Sensor and Simulation Notes

Note 58

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Positioning of Rods for Uniform TEM Waves

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Abstract

Conducting rods can be used to form a transmission line that supports a wave which closely resembles a uniform TEM wave. The problem of positioning these rods is considered, and the positions of the rods for symmetrical six and eight-rod transmission lines are determined. The impedances of these transmission lines are computed.
I. Introduction

The effect on buried structures due to the low-frequency content of an electromagnetic pulse emanating from a nuclear burst can be simulated by a buried parallel-plate transmission line. The purpose of the transmission line is to transport a uniform TEM wave to a test body which is located between the parallel plates. For the sake of generality it should be remarked that the medium in which the transmission line is immersed, and hence the concern of this note, is not limited to earth; instead the medium can be any isotropic, homogeneous, linear medium.

In a previous note, which deals with a related topic, the charge and field distributions have been computed for the case of a cylindrical test body with a diameter that is appreciable compared to the spacing between the parallel plates; these distributions are compared with those for the case in which the diameter of the test body is negligible.

From the standpoint of constructing a parallel-plate transmission line it is convenient to approximate the parallel plates by two correspondingly parallel linear-arrays composed of uniformly spaced conducting rods. If the arrays extend to infinity, containing an infinite number of rods, and if the two arrays are spaced far enough apart, the fields midway between the two arrays closely approximate those of a uniform TEM wave. The arrays, of course, cannot extend to infinity; they must be truncated and contain only a finite number of rods. This truncation causes the fields to distort from the uniform TEM distribution that is desired.

The problem with which the present note is concerned is that of positioning a finite number of conducting rods to form a transmission line which more nearly produces a uniform TEM distribution than a transmission line formed by two truncated, parallel linear-arrays containing the same finite number of rods. As might be expected, it turns out that the rods should be positioned in arrays which are symmetrical about the origin as shown in figure 1. The problem of positioning four rods was considered in a previous note.
II. Positioning of Rods

Figure 1 shows the cross-section of a transmission line formed by conducting rods which are arranged symmetrically about the origin and which extend to infinity along the z-axis. In order for the structure to be operated conveniently as a transmission line the potential difference between the upper rods \((y > 0)\) and the lower rods \((y < 0)\) must be uniform; i.e., all of the upper rods must be at one potential and all of the lower rods must be at the same magnitude of potential with opposite algebraic sign. Hence requiring that the structure operate as a transmission line imposes a restraint on the potentials of the rods.

As nearly as possible it is desired that the fields on the transmission line in figure 1 match those of a uniform TEM wave which has an electric field with only a \(y\)-component and whose magnitude is constant over the cross-section. If the rods are positioned symmetrically about the origin, the \(x\)-component of the electric field tends to cancel out in the vicinity of the origin, giving a \(y\)-component only. The magnitude of \(E_y\) can be made uniform in the vicinity of the origin by representing \(E_y\) in terms of a Taylor expansion in \(x\) about the origin and by forcing as many derivatives as possible to vanish at the origin.

An expansion in circular harmonics of the potential of a line charge \(q_i\), whose coordinates are \(r_i\) and \(\theta_i\), is

\[
\phi_i = \frac{q_i}{2\pi\epsilon} \left[ \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{r}{r_i} \right)^n (\cos n\theta_i \cos n\theta + \sin n\theta_i \sin n\theta) - \ln \frac{r}{r_i} \right]
\]

(1)

where \(r < r_i\). Along the \(x\)-axis the \(y\)-component of the electric field due to this line charge is

\[
E_y = \frac{q_i}{2\pi\epsilon} \sum_{n=1}^{\infty} \frac{x^{n-1}}{r_i^n} \sin n\theta_i \quad y=0
\]

(2)
The m-th partial derivative of $E_y$ with respect to $x$ evaluated at the
origin is

$$\frac{\partial^m E_y}{\partial x^m} \bigg|_{x=0, y=0} = \frac{q_i}{2\pi\varepsilon} \left[ \frac{m!}{r_i^{m+1}} \sin(m+1)\theta_i \right]$$

(3)

