Sensor and Simulation Notes
Note 59
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Some Considerations for Inductive Current Sensors

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Abstract

One way to measure a current or current density is to measure the line integral of the magnetic field around an area of interest by using an appropriate array of conducting loops. Such a sensor might be termed an inductive current sensor. In this note we consider designs for such sensors. One type of sensor design is appropriate for frequencies low enough that the sensor can be considered electrically small. However, for higher frequencies such that the sensor is not electrically small, the sensor should be viewed as a distributed system and appropriate techniques should be employed to improve its high-frequency performance.

I. Introduction and Some General Considerations

A common method of measuring transient currents uses an inductive current sensor. As discussed in a previous note an inductive current sensor can be used to measure a component of the total current density[1]. In the present note we discuss some designs for inductive current sensors. More specifically we consider inductive current sensors of the general shape illustrated in figure 1. The loop turns, oriented to be sensitive to the ϕ component of the magnetic field, are positioned on a surface of revolution, as illustrated, with height w, inner radius b, and outer radius a. We call the volume enclosed by this surface the sensor volume.

For this type of sensor we consider one of Maxwell's equations[2]

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \equiv \vec{J}_t \tag{1}$$

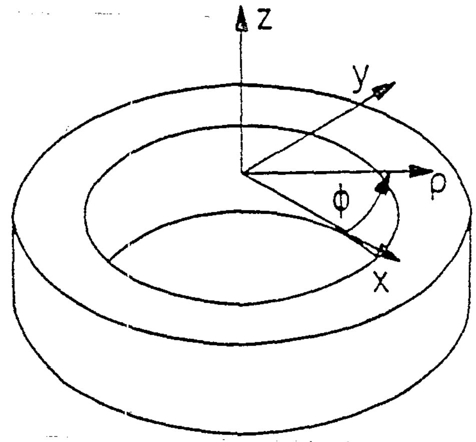
where we have defined the right side as $\vec{J}_t$, the total current density. In integral form this is

$$\oint_C \vec{H} \cdot d\vec{s} = \iint_S \vec{J}_t \cdot d\vec{S} \tag{2}$$

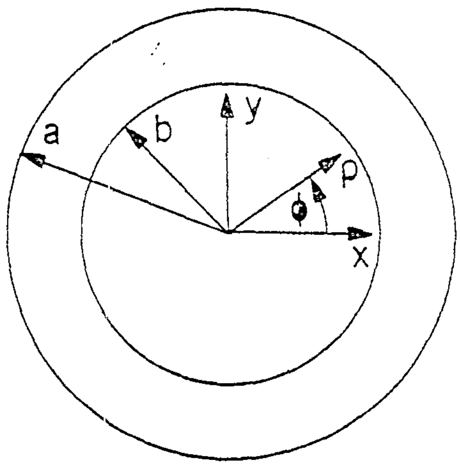
where the contour C bounds the surface S. Let the surface S be a circular disk of radius $\rho > 0$, lying on a plane of constant z. Define the surface integral as the total current I_t. The contour integral uses only the ϕ component of the magnetic field in the integrand giving

$$\rho \int_0^{2\pi} H_\phi \, d\phi = I_t \tag{3}$$

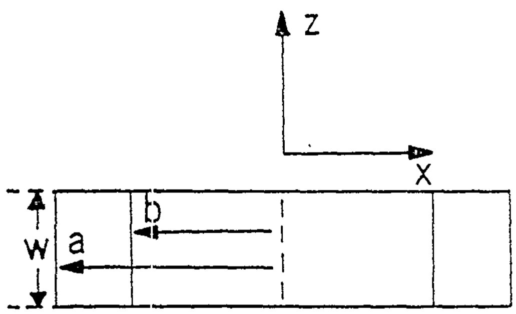
1. Capt. Carl E. Baum, Sensor and Simulation Note 33, Two Types of Vertical Current Density Sensors, February 1967.
2. All units are rationalized MKSA.



A. Perspective View



B. Top View



C. Side View (cross section)

Figure 1. Current Sensor Geometry

If we define an average H_ϕ on the circle of constant ρ and z as

$$H_{\phi_{avg}} \equiv \frac{1}{2\pi} \int_{o}^{2\pi} H_\phi \, d\phi \tag{4}$$

then equation (3) becomes

$$H_{\phi_{avg}} = \frac{I_t}{2\pi\rho} \tag{5}$$

If we can neglect any total current density on the loop conductors or within the sensor volume enclosed by the loop turns as defined in figure 1, then $H_{\phi_{avg}}$ is described inside this volume by equation (5) with one I_t applying to $H_{\phi_{avg}}$ throughout the sensor volume. I_t is then taken as the total current through a central circular area, of radius b, circumscribed by the sensor and perpendicular to the z axis. Or, more simply, I_t is the total current passing through the central hole through the sensor. I_t may be comprised of currents in various forms, including currents on wires, displacement current, conduction current, Compton current, etc.

