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The Planar, Uniform Surface Transmission Line Driven from A Sheet Source

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Abstract

The response characteristics are calculated for a transmission line consisting of a distributed planar source above and parallel to a ground surface. The medium between the sheet source and the ground surface can be made conducting to improve the characteristics of this type of transmission line as a simulator of the close-in nuclear electromagnetic pulse.

Foreword

The pulse shapes fall into two convenient cases depending on whether or not the medium above the ground surface is conducting. The figures for these two cases are grouped at the ends of their respective sections.

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Contents

I. Introduction

In a previous note we considered the response characteristics of a particular kind of surface transmission line which can be used for propagating fast-rising electromagnetic fields over a ground surface.[1] This transmission line consists of a wide conducting sheet above and parallel to the ground surface driven by a source at one end and terminated at the other end. The ground actually forms part of the transmission line. In calculating the response characteristics of this type of transmission line the horizontal electric field was constrained to be zero at the conducting sheet and, including the boundary conditions at the ground surface, the propagation constant of the transmission line was determined.

In the present note we consider a variation on this problem. As illustrated in figure 1 we replace the conducting sheet above the ground surface by a sheet source. This sheet source ideally has the capability of allowing the horizontal electric field to be of some desired form along the sheet. Ideally both the waveform and the propagation constant of the source can be chosen. Practically, such a sheet source might be an array of lumped elements, such as capacitors with switches between them which are triggered in a progressive manner so as to turn on the sheet source at a given velocity (ideally of magnitude c, the speed of light in vacuum) over the ground surface. The discrete sources and switches would ideally be placed quite close together in the sheet source so as to approximate a continuous distribution at the highest frequencies of interest, but such close spacing may make the number of sources and switches prohibitively large. Nevertheless, the approximation of a sheet source may be useful in the design of such a transmission line for use as a simulator of the nuclear electromagnetic pulse from a surface burst over a ground based facility.

There are two types of wave of interest on such a structure. The first type has a propagation constant matching that of the sheet source. The propagation velocity and attenuation with distance of this wave are the same as those of the sheet source. This same wave has a nonzero horizontal electric field at the sheet source. The second type of wave, derived in reference 1, has zero horizontal electric field at the sheet source and can be added to the first wave and still satisfy the boundary conditions. This kind of wave can be introduced at each end of the transmission line. It can be suppressed by providing appropriate sources and terminating impedances at each end to match the electromagnetic fields (or voltages and currents) in the wave associated with the sheet source. In this note we assume that such sources and terminations at the ends of the transmission line are provided so that we need consider only the wave with the propagation constant fixed by the sheet source. Associated with this sheet source there is also a wave produced above the source, but this wave is not considered here.

1. Capt Carl E. Baum, Sensor and Simulation Note XLVI, The Single-Conductor, Planar, Uniform Surface Transmission Line, Driven from One End, July 1967.

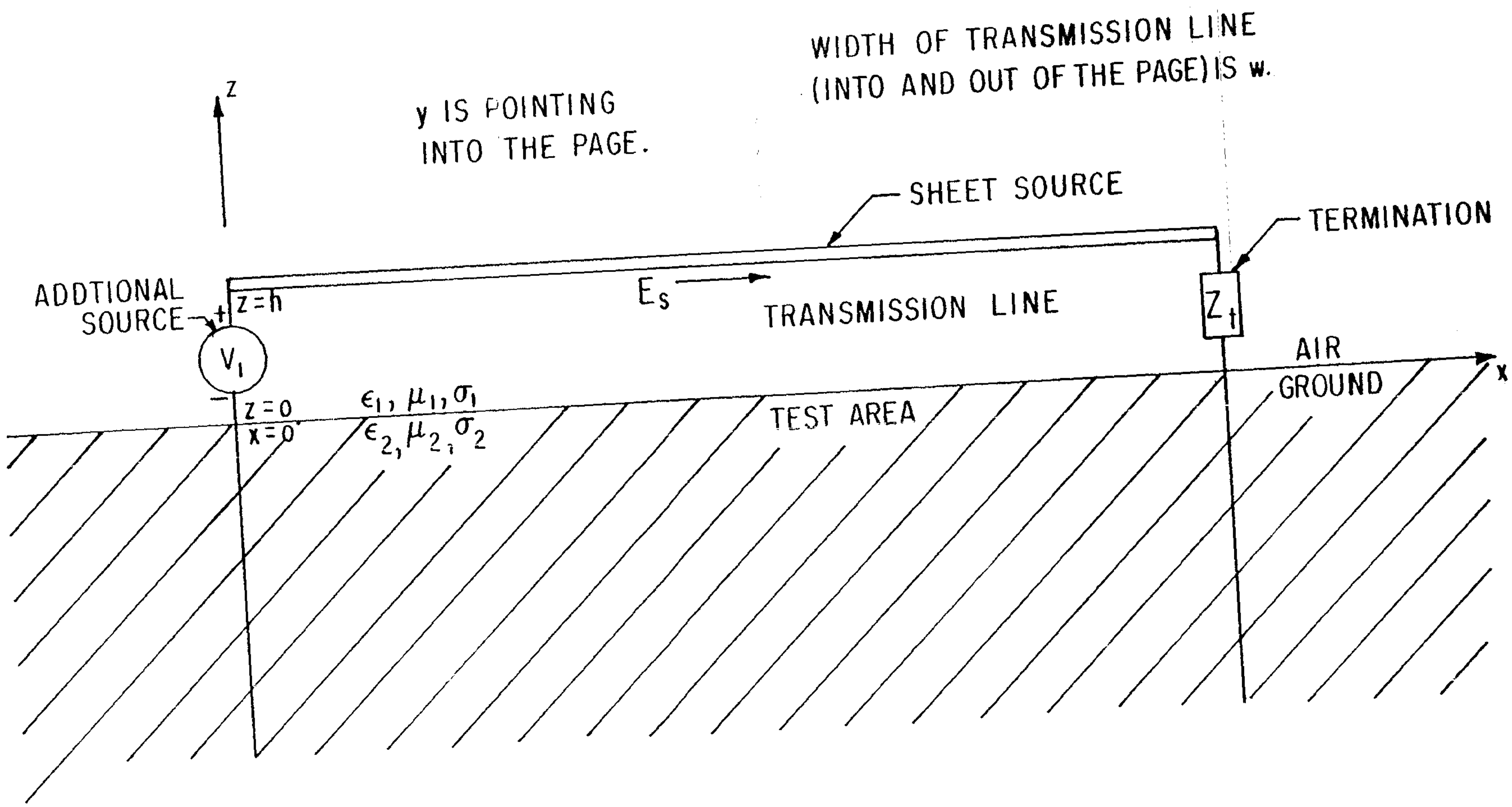


FIGURE 1 THE DISTRIBUTED – SOURCE, PLANAR, UNIFORM SURFACE TRANSMISSION LINE

With the propagation constant of the wave over the ground surface fixed by the sheet source we then have some flexibility in controlling its characteristics. For example, we can have the wave propagate over the ground surface at speed c, with no attenuation with distance, even if we make medium 1 (between the sheet source and the ground) conducting. Practically, it may be difficult to make medium 1 conducting, but if it can be done, then some of the mechanisms of the interaction of the nuclear electromagnetic pulse with a ground based facility in the nuclear source region can be better simulated. Perhaps medium 1 can be given a time-independent conductivity by filling it with some conducting dielectric, possibly solid or foamed. In this note we consider the cases of zero conductivity and nonzero time-independent conductivity for medium 1. With medium 1 conducting we can also have a long-time vertical current density if the propagation constant of the sheet source includes an attenuation of the source strength with distance along the transmission line.

Note the similarity between this type of simulation technique and the nuclear electromagnetic pulse. The sheet source forces current parallel to and above the ground surface, while in the nuclear source region the Compton current travels roughly parallel to the ground surface. The Compton current attenuates with distance according to the gamma-ray mean free path of a few hundred meters; the sheet source can perhaps be made to similarly decrease with distance. The air conductivity in the nuclear source region rapidly changes with time; it may be possible to use some conducting dielectric in place of the air to partly simulate the air conductivity. Note, however, that various aspects of the Compton current and air conductivity are not included in the simulation.

For the calculations in this note we extend the results of reference 1 to obtain the frequency domain solution for the electromagnetic fields with the propagation constant set by the sheet source. To consider pulse shapes the magnetic field just below the sheet source is made a step function, for convenience, and the various field components and related quantities (including the current density and the required sheet source) are calculated using numerical inverse Fourier transforms.[2] The quantities calculated are then time-domain response functions. We first consider the special case that medium 1 has the same propagation speed as free space and the sheet source has this same propagation speed with no attenuation. Then the case that medium 1 is a conducting dielectric is considered while the sheet source has the propagation speed, c, and an exponential attenuation with distance.

2. Frank Sulkowski, Mathematics Note II, FORPLEX: A Program to Calculate Inverse Fourier Transforms, November 1966.

II. Boundary Value Problem

As mentioned previously the calculations in this note are an extension of those in reference 1. First we summarize some of the things needed from that note. For convenience define

$$z' \equiv z-h \tag{1}$$

where z is the vertical distance above the ground surface. The sheet source is a distance, h, above the ground and has a width, w, in the y direction. We assume that w>>h so that we can neglect variation of the solution with y. Some common parameters are a wave impedance

$$Z \equiv \sqrt{\frac{s\mu}{\sigma+s\varepsilon}} \tag{2}$$

and a propagation constant

$$\gamma = \sqrt{s\mu(\sigma+s\varepsilon)} \tag{3}$$

where the permittivity, ε, permeability, μ, and the conductivity, σ, of both the ground and the air (or other medium) between the ground and the sheet source are assumed to be scalar constants, independent of position. A subscript, 1, is used for the medium above the ground, (medium 1) and a subscript, 2, is used for the ground (medium 2) in the case of all these various parameters. The solution of the wave equation is of the form, $e^{\pm\gamma_x x \pm\gamma_z z}$, where γ_x and γ_z are propagation constants pertaining to the x and z directions, respectively. These propagation constants are related as

$$\gamma_x^2 + \gamma_z^2 = \gamma^2 \tag{4}$$

Note that γ_x is the same in both regions while γ_z can differ between the two regions. For convenience we also define some combinations of the above parameters as

$$Q \equiv \gamma_{z_2} h \left(\frac{\gamma_1}{\gamma_2}\right)^2 \frac{\mu_2}{\mu_1} \tag{5}$$

and

$$Q_1 \equiv \gamma_{z_1} h \tag{6}$$

In the foregoing expressions s is the Laplace transform variable.

Next, summarize the electromagnetic field quantities from reference 1 in their forms before the horizontal electric field at z = h was set to zero. In medium 1 we have

$$\tilde{E}_{z_1} = \tilde{A}_1 e^{\gamma_{z_1} z' - \gamma_x x} + \tilde{B}_1 e^{-\gamma_{z_1} z' - \gamma_x x} \tag{7}$$

$$\tilde{E}_{x_1} = \tilde{A}_2 e^{\gamma_{z_1} z' - \gamma_x x} + \tilde{B}_2 e^{-\gamma_{z_1} z' - \gamma_x x} \tag{8}$$

and

$$\tilde{H}_{y_1} = \tilde{A}_3 e^{\gamma_{z_1} z' - \gamma_x x} + \tilde{B}_3 e^{-\gamma_{z_1} z' - \gamma_x x} \tag{9}$$

and in medium 2 we have

$$\tilde{E}_{z_2} = \tilde{A}_4 e^{\gamma_{z_2} z - \gamma_x x} \tag{10}$$

$$\tilde{E}_{x_2} = \tilde{A}_5 e^{\gamma_{z_2} z - \gamma_x x} \tag{11}$$

and

$$\tilde{H}_{y_2} = \tilde{A}_6 e^{\gamma_{z_2} z - \gamma_x x} \tag{12}$$

where the tilde, ∿, over a quantity is used to indicate the Laplace transform of the quantity. The coefficients are related as

$$\tilde{A}_3 = -\frac{\gamma_1}{\gamma_{z_1}} \frac{\tilde{A}_2}{Z_1} = -\frac{\gamma_1}{\gamma_x} \frac{\tilde{A}_1}{Z_1} \tag{13}$$

$$\tilde{B}_3 = \frac{\gamma_1}{\gamma_{z_1}} \frac{\tilde{B}_2}{Z_1} = -\frac{\gamma_1}{\gamma_x} \frac{\tilde{B}_1}{Z_1} \tag{14}$$

$$\tilde{A}_6 = -\frac{\gamma_2}{\gamma_{z_2}} \frac{\tilde{A}_5}{Z_2} = -\frac{\gamma_2}{\gamma_x} \frac{\tilde{A}_4}{Z_2} \tag{15}$$

$$\tilde{A}_3 = \frac{e^{Q_1}}{2}\left[1 + \frac{Q}{Q_1}\right]\tilde{A}_6 \tag{16}$$

and

$$\tilde{B}_3 = \frac{e^{-Q_1}}{2}\left[1 - \frac{Q}{Q_1}\right]\tilde{A}_6 \tag{17}$$

There are also the definitions

$$\tilde{A}_2 + \tilde{B}_2 \equiv \tilde{A} \tag{18}$$

and

$$\tilde{A}_3 + \tilde{B}_3 \equiv \tilde{B} \tag{19}$$

There is a convenient relationship between E_z and H_y. For medium 1 this is

$$\tilde{E}_{z_1} = -\frac{\gamma_x}{\gamma_1} Z_1 \tilde{H}_{y_1} \tag{20}$$

and for medium 2 this is

$$\tilde{E}_{z_2} = -\frac{\gamma_x}{\gamma_2} Z_2 \tilde{H}_{y_2} \tag{21}$$

For this note we use another field quantity, the vertical total current density. In medium 1 this is

$$\tilde{J}_{t_{z_1}} = (\sigma_1+s\varepsilon_1)\tilde{E}_{z_1} = \frac{\gamma_1}{Z_1}\tilde{E}_{z_1} \tag{22}$$

Relating this to the magnetic field gives

$$\tilde{J}_{t_{z_1}} = -\gamma_x \tilde{H}_{y_1} \tag{23}$$

In this note γ_x is taken as the propagation constant of the sheet source, which we call γ_s and choose to be of some desired form. In general, $\tilde{A}$ is then nonzero and can be related to $\tilde{B}$ as

$$\frac{\tilde{A}}{\tilde{B}} = \frac{\tilde{A}_2+\tilde{B}_2}{\tilde{A}_3+\tilde{B}_3} = -\frac{\gamma_{z_1}}{\gamma_1} Z_1 \frac{\tilde{A}_3-\tilde{B}_3}{\tilde{A}_3+\tilde{B}_3}$$

$$= -\frac{\gamma_{z_1}}{\gamma_1} Z_1 \frac{\sinh(Q_1)+\frac{Q}{Q_1}\cosh(Q_1)}{\cosh(Q_1)+\frac{Q}{Q_1}\sinh(Q_1)} \tag{24}$$

The field components in medium 1 are then

$$\tilde{H}_{y_1} = \frac{\tilde{A}_3 e^{\gamma_{z_1} z'} + \tilde{B}_3 e^{-\gamma_{z_1} z'}}{\tilde{A}_3+\tilde{B}_3} \tilde{B}\, e^{-\gamma_s x}$$

$$= \frac{\cosh(\gamma_{z_1} z)+\frac{Q}{Q_1}\sinh(\gamma_{z_1} z)}{\cosh(Q_1)+\frac{Q}{Q_1}\sinh(Q_1)} \tilde{B}\, e^{-\gamma_s x} \tag{25}$$

and

$$\tilde{E}_{x_1} = \frac{\tilde{A}_2 e^{\gamma_{z_1} z'} + \tilde{B}_2 e^{-\gamma_{z_1} z'}}{\tilde{A}_2+\tilde{B}_2} \tilde{A} e^{-\gamma_s x} = \frac{\tilde{A}_3 e^{\gamma_{z_1} z'} - \tilde{B}_3 e^{-\gamma_{z_1} z'}}{\tilde{A}_3-\tilde{B}_3} \tilde{A} e^{-\gamma_s x}$$

$$= \frac{\sinh(\gamma_{z_1} z)+\frac{Q}{Q_1}\cosh(\gamma_{z_1} z)}{\sinh(Q_1)+\frac{Q}{Q_1}\cosh(Q_1)} \tilde{A} e^{-\gamma_s x} \tag{26}$$

Finally $\tilde{E}_{z_1}$ is obtained from $\tilde{H}_{y_1}$ using equation (20), and $\tilde{J}_{t_{z_1}}$ is obtained from $\tilde{H}_{y_1}$ using equation (23).

