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A NUMERICAL STUDY ON MINIMIZATION OF INDUCED CURRENTS BY IMPEDANCE LOADING

by

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Abstract

The time behavior is obtained of the total current induced by a transient electromagnetic plane wave incident upon a finite, perfectly conducting, solid cylinder loaded at its center with an inductive-resistive insert. It is found that impedance loading has the effect of (1) decreasing the resonant frequencies of an otherwise unloaded cylinder and of (2) damping the amplitude of oscillation of the induced total current. The loading that maximizes the damping is discussed; the variation of the load resistance with the load inductance for such loading is deduced.

Acknowledgment

Thanks are due to Capt. Carl Baum and Capt. Sid Johnson for their helpful suggestions and discussions, to Georgene Peralta for typing the manuscript.

I. Introduction

Recently, two notes have been written on the problem of minimizing, by the technique of impedance loading, the induced currents on a scatterer exposed to an incident electromagnetic wave. One note deals with the mathematical formulation of the problem,[1] while the other discusses the practical means and desirability of reducing the field distortion caused by the sensor platform housing various electromagnetic sensors for EMP measurements.[2] The present note should be considered a continuation of the former one, since it presents numerical results that were obtained, with the aid of a CDC-6600 computer, from previous mathematical formulas.

The problem at hand is to compute the total current induced by a plane electromagnetic wave incident normally upon a finite, perfectly conducting, solid cylinder loaded at its center with an inductive-resistive (L-R) insert. The incident wave is taken to be linearly polarized with its electric-field vector parallel to the axis of the cylinder; the wave is also assumed to be a step function, reaching the nearest surface of the cylinder at time $t = 0$. The most useful information derived from the numerical solution of the problem is undoubtedly the variation of the decay time of the induced total current with the L-R parameters, whose values may be at one's disposal. With this information one can choose the value of R for a given value of L (or vice versa) to obtain the maximum possible damping of the induced total current on a cylinder of fixed radius-to-length and fixed radius-to-gap-width ratios.

II. Method of Computation

For easy reference we shall reproduce here the necessary mathematical formulas that were required for numerical computation, along with a few remarks on the method of computation.

By invoking the Lorentz reciprocity theorem and the principle of superposition it can be shown that

$$I_r(z) = I_p(z) - \frac{Y_T}{Y_T + Y_L} \frac{I_p(0)}{I_t(0)} I_t(z) \quad , \tag{1}$$

where I_r , the quantity of interest, is the total current induced by a given time-harmonic, incident electromagnetic wave on a cylinder loaded at its center, $z = 0$, with the admittance Y_L ; I_p is the total current induced by the same incident wave on the same cylinder with $Y_L = \infty$; I_t is the total current due to some specified distribution of tangential electric field over a circumferential gap at the center of the cylinder where the admittance is defined in the customary way.

Extensive data on $I_p(z)$ are available in a previous note[3] in which are also contained descriptions of the DIPOLE, the DIPLOTK, and the FORGE computer codes, which have been successfully employed in the past in computing scattering of a time-harmonic or transient electromagnetic wave from a finite, perfectly conducting, solid cylinder. Thus, the quantity that actually needs to be computed in (1) is $I_t(z)$, from which the determination of Y_T immediately follows.

By use of the representation theorem that the magnetic field at an interior point in a simple and source-free region bounded by a regular surface is expressible in terms of the surface values of the electric and magnetic fields, it can be shown that

$$\frac{1}{2} I_t(s) + \int K(s,s')I_t(s')ds' = - \frac{I_t(0)}{Y_T} \frac{1}{4w} \int_{-w}^{w} M(z-z')f(z')dz' \quad , \tag{2}$$

where

$$M(z) = \frac{4ika^2}{Z_o} \int_o^{\pi/2} \cos 2\psi \frac{e^{ik\sqrt{z^2 + 4a^2 \sin^2\psi}}}{\sqrt{z^2 + 4a^2 \sin^2\psi}} d\psi \quad , \tag{3}$$

Z_o and k being the free-space impedance and wave number, and a the radius of the cylinder. The variable s is orthogonal to the azimuthal angle ϕ, on both the cylindrical surface and the bottom and top flat plates. In general, the forcing function, i.e., the integral on the right side of (2), should influence not only the current on the cylindrical surface but also that on the top and bottom plates of the cylinder. However, in accordance with the above-mentioned computer codes one need not extend the forcing function to both end plates of the cylinder. For detailed discussions one must refer to a previous note.[3]

Our first concern in solving (2) numerically is to evaluate the integral on the right side of that equation. To this end, let us assume $f(z)I_t(0) = Y_T$. The implication of this assumption can be found in reference 1. Here it suffices to say that no appreciable errors will result from this assumption. The integral that must be evaluated is then given by

$$F(z) = -\frac{1}{4w} \int_{-w}^{w} M(z-z')dz' \quad . \tag{4}$$

From (3) one can easily deduce that

$$M(z) \to -2i \frac{ka}{Z_o} \ln k|z| \quad , \quad \text{for} \quad k|z| < ka < 1 \quad ,$$

and that

$$M(z) \to \pi \frac{(ka)^2}{Z_o} \left(\frac{a}{z}\right)^2 e^{ikz} \left(1 + \frac{i}{kz}\right) \quad , \quad \text{for} \quad |z| > a \quad .$$

Thus, the integral (4) exists for every $|z| \leq h$, h being the half length of the cylinder. After this integral has been numerically evaluated, (2) can be solved for I_t , and hence Y_T , by use of the computer codes mentioned above.

III. Impedance Loading by a Radial Transmission Line and a Resistive Sheet

In this section we shall discuss briefly how the load admittance Y_L (or the load impedance Z_L), which appears in (1), can be realized in practice. One way to achieve this is to cut a slot around the cylinder with a resistive sheet surrounding the slot and various appropriate materials filling it (Fig. 1).

Inside the slot $(b \le \rho \le a' , - w \le z \le w)$ let there be only an axial component (E_z) of the tangential electric field with no axial or circumferential variations. Consequently, the only non-vanishing component of the magnetic field is H_ϕ , which varies only with the axial distance ρ . Then, the simple theory of a short-circuited radial transmission line applies and gives, for the admittance Y_L' at $\rho = a'$,

$$Y_L' = \left(\frac{2\pi\rho H_\phi}{2wE_z}\right)_{\rho=a'} = \frac{\pi k_L}{i\omega\mu_L}\,\frac{a'}{w}\,\frac{J_1(k_La')Y_o(k_Lb) - J_o(k_Lb)Y_1(k_La')}{J_o(k_La')Y_o(k_Lb) - J_o(k_Lb)Y_o(k_La')} \quad , \quad (5)$$

where $k_L^2 = \omega^2\mu_L\varepsilon_L + i\omega\mu_L\sigma_L$, μ_L , ε_L and σ_L being the electrical parameters of the material within the line.

When k_L is real, the required k_L to make $Y_L' = 0$ is given by any of the roots of the numerator of (5) for a given b/a' . The variation of the roots $k_L^{(n)}$ with b/a' becomes obvious if one writes

$$k_L^{(n)} = \frac{(2n-1)\pi}{2(a'-b)}\,F_n\left(\frac{b}{a'}\right) \quad , \quad n = 1,2,\dots \quad ,$$

since curves of F_n versus b/a' are given in Jahnke and Emde.[4]

When $|k_La'| << 1$, equation (5) yields

$$Y_L' \approx \frac{1}{-i\omega L} \quad , \tag{6a}$$

where the equivalent inductance, L , is given by

$$L = \frac{\mu_L w}{\pi} \ln\left(\frac{a'}{b}\right) \quad . \tag{6b}$$

Similarly, the equivalent conductance, G , of the resistive sheet is defined by

$$G = \left(\frac{2\pi\rho H_\phi}{2wE_z}\right)_{\rho=a} - \left(\frac{2\pi\rho H_\phi}{2wE_z}\right)_{\rho=a'} = Y_L - Y_L' \quad . \tag{7}$$

Let the sheet be geometrically as well as electrically thin. Then, integrating the equation $\nabla \times \underline{H} = \sigma\underline{E}$ through the thickness of the sheet one has

$$G \approx \frac{\pi\Delta\sigma(a+a')}{2w} \quad , \tag{8}$$

where $\Delta = a - a'$.