This type of inductive current sensor measures I_t by measuring $H_{\phi_{avg}}$. This requires that the loop conductors be distributed around the sensor at sufficient values of ϕ to obtain an accurate average of H_ϕ. How many values of ϕ are needed depends on how H_ϕ vaires with ϕ inside the sensor volume. Also dimensional and other imperfections reduce the accuracy of this averaging. If the magnitude of the local H field is large compared to the magnitude of $H_{\phi_{avg}}$ in some frequency range of interest then an accurate average may be difficult to obtain. Typically the average is made by summing the measurements from positions equally spaced around the circle in order to get some high degree of symmetry in the sensor and thereby reject some undesired signals. Of course, any conductors used to bring the signals from the various measurement positions to some common collection position may also have undesirable interactions with various field components and so these conductors may need to be considered well.

A low-frequency equivalent circuit of an inductive current sensor is given in figure 2. The open circuit voltage can be obtained by use of equation (5). Suppose the material in the sensor volume has permeability $\mu_r \mu_o$, where μ_o is the permeability of free space. Then in terms of B we have

$$B_{\phi_{avg}} = \mu_r \mu_o \frac{I_t}{2\pi\rho} \tag{6}$$

Integrating $B_{\phi_{avg}}$ over a cross section of the sensor volume at a particular ϕ, multiplying by an effective number of loop turns, N, and differentiating with respect to time we obtain the open-circuit voltage to be

$$V_o = N \frac{\mu_r \mu_o}{2\pi} w \ln\left(\frac{a}{b}\right) \frac{\partial I_t}{\partial t} \tag{7}$$

From this we have a mutual inductance

$$M = N \frac{\mu_r \mu_o}{2\pi} w \ln\left(\frac{a}{b}\right) \tag{8}$$

so that we have

$$V_o = M \frac{\partial I_t}{\partial t} \tag{9}$$

As a special case of interest consider $\mu_r = 1$. Since by definition

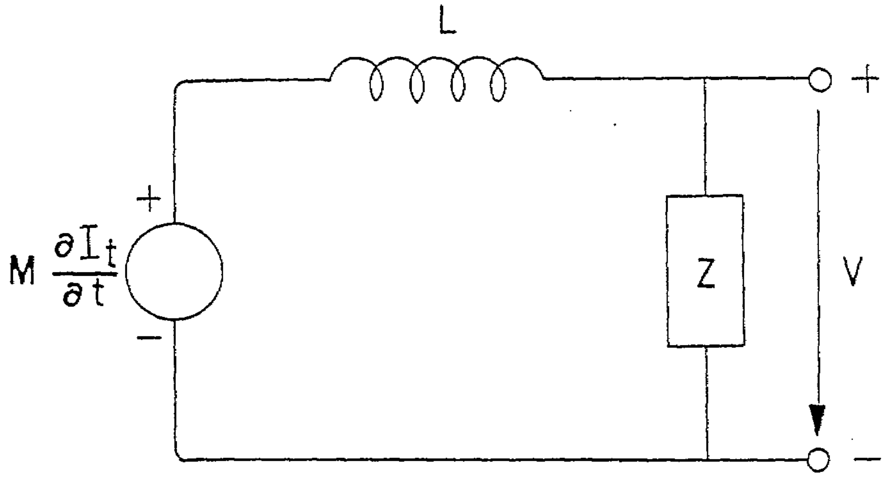


Figure 2. Equivalent Circuit for Inductive Current Sensor

we have

$$\mu_o \equiv 4\pi \times 10^{-7} \ \frac{\text{henries}}{\text{meter}} \tag{10}$$

then we have for this case

$$M = 2 \times 10^{-7} \ N \ w \ \ln\left(\frac{a}{b}\right) \ \text{henries} \tag{11}$$

Since M is the basic sensitivity of the device for measuring $\dot{I}_t$ we might make it some convenient number, such as 1, 2, or 5 times some integer power of 10. If we choose a and b such that $\ln\left(\frac{a}{b}\right)$ is some convenient number then N and w can be chosen as convenient numbers to give a convenient M. For this purpose we include the following table.