Other parameters of interest include what might be termed the voltages associated with the vertical electric field, both above and below the ground. For medium 1 define

$$\tilde{V}_1 \equiv - \int_o^h \tilde{E}_{z_1}\Big|_{x=0} dz$$

$$= \frac{\gamma_x}{\gamma_1} Z_1 \tilde{B} \left[\cosh(Q_1) + \frac{Q}{Q_1} \sinh(Q_1)\right]^{-1} \int_o^h \left[\cosh(\gamma_{z_1} z) + \frac{Q}{Q_1} \sinh(\gamma_{z_1} z)\right] dz$$

$$= \frac{\gamma_x Z_1}{\gamma_1 \gamma_{z_1}} \tilde{B} \frac{\sinh(Q_1) + \frac{Q}{Q_1} [\cosh(Q_1) - 1]}{\cosh(Q_1) + \frac{Q}{Q_1} \sinh(Q_1)} \tag{27}$$

From this we have the average vertical electric field in medium 1 as

$$\tilde{E}_{z_{1_{avg}}} = - \frac{\tilde{V}_1}{h} e^{-\gamma_s x} \tag{28}$$

Equation (27) or (28) might be used to define the desired output of additional sources at the beginning of the transmission line, as illustrated in figure 1. Similarly, for medium 2 define

$$\tilde{V}_2 \equiv - \int_{-\infty}^{o} \tilde{E}_{z_2}\Big|_{x=0} dz = - \frac{\tilde{A}_4}{\gamma_{z_2}} = \frac{\gamma_x Z_2}{\gamma_{z_2} \gamma_2} \tilde{A}_6$$

$$= \frac{\gamma_x Z_2}{\gamma_{z_2} \gamma_2} \frac{\tilde{B}}{\cosh(Q_1)} \tag{29}$$

An interesting ratio is

$$\frac{\tilde{V}_2}{\tilde{V}_1} = \frac{\gamma_{z_1}}{\gamma_{z_2}} \left(\frac{\gamma_1}{\gamma_2}\right)^2 \frac{\mu_2}{\mu_1} \frac{1}{\cosh(Q_1)} \frac{\cosh(Q_1) + \frac{Q}{Q_1} \sinh(Q_1)}{\sinh(Q_1) + \frac{Q}{Q_1} [\cosh(Q_1) - 1]} \tag{30}$$

If the magnitude of this ratio is small compared to one for frequencies of interest then the perturbation due to the conductors placed in the ground should be small. However, in some cases, such as when medium 1 is conducting, the magnitude of this ratio may not be small. In such cases one might change the manner of connecting the additional sources at the beginning of the surface transmission line so as to minimize the distortion of the fields in the ground. Similar field distortion problems may also be encountered at the termination end of the surface transmission line. However, such problems are not considered in this note.

For convenience in the analysis define some convenient parameters. The relative dielectric constants are

$$\varepsilon_{r_1} \equiv \frac{\varepsilon_1}{\varepsilon_o} \tag{31}$$

and

$$\varepsilon_{r_2} \equiv \frac{\varepsilon_2}{\varepsilon_o} \tag{32}$$

the relaxation times for the two media are

$$t_{r_1} \equiv \frac{\varepsilon_1}{\sigma_1} \tag{33}$$

and

$$t_{r_2} \equiv \frac{\varepsilon_2}{\sigma_2} \tag{34}$$

modifications of these relaxation times are

$$t'_{r_1} \equiv \frac{\varepsilon_o}{\sigma_1} \tag{35}$$

and

$$t'_{r_2} \equiv \frac{\varepsilon_o}{\sigma_2} \tag{36}$$

and the transit time from the ground to the sheet source (for $\varepsilon_1 = \varepsilon_o$ and $\mu_1 = \mu_o$) is

$$t_h \equiv \frac{h}{c} \tag{37}$$

III. Medium Above Ground Nonconducting.

As a special case consider first that

$$\gamma_s = \gamma_1 = \frac{s}{c} \tag{38}$$

Thus, we assume that $\varepsilon_1 = \varepsilon_o$, $\mu_1 = \mu_o$, and $\sigma_1 = 0$, or that medium 1 has the electromagnetic characteristics of free space. The sheet source propagates in the +x direction at the speed of light in vacuum with no attenuation with distance. For this special case we have

$$\gamma_{z_1}^2 = \gamma_1^2 - \gamma_s^2 = 0 \tag{39}$$

so that we need the limiting forms for $Q_1 = 0$ for the field components and related quantities. From some of equations (24) through (30) we have

$$\frac{\tilde{A}}{\tilde{B}} = - Z_1 \frac{Q}{\gamma_1 h} \left[1 + Q\right]^{-1} \tag{40}$$

$$\tilde{H}_{y_1} = \frac{1 + \frac{z}{h} Q}{1 + Q} \tilde{B} e^{-\gamma_s x} \tag{41}$$

$$\tilde{E}_{x_1} = \tilde{A} e^{-\gamma_s x} \tag{42}$$

$$\tilde{E}_{z_{1_{avg}}} = - Z_1 \frac{1 + \frac{Q}{2}}{1 + Q} \tilde{B} e^{-\gamma_s x} \tag{43}$$

and

$$\frac{\tilde{V}_2}{\tilde{V}_1} = \frac{1}{\gamma_{z_2} h} \left(\frac{\gamma_1}{\gamma_2}\right)^2 \frac{\mu_2}{\mu_1} \frac{1 + Q}{1 + \frac{Q}{2}} \tag{44}$$

From equation (23) the vertical total current density is

$$\tilde{J}_{t_{z_1}} = -\gamma_s \frac{1 + \frac{z}{h} Q}{1 + Q} \tilde{B} e^{-\gamma_s x} \tag{45}$$

For this special case also let $\mu_2 = \mu_o$. For use in this section introduce some parameters. There is a characteristic time

$$t_1 \equiv \left(t_h^2 t'_{r_2} \right)^{1/3} = \left(\frac{t_h^2 t_{r_2}}{\varepsilon_{r_2}} \right)^{1/3} \tag{46}$$

from which we define a normalized Laplace transform variable

$$s_1 \equiv s t_1 \tag{47}$$

and a normalized time

$$\tau_1 \equiv \frac{t - \frac{x}{c}}{t_1} \tag{48}$$

Also collect together some of the parameters in the form

$$p_4 \equiv \frac{t_{r_2}}{t_1} = \left[\varepsilon_{r_2} \left(\frac{t_{r_2}}{t_h} \right)^2 \right]^{1/3} \tag{49}$$

Then we have a common term in the various expressions as

$$Q = \gamma_1 h \frac{\gamma_1}{\gamma_2} \left[1 - \left(\frac{\gamma_1}{\gamma_2} \right)^2 \right]^{1/2}$$

$$= \frac{s t_h}{\sqrt{\varepsilon_{r_2}}} \left[\frac{s t_{r_2}}{1 + s t_{r_2}} \right]^{1/2} \left[1 - \frac{1}{\varepsilon_{r_2}} \frac{s t_{r_2}}{1 + s t_{r_2}} \right]^{1/2}$$

$$= \frac{s_1^{3/2}}{1 + s_1 p_4} \left[1 + \left(1 - \frac{1}{\varepsilon_{r_2}} \right) s_1 p_4 \right]^{1/2} \tag{50}$$

Roughly consider the ratio of the two voltages associated with the vertical electric field as in equation (44). Neglect the factor containing Q because of the manner in which it appears in both the numerator and the denominator giving

$$\frac{\tilde{V}_2}{\tilde{V}_1} \simeq \frac{1}{\gamma_{z_2} h} \left(\frac{\gamma_1}{\gamma_2} \right)^2 = \frac{1}{\gamma_2 h} \left(\frac{\gamma_1}{\gamma_2} \right)^2 \left[1 - \left(\frac{\gamma_1}{\gamma_2} \right)^2 \right]^{-1/2} \tag{51}$$

and for $\varepsilon_{r_2} >> 1$ this becomes

$$\frac{\tilde{V}_2}{\tilde{V}_1} \simeq \frac{1}{\gamma_2 h}\left(\frac{\gamma_1}{\gamma_2}\right)^2 = \left[\frac{st_{r_2}\left(\frac{t_{r_2}}{t_h}\right)^2}{\varepsilon_{r_2}^{\;3}(1 + st_{r_2})^3}\right]^{1/2} \tag{52}$$

This same expression appears in reference 1. The magnitude of this ratio is small for low enough frequencies, and as discussed in reference 1 the magnitude of this ratio can be small for all frequencies if appropriate restrictions are placed on ε_{r_2}, t_{r_2}, and t_h.

Now consider normalized response functions for some of the field components and related quantities. To do this we choose

$$\tilde{B} \equiv \frac{H_1}{s_1} \tag{53}$$

where H_1 is an arbitrary constant with dimensions of amperes/meter. This makes the magnetic field just under the sheet source a step function and the time-domain waveforms of the other quantities are consistent with this choice. This type of choice is somewhat arbitrary but perhaps useful in relating the time-domain waveforms of the various quantities. Then define a normalized magnetic field at z = 0 as

$$\tilde{h}_1(s_1) \equiv \left.\frac{\tilde{H}_{y_1} e^{\gamma_s x}}{H_1}\right|_{z=0} = \frac{1}{s_1}\left[1 + Q\right]^{-1} \tag{54}$$

Note that the time delay associated with x is removed from the normalized Laplace transform so that the normalized time of equation (48) is appropriate. The corresponding normalized horizontal electric field at z = h is then defined as

$$\tilde{e}_{s_1}(s_1) \equiv \left. -\frac{1}{Z_1 H_1}\frac{t_h}{t_1}\tilde{E}_{x_1} e^{\gamma_s x}\right|_{z=h} = \frac{Q}{s_1^2}\left[1 + Q\right]^{-1} \tag{55}$$

This electric field is that required of the sheet source to give the step magnetic field just beneath the sheet source. Note that since medium 1 is nonconducting, Z_1 is independent of frequency. The normalized average vertical electric field in medium 1 is defined as

$$\tilde{e}_{avg_1}(s_1) \equiv -\frac{1}{Z_1 H_1}\tilde{E}_{z_{1_{avg}}} e^{\gamma_s x} = \frac{1}{s_1}\,\frac{1 + \frac{Q}{2}}{1 + Q} \tag{56}$$

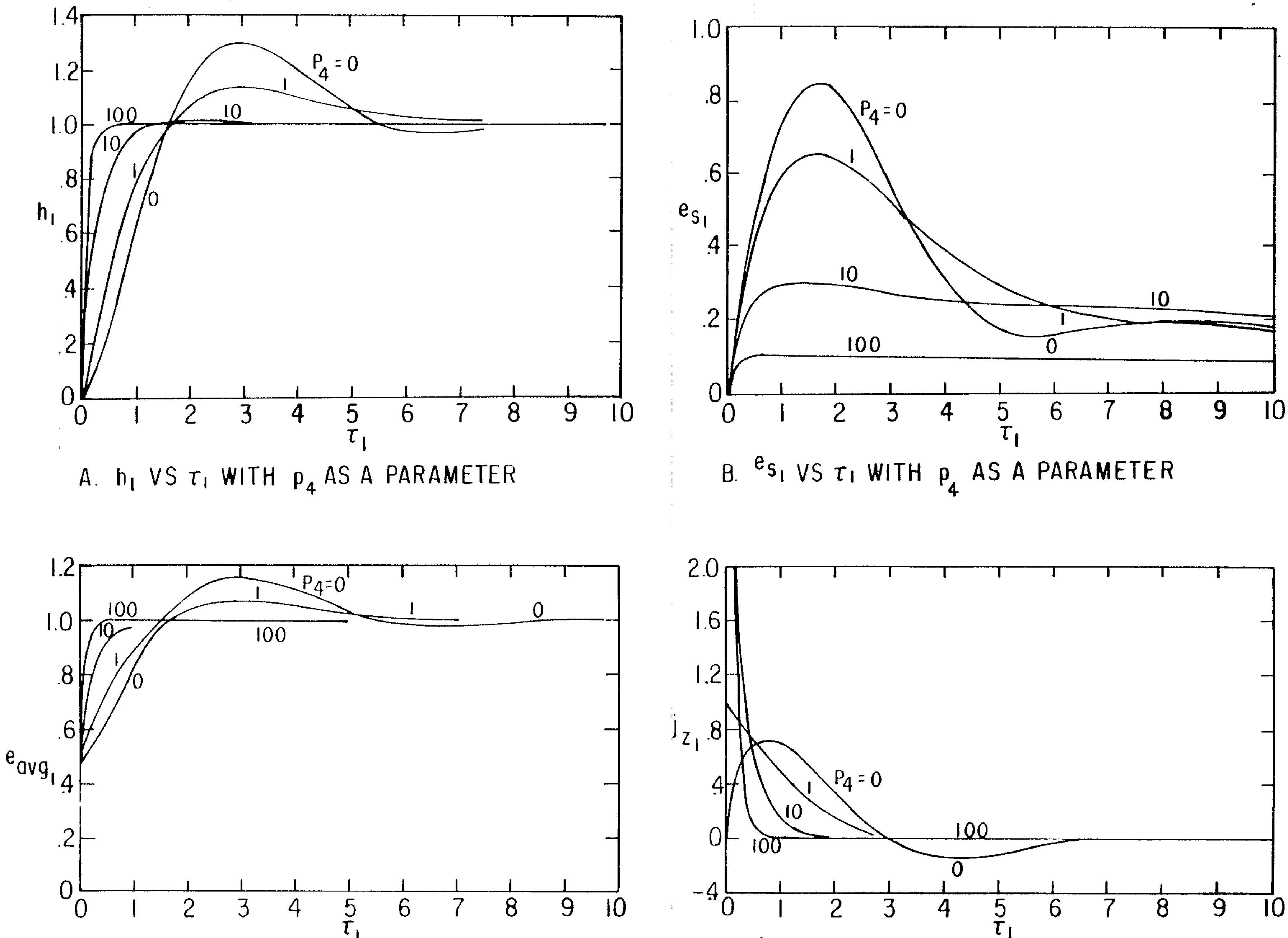
The normalized vertical total current density at $z = 0$ is defined as

$$\tilde{j}_{z_1}(s_1) \equiv -t_1 c \tilde{J}_{t_{z_1}} e^{\gamma_s x}\Big|_{z=0} = [1 + Q]^{-1} \tag{57}$$

The time-domain waveforms for the four quantities in equations (54) through (57) are plotted in figure 2 for several values of p_4 and for $\epsilon_{r_2} = 10$. These were also calculated for $\epsilon_{r_2} = 80$, but with little difference in the results; these results are not included. Note that both h_1 and e_{avg_1} go to one for large τ_1, while e_{s_1} and j_{z_1} go to zero for large τ_1. Both h_1 and e_{s_1} start at zero for $\tau_1 = 0$, while e_{avg_1} has a step rise to .5 at $\tau_1 = 0$. At $\tau_1 = 0+$ the normalized vertical total current density rises to

$$j_{z_1}(0+) = \lim_{s_1 \to \infty} s_1 \tilde{j}_{z_1}(s_1) = \left(\frac{p_4}{1 - \frac{1}{\epsilon_{r_2}}} \right)^{1/2} \tag{58}$$

Note for the present case that, since γ_s includes no attenuation with distance, j_{z_1} goes to zero for large τ_1. However, it may be desirable to have a total vertical current density which continues for larger times.