The m-th partial derivative of the electric field $E_y$ due to a set
of four line charges $q_i, q_i, -q_i$ and $-q_i$ that are located symmetrically at
$(r_i, \theta_i), (r_i, \pi - \theta_i), (r_i, \pi + \theta_i)$ and $(r_i, 2\pi - \theta_i)$, respectively, is

$$\frac{\partial^m E_y}{\partial x^m} \bigg|_{x=0, y=0} = \frac{4q_i}{2\pi\varepsilon} \left[ \frac{m!}{r_i^{m+1}} \sin(m+1)\theta_i \right]$$

(4)

for even values of $m$ and identically zero for odd values of $m$. From (4)
it is evident that for the case of four conducting rods arranged symmetrically
about the origin, the first three partial derivatives are zero at the origin
if $\theta_i = \pi/3$ which is the result reported in reference 3. The m-th partial
derivative for the special case of two line charges $q_o$ and $-q_o$ at $(r_o, \pi/2)$
and $(r_o, 3\pi/2)$, respectively, is

$$\frac{\partial^m E_y}{\partial x^m} \bigg|_{x=0, y=0} = \frac{2q_o}{2\pi\varepsilon} \left[ \frac{m!}{r_o^{m+1}} (-1)^{m/2} \right]$$

(5)

for even values of $m$ and identically zero for odd values of $m$.

The conducting rods can be arranged symmetrically with or without rods
being placed on the y-axis. For the case of $4K$ rods arranged symmetrically
in $K$ sets of four rods with no rods on the y-axis, (4) gives the m-th
derivative of $E_y$ which, if set equal to zero, is
\[ \sum_{i=1}^{K} \frac{q_i}{r_i^{m+1}} \sin(m+1) \theta_i = 0 . \]  \hspace{1cm} (6)

For the case of \(4K+2\) rods arranged symmetrically in \(K\) sets of four rods plus a pair of rods on the \(y\)-axis, (4) and (5) give

\[ \frac{q_o}{r_o^{m+1}} (-1)^{m/2} + 2 \sum_{i=1}^{K} \frac{q_i}{r_i^{m+1}} \sin(m+1) \theta_i = 0 \]  \hspace{1cm} (7)

for the \(m\)-th derivative equated to zero.

That the structure operate as a transmission line imposes an aforementioned restraint on the potentials of the rods. Due to symmetry the potential at each rod of a symmetrical set, which might be part of a larger symmetrical arrangement, satisfies the restraint; e.g., if the potential of the rods at \((r_i, \theta_i)\) and \((r_i, \pi - \theta_i)\) is \(\phi_i\), the potential of the rods at \((r_i, \pi + \theta_i)\) and \((r_i, 2\pi - \theta_i)\) is \(-\phi_i\). Thus the restraint on the potentials is met by requiring

\[ \phi_o = \phi_1 = \ldots = \phi_i = \ldots = \phi_K \]  \hspace{1cm} (8)

where these potentials pertain to the potentials of the rods at \((r_o, \pi/2)\), \((r_1, \theta_1)\), \ldots \((r_i, \theta_i)\), \ldots \((r_K, \theta_K)\), respectively. Clearly for an arrangement of \(4K\) rods, which has no rods on the \(y\)-axis, \(\phi_o\) is omitted from (8).
III. Impedance

The impedance of a multiple-rod transmission line can be written as

\[
Z_L = f_g Z_o
\]  

(9)

where \( f_g \) is a geometrical factor given by

\[
f_g = \frac{\varepsilon V}{Q}
\]  

(10)

and \( Z_o \) is the wave impedance of the medium in which the rods are immersed. In (10) \( \varepsilon \) is the permittivity of the medium, \( V \) is the potential difference between the upper and lower rods, and \( Q \) is the sum of the line charge of all of the rods either above or below \( y=0 \).

Values of \( f_g \) have been computed for the transmission lines considered in this note and are listed in Tables 1 and 2.
IV. Symmetrical Six-rod Transmission Line

A six-rod transmission line is shown in figure 2. For the case of six rods, which are arranged in a set of four rods plus two rods on the y-axis, it is possible to make the first five derivatives zero. In general for the case of \(4K+2\) rods it is possible to make the first \(4K+1\) derivatives vanish.