$\ln\left(\frac{a}{b}\right)$ (exact)		$\frac{a}{b}$	$\frac{b}{a}$
.05	$(= \frac{1}{20})$	1.05127	.95123
.0625	$(= \frac{1}{16})$	1.0645	.9394
.1	$(= \frac{1}{10})$	1.1052	.9048
.125	$(= \frac{1}{8})$	1.1331	.8825
.2	$(= \frac{1}{5})$	1.2214	.8187
.25	$(= \frac{1}{4})$	1.2840	.7788
.4	$(= \frac{2}{5})$	1.4918	.6703
.5	$(= \frac{1}{2})$	1.6487	.6065
.625	$(= \frac{5}{8})$	1.8682	.5353
1		2.718	.3679
1.25	$(= \frac{5}{4})$	3.490	.2865
2		7.389	.1353

Table 1. Some Convenient Values of $\ln\left(\frac{a}{b}\right)$

Such an inductive current sensor has a self inductance of approximately

$$L \simeq N^2 \frac{\mu_r \mu_o}{2\pi} w \ln\left(\frac{a}{b}\right) = NM \qquad (12)$$

This approximation requires that when the sensor is electrically driven from the load the currents on the sensor conductors approximate a current distribution which is independent of ϕ. Also in the equivalent circuit of figure 2 there is a load impedance Z across which the signal voltage, V, is measured. Typically Z is a simple resistance or a cable which is terminated in its characteristic resistive impedance. Of course, if a cable is used, a time delay should be included in calculating V. Letting Z be a resistance R we have a characteristic time constant, t_o, and corresponding characteristic radian frequency, ω_o, given by

$$t_o \equiv \frac{1}{\omega_o} \equiv \frac{L}{R} \qquad (13)$$

For radian frequencies, ω, such that $\omega \ll \omega_o$ and the sensor is electrically small then $V \simeq V_o$ and M gives the sensitivity as in equation (9). For $\omega \gg \omega_o$ but ω still small enough that the sensor is electrically small, then the current I into the load is approximately

$$I = \frac{M}{L} I_t \simeq \frac{1}{N} I_t \qquad (14)$$

so that in this case the output current is proportional to the total current through the sensor.

An inductive current sensor might then be designed to measure either $\dot{I}_t$ or I_t. In measuring $\dot{I}_t$ the mutual inductance, M, determines the sensitivity of the sensor. If H_ϕ associated with I_t is nearly independent of ϕ and if the frequency is sufficiently low and the currents induced on the sensor conductors are small compared to I_t, then M accurately describes the sensor sensitivity as in equation (9), and its formula in equation (8) is also quite accurate. This mode of sensor operation is similar to that of a $\dot{B}$ loop which can be designed to be characterized by an accurate equivalent area. On the other hand, the self inductance, L, given in equation (12) is often rather approximate, depending on the degree of current distribution in the sensor geometry. Then from equation (14) the formula for the sensitivity of the sensor for measuring I_t may also be rather approximate since it uses L. Of course, L and/or the sensitivity to I_t of the sensor can be determined experimentally to improve the accuracy.

In the following sections we go on to consider some designs for this type of inductive current sensor. We first consider some distribution techniques for the loop conductors for electrically-small sensors, followed by conducting shields for the loop turns. Then we consider a design for a high-frequency sensor where radian wavelengths of interest may be small enough to be of the order of the sensor dimensions.

II. Electrically-Small Inductive Current Sensors

For electrically-small inductive current sensors consider the distribution of the loop conductors as illustrated in figure 3A. This is shown as an example with 8 loops at equally spaced positions around the circular sensor geometry. Around the outside there are conductors connecting the loops together so as to give a signal at the output terminals proportional to the average of the signals from all the loops. There are many ways to connect the loops together, but no specific way is included in figure 3A. These connecting conductors are grouped closely together and do not form a closed conducting path completely around the sensor in order to minimize the areas of any conducting loops which could have signals induced on them associated with any component of the magnetic field which might be present. In figure 3A, these conductors connect to the loops around the outside of the sensor, but they could connect at some other position on the loops if it were desirable for some application.

Some ways of connecting the individual loops together are illustrated in expanded views in figure 3B and figure 4 (A and B). These connection techniques are similar to those discussed concerning $\dot{B}$ loops in a previous note[3]. Note in the expanded views that the individual loop turns have been broken to lay