A. h_1 VS τ_1 WITH p_4 AS A PARAMETER

B. e_{s_1} VS τ_1 WITH p_4 AS A PARAMETER

C. e_{avg_1} VS τ_1 WITH p_4 AS A PARAMETER

D. j_{z_1} VS τ_1 WITH p_4 AS A PARAMETER

FIGURE 2. PULSE SHAPES ON TRANSMISSION LINE: $\sigma_1 = 0$, $\epsilon_{r_2} = 10$

IV. Medium Above Ground Conducting

Now include an attenuation with distance in the propagation constant of the sheet source as

$$\gamma_s = \frac{s}{c} + \frac{1}{\chi} \tag{59}$$

where χ is a characteristic distance for the decrease of the magnetic field with increasing x. This characteristic distance might be comparable to the gamma-ray mean free path. We assume that $\mu_2 = \mu_1 = \mu_o$, but both media 1 and 2 are allowed to be conducting dielectrics. For this case define a characteristic time as

$$t_2 \equiv \frac{\mu_o \sigma_1 h^2}{4} \tag{60}$$

from which we define a normalized Laplace transform variable

$$s_2 \equiv s t_2 \tag{61}$$

and a normalized time

$$\tau_2 \equiv \frac{t - \frac{x}{c}}{t_2} \tag{62}$$

Also define

$$t_3 \equiv \frac{\chi}{c} \tag{63}$$

and

$$p_5 \equiv \frac{1}{2}\frac{h}{\chi} = \frac{t_2}{t_3}\sqrt{\frac{t_{r_1}}{\varepsilon_{r_1} t_2}} \tag{64}$$

Then for two common terms in the expressions we have

$$Q = \gamma_2 h \left[1 - \left(\frac{\gamma_s}{\gamma_2}\right)^2\right]^{1/2} \left(\frac{\gamma_1}{\gamma_2}\right)^2$$

$$= 2\left\{s_2 + \frac{t_{r_2}}{t_2}\left[s_2^2 - \frac{1}{\varepsilon_{r_2}}\left(s_2 + \frac{t_2}{t_3}\right)^2\right]\right\}^{1/2} \sqrt{\frac{\sigma_1}{\sigma_2}} \, \frac{1+s_2 \frac{t_{r_1}}{t_2}}{1+s_2 \frac{t_{r_2}}{t_2}} \tag{65}$$

and

$$Q_1 = \gamma_1 h \left[1 - \left(\frac{\gamma_s}{\gamma_1}\right)^2\right]^{1/2}$$

$$= 2\left\{s_2 + \frac{t_{r_1}}{t_2}\left[s_2^2 - \frac{1}{\varepsilon_{r_1}}\left(s_2 + \frac{t_2}{t_3}\right)^2\right]\right\}^{1/2} \tag{66}$$

Note the forms of Q and Q_1 for low frequency. In the limit of zero frequency we have

$$\lim_{s_2 \to 0} Q = 2\sqrt{\frac{\sigma_1}{\sigma_2}}\left(-\frac{t_{r_2} t_2}{\varepsilon_{r_1} t_3^2}\right)^{1/2} = \pm j2\frac{\sigma_1}{\sigma_2} p_5 = \pm j\frac{\sigma_1}{\sigma_2}\frac{h}{\chi} \tag{67}$$

and

$$\lim_{s_2 \to 0} Q_1 = 2\left(-\frac{t_{r_1} t_2}{\varepsilon_{r_1} t_3^2}\right)^{1/2} = \pm j2p_5 = \pm j\frac{h}{\chi} \tag{68}$$

These parameters are imaginary at zero frequency, as are γ_{z_1} and γ_{z_2}.

Referring back to equations (10) through (12) note the form chosen for the fields in medium 2. The z dependence is taken of the form $e^{\gamma_{z_2} z}$ to represent a wave propagating (and attenuating) in the -z direction. However, for sufficiently low frequencies, the inclusion of χ in γ_s changes the characteristics of γ_{z_2} such that an additional term of the form $e^{-\gamma_{z_2} z}$ is also needed to give a real valued function in the time domain. Such low frequencies correspond to field penetration into the ground a distance of the order of χ which might typically be a few hundred meters. By considering only certain limiting cases for the solutions, this mathematical difficulty is avoided. Note that both $e^{\gamma_{z_1} z'}$ and $e^{-\gamma_{z_1} z'}$ are used in equations (7) through (9) for the fields in medium 1 so that this kind of mathematical difficulty does not appear in connection with medium 1.

Another problem concerns the ratio of the two voltages associated with the vertical electric field as in equation (30). Consider the limiting form for low frequencies and for the special case of $\chi = \infty$, which is

$$\frac{\tilde{V}_2}{\tilde{V}_1} \simeq \frac{1}{\gamma_{z_2} h}\left(\frac{\gamma_1}{\gamma_2}\right)^2 \simeq \frac{\sigma_1}{\sigma_2}\frac{1}{h\sqrt{s\mu_o\sigma_2}} = \left(\frac{\sigma_1}{\sigma_2}\right)^{3/2}\frac{1}{2\sqrt{s_2}} \tag{69}$$

For sufficiently low frequencies the magnitude of this ratio can be of the order of or greater than one. Note that the magnitude of this ratio can be decreased by reducing σ_1/σ_2. Then for the case in which medium 1 is conducting there may be problems associated with the distortion of the fields by the conductors in the ground at the ends of the surface transmission line.

For the normalized response functions for the field components and related quantities we choose

$$\tilde{B} = \frac{H_2}{s_2} . \tag{70}$$

where H_2 is an arbitrary constant with dimensions of amperes/meter. Again this makes the magnetic field just under the sheet source a step function. The normalized magnetic field at $z = 0$ is then defined as

$$\tilde{h}_2(s_2) \equiv \frac{\tilde{H}_{y_1}}{H_2} e^{\gamma_s x} \bigg|_{z=0} = \frac{1}{s_2} \left[\cosh(Q_1) + \frac{Q}{Q_1} \sinh(Q_1) \right]^{-1} \tag{71}$$

The time delay and attenuation with x are both removed by the inclusion of $e^{\gamma_s x}$ in defining the normalized Laplace transforms of the various quantities. The normalized horizontal electric field at $z = h$ is defined as

$$\tilde{e}_{s_2}(s_2) \equiv - \frac{\sigma_1 h}{H_2} \tilde{E}_{x_1} e^{\gamma_s x} \bigg|_{z=h} = \frac{1}{s_2} \frac{Q_1}{1+s_2 \frac{t_{r_1}}{t_2}} \frac{\sinh(Q_1) + \frac{Q}{Q_1} \cosh(Q_1)}{\cosh(Q_1) + \frac{Q}{Q_1} \sinh(Q_1)} \tag{72}$$

This last field component is that required of the sheet source. The normalized average vertical electric field in medium 1 is defined as

$$\tilde{e}_{avg_2}(s_2) \equiv - \frac{\sigma_1 \chi}{H_2} \tilde{E}_{z_{1_{avg}}} e^{\gamma_s x} = \frac{1}{s_2 Q_1} \frac{1+s_2 \frac{t_3}{t_2}}{1+s_2 \frac{t_{r_1}}{t_2}} \frac{\sinh(Q_1)+\frac{Q}{Q_1}\left[\cosh(Q_1)-1\right]}{\cosh(Q_1)+\frac{Q}{Q_1} \sinh(Q_1)}$$

(73)

The normalized vertical total current density at z = 0 is defined as

$$\tilde{j}_{z_2}(s_2) \equiv -\frac{\chi}{H_2}\tilde{J}_{t_{z_1}} e^{\gamma_s x}\Big|_{z=0} = \frac{1}{s_2}\left(s_2\frac{t_3}{t_2}+1\right)\left[\cosh(Q_1)+\frac{Q}{Q_1}\sinh(Q_1)\right]^{-1} \quad (74)$$

For the case that $\chi = \infty$ both e_{avg_2} and j_{z_2} become infinite. For such a case we define another normalized average vertical electric field in medium 1 as

$$\tilde{e}'_{avg_2}(s_2) \equiv -\frac{\sigma_1 c t_2}{H_2}\tilde{E}_{z_{1avg}} e^{\gamma_s x} = \frac{1}{Q_1\left(1+s_2\frac{t_{r_1}}{t_2}\right)} \frac{\sinh(Q_1)+\frac{Q}{Q_1}[\cosh(Q_1)-1]}{\cosh(Q_1)+\frac{Q}{Q_1}\sinh(Q_1)} \quad (75)$$

and another normalized vertical total current density at z = 0 as

$$\tilde{j}'_{z_2}(s_2) \equiv -\frac{ct_2}{H_2}\tilde{J}_{t_{z_1}} e^{\gamma_s x}\Big|_{z=0} = \left[\cosh(Q_1)+\frac{Q}{Q_1}\sinh(Q_1)\right]^{-1} \quad (76)$$

Note that Q_1 appears as the argument of the exponential (sinh and cosh) terms. We can use the limiting form for high frequencies of e^{-Q_1} to calculate the transit time and attenuation of discontinuities in the waveforms for the field components. Expand Q_1 for large s_2 giving

$$Q_1 \simeq 2\, s_2\left[\frac{t_{r_1}}{t_2}\left(1-\frac{1}{\varepsilon_{r_1}}\right)\right]^{1/2}\left\{1+\frac{1}{s_2}\left[1-\frac{2}{\varepsilon_{r_1}}\frac{t_{r_1}}{t_2}\frac{t_2}{t_3}\right]\left[\frac{t_{r_1}}{t_2}\left(1-\frac{1}{\varepsilon_{r_1}}\right)\right]^{-1}\cdots\right\}^{1/2}$$

$$\simeq 2\, s_2\left[\frac{t_{r_1}}{t_2}\left(1-\frac{1}{\varepsilon_{r_1}}\right)\right]^{1/2} + \left[1-\frac{2}{\varepsilon_{r_1}}\frac{t_{r_1}}{t_2}\frac{t_2}{t_3}\right]\left[\frac{t_{r_1}}{t_2}\left(1-\frac{1}{\varepsilon_{r_1}}\right)\right]^{-1/2}\cdots \quad (77)$$

Then we have a normalized delay time given by

$$\Delta\tau_2^- = 2\left[\frac{t_{r_1}}{t_2}\left(1-\frac{1}{\varepsilon_{r_1}}\right)\right]^{1/2} \quad (78)$$

This is the time (at a fixed x) for discontinuities in a waveform to travel between $z = h$ and $z = 0$. There is also a transmission factor given by

$$f = e^{-\left[1 - \frac{2}{\varepsilon_{r_1}} \frac{t_{r_1}}{t_2} \frac{t_2}{t_3}\right]\left[\frac{t_{r_1}}{t_2}\left(1 - \frac{1}{\varepsilon_{r_1}}\right)\right]^{-1/2}} \tag{79}$$

This is the factor by which discontinuities are reduced (at a fixed x) in travelling between $z = h$ and $z = 0$. There is a common factor in various of the field quantities which is of the form

$$\left[\cosh(Q_1) + \frac{Q}{Q_1}\sinh(Q_1)\right]^{-1} = \left[\left(1 + \frac{Q}{Q_1}\right)\frac{e^{Q_1}}{2} + \left(1 - \frac{Q}{Q_1}\right)\frac{e^{-Q_1}}{2}\right]^{-1} \tag{80}$$

From this term we have the reflection coefficient at $z = 0$ for the magnetic field as

$$r_h = \frac{1 - \frac{Q}{Q_1}}{1 + \frac{Q}{Q_1}} \tag{81}$$

For the special limiting case of $\sigma_2 = \infty$ this becomes

$$r_h\Big|_{\sigma_2 = \infty} = 1 \tag{82}$$

In the more general case of finite σ_2 the limiting form of this reflection coefficient for high frequencies is

$$\lim_{s_2 \to \infty} r_h \equiv r_{h_o} = \frac{1 - \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}\left[\frac{\varepsilon_{r_2} - 1}{\varepsilon_{r_1} - 1}\right]^{1/2}}{1 + \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}\left[\frac{\varepsilon_{r_2} - 1}{\varepsilon_{r_1} - 1}\right]^{1/2}} \tag{83}$$

The reflection coefficient at $z = h$ for the magnetic field is -1 because of the constraint that the magnetic field be a step function there. The vertical electric field and vertical total current density have the same reflection coefficients as the magnetic field while the horizontal electric field has the sign of these reflection coefficients reversed. Combining the reflection coefficients, transmission factor, and delay time one can follow the first discontinuity in a waveform on through the pulse.

