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A Distributed Inductor for Use With a Two-Dimensional Simulator Structure

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Abstract

One technique for placing fast-rising pulses over a ground surface involves a surface transmission line with energy sources at one end and a termination resistance at the other end. If, for some cases, one is interested primarily in the magnetic-field waveform at the ground surface, then one can increase the efficiency of the simulator by reducing the magnitude of the low-frequency impedance of the termination. This can be done by placing an inductor in parallel with the termination resistance. In this note a design for such an inductor is considered. This inductor is a distributed structure which can be placed at the output of the surface transmission line. Of course, since this inductance significantly influences the impedance which the simulator presents to the energy sources, it influences the choice of the energy sources and other electrical elements for a desired magnetic-field waveform.

I. Introduction

In the design of EMP simulators it may be desirable to combine more than one simulator concept. For instance one part of the simulator structure may be chosen on the basis of its electromagnetic characteristics at the highest frequencies of interest for a desired pulsed waveform; another part of the structure may be chosen because of its characteristics at somewhat lower frequencies. In combining these two or even more parts something else may be needed to achieve a smooth transition among the various parts in going between various time regimes in the pulse waveform. Note that by the simulator structure we mean the electromagnetic structure with conductors, dielectrics, etc. used for guiding and propagating the fields in some fashion. Another part of the simulator is the energy sources which may also be divided to take care of different parts of the waveform. Some devices may be needed to transition between various energy sources, and/or between the energy sources and parts of the simulator structure. These devices may help to shape the resulting waveform and/or reduce unwanted interaction between the energy sources.

Various types of transition devices might be used. For example, a switch which connects an energy source to the simulator could be considered a transition device. This term could be appropriately applied to the switches used to switch in different capacitive energy sources at different times as discussed in a previous note[1]. Other types of transition devices might include passive circuit elements such as inductors, capacitors, and resistors. The impedances of these elements have different frequency dependences which can be used to allow or prevent energy transfer or absorb energy in different frequency ranges.

In this note we consider a design for an inductor which can be used as a transition circuit element in some two-dimensional simulator geometries. Specifically this inductor can be included in the termination of a surface transmission line such that the high frequencies are terminated with little reflection. However the low frequencies are purposely significantly reflected so as to lower the magnitude of the impedance which is presented to the energy sources at low frequencies. At sufficiently high frequencies the characteristic dimensions of the inductor are not small compared to a radian wavelength and the inductor is not a lumped element but a distributed one. One of the considerations in the design of this distributed inductor is to make it give only a small reflection to the incident high-frequency electromagnetic wave. In the following sections we first consider the combination of an inductor with a surface transmission line; we then consider a proposed design for a distributed inductor for such a use.

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1. Capt. Carl E. Baum, Sensor and Simulation Note 49, The Buried-Transmission-Line Simulator Driven by Multiple Capacitive Sources, August 1967.

II. Inductive-Resistive Termination of Surface Transmission Line

In a previous note we considered a uniform surface transmission line consisting of a flat perfectly conducting sheet of width, w, and length, d, which is placed a height, h, above a flat ground surface[2]. This is the central section in the examples in figure 1. We have assumed that $w \gg h$ and also that w is much larger than depths of significant field penetration into the ground so that the structure could be treated as a two-dimensional problem. By appropriate choice of h, based on d and the electromagnetic parameters of the ground, one can propagate a fast-rising pulse over the ground surface without too much loss of the high frequencies. One should also consider the launching and termination of the wave at opposite ends of the surface transmission line. For sufficiently high frequencies the impedance of the surface transmission line approximates that of a parallel plate transmission line of height h and width w giving an impedance[3]

$$Z \simeq \frac{h}{w} Z_1 \tag{1}$$

where Z_1 is the wave impedance in the air given by

$$Z_1 = \sqrt{\frac{\mu_o}{\varepsilon_o}} \tag{2}$$

Then, as schematically illustrated in figure 1A, we can use an input transmission line of length ℓ_1 and impedance Z leading from the electrical energy sources to the input of the surface transmission line; we can use an output transmission line of the same impedance and of length ℓ_2 leading from the output of the surface transmission line to the terminating impedances. Ignoring the inductor of value L for the moment, if we set the terminating resistor as