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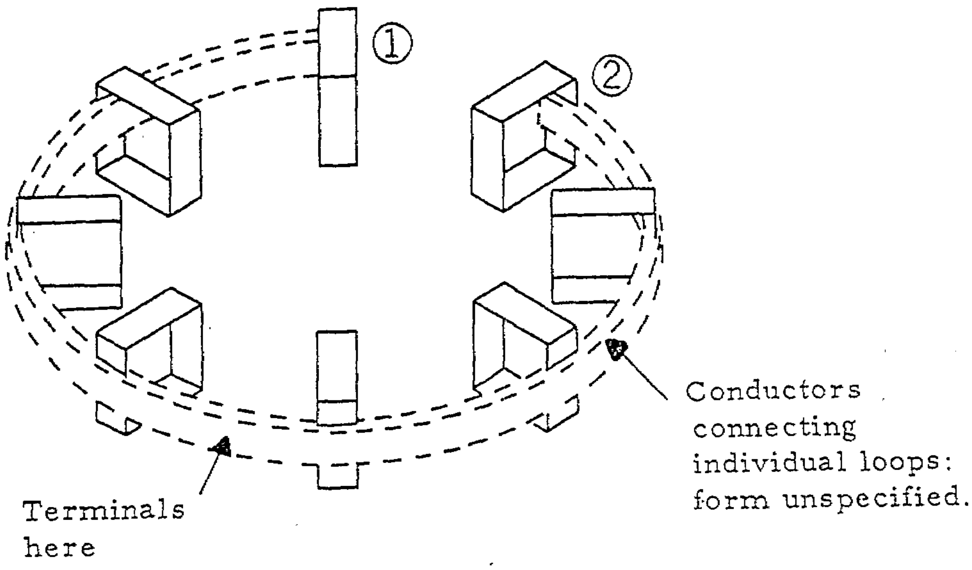
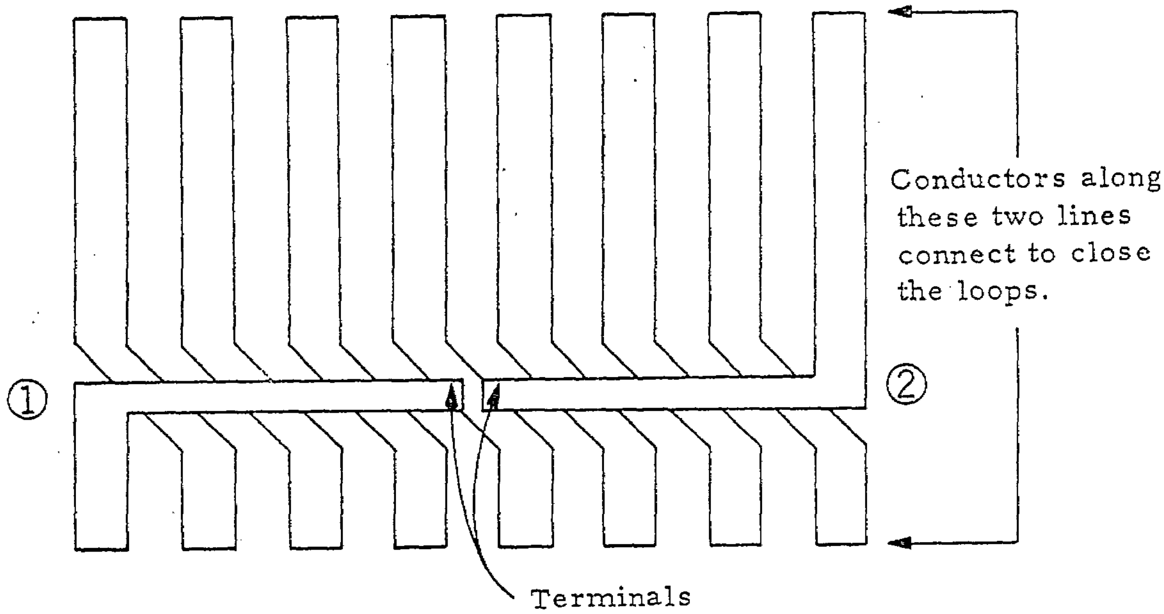
3. Capt. Carl E. Baum, Sensor and Simulation Note 43, Some Considerations for Electrically-Small Multi-Turn Cylindrical Loops, May 1967.