The normalized magnetic field has an initial rise at $\tau_2 = \Delta\tau_2$ for finite σ_2 given by

$$h_2(\Delta\tau_2+) = \lim_{s_2\to\infty} s_2\tilde{h}_2(s_2)e^{s_2\Delta\tau_2} = 2f \lim_{s_2\to\infty}\left[1 + \frac{Q}{Q_1}\right]^{-1}$$

$$= 2f\left\{1 + \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}\left[\frac{\varepsilon_{r_2} - 1}{\varepsilon_{r_1} - 1}\right]^{1/2}\right\}^{-1} \qquad (84)$$

and for the limiting case of $\sigma_2 = \infty$ this becomes

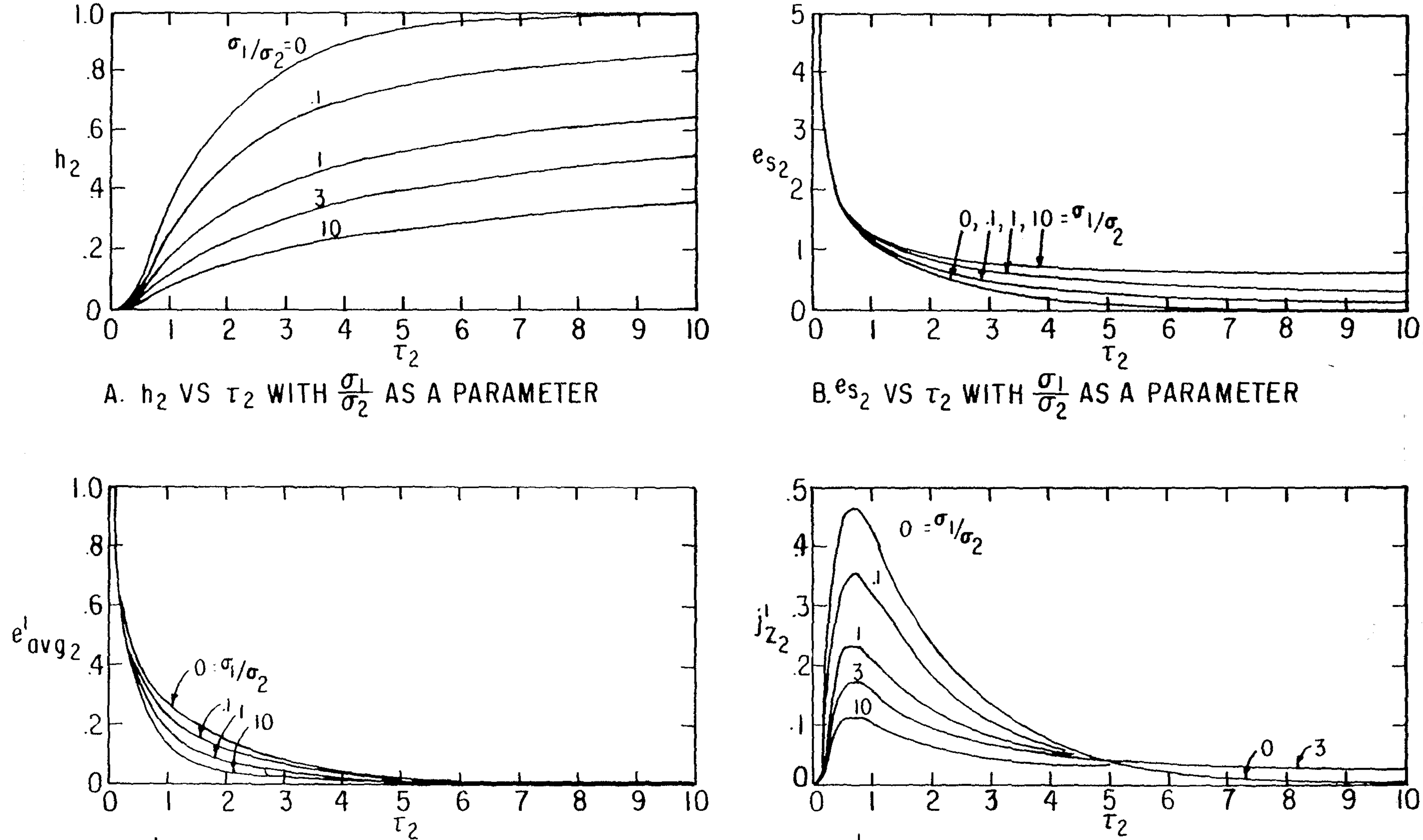
$$h_2(\Delta\tau_2+) = 2f \qquad (85)$$

The normalized horizontal electric field at $z = h$ has an initial rise at $\tau_2 = 0$ given by

$$e_{s_2}(0+) = \lim_{s_2\to\infty} s_2\tilde{e}_{s_2}(s_2) = \lim_{s_2\to\infty} \frac{Q_1}{1 + s_2\frac{t_{r_1}}{t_2}}$$

$$= 2\left[\frac{t_2}{t_{r_1}}\left(1 - \frac{1}{\varepsilon_{r_1}}\right)\right]^{1/2} \qquad (86)$$

The time-domain waveforms for the normalized field components are plotted versus τ_2 in figures 3 through 17. First the limiting case of $t_{r_1}/t_2 = t_{r_2}/t_2 = 0$ is considered in figures 3 through 5. In this case j'_{z_2} and j_{z_2} are included, whereas they are not subsequently included because of the recurring δ functions in the waveforms if $t_{r_1}/t_2 > 0$. Proceeding on, figures 6 through 11 are for the case that there is no attenuation of the source strength with x so that $p_5 = 0$. The two cases of $\varepsilon_{r_1} = 1$ and $\varepsilon_{r_1} = 10$ are included. Note that as σ_1/σ_2 is increased, the rise time of h_2 is increased. Finally, in figures 12 through 17 the effect of varying p_5 is considered. Note that $\sigma_1/\sigma_2 = 0$ is chosen for this last case so that difficulty is not encountered due to the mathematical form of the fields in medium 2. As illustrated in the figures, small p_5 has little effect on the waveforms for h_2 and e_{s_2}, but has a significant effect on e_{avg_2}. In any case that ε_{r_2} entered into the calculations for one of the figures in this note it was set equal to 10.



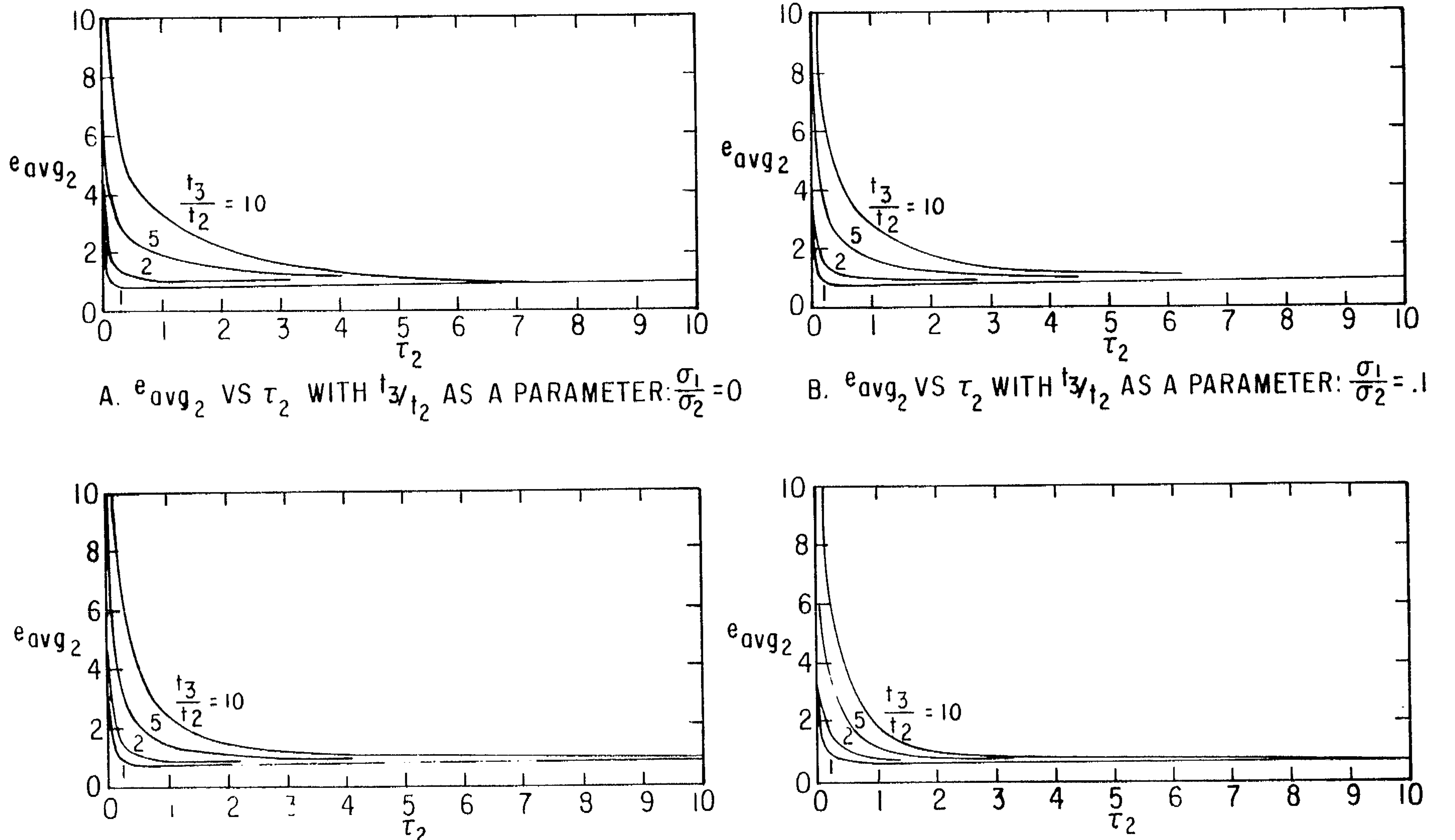
A. h_2 VS τ_2 WITH $\frac{\sigma_1}{\sigma_2}$ AS A PARAMETER

B. e_{s_2} VS τ_2 WITH $\frac{\sigma_1}{\sigma_2}$ AS A PARAMETER

C. e'_{avg_2} VS τ_2 WITH $\frac{\sigma_1}{\sigma_2}$ AS A PARAMETER

D. j'_{z_2} VS τ_2 WITH $\frac{\sigma_1}{\sigma_2}$ AS A PARAMETER

FIGURE 3. PULSE SHAPES ON TRANSMISSION LINE: $t_{r1}/t_2 = t_{r2}/t_2 = 0$



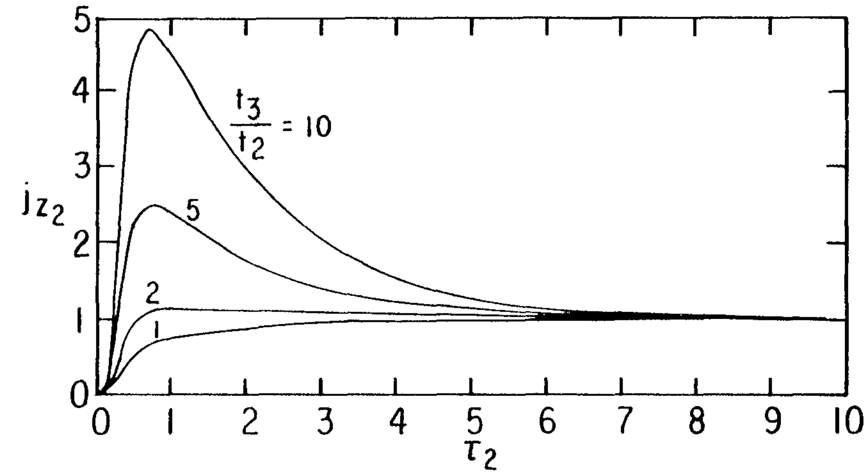
A. e_{avg_2} VS τ_2 WITH t_3/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 0$

B. e_{avg_2} VS τ_2 WITH t_3/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = .1$

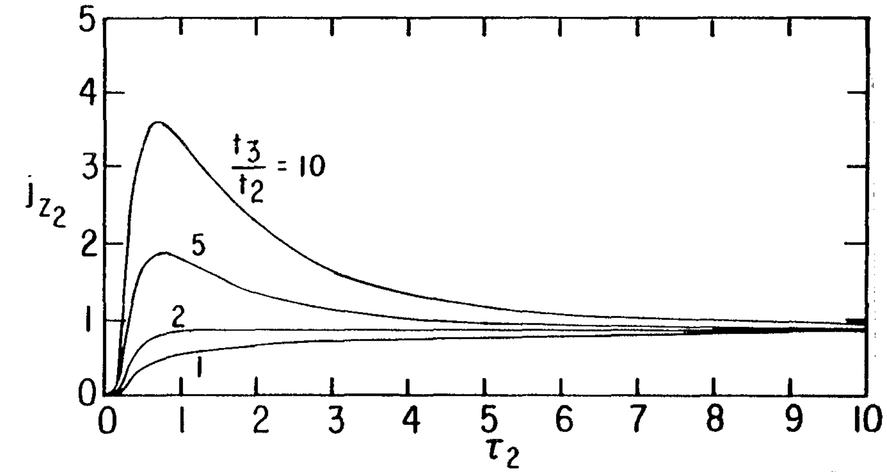
C. e_{avg_2} VS τ_2 WITH t_3/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 1$

D. e_{avg_2} VS τ_2 WITH t_3/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 10$

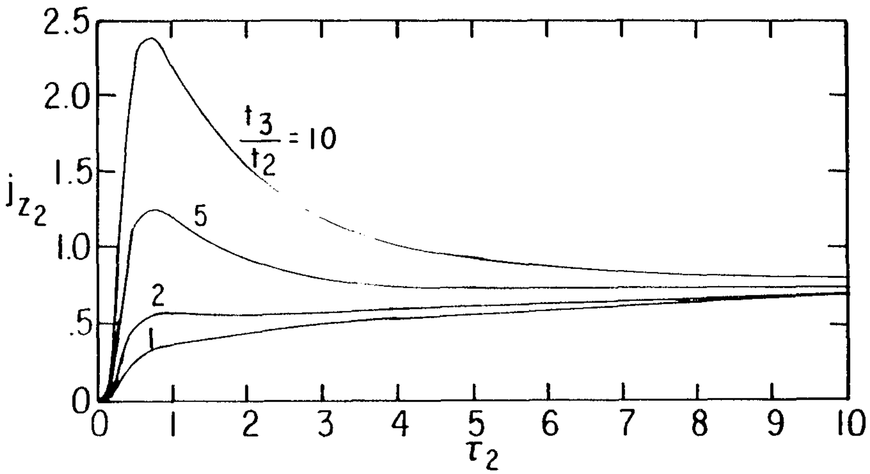
FIGURE 4. AVERAGE VERTICAL ELECTRIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $t_{r1}/t_2 = t_{r2}/t_2 = 0$



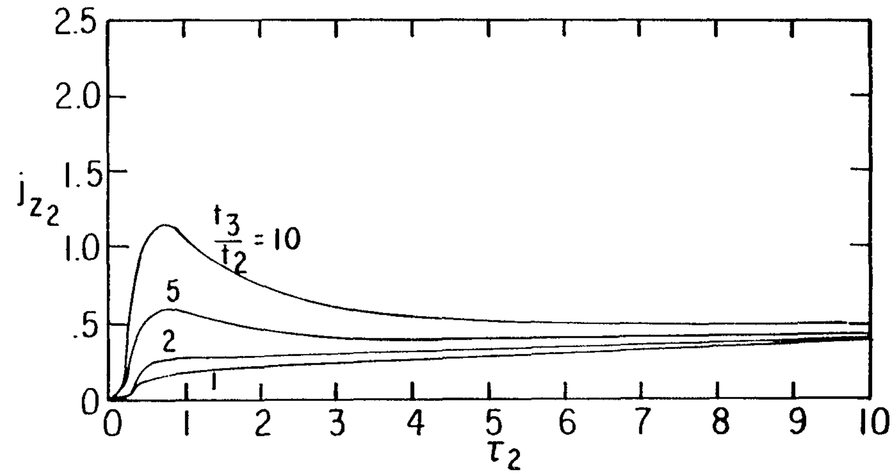
A. j_{z_2} VS τ_2 WITH t_3/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 0$



