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# Minimizing Transit Time Effects in Sensor Cables

## I. Introduction

In the case of various EM sensors it is desirable to drive either an infinite or zero impedance load. It will also be generally necessary to separate the sensor from its load by a section of cable (coax, twinax, etc.) These cables will have a finite, non-zero impedance and a finite, non-zero transit time causing ringing in the cable and errors in the response of the sensor.

In addition some of these sensors have the interesting property that the output impedance of the sensor is equal to the reciprocal of the desired load impedance, at least for short pulses. For example take the case of the electric field dipole antenna which can be represented as in figure 1. For short pulses it can be considered an electrical short. It is also desirable for the output of this sensor to drive a high impedance load so that the measured output will be proportional to the electric field strength. A dual situation is a loop antenna for measuring the magnetic field strength. For short pulses this sensor has effectively an infinite output impedance. Therefore both these sensor configurations have their impedances equal to the reciprocals of the desired load impedances.

## II. Elimination of Cable Ringing

The general procedure used to eliminate the cable ringing is to reduce the pulse amplitude arriving at the load to a value one half that at the sensor. The reflection at the load serves to double the voltage or current back to the value at the sensor. This reflection is then absorbed in the same network which originally cut the pulse size in half.

Some general restrictions which are placed on these added impedances are that in the case of an infinite load impedance there must be no shunt impedances introduced and that in the case of a zero load impedance there must be no series impedances introduced. These somewhat restrict the complexity of the problem. Similarly these added impedances are restricted to be resistors in an attempt to make the cancellation of ringing hold for all frequencies. The cable considered is twinax and the circuits are balanced but the technique can be used just as easily for coax.

Consider the technique used in figure 3 for the case of the low impedance sensor. The voltage output from the sensor is immediately cut in half by series resistors. When the voltage pulse reaches the load it is reflected with a voltage reflection coefficient of + 1, thus doubling the voltage at the load and compensating for the initial attenuation by the series resistors at the sensor. When the reflected pulse returns to the sensor it is terminated in its characteristic impedance. The dual situation for the high impedance sensor is shown in figure 4 where everything applies as before if voltage is replaced by current.

The technique used in figure 5 is a little more complicated. The resistors located in the middle of the cable give the property at plane B that a voltage pulse incident on this plane from either side has voltage transmission and reflection coefficients both equal to + 1/2. Thus a pulse coming from the sensor (plane A), (1), will drop to half value passing through plane B, (2), and will reflect with a reflection coefficient of +1 at plane C, (3). As before the voltage at the load will be the same as the voltage at the sensor. Meanwhile the reflection from plane B will be reflected from plane A with a voltage reflection coefficient of -1, (3). Thus the reflections from planes A and C will arrive at plane B at the same time with the same amplitudes but opposite polarities. The transmitted and reflected pulses which each of these pulses produce at plane B will exactly cancel, (4). For example, the pulse coming from plane C will produce a reflection of one half original amplitude heading toward plane C but the pulse coming from plane A will also produce a transmitted pulse heading toward plane C again of one half the original amplitude. However being of opposite sign these two pulses will cancel. This sequence is indicated at the top of figure 5.

The dual situation to that just described is one with a shunt resistor in the middle of the cable as in figure 6. The entire analysis above applies if everything is done in current instead of voltage.

## III. Possibility of Other Resistor Configurations

A. Consider the case of the high impedance load. The success of any configuration which has resistors in the cable depends on the cancellation of reflections at the resistors. Continuity of current requires that the sum of the voltage reflection and transmission coefficients at the resistors in the cable be +1.0. It is also necessary that the product of the transmission coefficients through all the resistor configurations (including those at the sensor) be 0.5 so that reflection at the load brings the voltage back up to the original value.