Thus, one can see from (6), (7) and (8) that the load admittance, Y_L , is a parallel combination of an inductance and a resistance ----- the so-called inductive-resistive insert. Before concluding this section, perhaps it is worth pointing out that the assumptions that are required for the derivations of (6) and (8) in no way violate the condition $f(z)I_t(0) = Y_T$, which is the basic assumption in the present theory.

IV. Time Behavior of the Induced Total Current

Figures 2-17 illustrate the time behavior of the induced total current at the midpoint of the cylinder for two values of a/h ratio, two values of w/a ratio, and various values of L and G (or R). The magnetic vector of the incident wave is assumed to have the form (Fig. 1A)

$$\underline{H}^{inc}(x,t) = \underline{e}_y H_o U\left(t - \frac{x+a}{c}\right) \quad ,$$

where U is the unit step function, c the vacuum speed of light, and $\underline{e}_y$ the unit vector in the y direction. Studying these curves, we find that the period of oscillation, which can roughly be determined by the zero-point crossings on the t axis, increases with decreasing G for a fixed L, and increases with increasing L for a given G. From this finding we immediately deduce that the shortest period of oscillation occurs when L, or R, or both have zero values, that is to say, when the cylinder is unloaded. Our deduction is, in fact, consistent with the information one may draw from figures 8 and 10 of a previous note.[3] Thus we conclude that impedance loading has indeed the effect of decreasing at least the lowest resonant frequency of an otherwise unloaded cylinder.

In figures 18-21, the time history is given of the induced total current at $z = h/2$ rather than at $z = 0$ on the cylinder. These curves are similar to the corresponding ones at $z = 0$.

In addition to having the effect of decreasing the resonant frequencies of a cylinder, impedance loading may result in a significant reduction in the amplitude of oscillation. Further discussions of this point will be left to the next section.

V. Decay Time of the Induced Total Current

The curves presented in figures 2-17 take approximately the analytical form $I(0,t) = F(t)\exp(-\alpha \frac{ct}{h})$, $F(t)$ being an almost sinusoidal function and α^{-1} the decay time constant. By fitting the envelope of any particular curve in figures 2-18 to an exponential function, the value of α can be determined for that particular curve. Figures 22 and 23 give the variation of the decay time constant with the normalized load inductance $\frac{L}{\mu_o h}$, the value of G being chosen to maximize α for a given L. These curves show that α varies roughly as $\ell n(1 + \frac{L}{\mu_o h})$. Referring to figure 27 of a previous note,[3] which shows the variation of the decay time constant with a/h for an unloaded cylinder, one can easily see that α can indeed be increased significantly by inductive-resistive loading. For example, when $L \approx 5\mu_o h$ henries and $R \approx 60\pi$ ohms for $a/h = 0.1$ (or $R \approx 120\pi$ ohms for $a/h = 0.01$), α is about one order of magnitude larger than it would be in the unloaded case.

VI. Optimum Loading for Maximum Damping

From a designer's viewpoint, it is useful to have the information available regarding what value the load conductance should take for a given value of the load inductance so that the induced total current will be most rapidly damped out. This value of conductance will be referred to as the optimum load conductance G_{op} . In figures 24 and 25, $Z_o G_{op}$ is plotted against $\frac{L}{\mu_o h}$ for two values of a/h and two values of w/a . These curves were obtained from figures 2-17 by using the criterion that α be a maximum. Since there are insufficient numerical data for accurately determining the maximum α , these curves are rather approximate in nature. Nevertheless, it is felt that they would provide useful, though rough, information concerning (1) the range of G_{op} and (2) the dependence of G_{op} on L . For more accurate information on these two points one may first interpolate the data available from figures 2-17. For example, one may first plot α versus $Z_o G$ for a fixed L . From this plot the maximum α , and hence G_{op} , may be determined more accurately for that L than the optimum load resistance given in figures 24 and 25.

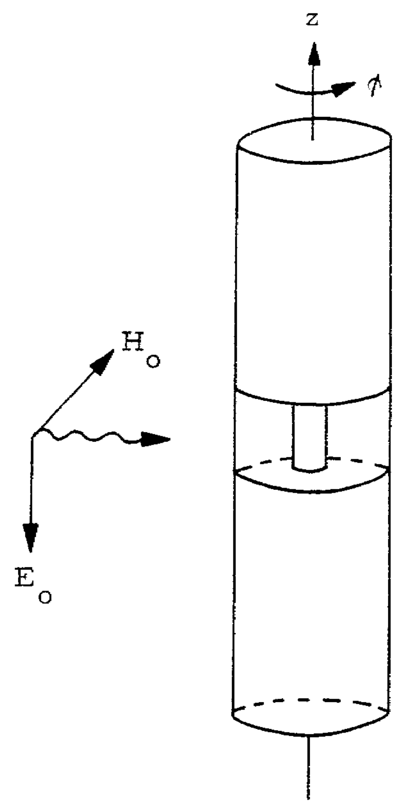

Figure-1A. EM scattering by an impedance-loaded, perfectly conducting, finite, solid cylinder.

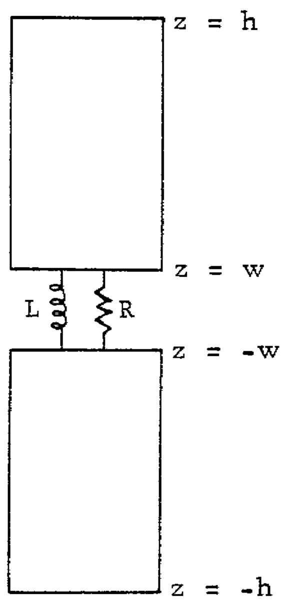

Figure-1C. Equivalent circuit.

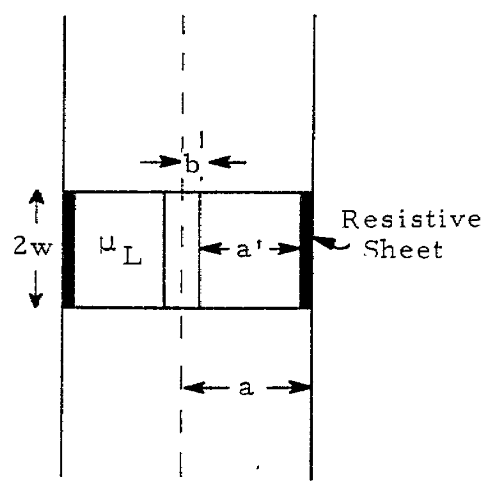

Figure 1B. The gap.

