1}{2} \frac{k_2}{k} - \frac{jG_{s_3} Z_2}{2kr_3} \right] \\ & + \left\{ \left(\frac{r_2}{r_3} \right)^2 \frac{jG_{s_2} Z_2}{2k_2 r_2} - \left(\frac{r_1}{r_3} \right)^2 \left[1 + \frac{jG_{s_2} Z_2}{2k_2 r_2} \right] \right\} \left[\frac{1}{2} \frac{k}{k_2} - \frac{1}{2} \frac{k_2}{k} - \frac{jG_{s_3} Z_2}{2kr_3} \right] \Bigg\}^{-1} \end{aligned} \tag{66}$$

Specialize this case further by letting $\sigma >> \omega\epsilon$ and $\sigma_2 >> \omega\epsilon_2$ and $\sigma_2 << \sigma$. This set of assumptions might apply to such a shielded loop in the presence of intense ionizing nuclear radiation such as found in the source region of the nuclear electromagnetic pulse. Media 2 and 3 might be insulators which have been made conducting (less conducting than the surrounding air) by the ionizing nuclear radiation. Under these assumptions a' becomes

$$a' \approx j4kr_1 \left[\frac{\sigma}{\sigma_2} + \frac{G_{s_3}}{\sigma_2 r_3} \right]^{-1} \left\{ 1 - \left(\frac{r_1}{r_3} \right)^2 + \frac{G_{s_2}}{2\sigma_2 r_2} \left[1 - \left(\frac{r_1}{r_2} \right)^2 - \left(\frac{r_2}{r_3} \right)^2 + \left(\frac{r_1}{r_3} \right)^2 \right] \right\}^{-1} \tag{67}$$

For $G_{s_2}/2\sigma_2 r_2 >> 1$ this reduces to

$$a' \simeq j4kr_1 \frac{2\sigma_2 r_2}{G_{s_2}} \left[\frac{\sigma}{\sigma_2} + \frac{G_{s_3}}{\sigma_2 r_2}\right]^{-1} \left[1 - \left(\frac{r_1}{r_2}\right)^2 - \left(\frac{r_2}{r_3}\right)^2 + \left(\frac{r_1}{r_3}\right)^2\right]^{-1} \tag{68}$$

Note that even with the outer shield removed so that $G_{s_3} = 0$ there is still some reduction in the electric field sensitivity due to the presence of medium 3 which is less conducting than the external medium. We consider the case of a single shield which is also in contact with the external medium by setting G_{s_2} to zero, giving

$$a' \simeq j4kr_1 \left[\frac{\sigma}{\sigma_2} + \frac{G_{s_3}}{\sigma_2 r_3}\right]^{-1} \left[1 - \left(\frac{r_1}{r_3}\right)^2\right]^{-1} \tag{69}$$

Now remove the last shield by setting $G_{s_3} = 0$. Note that some reduction in the electric field sensitivity is still obtained just by the addition around the loop of a medium of lower conductivity than the external medium.

Another interesting case is obtained from equation (66) by letting $\sigma \gg \omega\varepsilon$ but $\omega\varepsilon_2 \gg \sigma_2$. Also assume that $\sigma \gg \omega\varepsilon_2$. This set of assumptions might apply to a shielded loop in some conducting medium such as soil or sea water. Media 2 and 3 could be insulating dielectrics. Under these assumptions a' becomes

$$a' \simeq j2kr_1 \left[\frac{\sigma}{j2\omega\varepsilon_2} + \left(\frac{j\omega}{\omega_{e_3}}\right)^{-1}\right]^{-1} \left\{1 - \left(\frac{r_1}{r_3}\right)^2 + \left(\frac{j\omega}{\omega_{e_2}}\right)^{-1} \left[1 - \left(\frac{r_1}{r_2}\right)^2 - \left(\frac{r_2}{r_3}\right)^2 + \left(\frac{r_1}{r_3}\right)^2\right]\right\}^{-1} \tag{70}$$

This equation has a form similar to equation (67). Further simplifications can be introduced into equation (70) in the same manner as for the previous case with media 2 and 3 conducting.

Then not only can the extra shields be made to have negligible effect on the response of the cylindrical $\dot{B}$ loop to the magnetic field; these same shields can reduce the response of the loop to the electric field. As indicated by the above analysis the electric field response can be reduced beyond that attained with a single conducting shield by the use of two conducting shields. The additional media added outside the loop also affect the electric field response; in the case where the external medium is highly conducting the addition of an insulating medium outside the loop can significantly reduce the electric field sensitivity.

IV. Summary

Conducting shields can be used to reduce the electric field sensitivity of a cylindrical $\dot{B}$ loop without significantly affecting the magnetic field sensitivity of the loop. One might roughly think in terms of the magnetic field penetrating through the shields and the electric field being shorted out by the shields. If the external medium is highly conducting, the addition, next to the loop, of media which are comparatively good insulators also reduces the electric field sensitivity of the loop. In this case one might roughly think in terms of an insulator blocking the current density in the external medium from reaching the loop conductors.

There are several limitations on the analysis used in this note. The cylindrical $\dot{B}$ loop is only roughly approximated as a conducting sheet. Considering the short circuit surface current density removes this limitation to some extent. However, a shorted multi-turn cylindrical loop still is not quite a perfectly conducting cylindrical sheet for currents in the azimuthal (ϕ) direction. A practical cylindrical loop also has finite length. Note that if signal leads are brought from the loop out through the shield(s) the current patterns on both the loop and the shields will be changed and other problems such as common mode signals on the signal leads may appear. In such a case the conducting shield may still prove advantageous but one may have to be careful in how the shield(s) and loop are joined to the signal leads and any signal-lead shields.

The analysis in this note presumes linear parameters which are constant in time for the various media and also presumes no sources for the electromagnetic fields in the vicinity of the shielded loop. In the source region for the nuclear electromagnetic pulse these assumptions do not apply. For such a case the results of the present calculation can only be applied in a very approximate sense.

A final observation might be that the separation of the loop response into magnetic and electric field sensitivities is only an approximation which has some validity for electrically-small loops. For wavelengths or skin depths, whichever is appropriate, of the order of the loop dimensions, more than the first two terms in a cylindrical coordinate expansion of the electromagnetic fields enter into the loop response. However, electric field sensitivity can be a useful concept in some cases in the design of electrically-small loops. Note also that, while the incident electromagnetic fields used for these calculations propagate in a direction perpendicular to the loop axis, the incident fields can actually have a more general form.