$$R = Z \tag{3}$$

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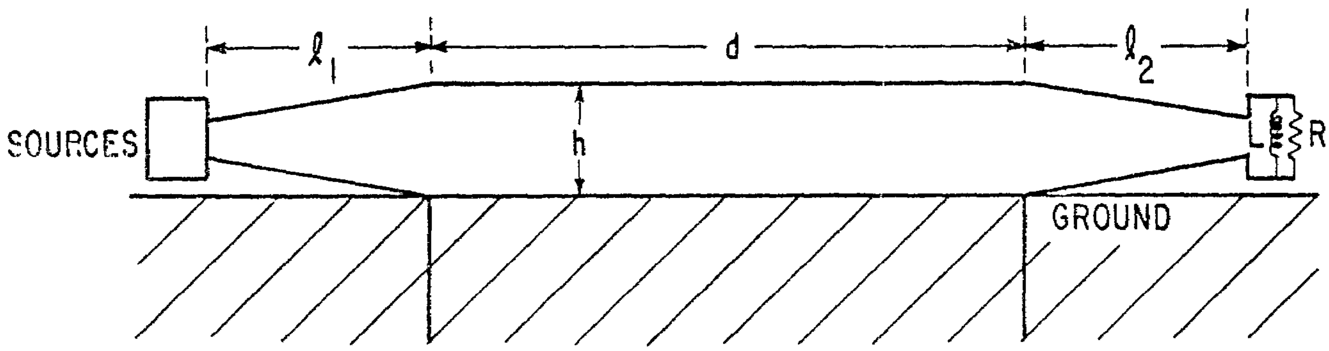
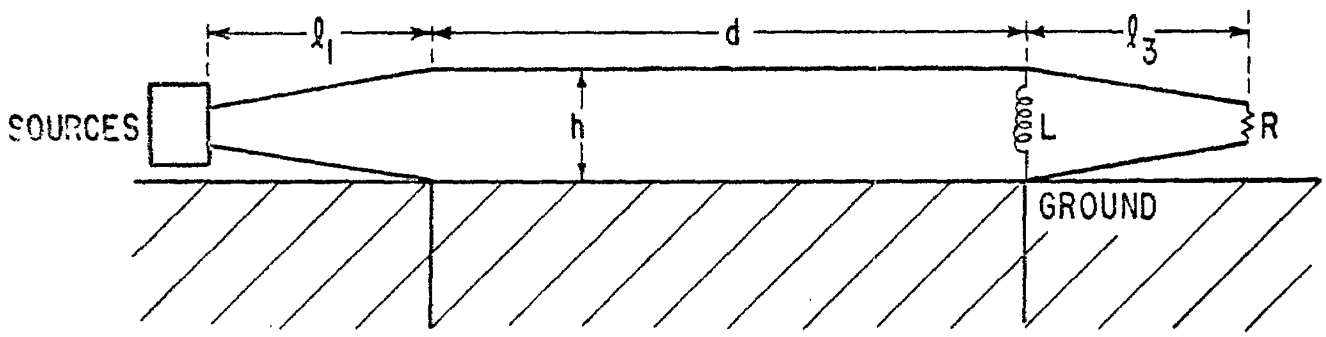
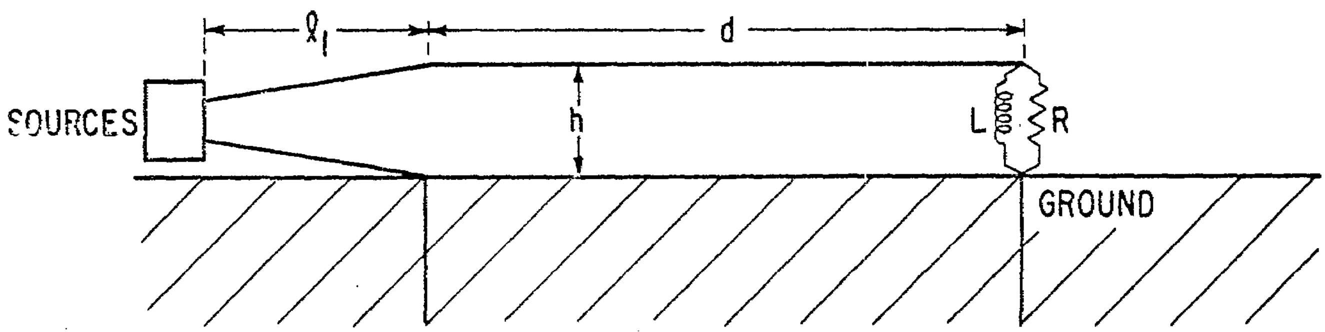
2. Capt. Carl E. Baum, Sensor and Simulation Note 46, The Single-Conductor, Planar, Uniform Surface Transmission Line, Driven from One End, July 1967.
3. All units are rationalized MKSA.



A. INDUCTOR AND RESISTIVE TERMINATION AT OUTPUT OF OUTPUT TRANSMISSION LINE



B. INDUCTOR AT OUTPUT OF SURFACE TRANSMISSION LINE, RESISTIVE TERMINATION AT OUTPUT OF OUTPUT TRANSMISSION LINE



C. INDUCTOR AND RESISTIVE TERMINATION AT OUTPUT OF SURFACE TRANSMISSION LINE

FIGURE 1. INDUCTIVE-RESISTIVE TERMINATION OF SURFACE TRANSMISSION LINE

then the output transmission line is terminated in its characteristic impedance. Then, for frequencies high enough that Z approximates the impedance of the surface transmission line, the whole transmission line structure of length $\ell_1 + d + \ell_2$ has the same impedance and behaves as a terminated transmission line with only small reflections. The input and output transmission lines might each consist of several parallel transmission lines[4]. In this case R and Z would apply to the collection of parallel transmission lines and not to each separately.

While it is desirable to terminate the transmission line at high frequencies in its characteristic impedance to avoid fast-rising reflections, this does not necessarily apply for low frequencies. For frequencies low enough that the radian wavelength is much larger than $\ell_1 + d + \ell_2$, reflections on the transmission line do not appear distinct from the initial wave, but rather blend together with it to produce a smooth waveform. Thus we might consider terminations which have the characteristic impedance for high frequencies but some other impedance for low frequencies.

