Sensor and Simulation Notes

Note LIII

18 April 1968

Admittance Sheets for Terminating High-Frequency Transmission Lines

Capt. Carl E. Baum
California Institute of Technology
and
Air Force Institute of Technology

Abstract

A simple resistive termination, while adequate for low frequencies, introduces reflections on TEM transmission lines at frequencies high enough that the characteristic dimensions are not electrically small. A more general termination can be defined in the form of an admittance sheet from the solution of an electromagnetic boundary value problem. By approximating the characteristics of this admittance sheet, reflections on the transmission line can be reduced.

I. Introduction

In using a cylindrical transmission line as a simulator for a free-space plane electromagnetic wave one has the general problems of launching and terminating the desired TEM mode without introducing reflections in the TEM and higher order modes. One approach to solving these problems involves the use of conical transmission lines with cross section geometries approximating the cross section geometry of the cylindrical transmission line[1]. The purpose of this note is to propose an alternative scheme for terminating the TEM mode on such transmission lines.

Basically the concept involves designing a thin admittance sheet at the end of the cylindrical transmission line. Its surface admittance (or impedance if one prefers) is to be chosen such that on the side of the transmission line there is only the desired TEM mode (without reflection) and on the opposite side the field distribution is more general so as to satisfy Maxwell's equations together with boundary and radiation conditions. Besides applying this termination approach to cylindrical transmission lines one might apply it to conical transmission lines as well. For example one might just use a single conical transmission line driven from its apex as the simulator. In another case one might terminate a cylindrical transmission line by first attaching to it a conical transmission line to reduce the cross-section dimensions, and then terminate the conical transmission line at some position before reaching the apex. At this position, however, the wave lengths corresponding to the highest frequencies of interest may still be smaller than the cross-section dimensions.

In this note we first discuss some approximate lumped element concepts related to such terminations. This is followed by a discussion of the admittance sheet approach, both as an electromagnetic boundary value problem and with respect to some approximations involved in realizing such sheets.

- - - - - - - - - - - - -

1. Capt. Carl E. Baum, Sensor and Simulation Note XXXI, The Conical Transmission Line as a Wave Launcher and Terminator for a Cylindrical Transmission Line, January 1967.

II. Approximate Termination Characteristics

At frequencies low enough that the associated wavelengths are much larger than the characteristic cross-section dimensions one can terminate a transmission line in its characteristic admittance without introducing significant reflections. For simplicity consider the parallel plate transmission line with infinitely wide plates as illustrated in figure 1A. In this case the characteristic admittance per unit width is just $\sqrt{\varepsilon_o/\mu_o}$ divided by the plate spacing. One might then try to terminate the transmission line with an admittance sheet placed perpendicular to the direction of propagation of the incident wave (the z direction) and extending between the plates. Then an admittance sheet with a surface admittance, $Y_s = \sqrt{\varepsilon_o/\mu_o}$, gives the characteristic admittance as a termination for sufficiently low frequencies. However, for high frequencies such that the wavelengths are much less than the plate spacing the situation is somewhat altered. For our assumed free space medium the wave admittance is also $\sqrt{\varepsilon_o/\mu_o}$. An incident high-frequency plane wave undergoes a reflection (referred to the electric field) at such a planar sheet with a reflection coefficient given by a simple calculation as

$$r_e = \frac{\sqrt{\varepsilon_o/\mu_o} - 2\sqrt{\varepsilon_o/\mu_o}}{\sqrt{\varepsilon_o/\mu_o} + 2\sqrt{\varepsilon_o/\mu_o}} = -\frac{1}{3} \qquad (1)$$

One can simply think of the admittance of the sheet as being in parallel with the characteristic admittance of the space to the right of the planar surface admittance, presenting an admittance of $2\sqrt{\varepsilon_o/\mu_o}$ to the incident plane wave.

In order to improve on this termination sheet we would like $Y_s \to 0$ as $\omega \to \infty$ where ω is the radian frequency of the incident wave. Then in the high-frequency limit the incident wave would not be reflected. Thus to minimize reflections at both low and high frequencies we could make Y_s frequency dependent. One way to do this might be to include a series inductance, $L > 0$, so that

$$Y_s = \left[sL + \sqrt{\frac{\mu_o}{\varepsilon_o}} \right]^{-1} \qquad (2)$$

where s is the Laplace transform variable and can be taken as $j\omega$. This simple form of Y_s matches the required Y_s at both high and low frequency limits, at least for positions away from the conducting plates.