Within the cable any individual resistor configuration with non-zero resistance will at least partially reflect a pulse. The only possible way of removing these reflections is by termination or cancellation with other reflections. For two pulses to cancel each other at a resistor configuration it is necessary that

$$V_1 T + V_2 R = 0 \qquad (1)$$

and

$$V_1 R + V_2 T = 0 \qquad (2)$$

where $V_1$ and $V_2$ are the two voltage pulses, and T and R are the voltage transmission and reflection coefficients respectively (equal in both directions by symmetry). Solving these equations requires that $R = \pm T$ and since in this case $R > 0.0$ and $T > 0.0$, $R = T$. Also since $R + T = 1.0$, $R = T = 0.5$. This then also requires that $V_1 = -V_2$, showing the necessity for reciprocal impedances at the ends of the cables.

Since there must be at least one resistor configuration (including resistors at the sensor) which can cancel reflections (or terminate them), so that ringing will not go on forever, there are two possible cases to consider. First a configuration within the cable with $R = T = 0.5$ can cancel reflections. However since the product of all the transmission coefficients must be 0.5, all the remaining resistor configurations (including these at the sensor) must have the product of their transmission coefficients equal to 1.0. Since for this case no transmission coefficients can be greater than 1.0 they must all be 1.0 and therefore any extra resistor configurations are non existent. This is exactly the case shown in fig. 5.

Second a cable with resistors at the sensor can cancel reflections by termination of the pulse in the characteristic impedance of the cable. However this makes the voltage transmission coefficient of the pulse from the sensor equal to 0.5. As before then all resistor configurations in the cable must have $T = 1.0$ and thus be non existent. This is exactly the case shown in fig. 3.

Summarizing, then, to eliminate cable ringing (in the case of a high load impedance with zero source impedance) for a finite number of series resistors in the cable (and at the sensor) there are only two possible configurations which will exactly eliminate ringing, and thus signal distortion at the load. These are the configurations listed in figures 3 and 5.

B. The argument in A also applies for the case of a sensor with infinite impedance driving a zero impedance load. The situation is the exact dual. Thus the only possible configurations with a finite number of shunt resistors are those shown in figures 4 and 6.

## IV. Extension to Cables with Distributed Resistance

The extension of the above analysis to completely distributed systems, i.e., resistance per unit length as a function of position, becomes prohibitive except for certain approximate cases. Nevertheless from an engineering viewpoint it may be advantageous to use wire with a constant resistance per unit length in the construction of the twinax (or coax) cables leading from the sensors to their loads. This case applies to the sensor with zero output impedance driving an infinite impedance load. The dual case would require a shunt resistance per unit length as a property of the dieclectric which may be somewhat difficult to achieve.

A transmission line has a propagation constant (in the frequency domain) given by

$$K = -j\sqrt{(\mathcal{R}+j\omega\mathcal{L})(\mathcal{G}+j\omega\mathcal{C})} \tag{3}$$

where $\mathcal{R}$ , $\mathcal{L}$ , $\mathcal{G}$ , and $\mathcal{C}$ are respectively the series resistance, inductance, shunt conductance, and capacitance per unit length. Since it is assumed that $\mathcal{G} = 0$ eqn. (3) can be rewritten as

$$K = \omega\sqrt{\mathcal{L}\mathcal{C}}\sqrt{1 + \mathcal{R}/j\omega\mathcal{L}} \tag{4}$$

If $\quad \omega \gg \mathcal{R}/\mathcal{L} \quad$ then

$$K = \omega\sqrt{\mathcal{L}\mathcal{C}}\left(1 + \mathcal{R}/j2\omega\mathcal{L} + \cdots\right) \tag{5}$$

or
$$K \simeq \omega\sqrt{\mathcal{L}\mathcal{C}}\left(1 + \mathcal{R}/j2\omega\mathcal{L}\right)$$

Thus the frequency components of a wave on the line have the form

$$e^{j(\omega t - Kx)} = e^{j\omega(t-\sqrt{\mathcal{L}\mathcal{C}}\,x)}\; e^{-\frac{\mathcal{R}}{2}\sqrt{\frac{\mathcal{C}}{\mathcal{L}}}} \tag{6}$$