Suppose that one wishes a certain magnetic-field waveform at the ground surface and is not as concerned about the vertical electric field above the ground surface, at least for low frequencies and this kind of simulator. Then, in order to minimize the energy expended for a given magnetic-field waveform (implying a given current), one might reduce the real part of the terminating impedance for low frequencies. A simple way to do this is to add an inductance, L, in parallel with R as schematically illustrated in figure 1A. We have a characteristic time constant for the termination defined by

$$t_o \equiv \frac{L}{Z} \tag{4}$$

The transit time for the complete transmission line is

$$t_1 = \frac{\ell_1 + d + \ell_2}{c} \tag{5}$$

and for the surface transmission line it is

4. Guy W. Carlisle, Sensor and Simulation Note 54, Matching the Impedance of Multiple Transitions to a Parallel-Plate Transmission Line, April 1968.

$$t_d = \frac{d}{c} \tag{6}$$

where

$$c \equiv \frac{1}{\sqrt{\mu_o \varepsilon_o}} \tag{7}$$

Then if $t_o >> t_1$ the reflections from the termination will be small for radian frequencies of the order of $1/t_1$ and higher. One could reduce L, and thereby reduce t_o, in order to reduce the magnitude of the terminating impedance for low frequencies. However t_o can only be reduced to the order of t_1 and still have a magnetic-field waveform which has no significant distinct reflections. Perhaps a future note will contain detailed calculations of such waveforms for various values of t_o/t_1, including the influence of the ground parameters.

In order to further reduce L, while retaining an acceptable waveform, one could reduce t_1 as much as possible. This could be done by shortening the total length of the transmission line by reducing ℓ_1 and/or ℓ_2, still keeping the configuration of figure 1A. Also we could move the inductor to the output of the surface transmission line. In this latter case the length of the output transmission line, now called ℓ_3, does not enter into t_1 since the output transmission line is terminated in its characteristic impedance, thus presenting this R effectively at its input. This is shown schematically in figure 1B. Of course now the inductor spans the height h and width w of the surface transmission line and for the highest frequencies of interest should be considered as a distributed element. A design for such a distributed inductor is considered in section III. Similarly one might bring the termination resistance R up to the output of the surface transmission line and remove the output transmission line. Then for sufficiently high frequencies the resistive termination should be considered as a distributed element. For such a distributed resistive termination we might include some series inductance as well, so as to improve its high-frequency characteristics[5].

5. Capt. Carl E. Baum, Sensor and Simulation Note 53, Admittance Sheets for Terminating High-Frequency Transmission Lines, April 1968.

For convenience we define some inductances

$$L_o \simeq \frac{w}{h} L \quad , \quad L_d \equiv \mu_o d \quad , \quad L_1 \equiv \mu_o \ell_4 \tag{8}$$

where ℓ_4 is the distance from the electrical energy sources to the inductor, L. In this notation we have

$$t_o = \frac{L_o}{Z_1} \quad , \quad t_1 = \frac{\ell_4}{c} = \frac{L_1}{Z_1} \quad , \quad t_d = \frac{L_d}{Z_1} \tag{9}$$

These inductances are surface inductances referred to a cross-section plane on the surface transmission line. The low-frequency inductance of the surface transmission line, only including the volume above the ground surface, is approximately $(h/w)L_d$. Including the other transmission line(s) raises the low-frequency inductance of the above-ground structure to approximately $(h/w)L_1$.

Assume that ℓ_4, d, h and the ground parameters are specified. Then we can choose a value of t_o/t_1 or t_o/t_d to obtain as small a value of t_o as possible without introducing significant high-frequency reflections into the magnetic-field waveform. These time ratios can be converted to ratios of surface inductances as

$$\frac{t_o}{t_1} = \frac{L_o}{L_1} = \frac{L_o}{\mu_o \ell_4} \quad , \quad \frac{t_o}{t_d} = \frac{L_o}{L_d} = \frac{L_o}{\mu_o d} \tag{10}$$

These forms may be more convenient for choosing the parameters for the distributed inductor considered in section III.

III. Distributed Inductor

As a design for such a distributed inductor consider the one illustrated in figures 2 and 3. We assume that the incident wave approximates a uniform TEM plane wave propagating in the $+x$ direction with the electric field parallel to the z axis (vertical) and the magnetic field parallel to the y axis. Except for the inductor, the structure is assumed independent of the y coordinate out to the full width, w. The inductor has dimensions b in the x direction, h in the z direction and approximately w in the y direction. The inductor structure is not independent of y but is periodic in y with a period $2a$ over the distance w with $w >> a$.

The inductor can be considered to be made of three sets of wires. The front and back wires are on two planes of constant x ; the front-to-back wires are on many planes of constant y , the planes being spaced a constant distance, a , apart. We can divide the distributed inductor into what one could term solenoids by dividing along the planes of front-to-back wires. The front-to-back wires in any one plane are considered as belonging to both adjacent solenoids. Each solenoid then has height h and cross sectional dimensions a and b .