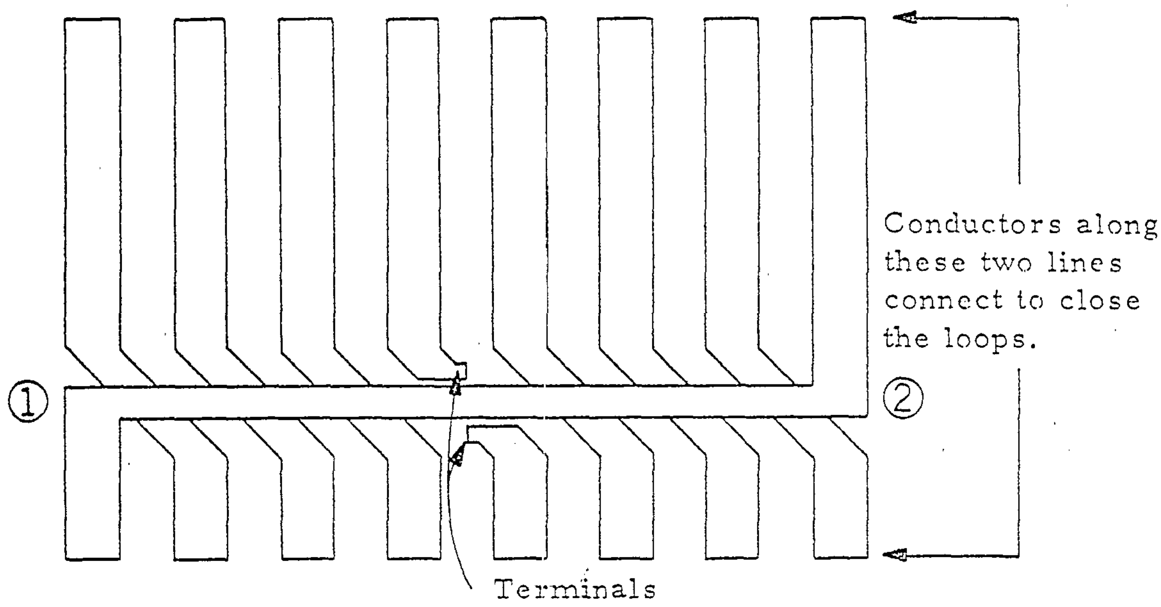


A. Perspective View

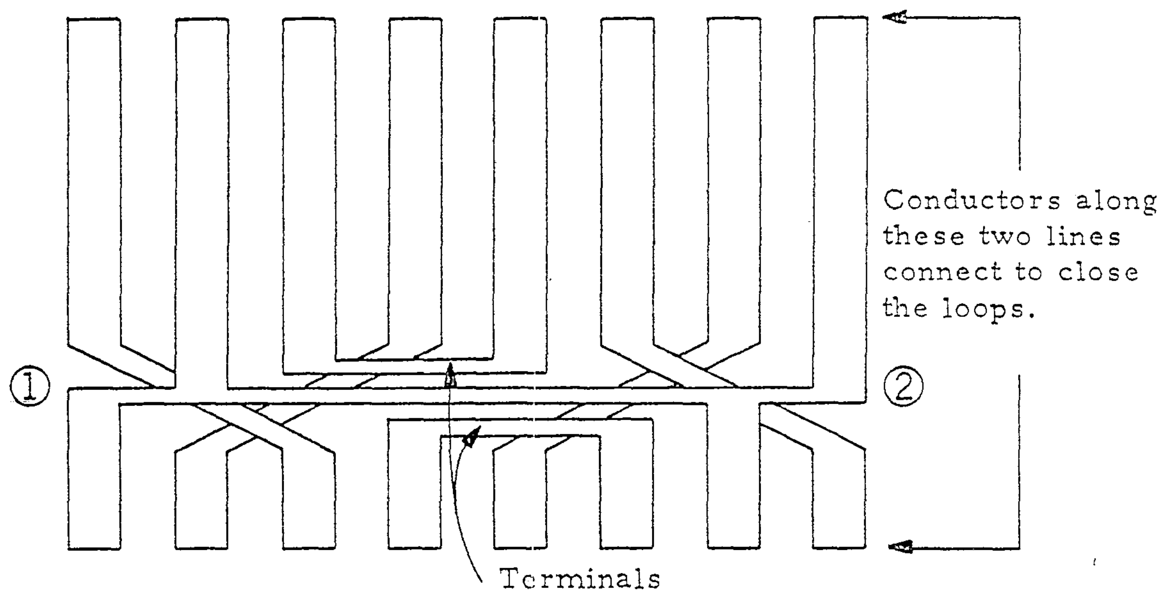


B. Terminals at ends of single winding: N = 8: Expanded View

Figure 3. Geometry for Electrically-small Current Sensor



A. Terminals in center of single winding: N = 8: Expanded View



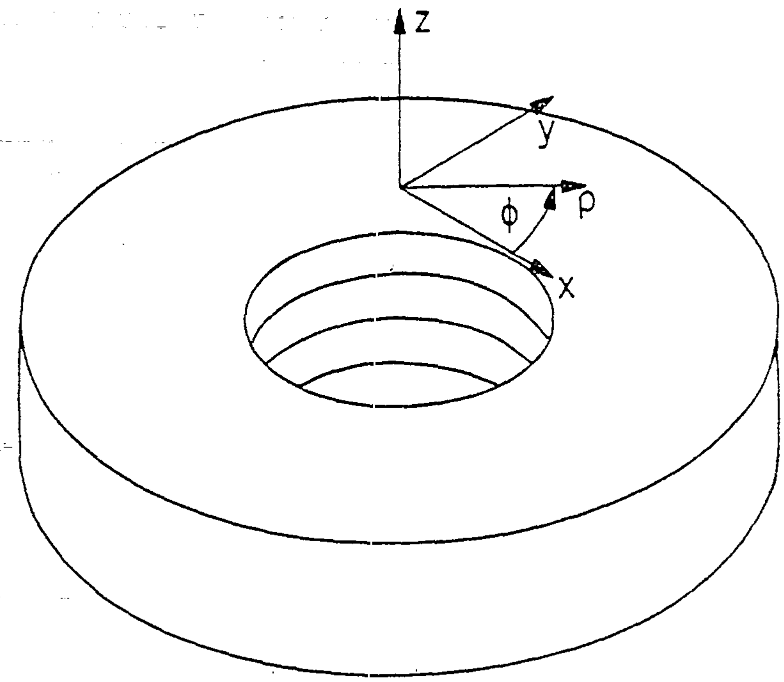
B. Terminals in center of two windings (counterwound): N = 4: Expanded View