The first term, $e^{j\omega(t-\sqrt{\mathcal{L}\mathcal{C}}\,x)}$, represents a wave propagating down the transmission line with no phase distortion assuming $\omega \gg \mathcal{R}/\mathcal{L}$ or a pulse width,

$$\Delta t \ll \frac{\mathcal{L}}{\mathcal{R}} = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}\,\frac{\sqrt{\mathcal{L}\mathcal{C}}}{\mathcal{R}} = \frac{1}{V_p}\,\frac{Z}{\mathcal{R}} \tag{7}$$

where Vp is the propagation velocity of the wave (neglecting $\mathcal{R}$) and Z is the characteristic impedance of the cable (neglecting $\mathcal{R}$.) The second term, $e^{-\frac{\mathcal{R}}{2}\sqrt{\frac{\mathcal{C}}{\mathcal{L}}}}$, represents an attenuation of the wave which is again constant with frequency and thus does not distort the pulse, again with the criterion of eqn. (7). The attenuation per unit length is thus

$$\alpha = \frac{\mathcal{R}}{2}\sqrt{\frac{\mathcal{C}}{\mathcal{L}}} = \frac{1}{2}\frac{\mathcal{R}}{Z} \tag{8}$$

For a cable of length, d, then one might set

$$e^{-\alpha d} = 0.5 \tag{9}$$

or

$$\mathcal{R} = \frac{2Z}{d}\ln 2 \tag{10}$$

By this technique the voltage at the load would initially be that at the sensor delayed by the transit time of the cable. However the pulse would still ring in the cable decreasing in amplitude by a factor of four on each round trip. Thus this technique would only be approximate but perhaps useful.

## V. Effects of Sensor Capacitance and Inductance

### A. Initial Effects

The techniques outlined in section II are exact solutions to the problem assuming that the capacitance of the electric field sensor and the inductance of the magnetic field sensor are infinite. In practice, of course, this is not true. Consider the case of the electric field sensor and let V be a unit step function. In the case in fig. 3 the sensor is effectively driving an impedance 2Z until the reflection returns at a time 2 $t_r$. At this time the voltage into the cable, $V'$, which will arrive at the load a time $t_r$ later, is given by

$$V' = Ve^{-2t_r/2ZC} = Ve^{-t_r/ZC} \tag{11}$$

In the case of fig. 5 the sensor is effectively driving an impedance Z until the reflection returns at a time $t_r$. Again the voltage into the cable is given by

$$V' = Ve^{-t_r/ZC} \tag{12}$$

In each case the voltage decrease is the same except that with the resistors at the sensor it takes twice as long to decrease this amount.

Since it is generally desirable to have no decrease in the voltage at all then it is necessary that for either configuration

$$\frac{ZC}{t_r} >> 1 \tag{13}$$

The dual situation for the magnetic field loop leads to the analogous criterion

$$\frac{L}{Z t_r} >> 1 \tag{14}$$

where

$$I' = I e^{-t_r Z/L} \tag{15}$$

B. Longer Time Effects

Eventually any second order ringing will settle to an asymptotic value. For low frequencies the cable can be considered as a lumped capacitance or inductance as appropriate. For the electric field sensor then the asymptotic voltage at the load, V", from the unit step input can be calculated by considering a capacitive divider network giving

$$V'' = \frac{V}{1 + \frac{C_d}{C}} = \frac{V}{1 + t_r/ZC} \tag{16}$$

Ideally V" = V' = 1 and so the design criteria are the same as before in eqns. (13) and (14).