Referring to figures 2 and 3 note that the back wires are dashed in order to distinguish them. Now follow the wires in each solenoid; they form a helix which advances a distance h_o in the z direction for each turn of the helix. This distance is given by

$$h_o = a\left[\tan(\psi_1) + \tan(\psi_2)\right] + 2b\ \tan(\psi_o) \tag{11}$$

where the pitch angles for the front wires, back wires, and front-to-back wires are respectively ψ_1 , ψ_2 , and ψ_o as indicated in figure 2 (B and C). Note that adjacent solenoids are mirror images of each other and that two adjacent solenoids form one period of length $2a$ in the y direction. The structure also has a period of h_o in the z direction, within its height h . As indicated in figure 3 the wires in each solenoid contact the transmission line conductors at both top and bottom. The top conductor is a perfectly conducting sheet or wire grid with conductors parallel to the x axis. If a wire grid is used then contact is made to all the wires in the grid, connecting them together near the inductor. The bottom conductor may be similar to the top conductor near the distributed inductor. Of course, the bottom conductor begins at the ground contact at the end of the surface transmission line.

If we were to determine the characteristics of the distributed inductor from the solution of an electromagnetic boundary value problem we might approximate the currents in the wires by current sheets, as indicated in

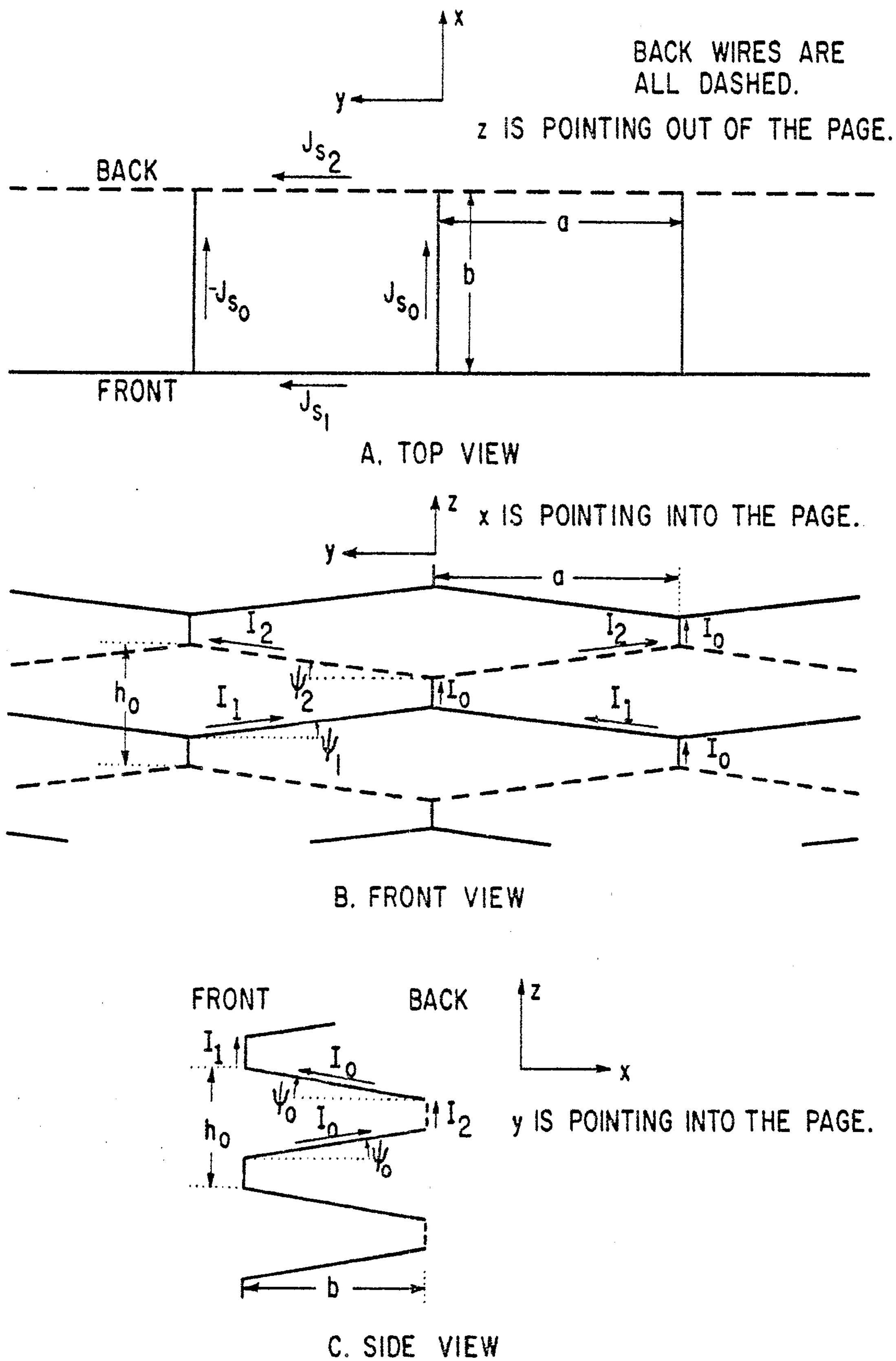


FIGURE 2. DISTRIBUTED INDUCTOR

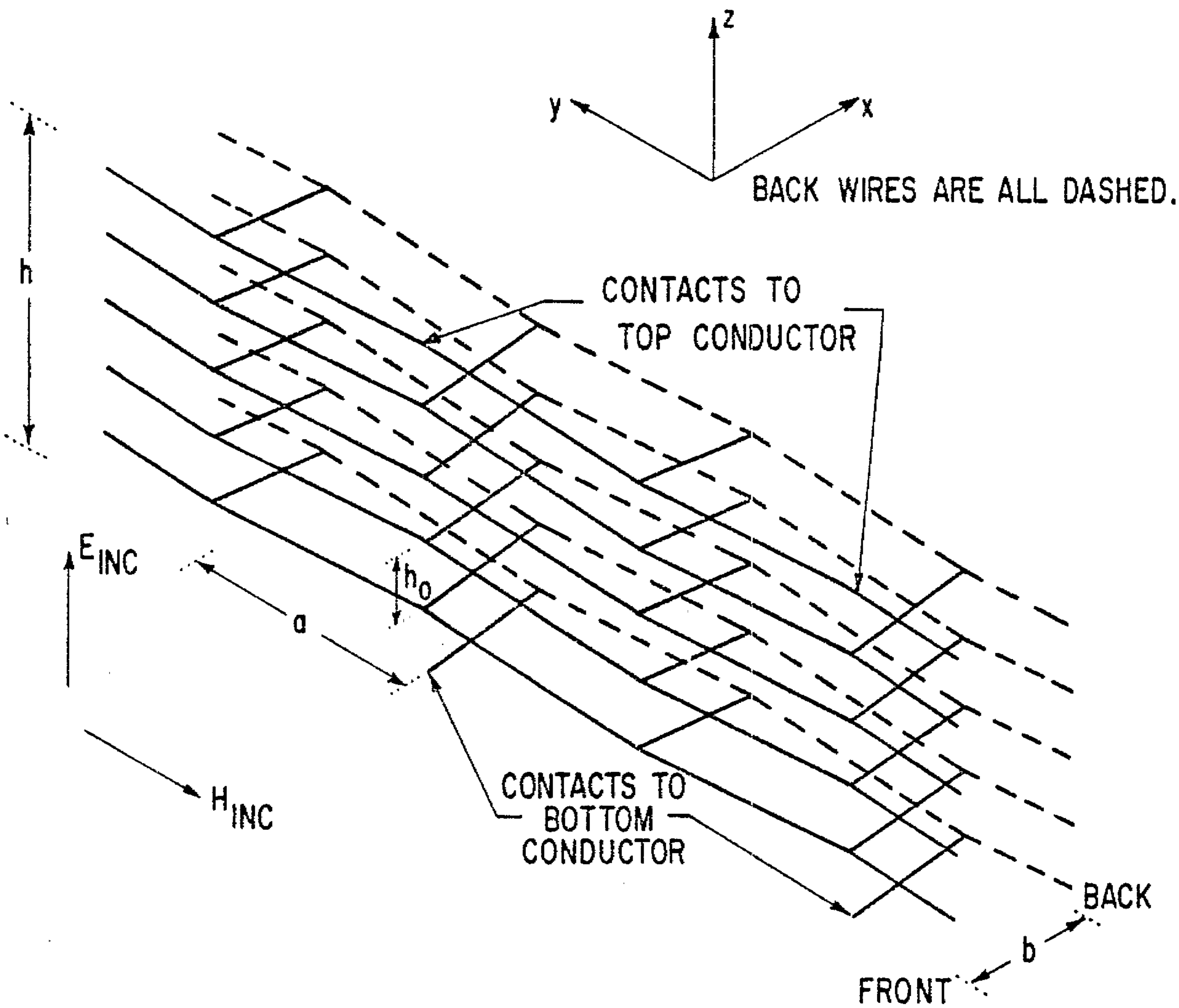


FIGURE 3. DISTRIBUTED INDUCTOR: PERSPECTIVE VIEW

figure 1. The surface current densities corresponding to the front wires, back wires, and front-to-back wires are respectively J_{s_1} , J_{s_2} , and J_{s_o} . These surface current densities are constrained to flow parallel to the directions of the wires given by the pitch angles; the component of the electric field parallel to the direction of current flow is zero on the current sheet. For high frequencies the surface current densities need not be uniform but they possess certain symmetry and periodicity properties related to the symmetry and periodicity properties of the structure. For the current sheet approximation to hold we should have $h_o << a$ and $h_o << b$ which in turn requires the pitch angles to be small.