B. j_{z_2} VS τ_2 WITH t_3/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = .1$

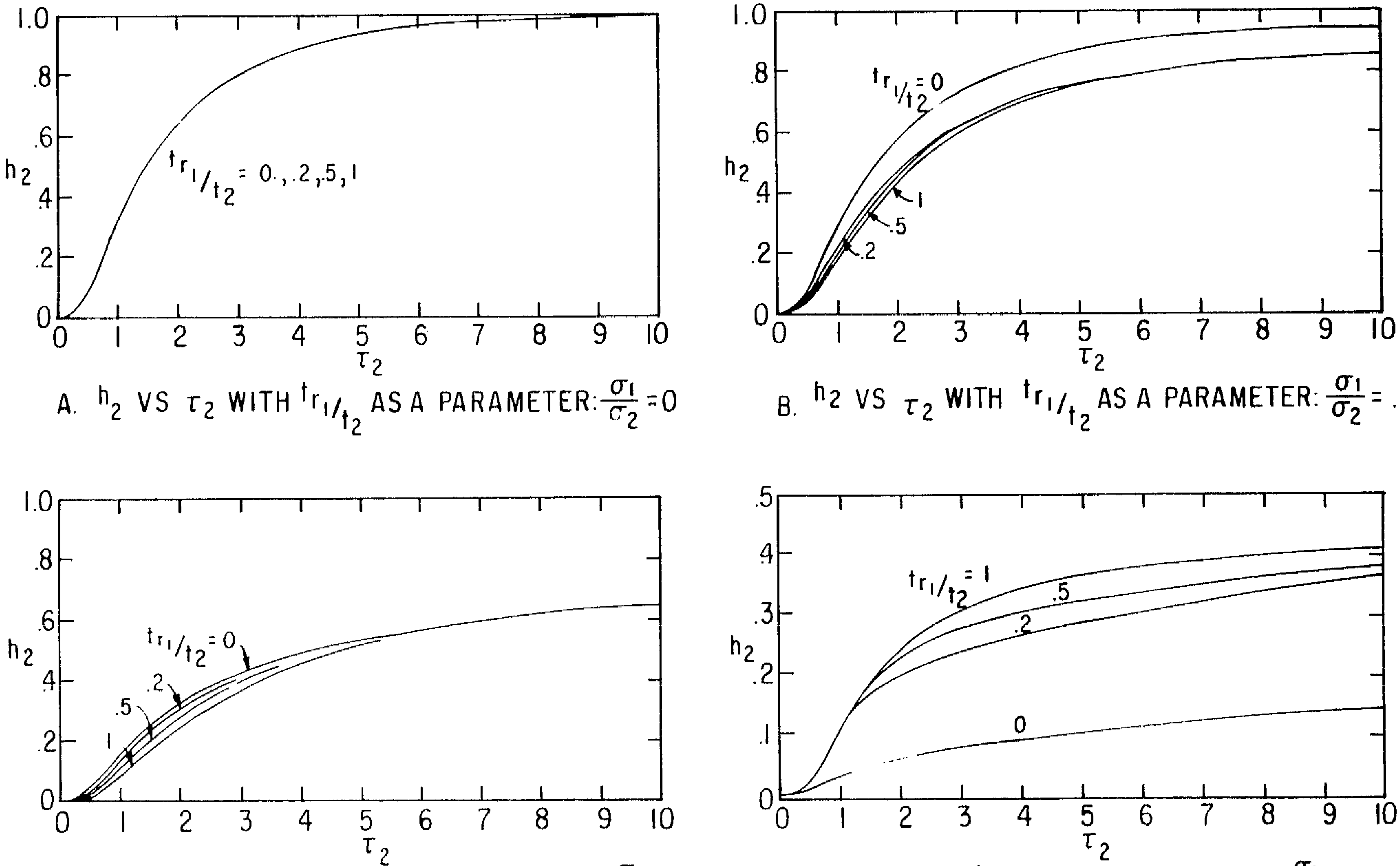


C. j_{z_2} VS τ_2 WITH t_3/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 1$



j_{z_2} VS τ_2 WITH t_3/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 10$

FIGURE 5. VERTICAL CURRENT DENSITY PULSE SHAPE ON TRANSMISSION LINE: $t_{r_1}/t_2 = t_{r_2}/t_2 = 0$



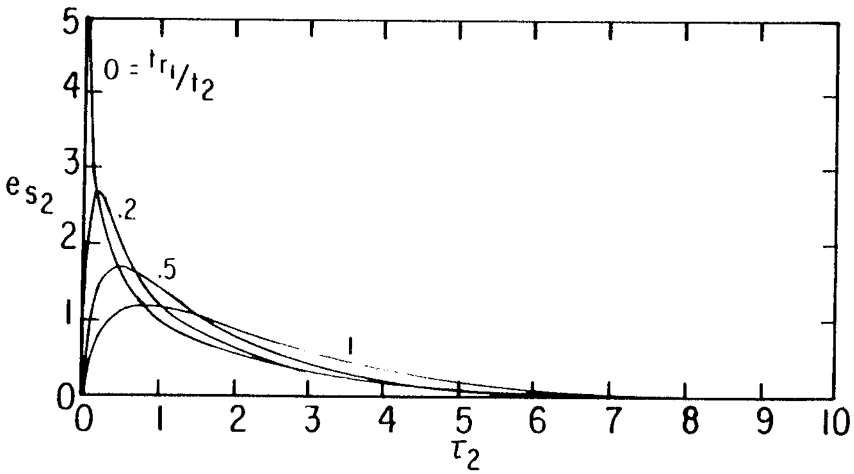
A. h_2 VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 0$

B. h_2 VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = .1$

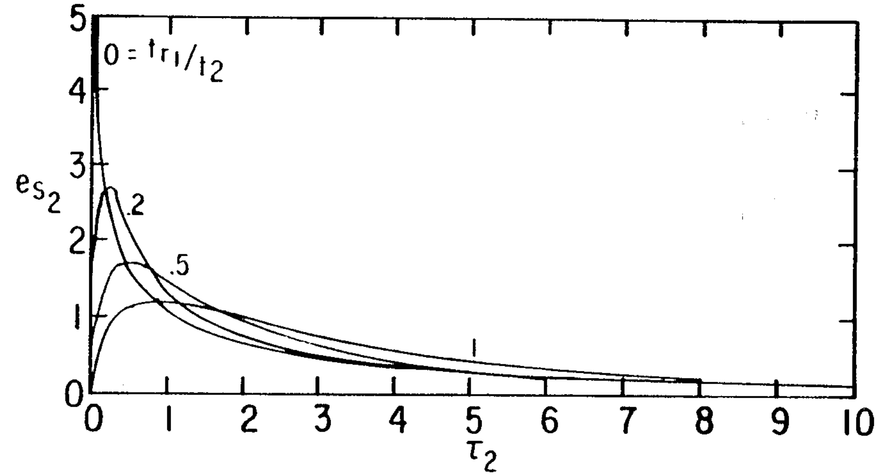
C. h_2 VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 1$

D. h_2 VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 10$

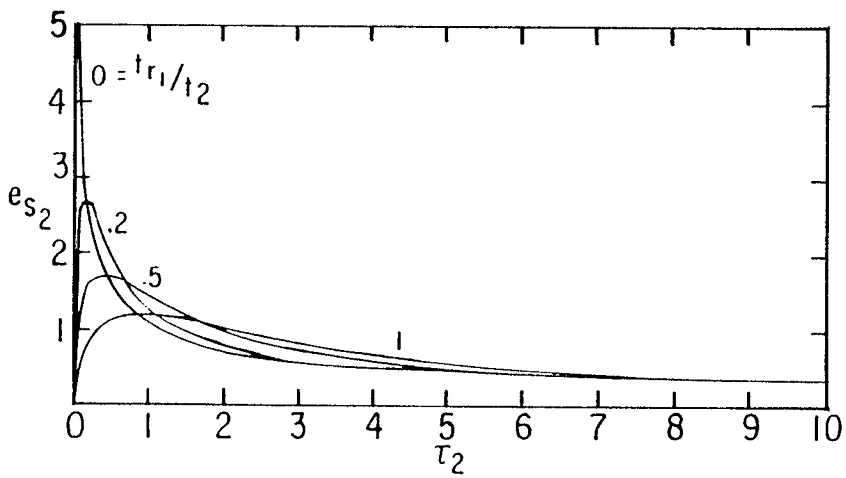
FIGURE 6. MAGNETIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\epsilon_{r_1} = 1$, $\epsilon_{r_2} = 10$, $\rho_5 = 0$



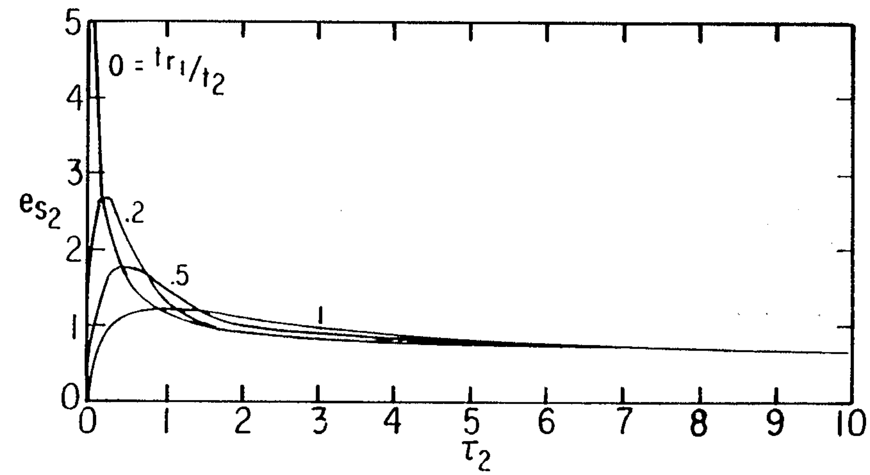
A. e_{s_2} VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 0$



B. e_{s_2} VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = .1$

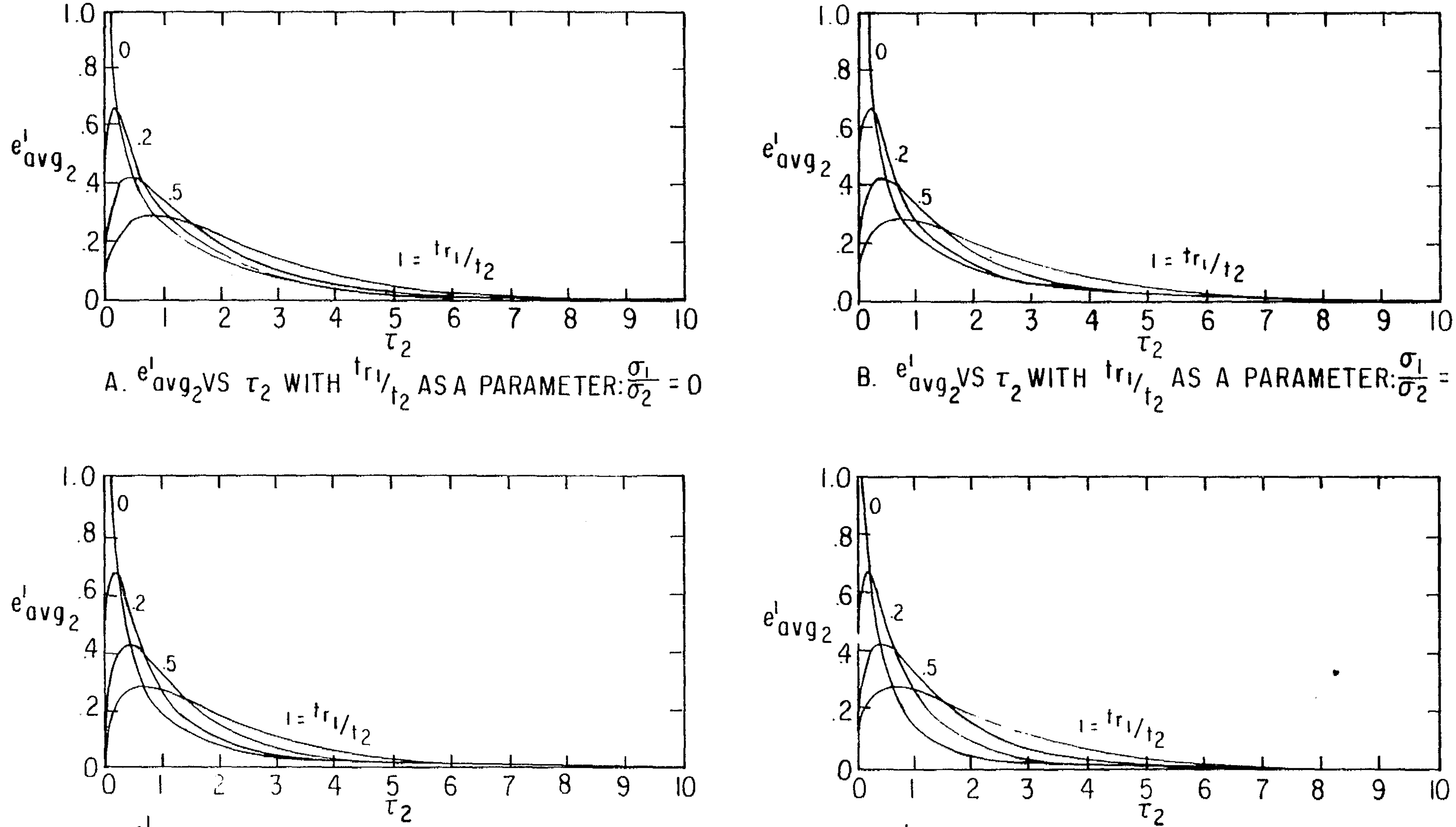


C. e_{s_2} VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 1$



D. e_{s_2} VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 10$

FIGURE 7. HORIZONTAL ELECTRIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\epsilon_{r_1} = 1, \epsilon_{r_2} = 10, p_5 = 0$