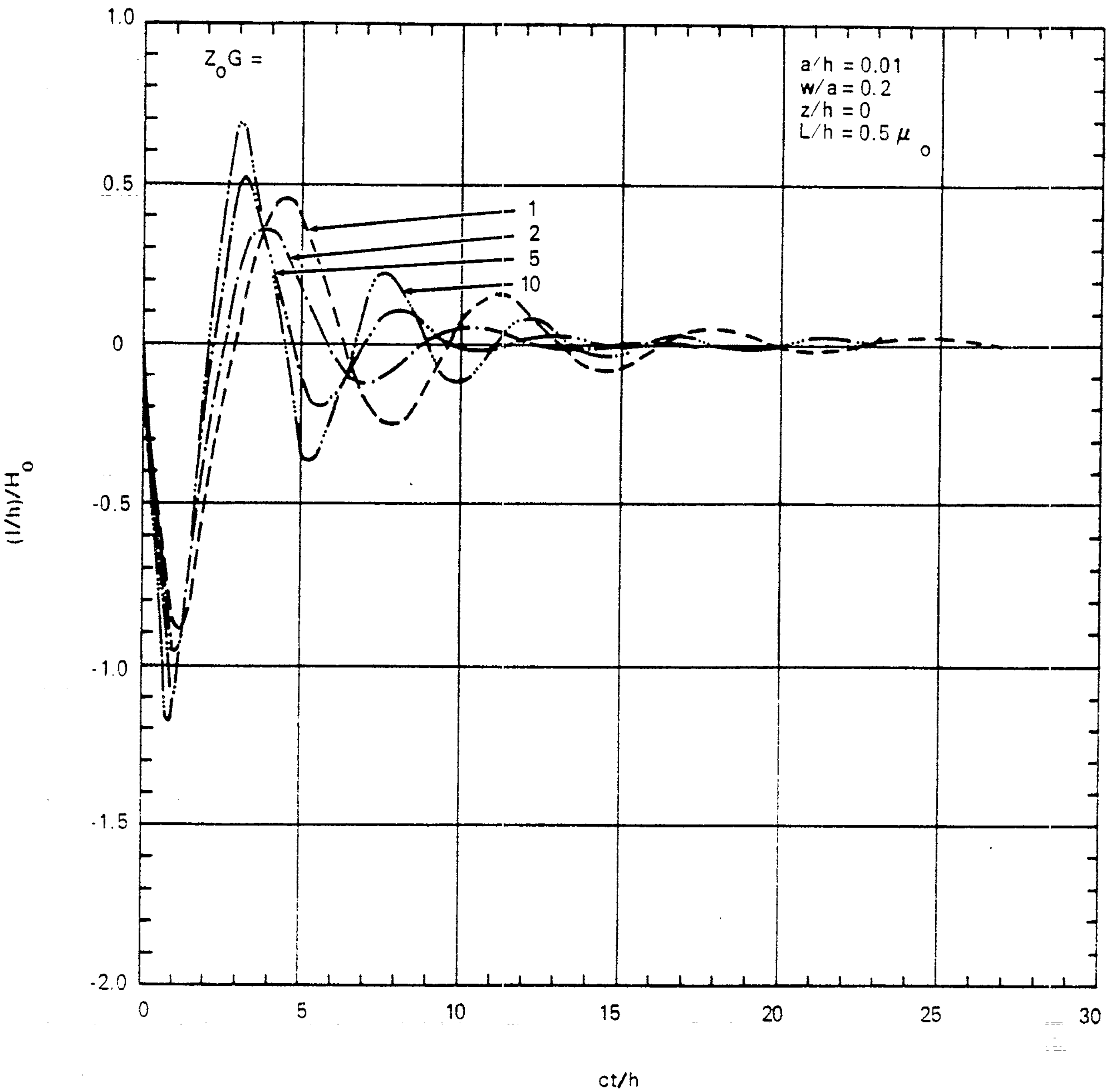

Figure 2. Current on a Loaded Cylinder versus ct/h

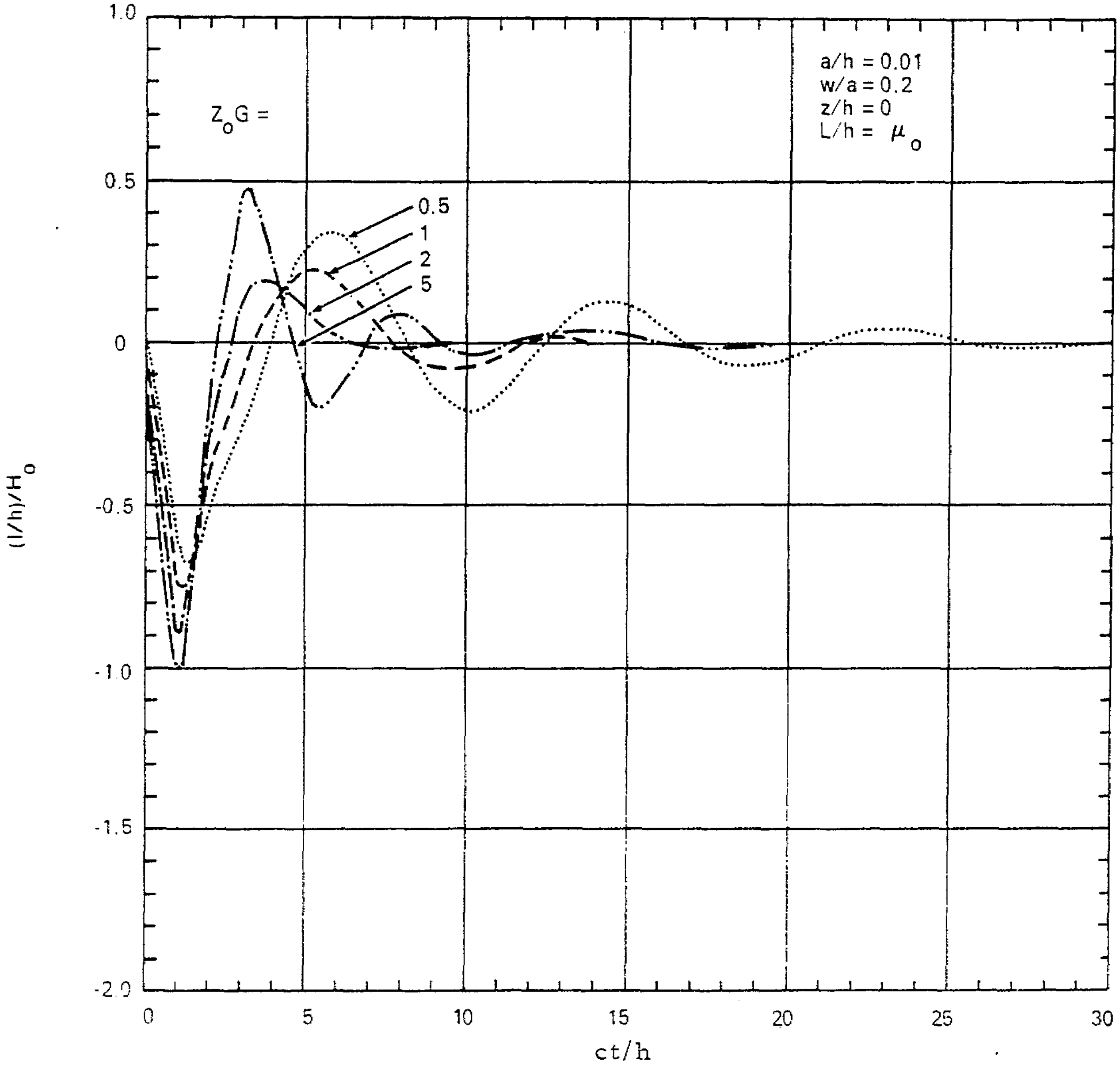

Figure 3. Current on a Loaded Cylinder versus ct/h