Figure 4. Geometry for Electrically-small Sensor

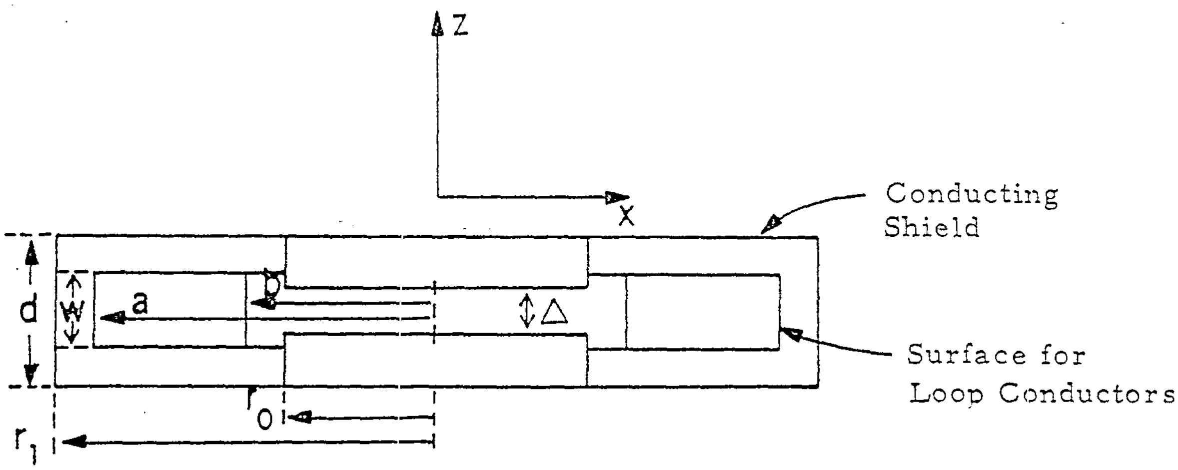
them out. Also the symbols 1 and 2 are used to relate the positions of the conductors in the expanded views to figure 3A. Figures 3B and 4A show the conductors in the form of a single winding but with the output terminals either at the ends of the winding or in the center of the winding. These two cases are illustrated with an effective number of loop turns of eight. Figure 4B shows the conductors in the form of two windings wound in opposite directions (counter winding) with output terminals in the center of the windings; the two windings are joined at their ends and the opposite ends are shorted together by an additional conductor. This counter winding adds some symmetry to the conductors, improving the rejection of signals one might associate with unwanted electric field components. This last case is illustrated with four effective loop turns, although there are eight individual loop turns because of the way in which the turns are connected.

With the present type of electrically-small inductive current sensor we might also use a conducting shield with a gap as illustrated in figure 5. The shield is doughnut shaped, like the sensor geometry in figure 1, but has dimensions chosen so that the loops conductors and associated connecting conductors can fit inside the shield as shown in figure 5B. Note that the shield has a gap of width Δ completely around the hole through the center to allow H_ϕ to penetrate into the interior of the shielded volume. Note that since the shield geometry is independent of ϕ then $H_{\phi_{avg}}$ will not be distorted at low frequencies. The higher order magnetic-field terms in ϕ, however, may be distorted but we are not trying to measure these.

This type of shield has the advantage of largely preventing the x and y components of the electric field from inducing currents on the loop conductors, corresponding currents being induced on the shield. The z component of the electric field is associated with the z component of the displacement current density, D_z, (or possibly also conduction current density) and so contributes to I_t passing through the hole in the center of the sensor and to the resulting $H_{\phi_{avg}}$. The gap in the shield need not be placed around the inside of the hole through the center; it could be placed around the outside of the shield or in some other position on the shield, still maintaining a shield geometry which is independent of ϕ. Note that the position of the gap does, however, affect the sensitivity of the sensor to D_z by determining how much of the charge induced on the shield passes through the hole through the sensor, thereby contributing to I_t. With the gap around the hole in the center of the shield the sensitivity to D_z is then minimized. This sensitivity can be further reduced by making the hole through the center smaller, thereby decreasing the integral of D_z over this hole. One might follow this approach if he were measuring the current on a wire passed through the sensor and wished to minimize the addition of D_z