Another approach to approximating the required form of the termination admittance is to look more closely at some of the low frequency characteristics of the transmission line. In particular we can attribute a capacitance to the electric field near the ends of the conductors. If we consider the static problem of a potential difference between the conductors, then we can attempt to calculate this fringe capacitance. For the two dimensional problem in figure 1A the fringing electric field falls off as $1/r$ outside the plates for distances far from the plates, where r is here the distance from the plate edge. Such a capacitance, per unit width of the transmission line, diverges logarithmically in this case[2]. However, for cases such as in figure 1B, a three dimensional problem, the static electric field can fall off as $1/r^3$ or faster at large distances, r, from the open end of the transmission line, leading to a definite fringe capacitance. We might then try to represent the end of the transmission line, not as an open circuit, but as a capacitance for low frequencies.

In the high frequency limit with no admittance sheet the incident TEM wave is radiated out the end of the transmission line without reflection. Thus in the high frequency limit we can represent the end of the transmission line as presenting a conductance. Then combining this capacitance and conductance in series in the form of an equivalent surface admittance gives a surface admittance, Y_o, which represents the termination characteristics of the transmission line as

$$Y_o = \left[\frac{1}{sC} + \sqrt{\frac{\mu_o}{\varepsilon_o}}\right]^{-1} \tag{3}$$

Here C is the capacitance as included in the equivalent surface admittance.

Now we would like to add a terminating sheet admittance, Y_s, to Y_o

- - - - - - - - - - - - - -

2. Lt. Carl E. Baum, Sensor and Simulation Note XXI, Impedances and Field Distributions for Parallel Plate Transmission Line Simulators, June 1966.

to approximate $\sqrt{\varepsilon_o/\mu_o}$, the wave admittance of free space at both high and low frequencies. If we take a Y_s of the form in equation (2) we get a termination admittance, Y_t , normalized as a surface admittance as

$$Y_t = Y_s + Y_o = \left[sL + \sqrt{\frac{\mu_o}{\varepsilon_o}}\right]^{-1} + \left[\frac{1}{sC} + \sqrt{\frac{\mu_o}{\varepsilon_o}}\right]^{-1} \tag{4}$$

If now we choose L such that

$$\frac{L}{\mu_o} = \frac{C}{\varepsilon_o} \tag{5}$$

we then find that, independent of frequency,

$$Y_t = \sqrt{\varepsilon_o/\mu_o} \tag{6}$$

which is the desired form. Of course our form for Y_o is rather approximate making this last result fortuitous.

III. Admittance Sheet Approach

We now proceed to discuss a general procedure for calculating Y_s . Consider transmission-line geometries such as illustrated in figure 1. Let space be divided into two regions of interest by a surface, S . Call one of these regions the TEM region or transmission line region. In this region we hypothesize that there is only a particular desired TEM mode, propagating in a single direction. In figure 1A this is the z direction; in figure 1B this is the ρ (or radial) direction. An appropriate cylindrical or conical transmission line provides appropriate perfectly conducting boundaries for the TEM mode in this region. The second region of interest we call the radiation region.

Dividing the two regions we have the surface, S . On this surface we specify the tangential component of the electric field. Part of this surface may consist of perfect conductors along which the tangential component of the electric field is zero. The remainder of this surface consists of the admittance sheet Y_s . On the admittance sheet we specify the tangential component of the electric field to be exactly the tangential component of the electric field of the TEM mode in the TEM region. With the tangential component of the electric field now specified along S we can in principle solve a boundary value problem for the electromagnetic fields in the radiation region satisfying the radiation condition at infinity. From this we can calculate the tangential component of the magnetic field along the admittance sheet in the radiation region; it will in general differ from the tangential component of the magnetic field along the admittance sheet in the TEM region, due to the desired TEM mode in this latter region.