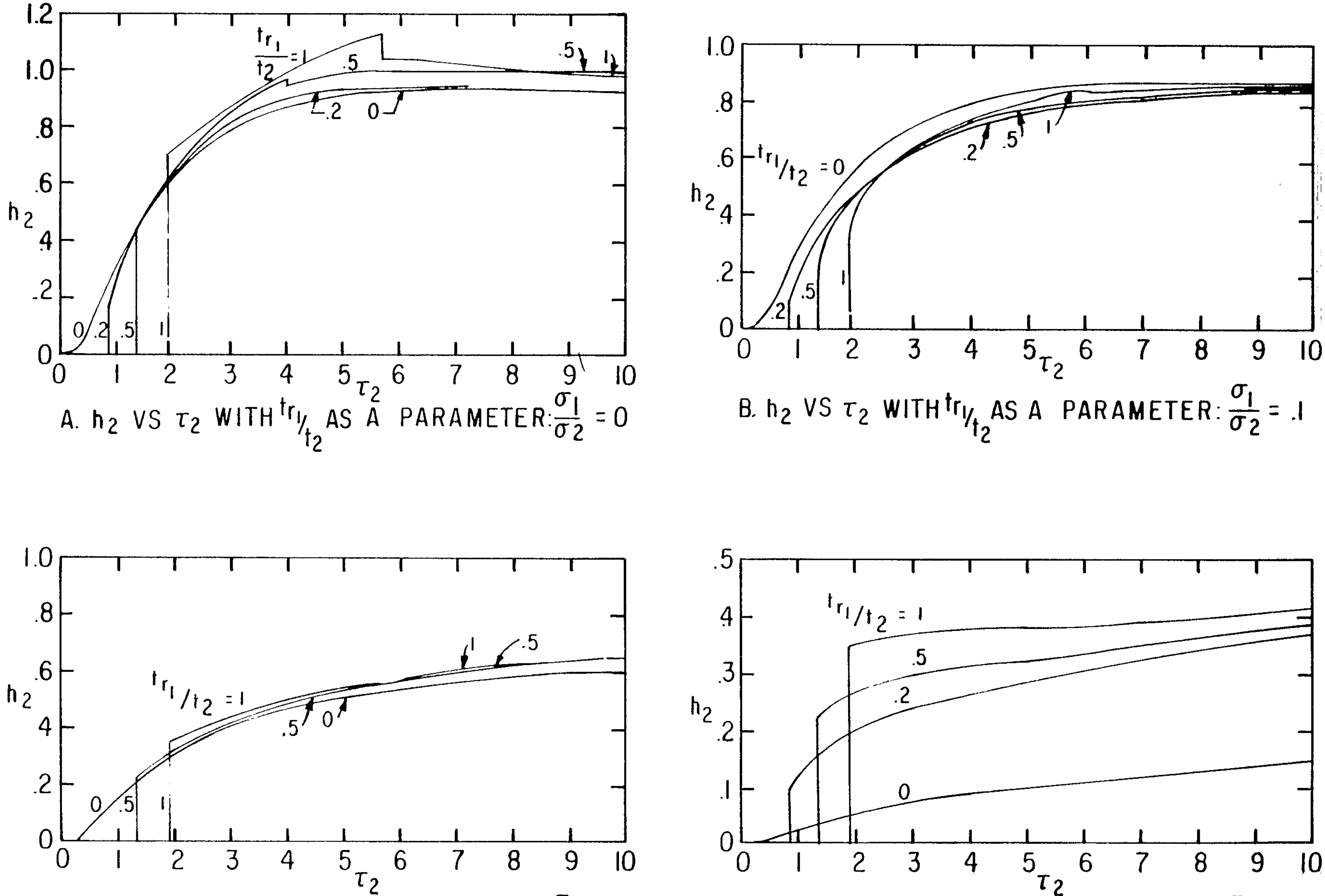
A. e'_{avg_2} VS τ_2 WITH ${t_{r1}}/{t_2}$ AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 0$

B. e'_{avg_2} VS τ_2 WITH ${t_{r1}}/{t_2}$ AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = .1$

C. e'_{avg_2} VS τ_2 WITH ${t_{r1}}/{t_2}$ AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 1$

D. e'_{avg_2} VS τ_2 WITH ${t_{r1}}/{t_2}$ AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 10$

FIGURE 8. AVERAGE VERTICAL ELECTRIC PULSE SHAPE ON TRANSMISSION LINE: $\epsilon_{r_1} = 1$, $\epsilon_{r_2} = 10$, $p_5 = 0$



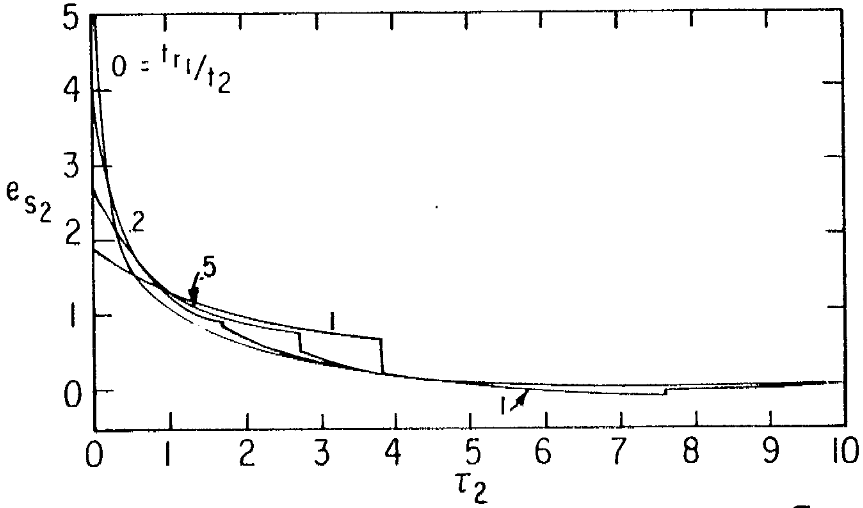
A. h_2 VS τ_2 WITH t_{r1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 0$

B. h_2 VS τ_2 WITH t_{r1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = .1$

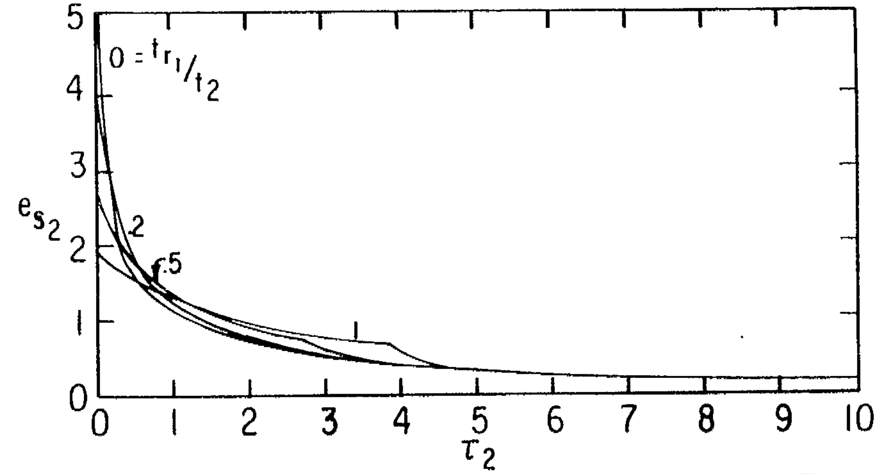
C. h_2 VS τ_2 WITH t_{r1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 1$

D. h_2 VS τ_2 WITH t_{r1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 10$

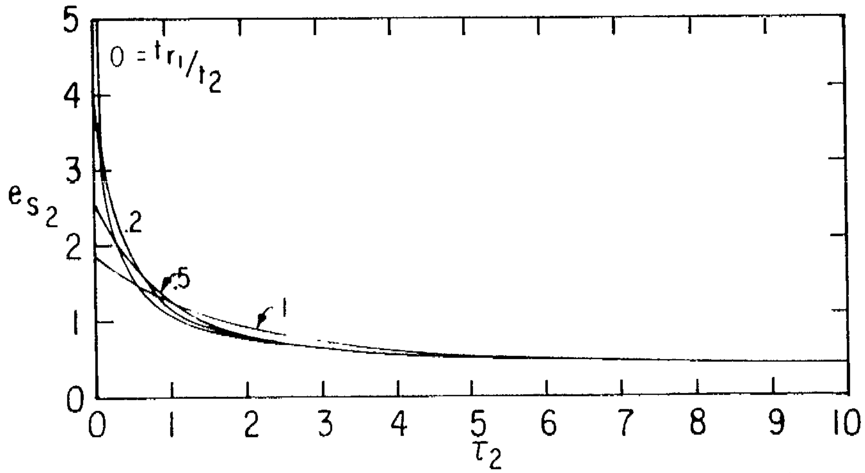
FIGURE 9. MAGNETIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\epsilon_{r_1} = 10$, $\epsilon_{r_2} = 10$, $P_5 = 0$



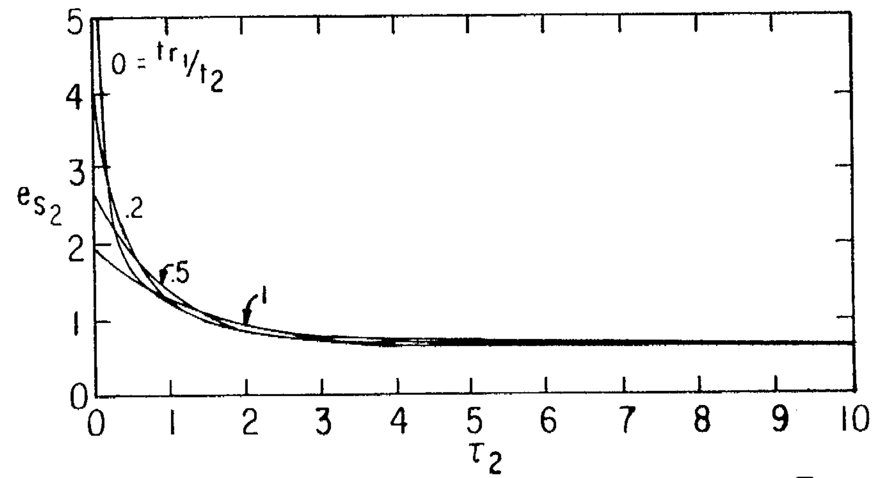
A. e_{s_2} VS τ_2 WITH ${}^{t_{r1}}/_{t_2}$ AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 0$



B. e_{s_2} VS τ_2 WITH ${}^{t_{r1}}/_{t_2}$ AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = .1$

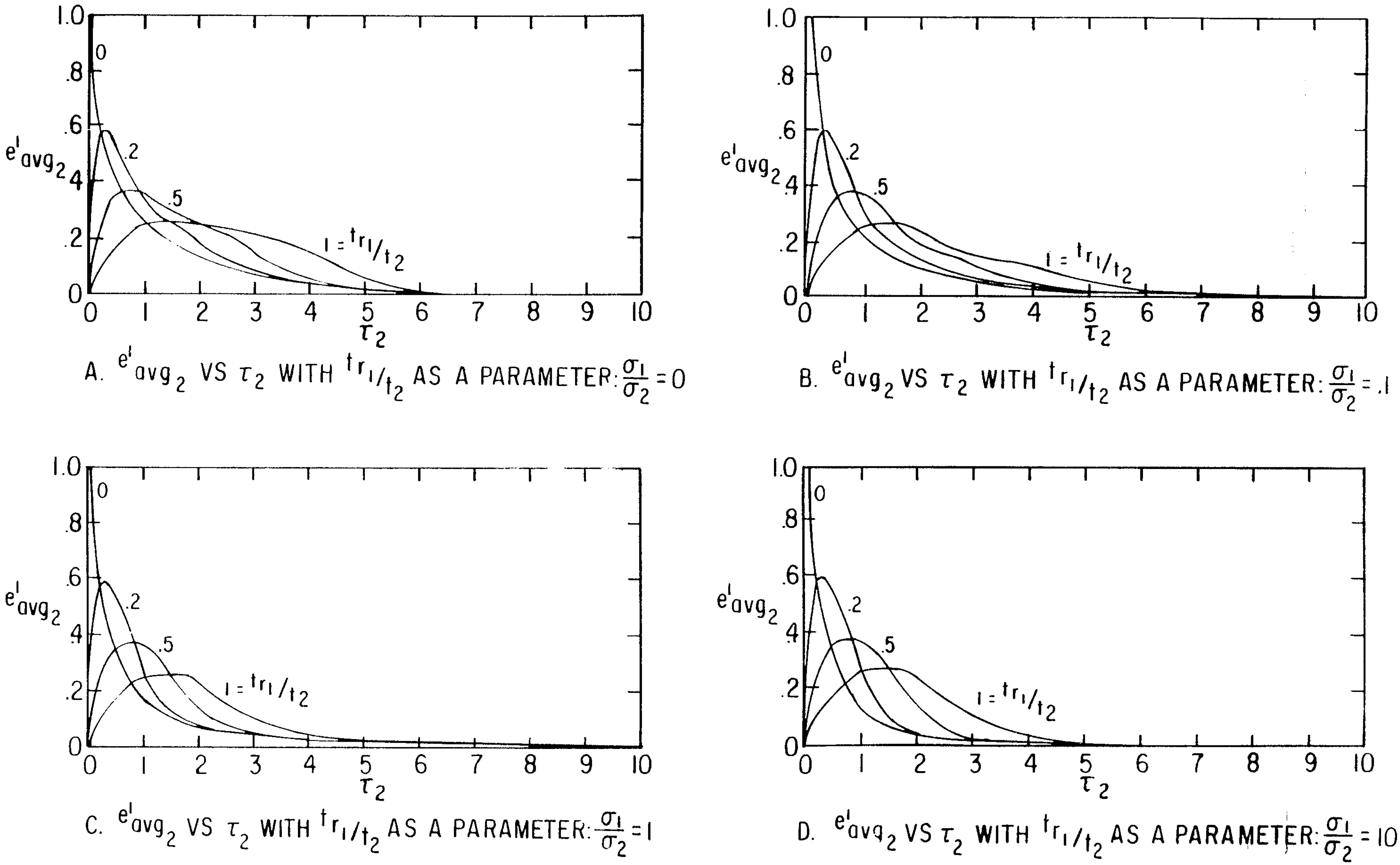


C. e_{s_2} VS τ_2 WITH ${}^{t_{r1}}/_{t_2}$ AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 1$



D. e_{s_2} VS τ_2 WITH ${}^{t_{r1}}/_{t_2}$ AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 10$

FIGURE 10. HORIZONTAL ELECTRIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\epsilon_{r_1} = 10$, $\epsilon_{r_2} = 10$, $P_5 = 0$



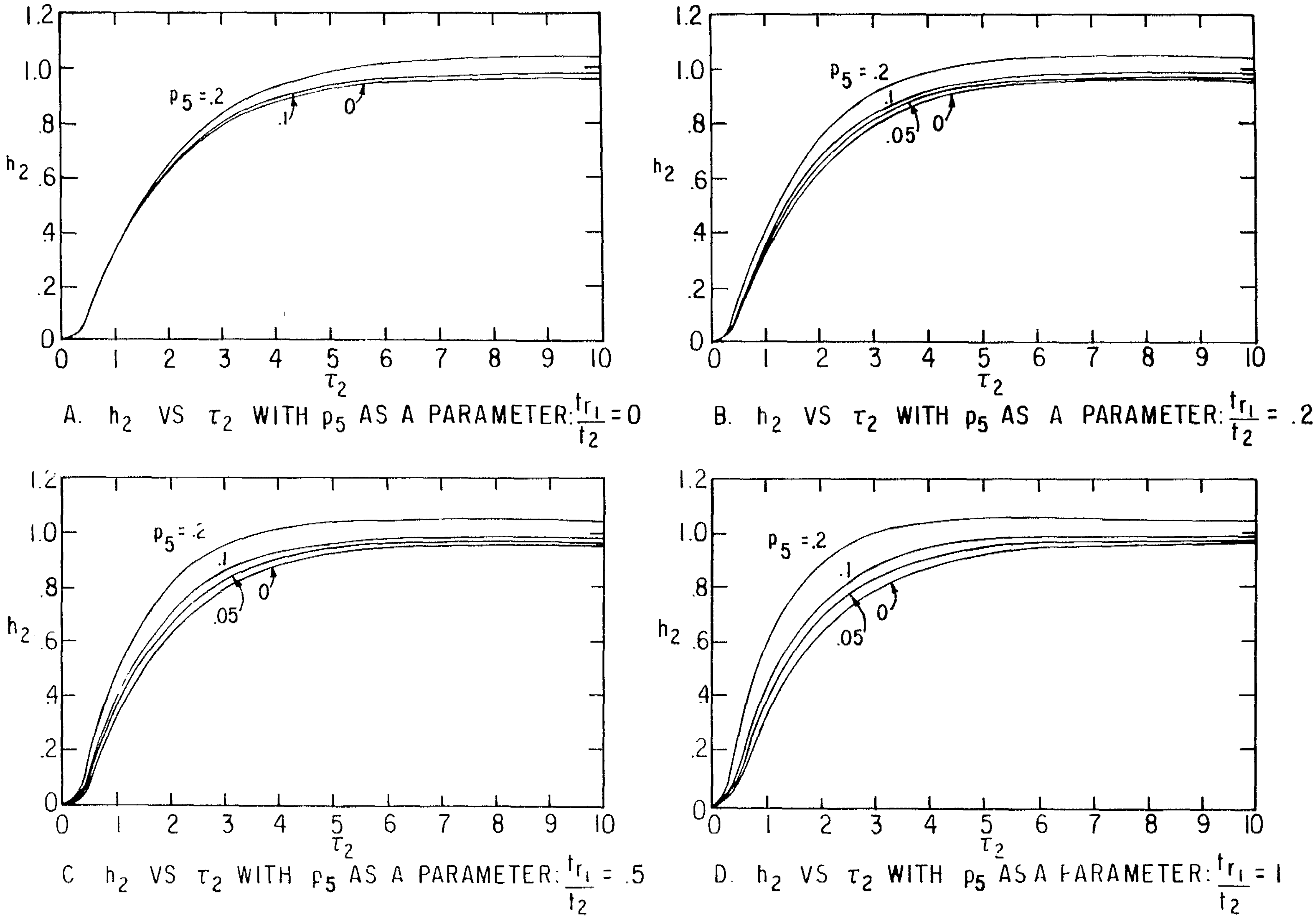
A. e'_{avg_2} VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 0$