A. Perspective View



B. Side View (cross section)

Figure 5. Shield Geometry

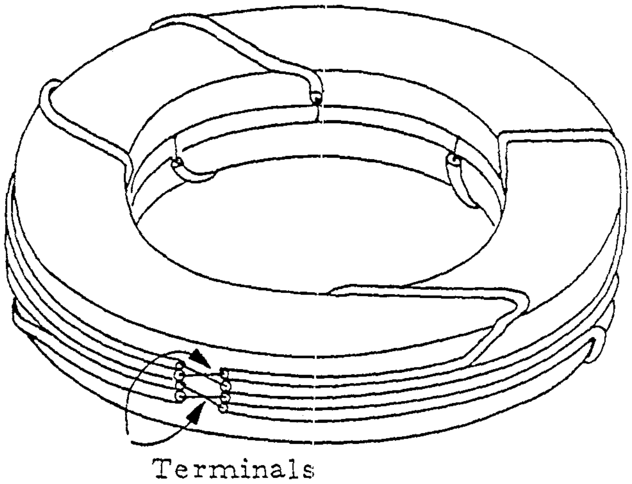
to the resulting signal. In other cases one may wish to use this type of sensor to measure D_z , or in general J_{t_z} , the z component of the total current density. In such cases one might wish to position the shield gap somewhat differently and perhaps enlarge the hole through the sensor.

Finally, note that we may typically desire to remove the signal some distance from the sensor, say to an oscilloscope, via a cable. In connecting this cable to the sensor we may wish to cut a hole in the shield at some point around the outside. If this cable is, say, a twinax in order to carry a differential signal, then we could connect the cable shield to the sensor shield at this hole, passing the two center conductors inside the sensor shield to connect to the sensor terminals. If the cable shield has a significant current induced on it by the external fields, then this current can pass onto the sensor shield and the associated currents on the loop conductors inside the shield can thereby be reduced by the presence of the shield.

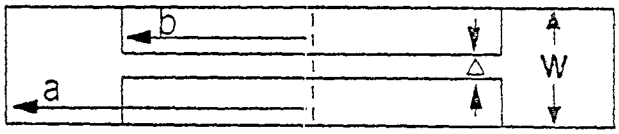
III. High-Frequency Inductive Current Sensors

Now consider a design for high-frequency inductive current sensors. In this case we want the sensor to have a response proportional $\dot{I}_t$ or I_t for frequencies with corresponding wavelengths of the order of the outer sensor radius, a , or larger. At such high frequencies the types of loop-turn geometries discussed in the previous section may have problems with capacitance between the various conductors which link the loop turns together, transit times between the loop turns, and other high-frequency effects. The sensor is not electrically small at such high frequencies and quasi-static calculations using the magnetic field do not apply as well. The sensor should then be considered as a distributed system with a view to reducing these high-frequency problems.

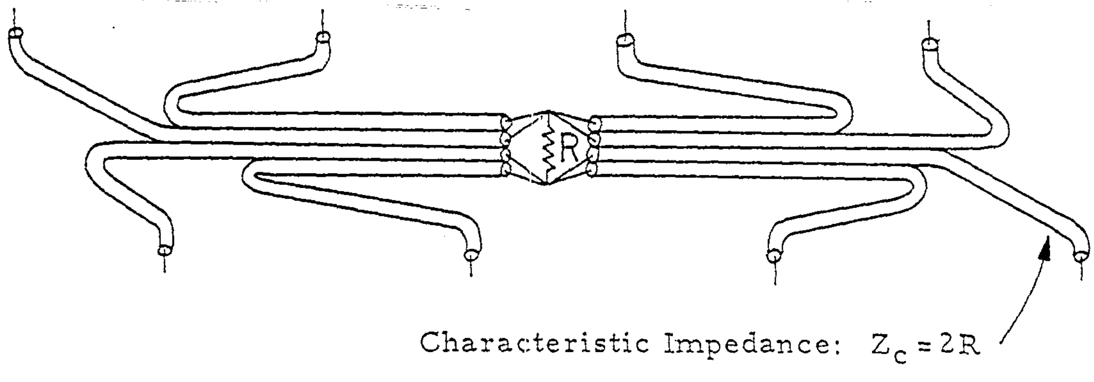
As an example consider the design of a high-frequency current sensor for measuring $\dot{I}_t$ as illustrated in figure 6. This example has two effective loop turns and averages the signals from eight equally-spaced positions around the sensor. For this design the individual loop turns are all joined together to form a single structure consisting of a highly conducting shell of inner radius b , outer radius a , and height w , with a gap of width Δ around the inside of the central hole through the sensor. We are using a structure for the loop turns of this high-frequency sensor which has the same geometry as the conducting shield discussed in the previous section. The geometry of this structure is independent of ϕ and does not distort $H_{\phi_{avg}}$ for frequencies low enough that the magnetic field can penetrate through the gap into the loop structure. Considering some small part of the sensor circumference between two values of ϕ , note that this roughly approximates a cylindrical loop with a rectangular cross section. Take the media both inside and outside the sensor