Consider now a small section of the admittance sheet as in figure 2A. The characteristics of this sheet are defined by

$$\tilde{J}_s = Y_s\tilde{E}_1 = Y_s\tilde{E}_2 = \tilde{H}_1 - \tilde{H}_2 \tag{7}$$

where $\sim$ indicates the Laplace transform. The tangential electric field is continuous across the sheet but the component of the tangential magnetic field perpendicular to J_s is discontinuous by an amount J_s . With $\vec{n}$ as a unit normal to the surface we can more generally write the characteristics of the sheet in vector form as

$$\vec{n} \times (\vec{H}_{in} - \vec{H}_{out}) = \vec{J}_s \tag{8}$$

$$\vec{n} \times \vec{E}_{in} = \vec{n} \times \vec{E}_{out} \tag{9}$$

and

$$Y_s \vec{e} \cdot (\vec{n} \times \tilde{\vec{E}}_{in}) = \vec{e} \cdot (\vec{n} \times \tilde{\vec{J}}_s) \tag{10}$$

where $\vec{e}$ is defined as a unit vector parallel to $\vec{n} \times \vec{E}_{in}$. In equation (10) we include the possibility that the required $\vec{J}_s$ may not be parallel to the tangential component of $\vec{E}_{in}$. In such a case we then define Y_s as related only to the component of $\vec{J}_s$ parallel to the tangential component of $\vec{E}_{in}$. If there is a component of $\vec{J}_s$ perpendicular to the tangential component of $\vec{E}_{in}$, then this component of $\vec{J}_s$ should flow as on a perfect conductor and allowance should be made for it. Note that because of the assumed TEM mode on the transmission line then $\vec{E}_{in}$ at any fixed position is always parallel to a fixed direction. Thus on the admittance sheet $\vec{e}$ is a fixed direction at each position, independent of frequency. In some particular cases due to symmetry we can have $\vec{J}_s$ always parallel to the tangential component of $\vec{E}_{in}$ simplifying matters somewhat.

The procedure for finding Y_s is then to first find $\vec{E}_{in}$ and $\vec{H}_{in}$ from the assumed TEM mode. Use equation (9) to establish the tangential component of $\vec{E}_{out}$ on S . Then solve the boundary value problem in the radiation region. This determines $\vec{H}_{out}$ so that we can find the required $\vec{J}_s$ from equation (8). Finally from equation (10) we can determine Y_s and see whether we need to provide for a component of $\vec{J}_s$ perpendicular to the tangential electric field. The Y_s thus found will in general be a function of both s and position on the admittance sheet.

For a given transmission line geometry and shape of the surface, S, we now have defined in principle a way of calculating Y_s, consistent with having only the single TEM mode propagating in one direction on the transmission line. However the Y_s thus calculated may have some undesirable features. For example one might wish to realize it with passive lumped resistors, inductors, and capacitors. However, the required Y_s might be active. An example of an active Y_s is given in figure 2B. To illustrate this point consider a TEM wave propagating between the infinitely wide parallel plates and let its time dependence be a step function of the form $u(t - z/c)$. Then at the time when the TEM wave reaches the edges of the parallel plates part of the sheet admittance (labelled "active region") must launch the part of the TEM wave above the top plate and below the bottom plate. The "active regions" must then supply energy to the fields. In the case being considered we would expect Y_s in these "active regions" to be unstable since it is required to send out energy before the signal, which it must match, can propagate to it. This example shows that not all choices of part of the surface, S, for Y_s lead to passive Y_s over the entire admittance sheet. Of course the above argument for the active character of Y_s can be removed by moving the admittance sheet to the position it occupies in figure 1A. This does not show that no part of the admittance sheet in figure 1A is active; but it shows that there may be some advantage gained by constructing the admittance sheet in a shape which in some sense optimizes the characteristics of Y_s.

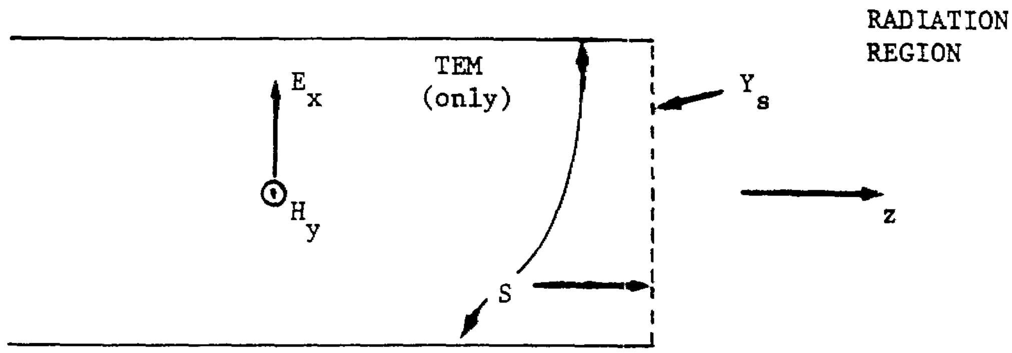
Once we have found a particular Y_s as a function of s and position on the sheet we can consider an approximation problem, namely the approximation of Y_s by a finite number of resistors, capacitors, and inductors, with perhaps the constraint that these elements all be passive. One might try to approximate Y_s over a certain range of ω, or for convenient pulse shape of the incident wave try to approximate J_s as a function of time. As a result of such an approximation one would have one or more equivalent networks with the element values as functions of position on the admittance sheet.