For sufficiently low frequencies the currents in the wires are approximately constant between nodes (wire junctions). The currents in the front wires, back wires, and front-to-back wires are respectively I_1 , I_2 , and I_o with polarities as indicated in figure 2 (B and C). Note, with respect to the direction of the y coordinate, that I_1 and I_2 have opposite directions in adjacent solenoids. Similarly I_o reverses with respect to the x direction in going between adjacent planes for the front-to-back wires. The currents are related as

$$I_1 = I_2 = \frac{I_o}{2} \tag{12}$$

Associated with these currents there is a vertical magnetic field in each solenoid which has opposite polarity in adjacent solenoids. Define the turn density, n , as

$$n \equiv \frac{1}{h_o} \tag{13}$$

The magnitude of the vertical magnetic field inside each solenoid, for $h >> a$ and $h >> b$, is approximately

$$|H_z| \simeq n|I_1| = \frac{1}{h_o} |I_1| \tag{14}$$

The inductance of each solenoid can be thought of as having two contributions. Assuming the current spread into uniform current sheets gives an inductance for the solenoid as

$$L_2 = \mu_o N^2 \frac{ab}{h} f_2 = \mu_o n^2 \, abh \, f_2 \tag{15}$$

where N is the total number of turns in the solenoid given by

$$N \equiv nh \tag{16}$$

and where $f_2 \simeq 1$ for $h >> a,b$ and $h_o << a,b$. In general f_2 is a function of a , b , h , the pitch angles, and the conductors located at the top and bottom of the inductor. A second contribution is a correction to account for the finite number of wires instead of a uniform current sheet. Using the results of a previous note, we have an effective displacement of the front and back wires, separately, given by[6]

$$\Delta b \simeq \frac{h_o}{2\pi} \ln\left(\frac{h_o}{2\pi r_o}\right) \tag{17}$$

and for the front-to-back wires, separately for each side, given by

$$\Delta a \simeq \frac{h_o}{\pi} \ln\left(\frac{h_o}{2\pi r_o}\right) \tag{18}$$

where r_o is the wire radius and where we have assumed $h_o << a,b$. The factor of 2 larger correction for the front-to-back wires is related to the equal but opposite vertical magnetic fields in adjacent solenoids, compared to the relatively small vertical magnetic fields outside the volume enclosed by the distributed inductor, but not too close to the wires. The finite number of wires then increases the inductance of each solenoid by

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6. Lt. Carl E. Baum, Sensor and Simulation Note 21, Impedances and Field Distributions for Parallel Plate Transmission Line Simulators, June 1966.

$$L_3 \simeq \mu_o N^2 \frac{2b\Delta a + 2a\Delta b}{h} = \mu_o n^2 h(2b\Delta a + 2a\Delta b)$$

$$\simeq \mu_o n^2 h \frac{h_o}{\pi} (2b + a)\ln\left(\frac{h_o}{2\pi r_o}\right)$$

$$= \mu_o \frac{N}{\pi} (2b + a)\ln\left(\frac{h_o}{2\pi r_o}\right) \tag{19}$$

The surface inductance of the distributed inductor is then

$$L_o = \frac{a}{h} (L_2 + L_3) \tag{20}$$

For the total inductance all the solenoids are in parallel giving

$$L \simeq \frac{a}{w} (L_2 + L_3) \tag{21}$$

Assuming $L_2 \gg L_3$ then we have the approximation

$$L_o \simeq \frac{a}{h} L_2 = \mu_o n^2 a^2 b \; f_2 = \mu_o \frac{a^2 b}{h_o^2} f_2 \tag{22}$$

where f_2 is of the order of one. Equations (10) can then be written

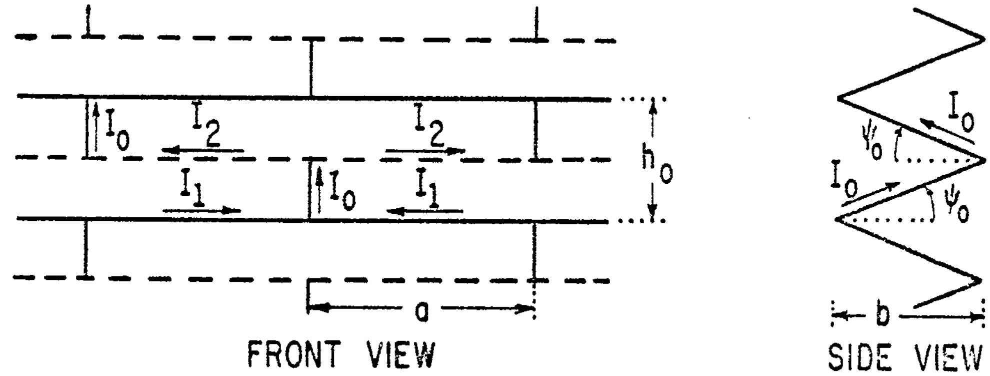
$$\frac{a^2 b}{h_o^2 \ell_4} f_2 \simeq \frac{t_o}{t_1} \quad , \quad \frac{a^2 b}{h_o^2 d} f_2 \simeq \frac{t_o}{t_d} \tag{23}$$

Assume t_o/t_1 or t_o/t_d is fixed. Also let ℓ_4 or d be fixed. This establishes a relation among a , b , and h_o . If we want $b/d \ll 1$ and t_o/t_d is of the order of one or larger, this would imply $h_o/a \ll 1$ which is consistent with small pitch angles. Note that the distributed inductor is not located on a plane of constant x because of its dimension b in the x direction. Thus it is not located exactly at the end of the surface transmission line. This gives some imprecision in ℓ_4 , the effective distance from the electrical energy sources to the distributed inductor. However, if $b << d$ and $a << d$, then the imprecision in ℓ_4 should be small compared to d .