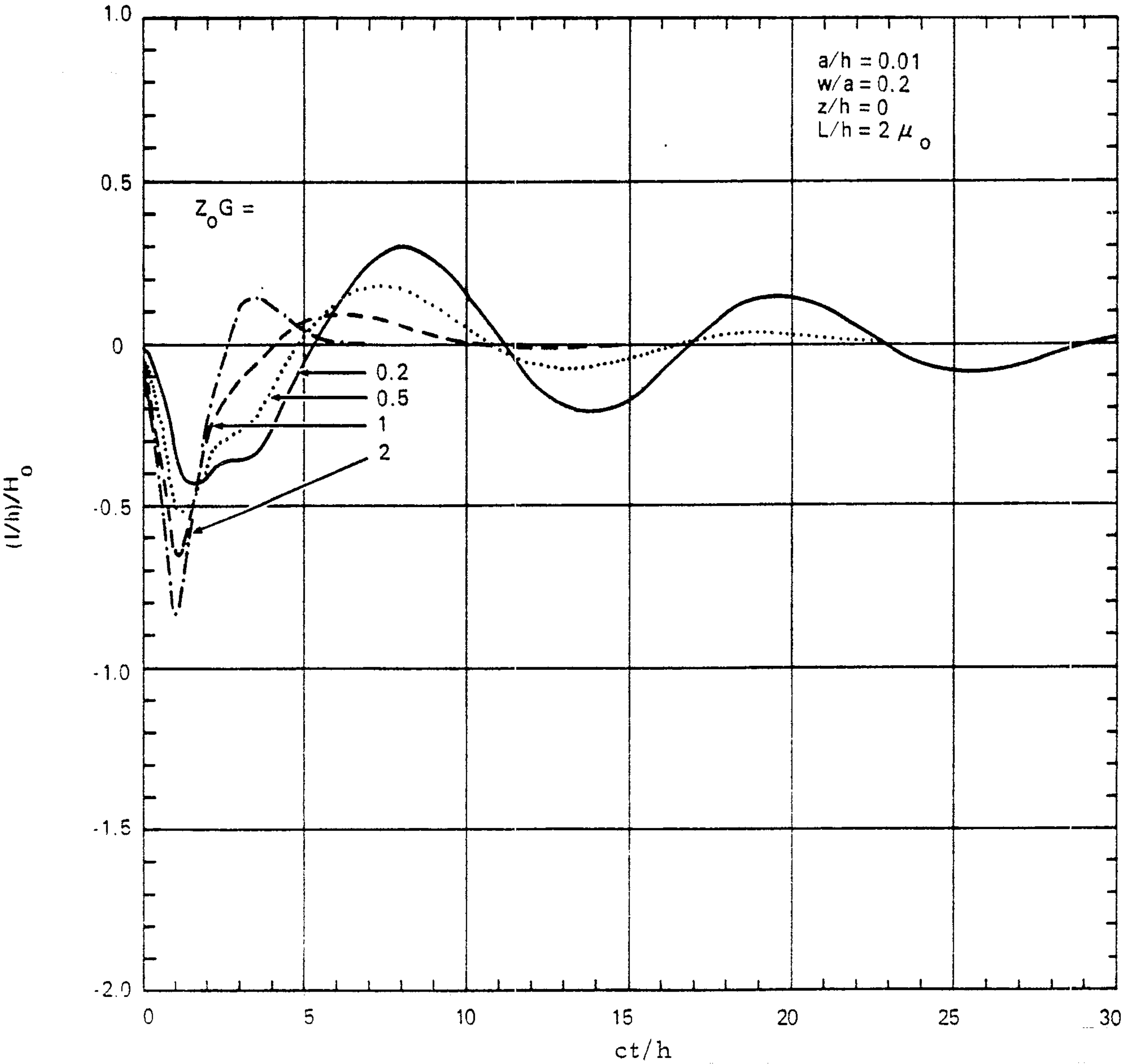

Figure 4. Current on a Loaded Cylinder versus ct/h

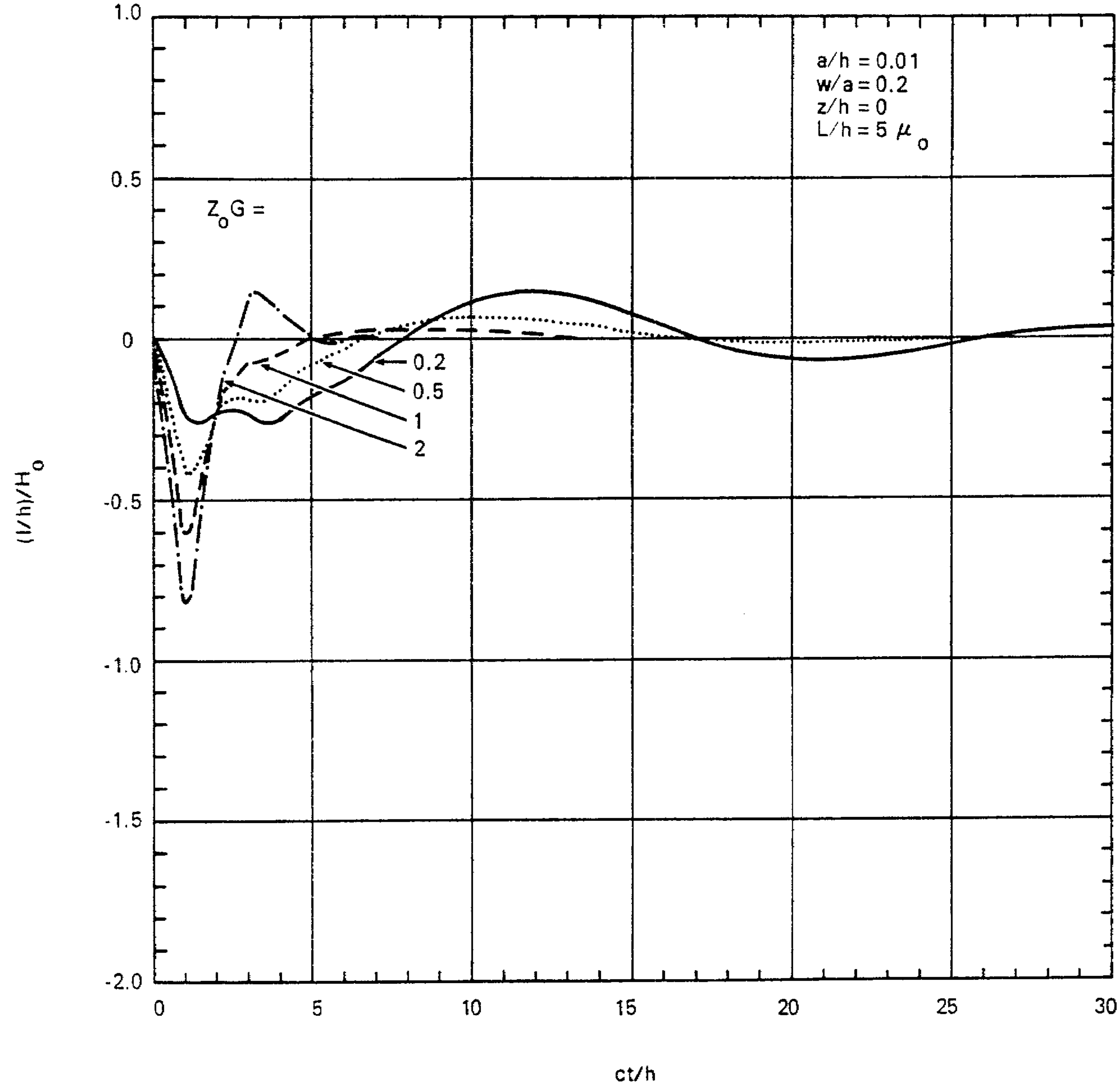

Figure 5. Current on a Loaded Cylinder versus ct/h

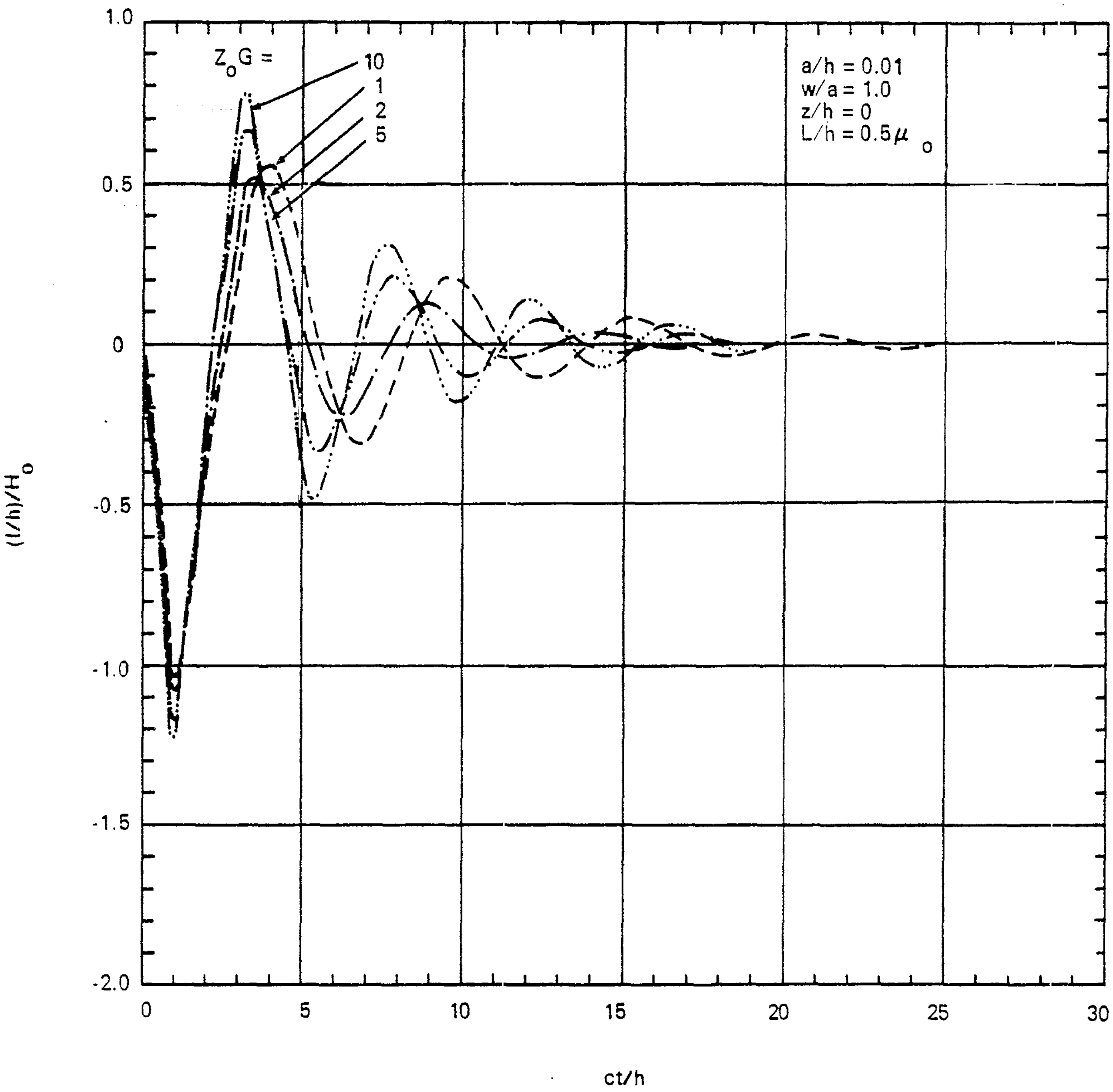