If the line charge is normalized with respect to \(q_0\) and the distances are normalized with respect to \(r_0\), (7) gives

\[-1 + \frac{2q_1}{r_1^3} \sin 3\theta_1 = 0\]

and (8) requires

\[\phi_0 = \phi_1\]

(11)

(12)

giving three equations in terms of three independent variables \(\theta_1, r_1\) and \(q_1\).

The potential at the surface of a rod centered at \((r_1, \theta_1)\) with radius \(a\) and line charge \(q_1\) has the form

\[\phi_i = S_i + M_i\]

(13)

where \(S_i = q_i \ln a\) is a "self-potential" due to the line charge at the center of the rod and \(M_i\) is a "mutual-potential" due to all of the other line charges.
Since $a$ is very small compared to the spacing of any pair of rods, $S_1$ is the dominate term. If all of the rods have the same radius and if (12) is to be satisfied, in the first approximation it is reasonable to compute first-order values for $r_1$ and $\theta_1$ by means of (11) with $q_1 = q_o = 1$. Substitution of these values for $r_1$ and $\theta_1$ into (12) gives a second-order value for $q_1$ which, in turn, if substituted back into (11) gives second-order values for $r_1$ and $\theta_1$ and hence establishes an iterative procedure.

A summary of data which are pertinent to a six-rod transmission line is presented in Table 1. In figures 4 and 5 $E_y$ is plotted versus $x/r_{\min}$ and $y/r_{\min}$, respectively, where $r_{\min} = r_o$ is the distance from the origin to the nearest conducting rod. These figures show that $E_y$ is virtually uniform along both the $x$ and $y$ axes for values of $x/r_o$ and $y/r_o$ less than 0.5. At $x/r_o = 0.7$ $E_y$ has dropped off less than four percent. From these considerations it is reasonable to state that $E_y$ is uniform in the region $r \leq 0.5 r_o$. 


V. Symmetrical Eight-rod Transmission Line

An eight-rod transmission line is shown in figure 3. For the case of eight rods, arranged in two sets of four, it is possible to make the first seven derivatives zero. In general for the case of $4K$ rods it is possible to make the first $4K-1$ derivatives vanish.

Normalized with respect to $q_1$ and $r_1$, (6) gives

$$\sin 3\theta_1 + \frac{\sin 3\theta_2}{r_2^3} = 0$$

$$\sin 5\theta_1 + \frac{\sin 5\theta_2}{r_2^5} = 0$$

$$\sin 7\theta_1 + \frac{\sin 7\theta_2}{r_2^7} = 0$$

and (8) requires

$$\phi_1 = \phi_2$$

These equations can be solved by using the iterative procedure outlined previously for the six-rod transmission line.

Data for an eight-rod transmission line are listed in Table 2. Figures 4 and 5 show curves of $E_y$ versus $x/r_{\text{min}}$ and $y/r_{\text{min}}$ where $r_{\text{min}} = r_1$ for the case of an eight-rod transmission line. These curves show that $E_y$ is virtually uniform for $x/r_1$ and $y/r_1$ less than 0.6 and suggest that $E_y$ is uniform within the region $r \leq 0.6 r_1$. 
VI. Concluding Remarks

Although an eight-rod transmission line provides a uniform TEM wave over a fairly large area, transmission lines containing more than eight rods may be needed in practical application due to power-density requirements. Equations for transmission lines with more than eight rods can be obtained easily from the preceding development; however, these equations are not readily solvable. A systematic computer solution is not evident -- at least not to this investigator. Nevertheless it should be remarked that, if the need warranted sufficient time and effort, solutions probably could be obtained.
Acknowledgement

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References


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Figure 1. Cross-section of a Multiple-Rod Transmission Line.
Figure 2. Symmetrical Six-Rod Transmission Line.
Figure 3. Symmetrical Eight-Rod Transmission Line.
NOTE: 1. $r_{\text{min}}$ is the distance from origin to nearest rod
2. $\ln \left( \frac{a}{r_{\text{min}}} \right) = -7.0$

Figure 4. Normalized $E_y$ Along the X-Axis for Symmetrical Multiple-Rod Transmission Lines.
Figure 5. Normalized $E_y$ Along the Y-Axis for Symmetrical Multiple-Rod Transmission Lines.