A. Perspective View



B. Side View (cross section, without cables)



C. Signal cables laid out with termination

Figure 6. High-frequency Current Sensor: $N = 2$

to have the electromagnetic properties of free space. Then, for a sufficiently large resistance loading the loop gap, the upper frequency response of the loop occurs at a frequency with a radian wavelength of the order of the largest cross-section dimension of the loop (w or $b-a$)[4]. The signals from all such loops are then averaged to give a signal which is approximately proportional to the time derivative of $H_{\phi_{avg}}$ with about the same upper frequency response. However for radian wavelengths significantly smaller than the sensor radius, a, there will typically be significant displacement current through the sensor, compared to, say, the current on a wire through the sensor which one may wish to measure. Thus there may often be no reason to consider radian wavelengths any smaller than those of the order of a.

A network of coaxial cables, each of characteristic impedance Z_c, is used to take the signals from the positions around the gap in the sensor structure to a collection point for averaging. The outer shields of the cables are laid in continuous electrical contact with the conducting sensor structure so that the cables can be considered as small perturbations of the sensor geometry. All cables have the same transit time from the sensor gap to the terminals at the collection point so that the averaging of the signals has a common time applying to all signal introduction positions around the sensor. The network of coaxial cables for the example is shown laid out in figure 6C. One half of the cables have their center conductors connected to the bottom of the sensor gap while the other half have their center conductors connected to the top of the gap. This results in a differential signal for $H_{\phi_{avg}}$ at the output terminals and in an effective two-turn sensor. The average signal (differential in this case) from the signal introduction positions is terminated in a resistance R (which equals $2Z_c$ in this case) so as to give no reflection. Signals other than the average signal are not terminated without reflection but have no effect on the output signal across R. Note that R may represent the impedance of another cable (balanced twinax in this case) which carries the resulting average signal from the output terminals to some other instruments located away from the sensor. There are also various other ways to design cable networks for the present kind of sensor such as by using some cable networks similar to those which can be used for $\dot{B}$ loops[5].

Note that in the example in figure 6 we have placed the sensor gap around the inside of the hole through the sensor. For some applications we may wish to place this gap elsewhere. For example, for a current sensor built into a conducting cylinder and used for measuring the current along the cylinder one might want the gap around the outside of the sensor[6]. In some cases we

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4. Lt. Carl E. Baum, Sensor and Simulation Note 8, Maximizing Frequency Response of a $\dot{B}$ Loop, December 1964.
5. Lt. Carl E. Baum, Sensor and Simulation Note 23, A Technique for the Distribution of Signal Inputs to Loops, July 1966.
6. Capt. Carl E. Baum, Sensor and Simulation Note 56, Some Electromagnetic Considerations for a Rocket Platform for Electromagnetic Sensors, June 1968.

may position this gap with a view to adjusting the sensitivity of the sensor to the vertical displacement current density in some desirable manner, appropriate to the particular case.

IV. Summary

In measuring the total current, I_t , through a circular inductive current sensor one tries to measure the average value of H_ϕ around the hole through the sensor by combining signals proportional to H_ϕ (or its time derivative) from many equally spaced positions around the sensor. If H_ϕ is approximately independent of ϕ , or has other degrees of symmetry appropriate to the particular sensor then the mutual inductance, M , expressing the sensitivity of the sensor to $\dot{I}_t$, can be calculated fairly accurately. The self inductance, L , of the sensor may be only known somewhat less accurately.

For frequencies low enough that the sensor is electrically small the individual loop turns distributed around the sensor can be directly interconnected as in a multi-turn loop with appropriate care given to the positioning of the connecting conductors. An appropriate conducting shield with a gap can be added to this sensor to improve the rejection of some undesired signals. For higher frequencies such that the sensor is not electrically small the sensor should be considered as a distributed system with a view toward minimizing the high-frequency problems. One technique useful for this high-frequency case is to use transmission lines such as coaxial cables to collect the signals from around the sensor structure.

In this note we have considered inductive current sensors with geometries based on a circle. Because of their axial symmetry such geometries are convenient both from practical and analytical points of view. However inductive current sensors are not limited to such geometries. The sensor could have a variety of shapes to fit particular measurement requirements. In each case the positioning of the sensor conductors must be chosen to give a resulting signal proportional to the line integral of the magnetic field around the area of interest.