Again for the magnetic field loop

$$I'' = \frac{I}{1 + t_r Z/L} \tag{17}$$

## VI. Optimization

### A. E Field Sensor

Using the techniques outlined in this note it is first necessary to consider eqns. (13) and (14). To get a feeling of the numbers involved consider an E field sensor consisting of two parallel plates 1.0 $m^2$ in area with about 0.1 meter spacing driving a twinax cable with 10 nanoseconds transit time. The antenna capacitance is about 88.5 pf. This requires that the cable impedance

$$Z \gg 113\,\Omega \tag{18}$$

For $V'' > .9V$ it would then be necessary from eqn. (16) that

$$Z > \frac{9t_r}{C} = 1020\,\Omega \tag{19}$$

This impedance is not impossible but definitely greater than that standardly used. This indicates that there is a premium attached to making cables as short as possible and raising the impedance as high as possible. Of course to raise the impedance the center conductors will be smaller increasing the losses. However this effect, indicated in section IV except that skin effect now also plays a role, can be used to lower the initial voltage at the load to the reduced voltage, $V''$, making the step response even a little flatter. Another possibility is to introduce some negative capacitance into the system equal to the cable capacitance. However this cannot compensate at the sensor itself for times shorter than twice the transit time of the cable. Negative capacitance can help but it has its limitations. Thus the parameters in eqn. (13) are still of the utmost importance.

### B. H Field Sensor

To get a feeling of the numbers involved in a shorted H field loop consider a loop with radius 0.5 meter and .01 meter radius wire again driving a 10 nanosecond cable. The loop inductance is about $.2\times10^{-5}$ henries. This requires that the cable impedance

$$Z \ll 200\,\Omega \tag{20}$$

For I" > .9 I eqn (17) implies

$$Z < \frac{L}{9 t_r} = 22.2\ \Omega \tag{21}$$

This impedance cable can also be made even in twinax, although it will also have to be specially constructed. The tradeoffs are similar to those concerning the E field sensor and analogous secondary schemes can be brought into play to compensate for the cable inductance.

## VII. Summary

In summary then it can be said that for the E field antenna and the H field loop, the transit time effects in the cable from the sensor to the load can be significantly reduced by:

1. The proper placing of discrete resistors.
2. Reduction of total cable capacitance or inductance as appropriate.
3. Increase of sensor capacitance or inductance as appropriate.

Ideally then the frequency response of these sensor configurations is limited only by the sensor itself.

CARL E. BAUM, 1/LT, USAF
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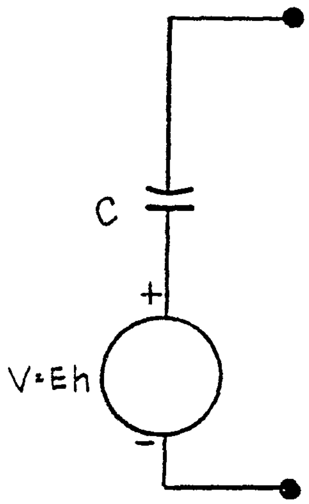
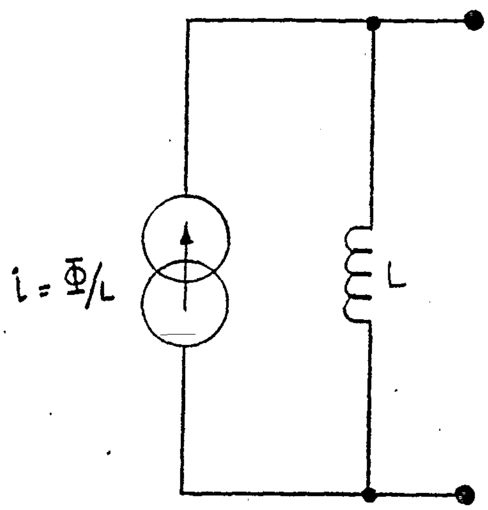



C = antenna capacitance
E = electric field strength
h = effective height of antenna
V = equivalent voltage source