Figure 6. Current on a Loaded Cylinder versus ct/h

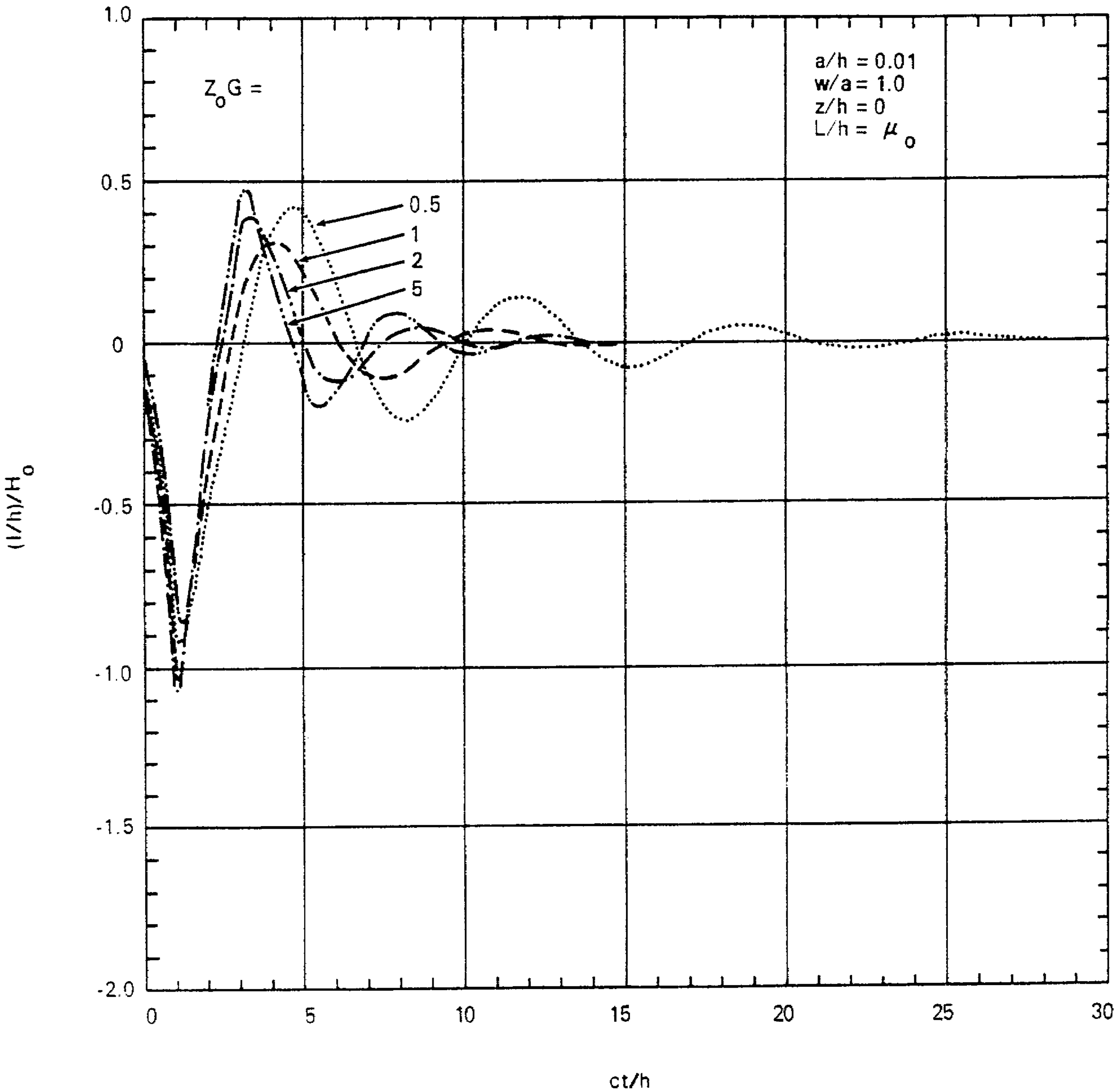

Figure 7. Current on a Loaded Cylinder versus ct/h

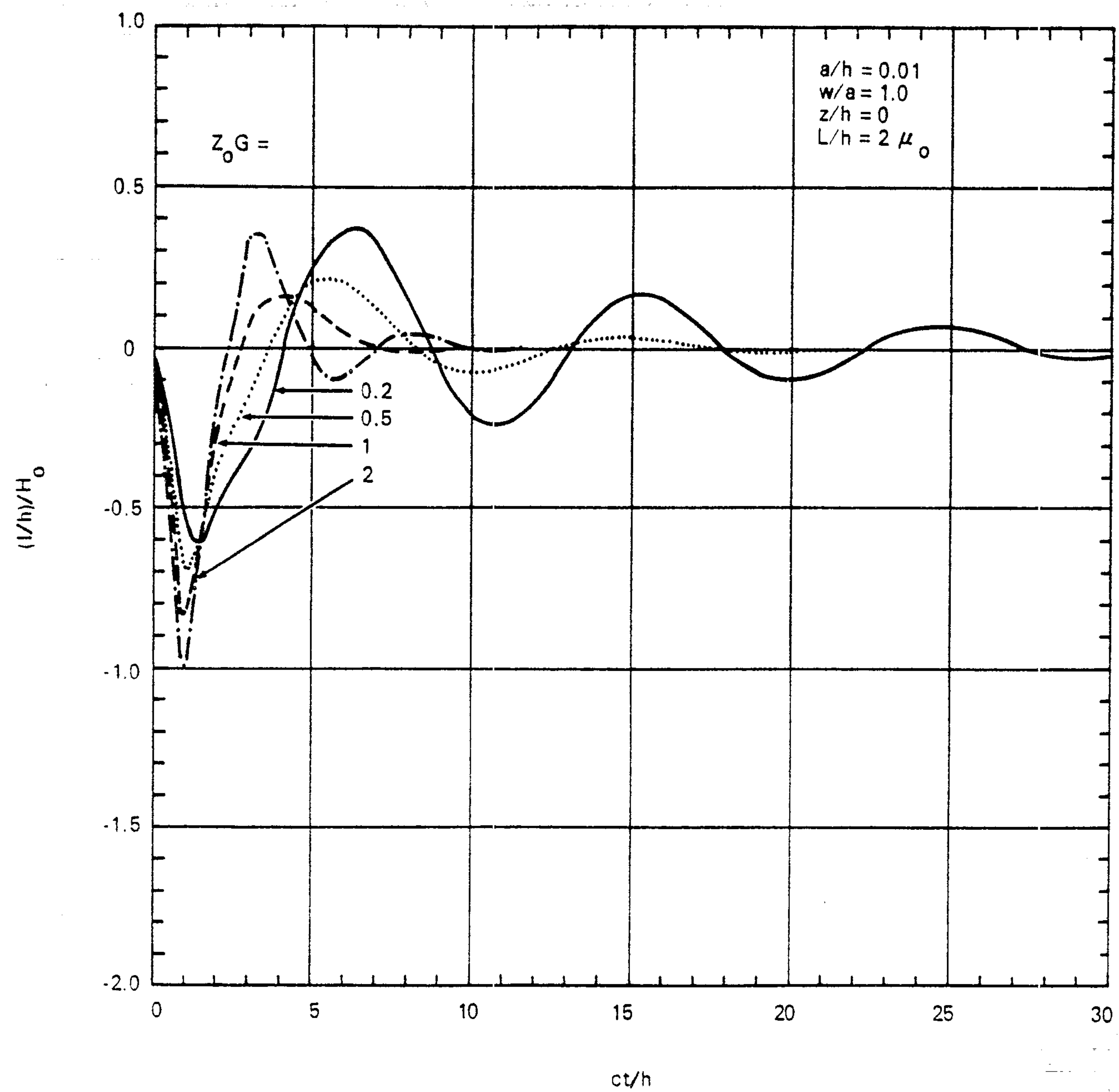