B. e'_{avg_2} VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = .1$

C. e'_{avg_2} VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 1$

D. e'_{avg_2} VS τ_2 WITH t_{r_1}/t_2 AS A PARAMETER: $\frac{\sigma_1}{\sigma_2} = 10$

FIGURE 11. AVERAGE VERTICAL ELECTRIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\epsilon_{r_1} = 10, \epsilon_{r_2} = 10, \rho_5 = 0$



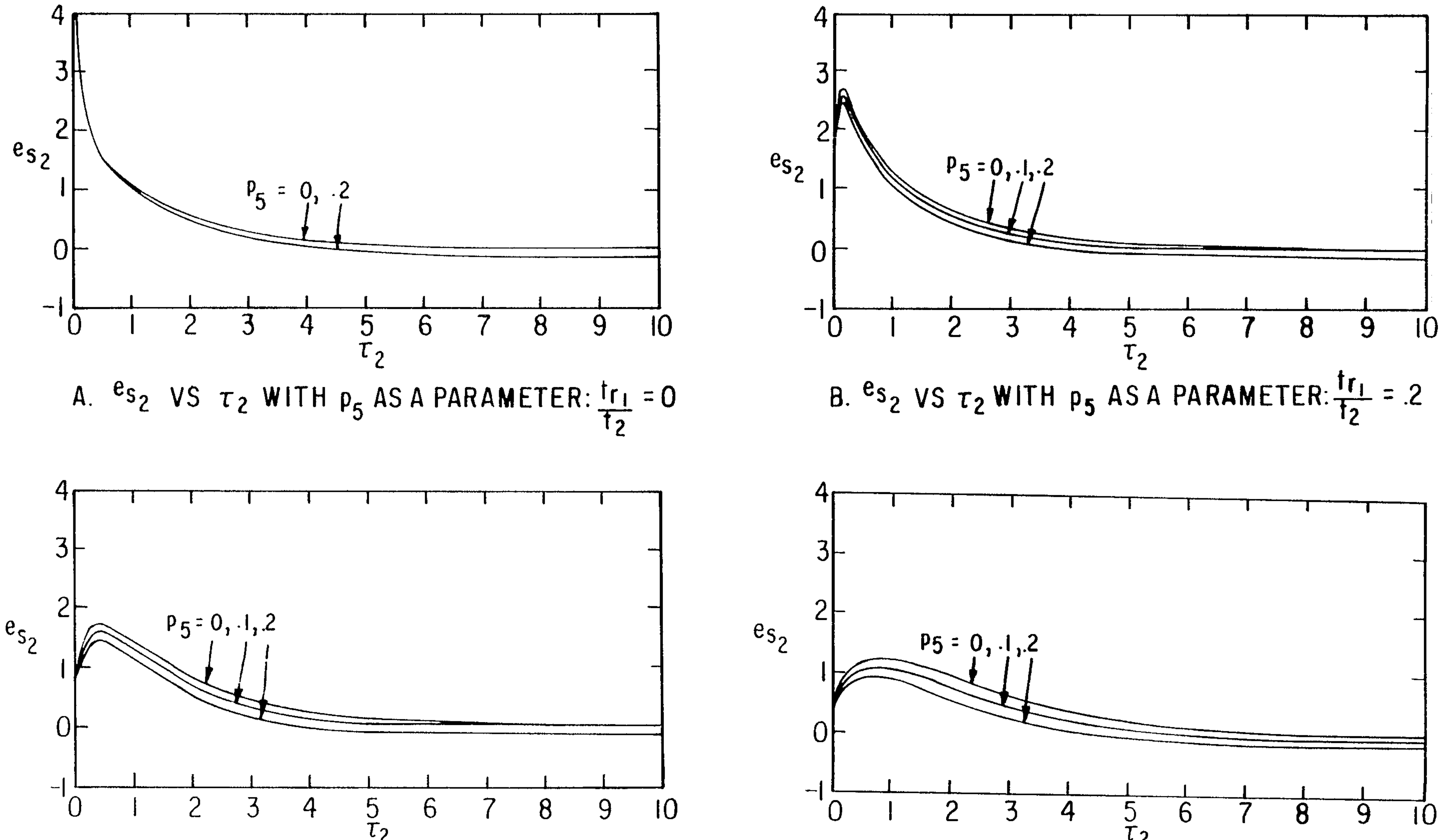
A. h_2 VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 0$

B. h_2 VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .2$

C h_2 VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .5$

D. h_2 VS τ_2 WITH p_5 AS A FARAMETER: $\frac{t_{r_1}}{t_2} = 1$

FIGURE 12. MAGNETIC FIELD PULSE SHAFE ON TRANSMISSION LINE: $\frac{\sigma_1}{\sigma_2} = 0$, $\epsilon_{r_1} = 1$



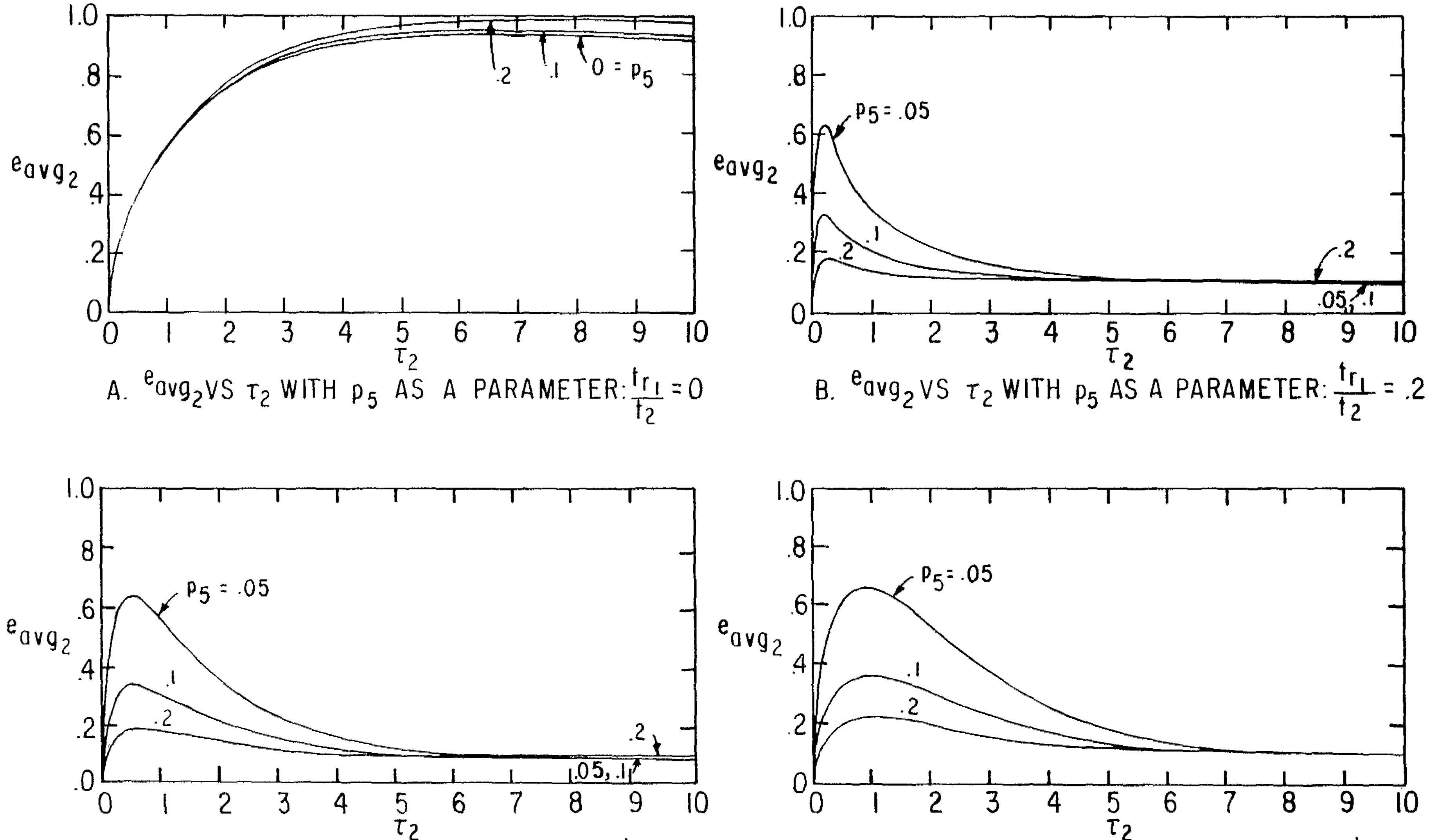
A. e_{s_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 0$

B. e_{s_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .2$

C. e_{s_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .5$

D. e_{s_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 1$

FIGURE 13. HORIZONTAL ELECTRIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\frac{\sigma_1}{\sigma_2} = 0, \epsilon_{r_1} = 1$



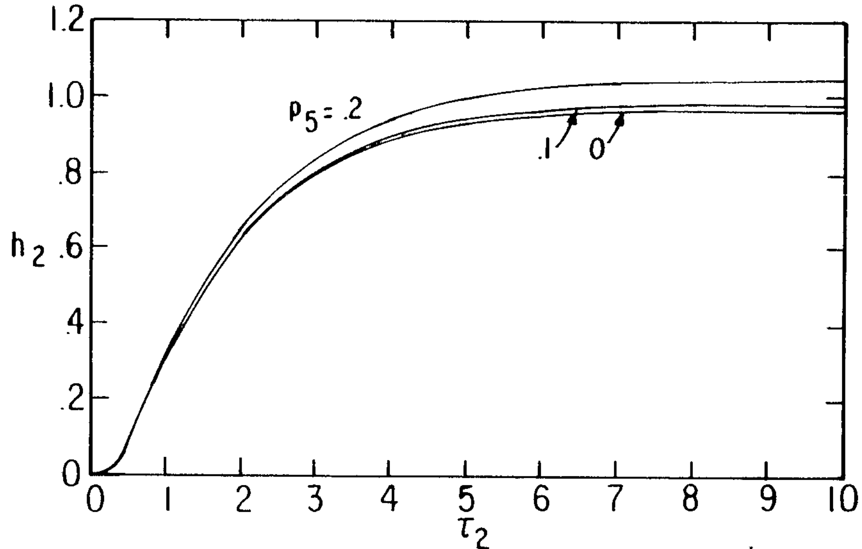
A. e_{avg_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 0$

B. e_{avg_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .2$

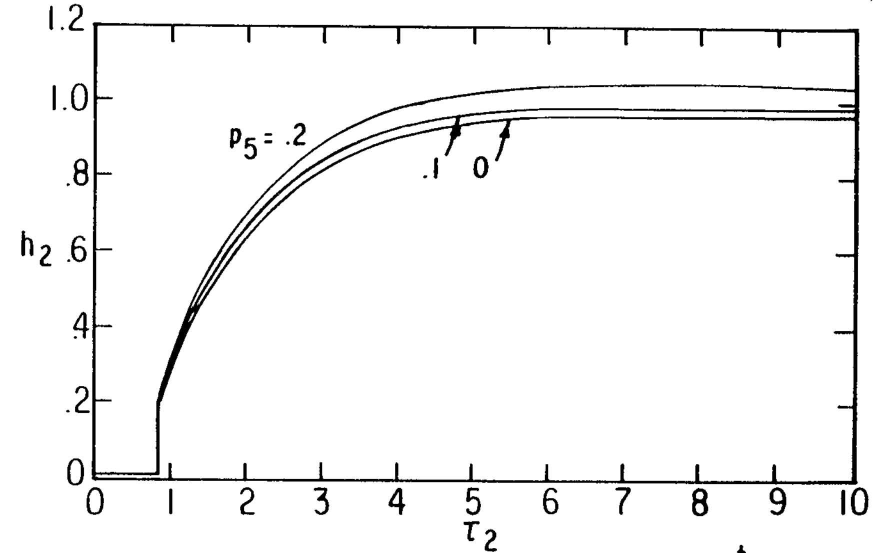
C. e_{avg_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .5$

D. e_{avg_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 1$

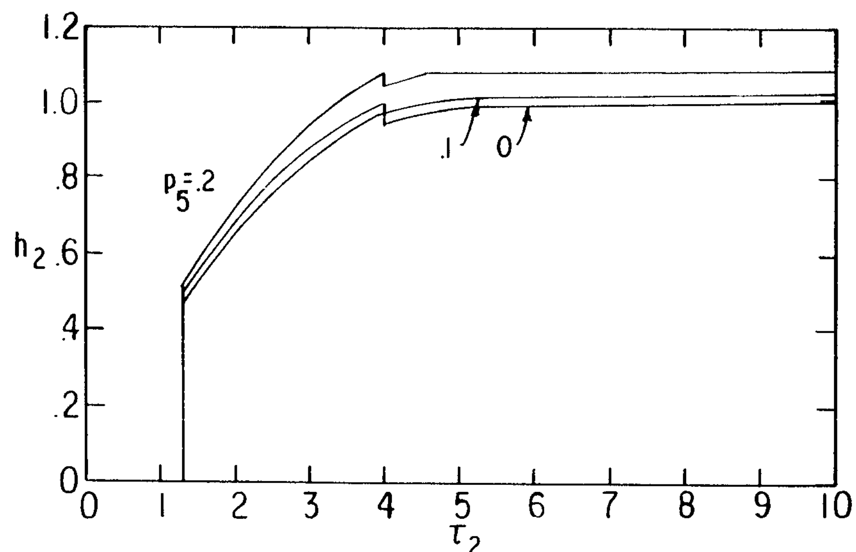
FIGURE 14. AVERAGE VERTICAL ELECTRIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\frac{\sigma_1}{\sigma_2} = 0, \epsilon_{r_1} = 1$