Fig. 1 ELECTRIC FIELD DIPOLE ANTENNA




L = loop inductance
Φ = magnetic flux linking loop
i = equivalent current source

Fig. 2 MAGNETIC FIELD LOOP ANTENNA

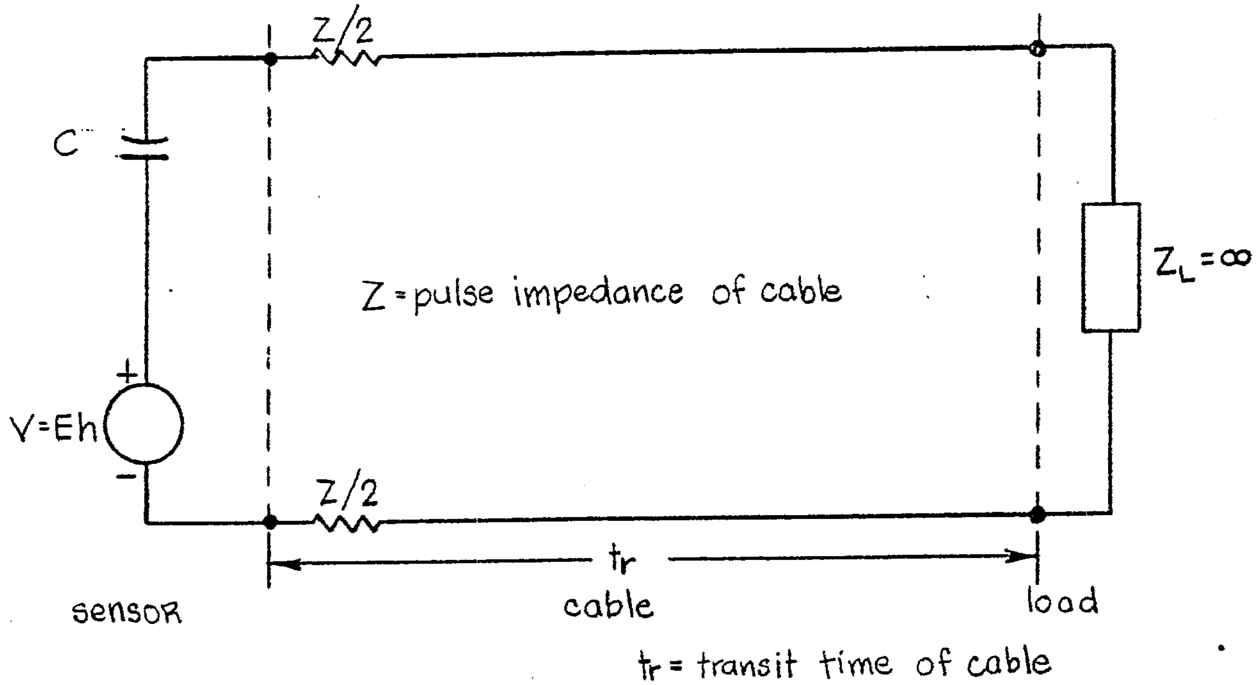



Fig.3 SERIES RESISTORS AT SENSOR

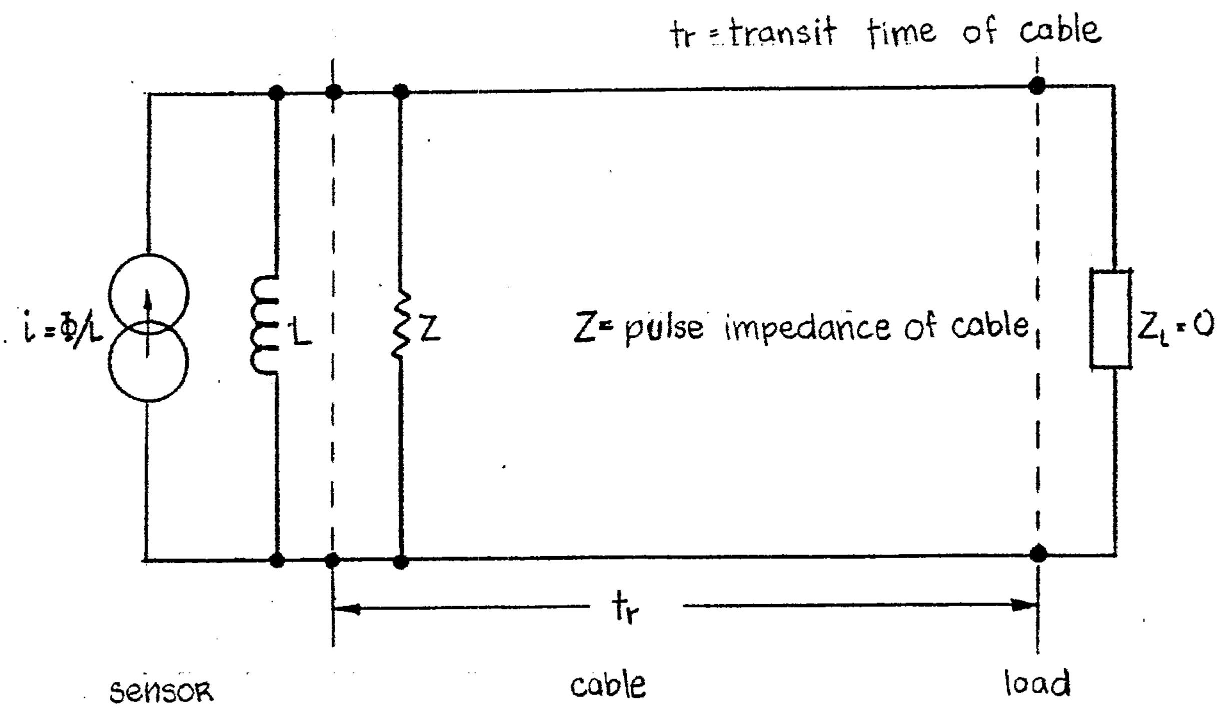
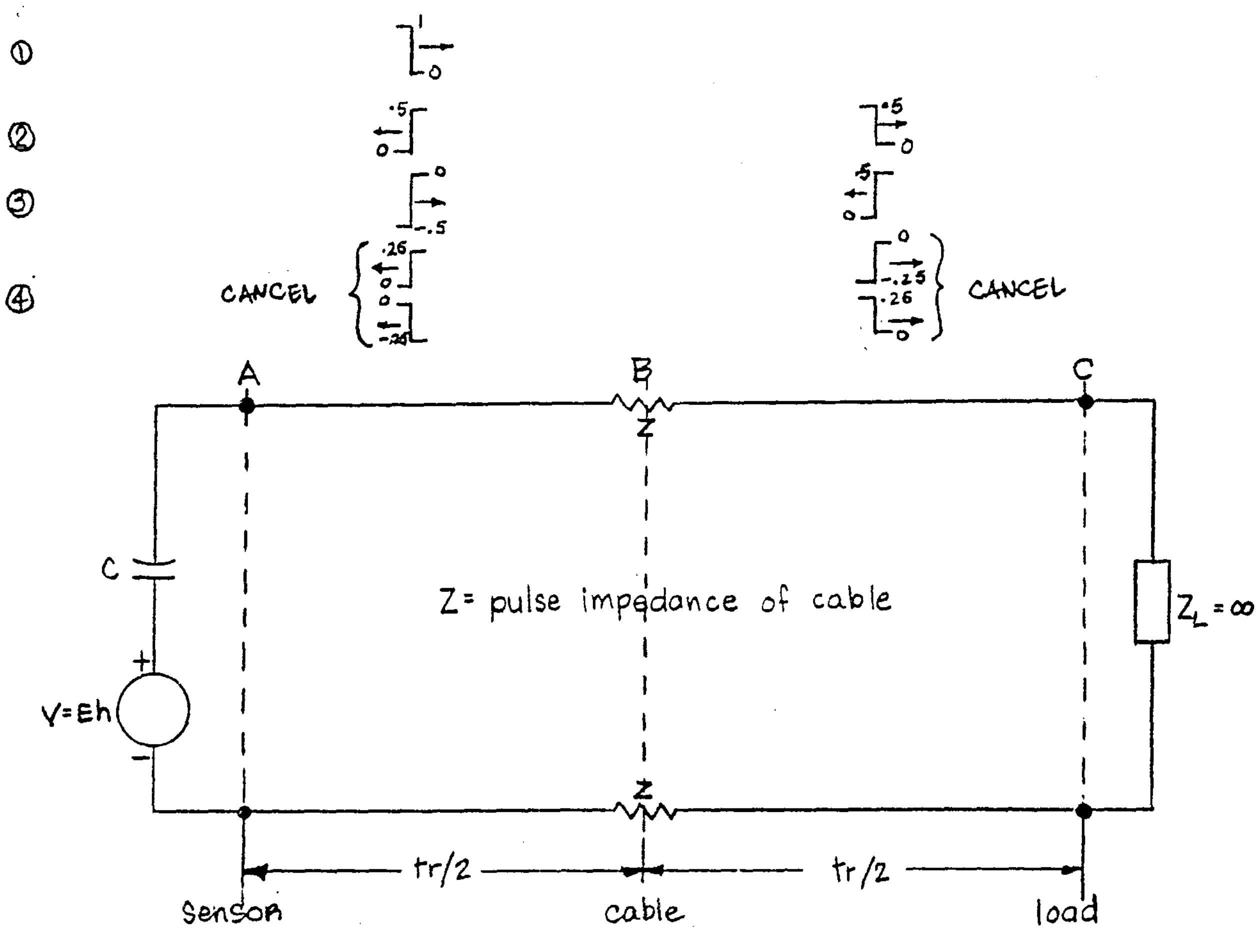
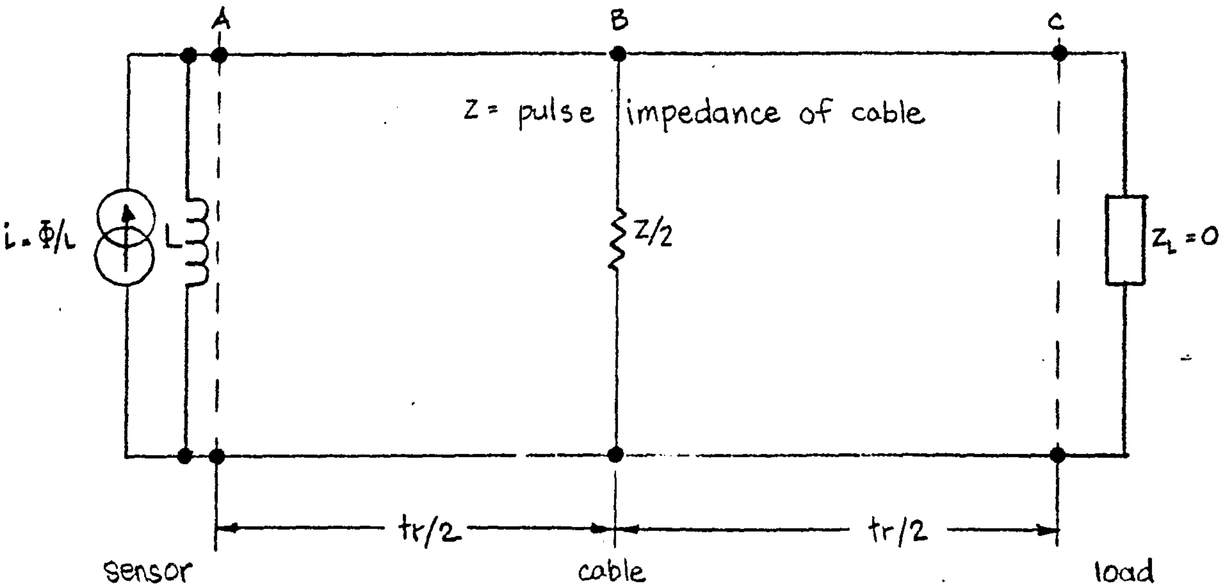



Fig. 4 SHUNT RESISTOR AT SENSOR




tr = transit time of cable

Fig.5 SERIES RESISTORS IN CENTER OF CABLE




tr = transit time of cable

Fig 6. SHUNT RESISTOR IN CENTER OF CABLE