Figure 8. Current on a Loaded Cylinder versus ct/h

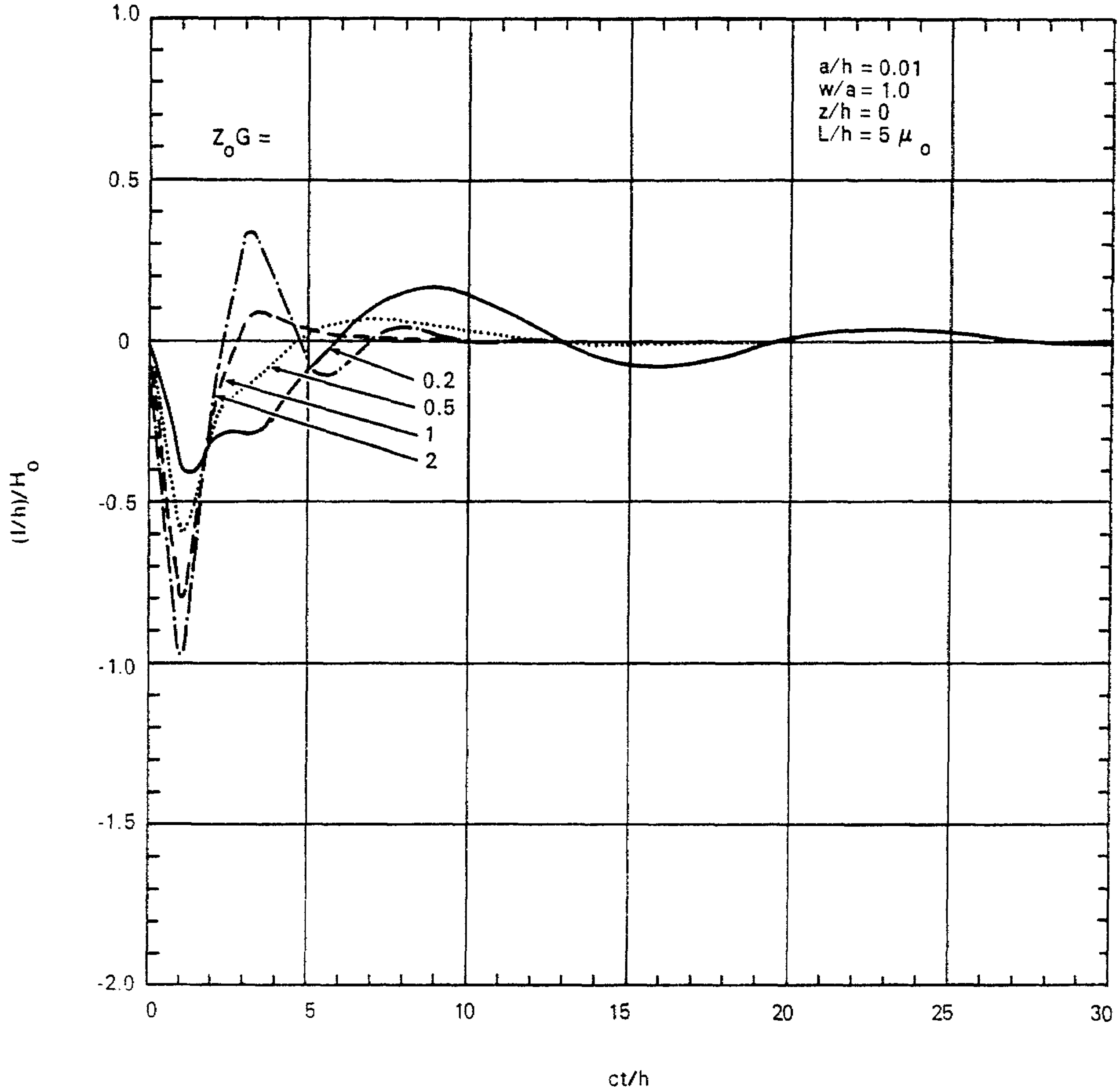

Figure 9. Current on a Loaded Cylinder versus ct/h

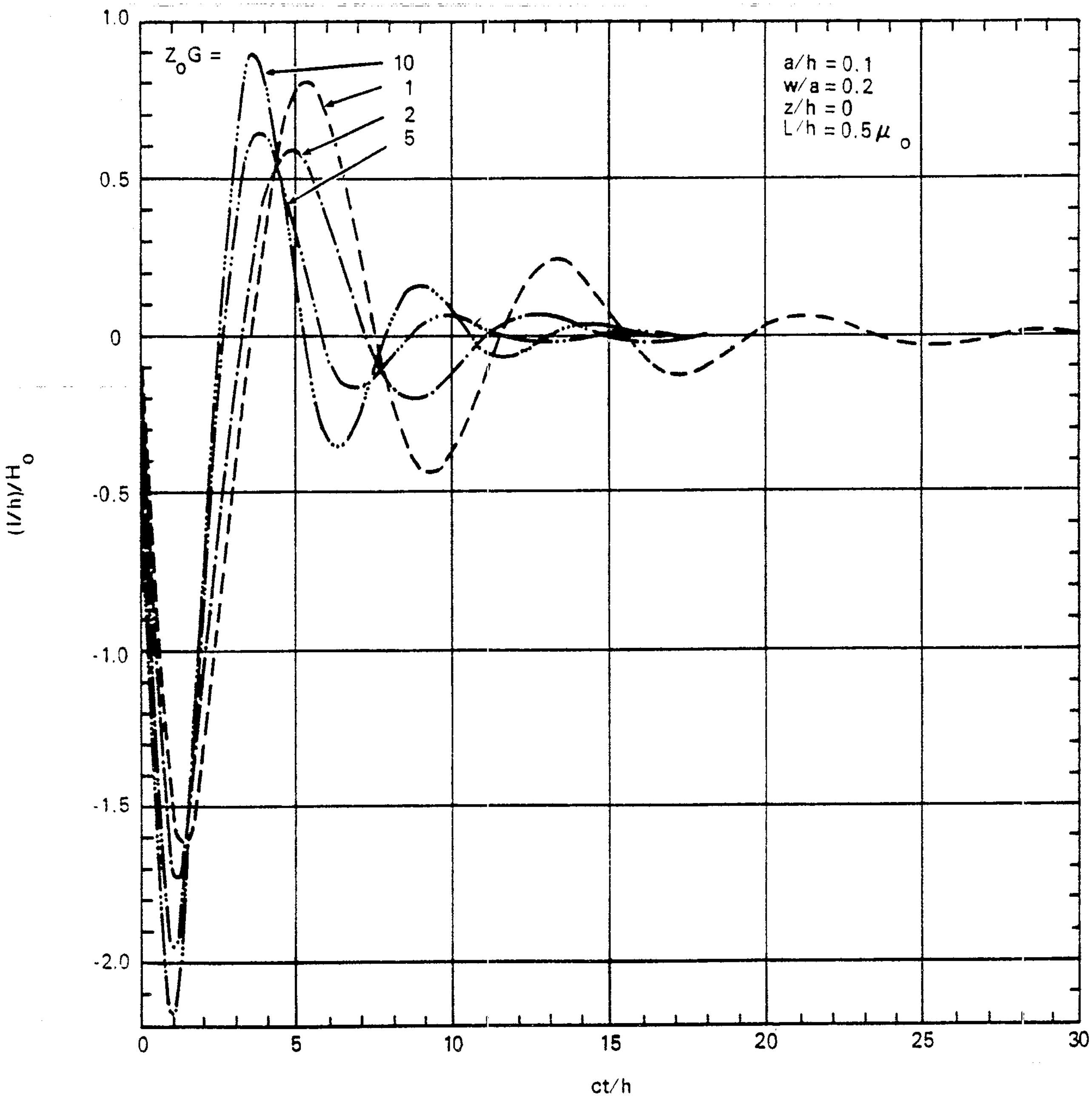