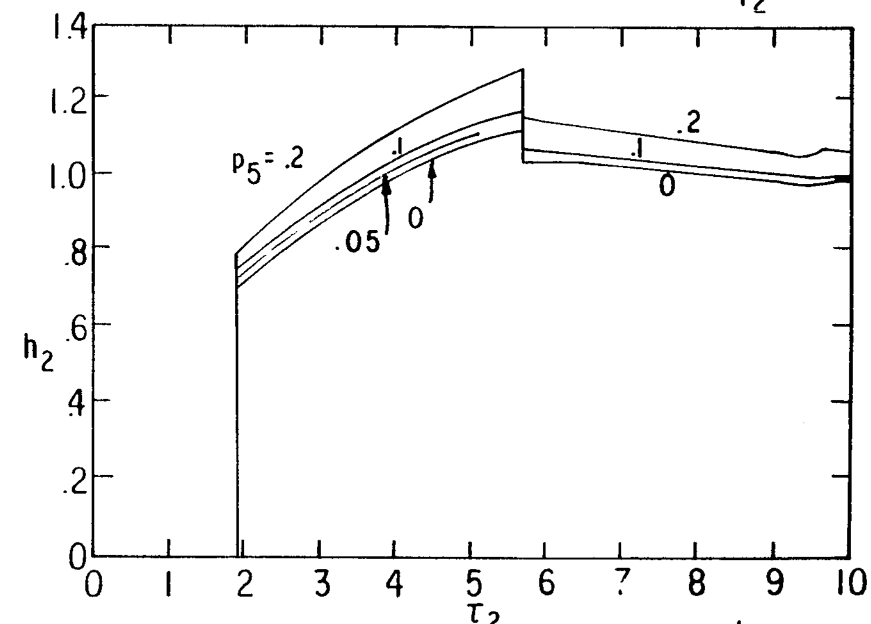
A. h_2 VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 0$



B. h_2 VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .2$

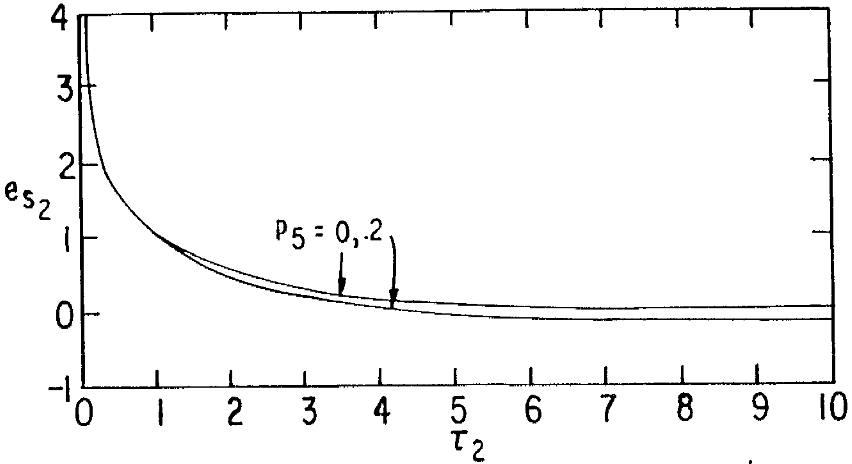


C. h_2 VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .5$

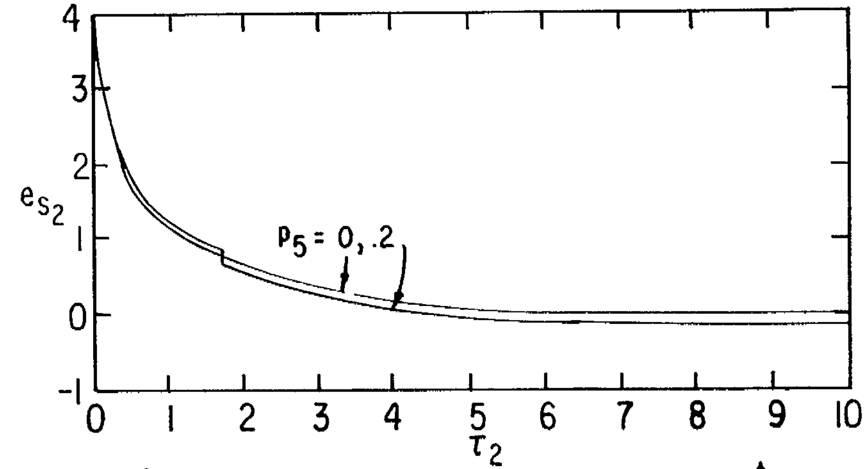


D. h_2 VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 1$

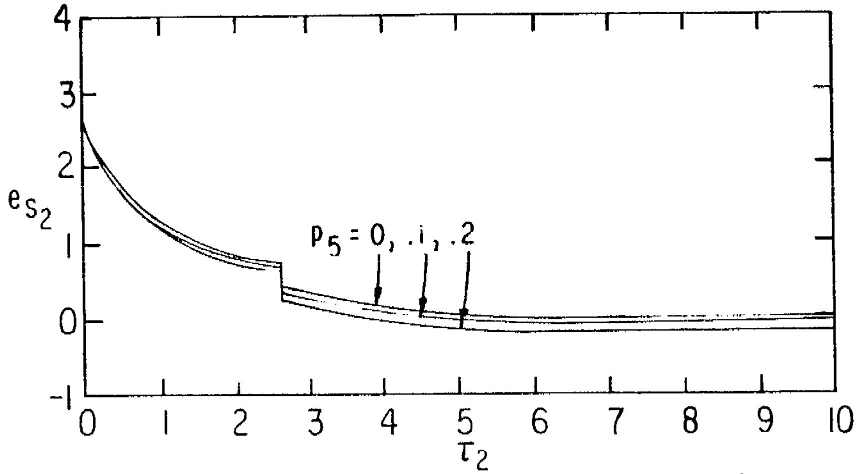
FIGURE 15. MAGNETIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\frac{\sigma_1}{\sigma_2} = 0, \epsilon_{r_1} = 10$



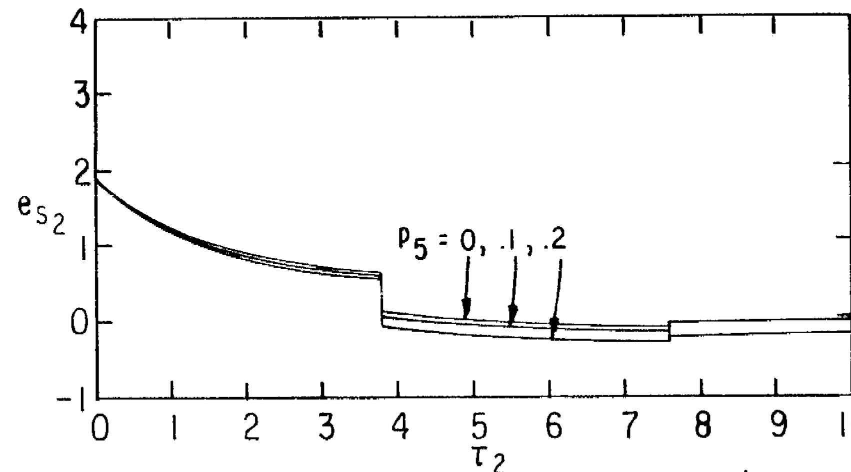
A. e_{s_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 0$



B. e_{s_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .2$

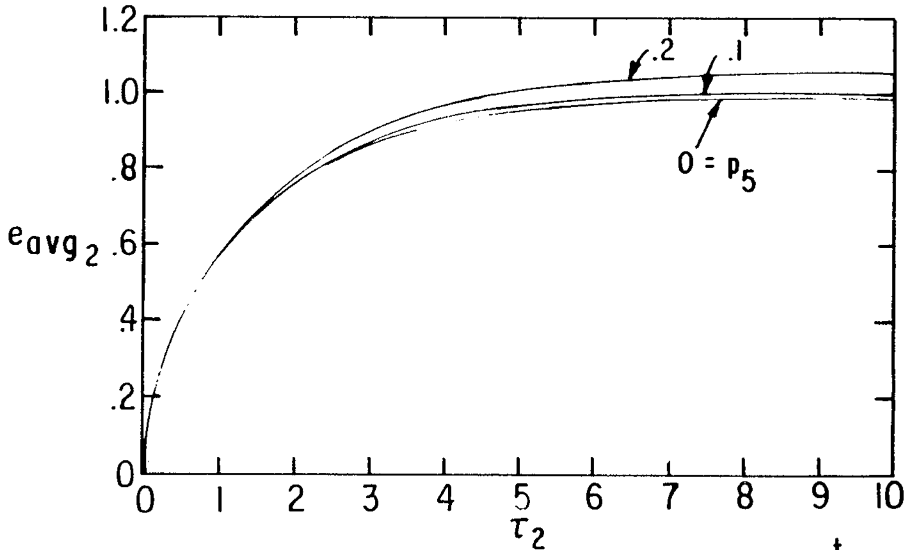


C. e_{s_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .5$

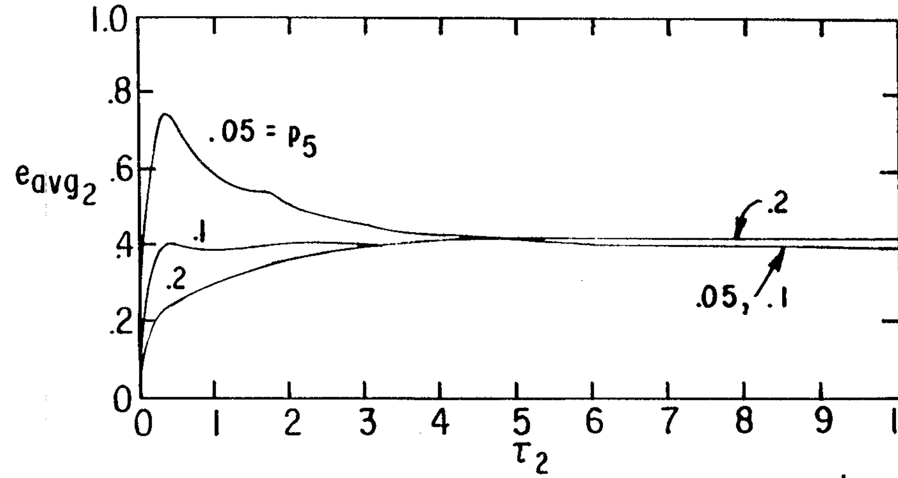


D. e_{s_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 1$

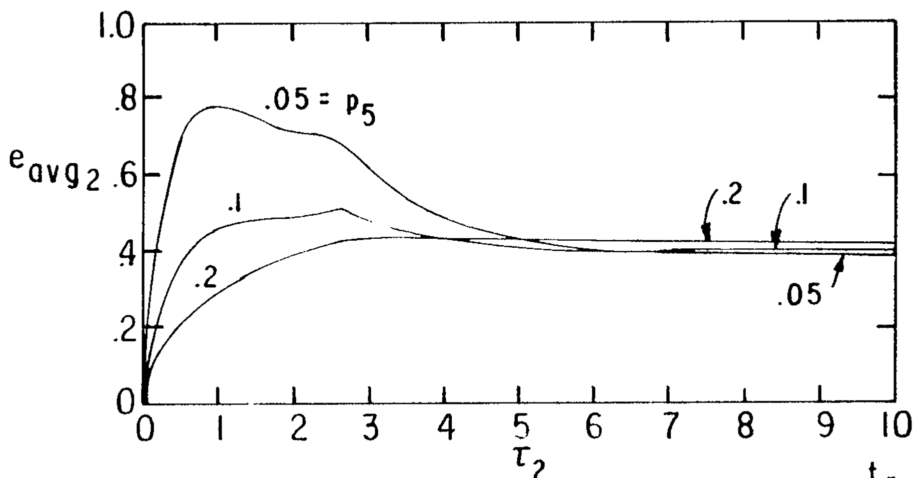
FIGURE 16. HORIZONTAL ELECTRIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\frac{\sigma_1}{\sigma_2} = 0, \epsilon_{r_1} = 10$



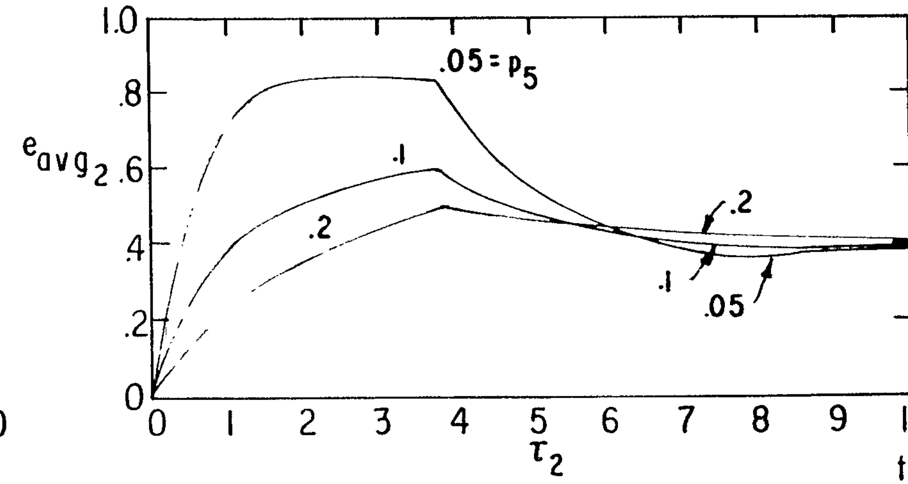
A. e_{avg_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 0$



B. e_{avg_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .2$



C. e_{avg_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = .5$



D. e_{avg_2} VS τ_2 WITH p_5 AS A PARAMETER: $\frac{t_{r_1}}{t_2} = 1$

FIGURE 17. AVERAGE VERTICAL ELECTRIC FIELD PULSE SHAPE ON TRANSMISSION LINE: $\frac{\sigma_1}{\sigma_2} = 0, \epsilon_{r_1} = 10$

V. Summary

We have considered the response characteristics of a transmission line consisting of a distributed planar source above a ground surface. The response characteristics have been considered for two cases: first, medium 1 nonconducting, and, second, medium 1 conducting. With medium 1 conducting, the amplitude of the source can be attenuated along the transmission line so that there is a long-time vertical current density. Making medium 1 conducting may provide a more valid simulation of the close-in nuclear electromagnetic pulse. In addition, the conductivity of medium 1 lowers the ratio of the electric to the magnetic fields for times long compared to t_2, the characteristic diffusion time for medium 1. Thus, large magnetic fields may be attained without unduly large electric fields.

The approach taken in this note is somewhat idealistic in certain assumptions that have been made. In particular, it may be rather difficult to closely approximate the sheet source in practice. There is also the problem of providing the proper sources, terminations, and ground contacts at each end of the transmission line. There are also various other things to consider such as the wave propagating above the sheet source. Hopefully, these problems may not be too difficult so that this type of simulation technique may be useful, at least in an approximate form.