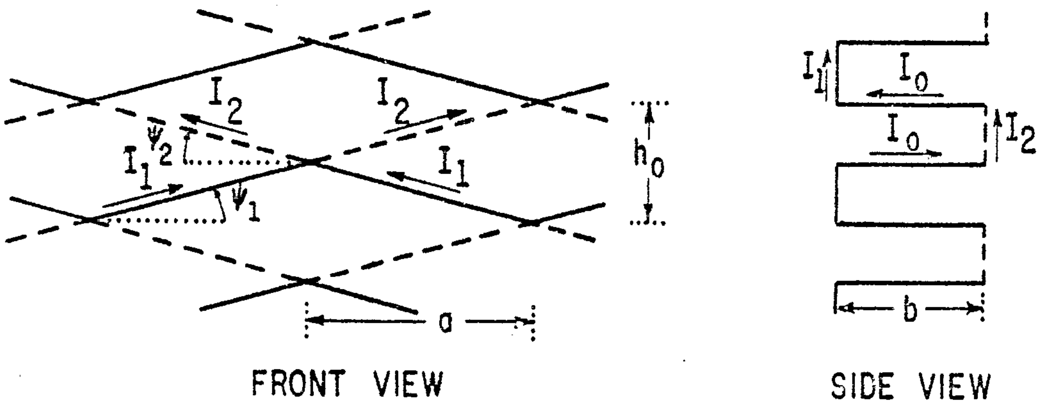
There are various special cases for the geometry of the distributed inductor; a few of these cases are illustrated in figure 4. Certain cases may be desirable from a construction standpoint. Some cases may have good high-frequency characteristics, i.e. give only small reflection of the incident wave at high frequencies. One might compare some of the high-frequency characteristics of various cases by approximating the wires by current sheets as discussed before. On the current sheets the component of the electric field parallel to the wires is made zero. Considering the incident electric field as approximately parallel to the z axis, then the cases in figure 4 have various wires perpendicular to the incident electric field so that the corresponding current sheets do not interact with the incident electric field. Perhaps some detailed calculations using a current-sheet model could be made in the future.

Figure 5 shows a possible location of the distributed inductor at the end of the surface transmission line. This figure also shows a possible location for a distributed resistive termination just behind the distributed inductor. As noted previously such a distributed resistive termination can have improved high-frequency characteristics by including some distributed series inductance. One way to design such a termination would be to use a structure similar to the distributed inductor as in figure 3, but with resistive wires instead of highly conducting wires, thereby having both resistance and inductance. However, it may be desirable to have a nonuniform surface inductance for the termination making the turn density and/or dimension in the x direction vary as a function of z . In such a case it may also be desirable to vary the resistance per unit length of the resistive wires.

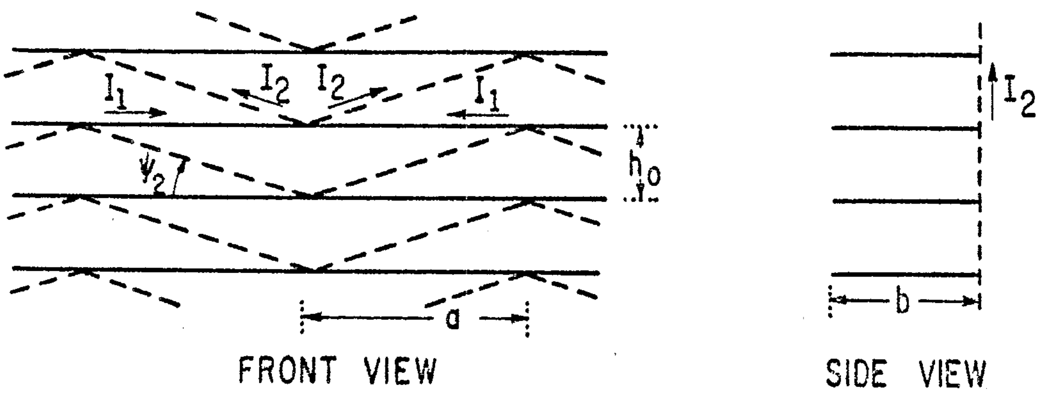
BACK WIRES ARE ALL DASHED.