Figure 10. Current on a Loaded Cylinder versus ct/h

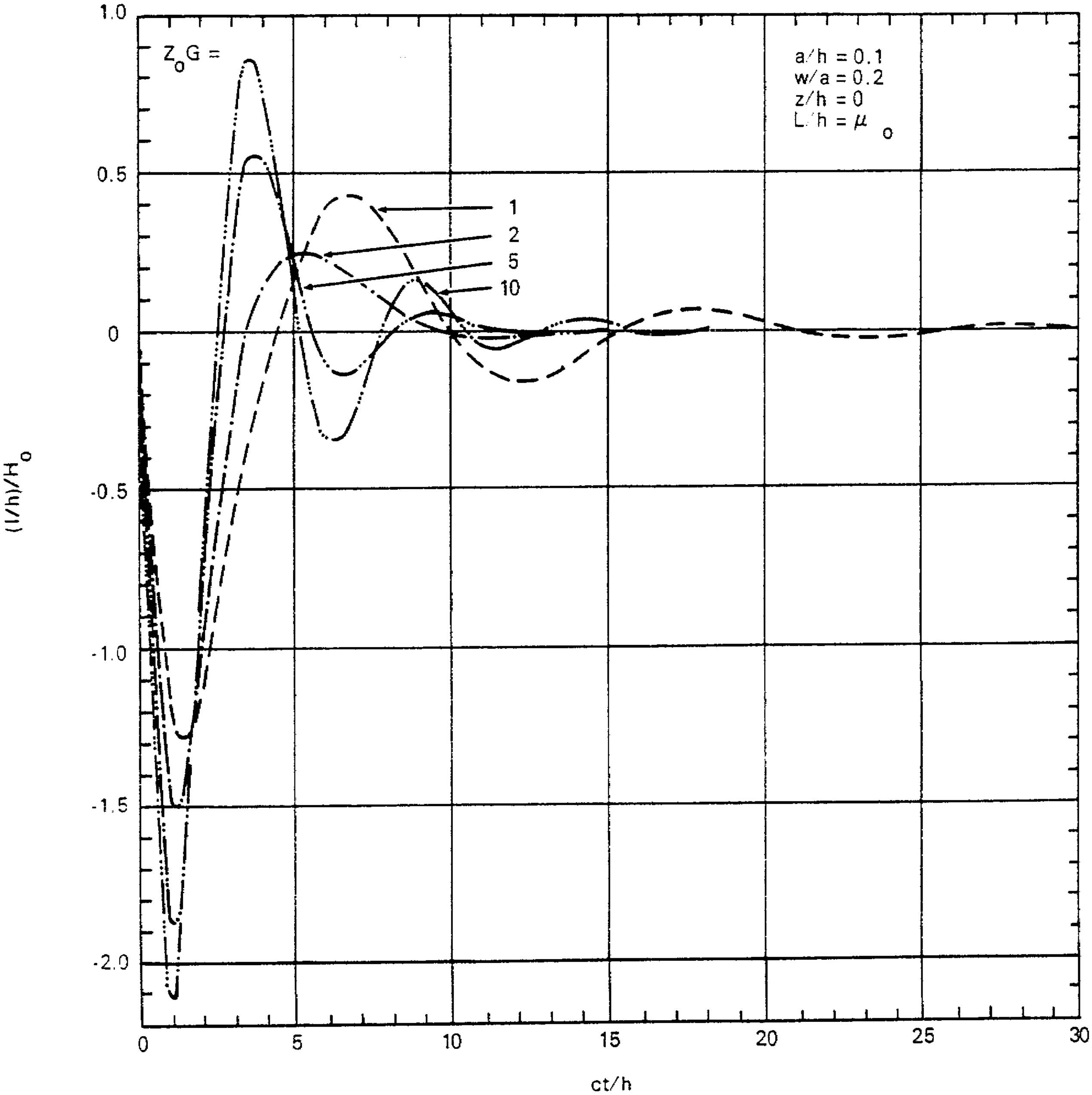

Figure 11. Current on a Loaded Cylinder versus ct/h

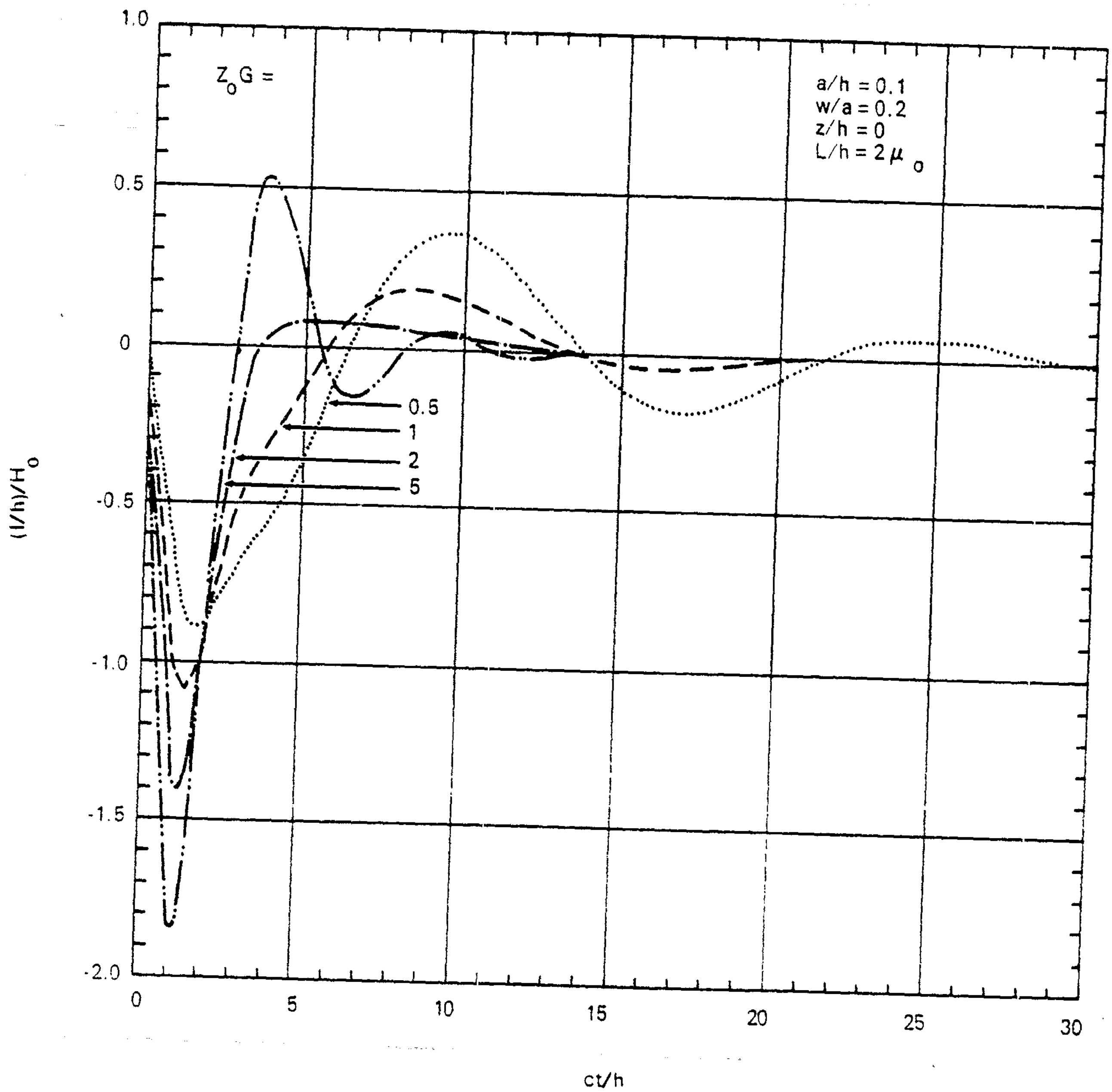

Figure 12. Current on a Loaded Cylinder versus ct/h