Next there is the problem of realizing these equivalent circuits in a form approximating the desired continuous admittance sheet. Considering a small curvilinear square (or rectangle) on this sheet we can replace it by a small two terminal network approximating Y_s and passing current parallel to the direction of tangential $\vec{E}_{in}$. If needed, wires can be added to pass current perpendicular to this direction of tangential $\vec{E}_{in}$. The entire admittance sheet is then a spatial grid of such two-terminal networks and wires. Or, perhaps the lumped elements can be themselves distributed somewhat so as to blend the two-terminal networks and wires together. Of course in using an array of lumped elements the desired continuous admittance sheet is only approximated.

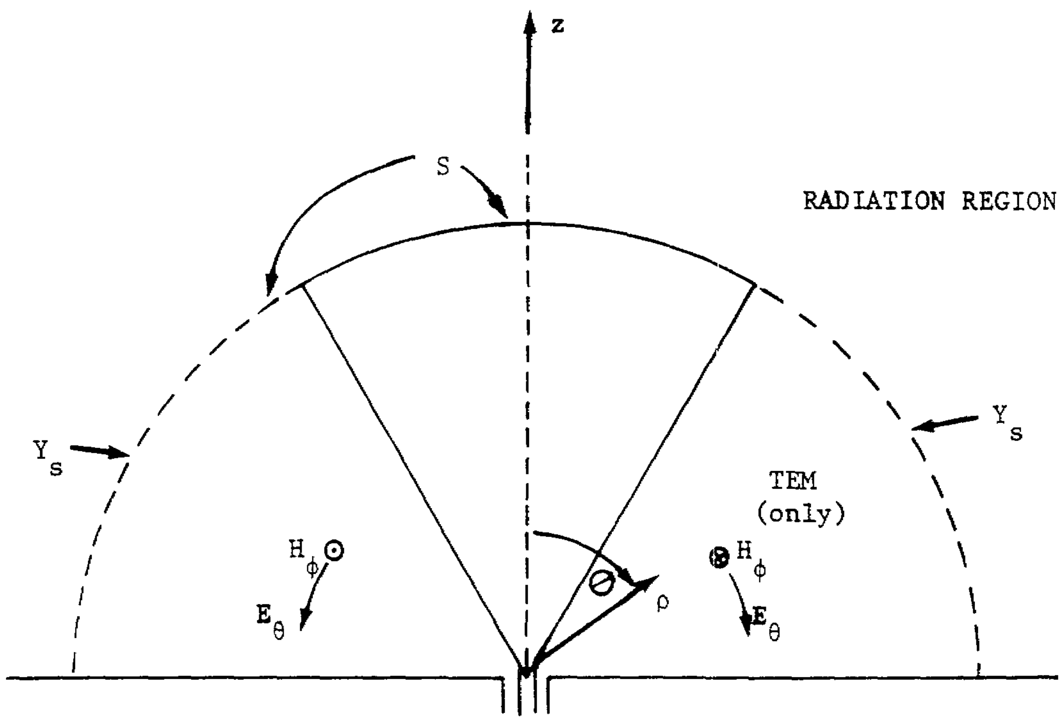
IV. Summary

The characteristic impedance of a TEM transmission line can be used for a terminating resistance for low frequencies. However for frequencies with wavelengths of the order of the cross section dimensions or smaller such a resistive termination introduces reflections on the transmission line. Such reflections may be reduced by making a more general termination structure which approximates an admittance sheet determined by the solution of an electromagnetic boundary value problem in which only the single desired TEM mode is on the transmission-line structure. Hopefully specific examples will be considered in future notes.