A. FRONT WIRES AND BACK WIRES HORIZONTAL



B. FRONT - TO - BACK WIRES HORIZONTAL



C. FRONT WIRES AND FRONT-TO-BACK WIRES HORIZONTAL

FIGURE 4. DISTRIBUTED INDUCTOR : SPECIAL CASES

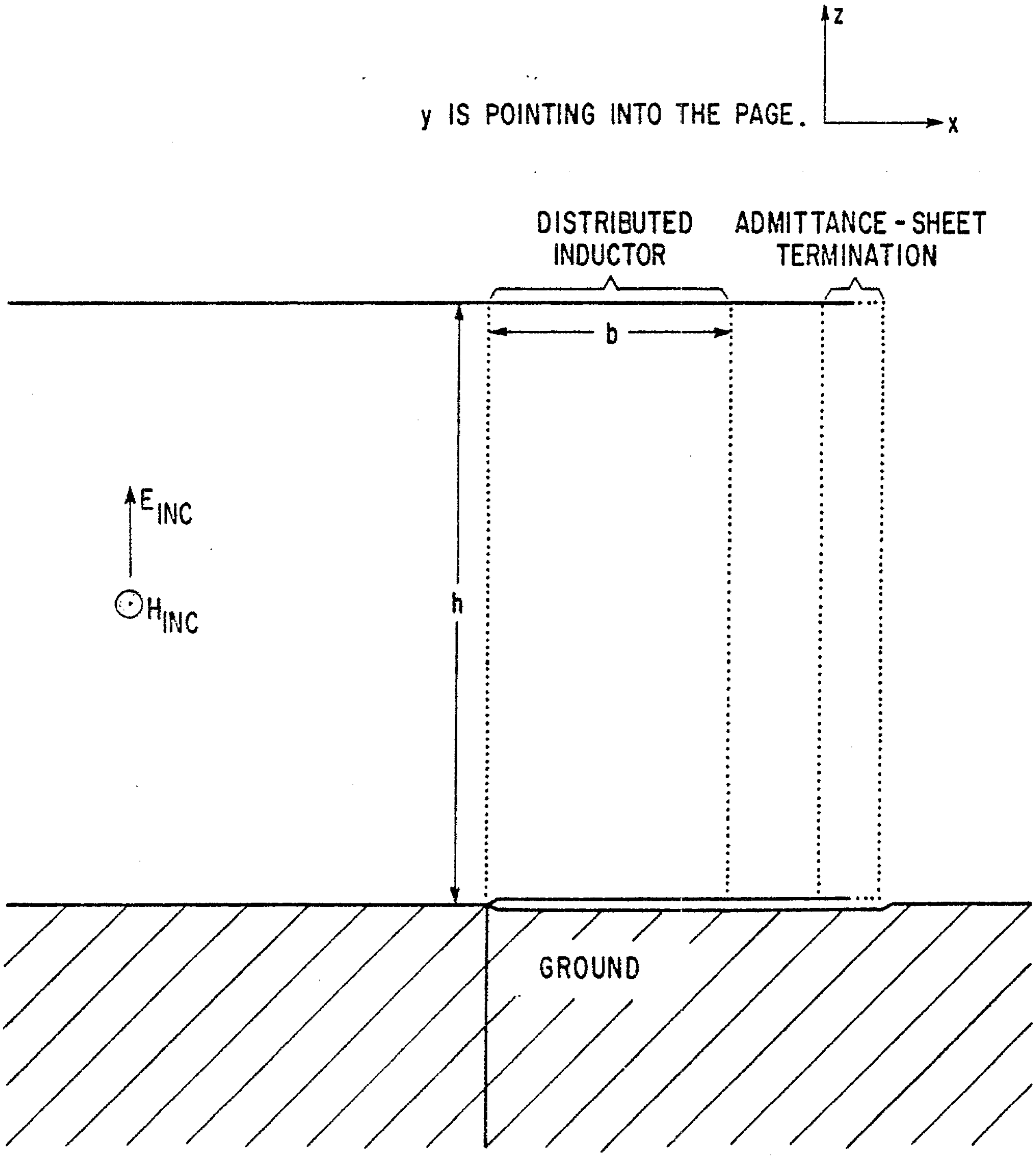


FIGURE 5. DISTRIBUTED INDUCTOR AND ADMITTANCE-SHEET TERMINATION WITH SURFACE TRANSMISSION LINE

IV. Summary

At high frequencies it is desirable to terminate a uniform surface transmission line in a resistance which approximates the transmission-line impedance, thereby avoiding significant fast-rising reflections. However, for low frequencies with wavelengths much longer than the transmission line, reflections from the termination blend with the incident wave. For such low frequencies the termination impedance then need not be the same as for high frequencies if we are willing to sacrifice the vertical electric field above the ground associated with a desired magnetic field. One could then reduce the magnitude of the termination impedance for low frequencies in order to reduce the energy required for a given magnetic-field waveform. One way to reduce the magnitude of the termination impedance for low frequencies is to add an appropriate inductance in parallel with the termination resistance.

In order to minimize the inductance required we could place it at the end of the surface transmission line. One might also place it there in order to avoid the necessity of adding an output transmission line to the surface transmission line. In this position the inductor occupies a large volume, and for frequencies with radian wavelengths on the order of the inductor's characteristic dimensions the inductor should be considered as a distributed element and designed to avoid significant reflections at such frequencies. In this note we have discussed one type of distributed inductor which approximates a row of parallel solenoids with uniform turn densities and with adjacent solenoids wound in opposite directions. The wires are all kept nearly horizontal to reduce high-frequency reflection from the inductor.

The distributed inductor considered in this note applies to planar surface transmission lines and similar structures which can be approximated as two-dimensional geometries. Perhaps similar inductor distribution techniques can be used in some curved fashion for some three-dimensional geometries. In this note we discussed the distributed inductor in the context of a uniform surface transmission line. One might also use such an inductor with a non-uniform surface transmission line, e.g. one for which the height of the top conducting sheet varies along the transmission line from beginning to end. We have discussed the inductance of this inductor in terms of avoiding significant fast-rising reflections on the surface transmission line. However, another consideration is the interaction of this inductive-resistive termination with the electrical energy source in determining the resulting magnetic-field waveform. This type of termination may or may not combine with various sources to give a desired waveform. In some cases additional elements may be useful in shaping the waveform as desired.