A procedure similar to that for calculating Y_s could be used to define the characteristics of a sheet source for launching a TEM wave on a transmission line. However such a source may be rather complex. On the other hand, in the termination case, to the extent that one uses lumped passive elements, the practical realization of such terminations should be somewhat simpler.

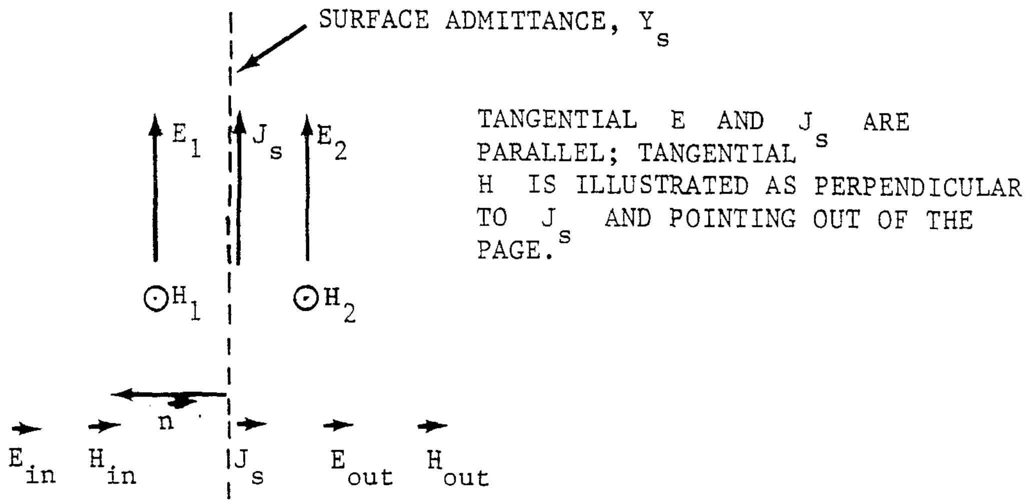


A. Infinitely wide parallel plates

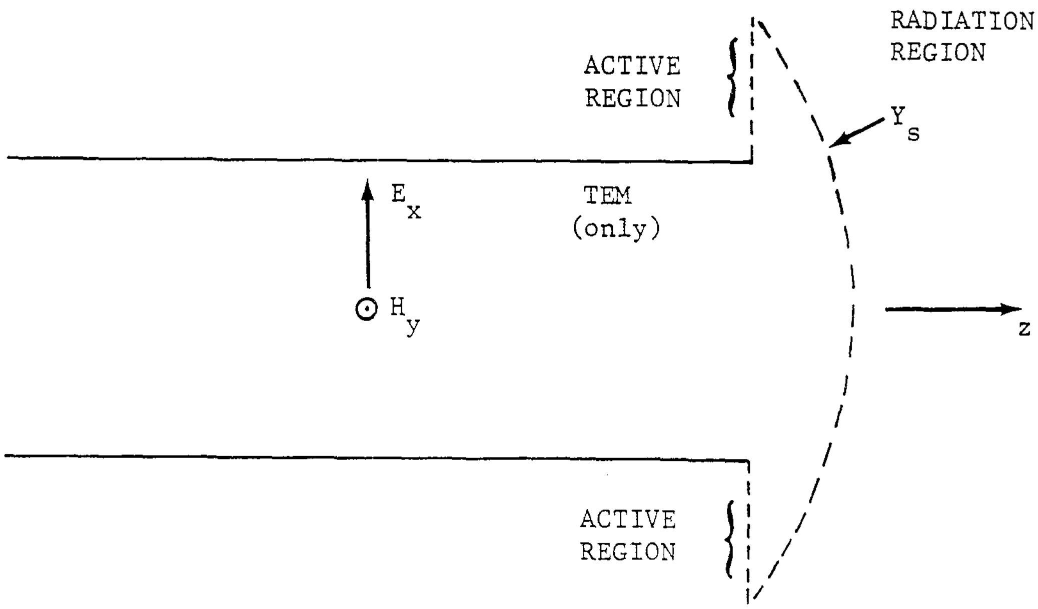


B. Symmetrical cone with ground plane

FIGURE 1: EXAMPLES OF TERMINATION-SHEET GEOMETRIES



A. Boundary conditions for admittance sheet



B. Example of active admittance sheet

FIGURE 2: ADMITTANCE SHEET