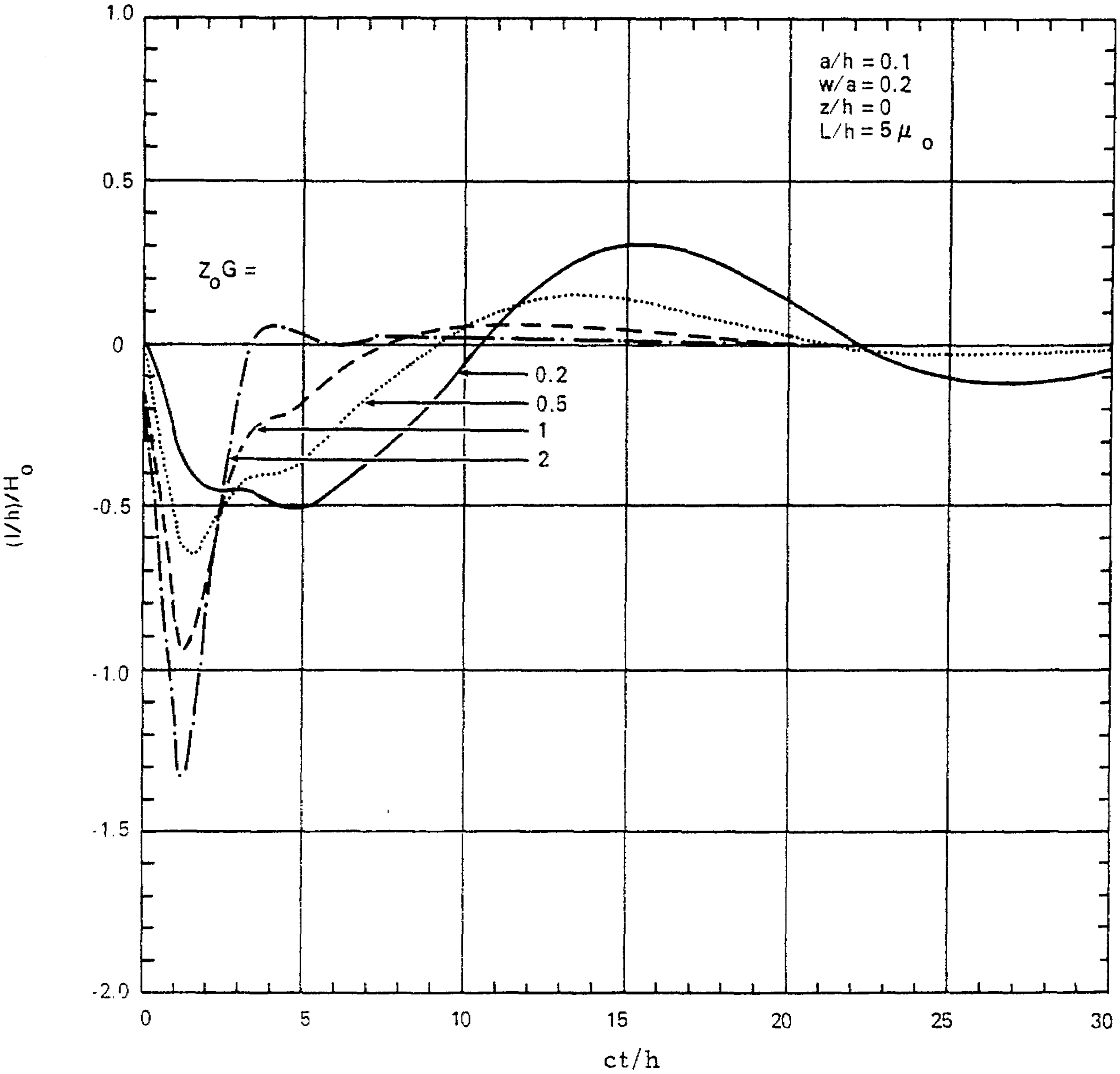

Figure 13. Current on a Loaded Cylinder versus ct/h

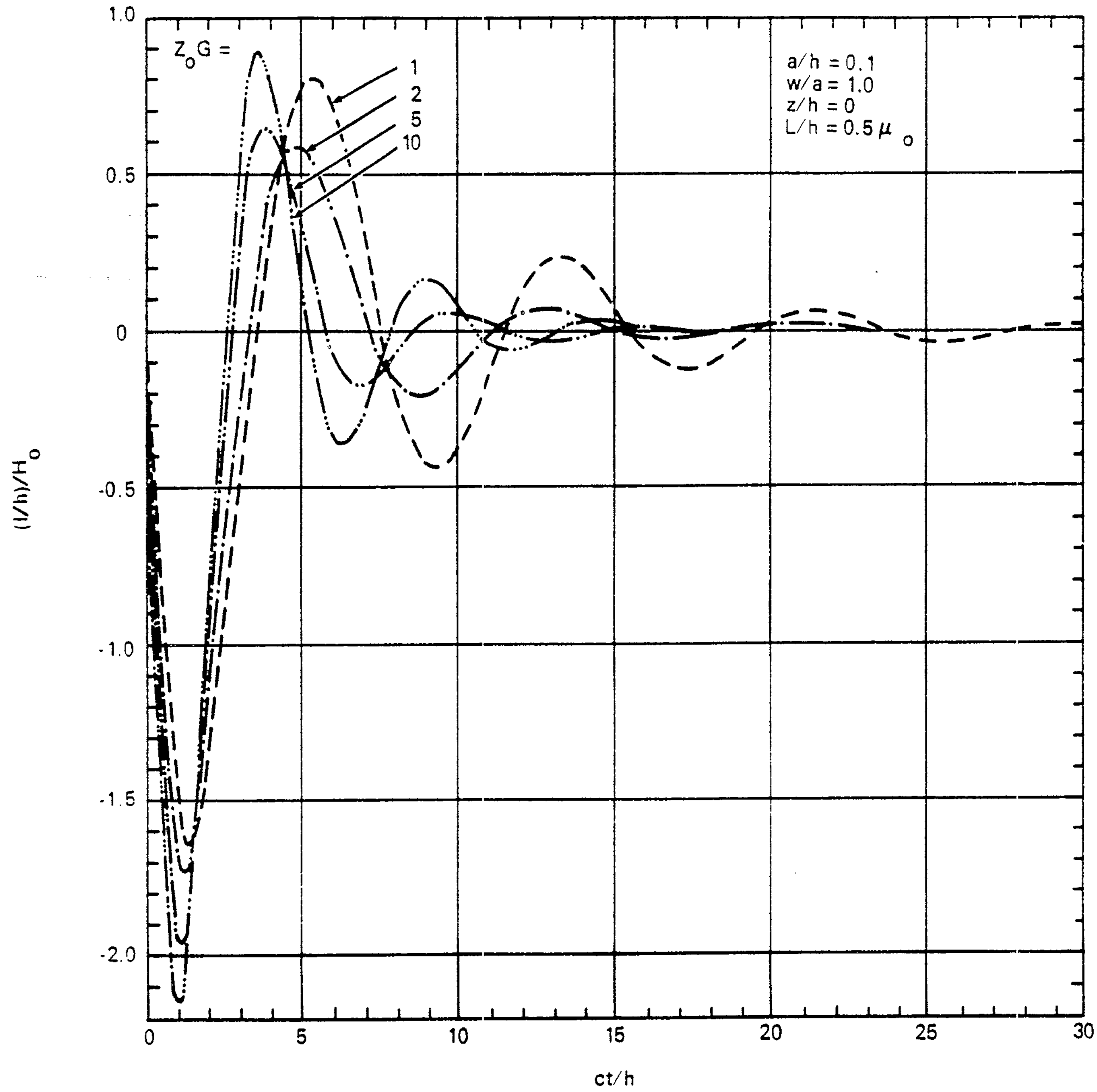

Figure 14. Current on a Loaded Cylinder versus ct/h

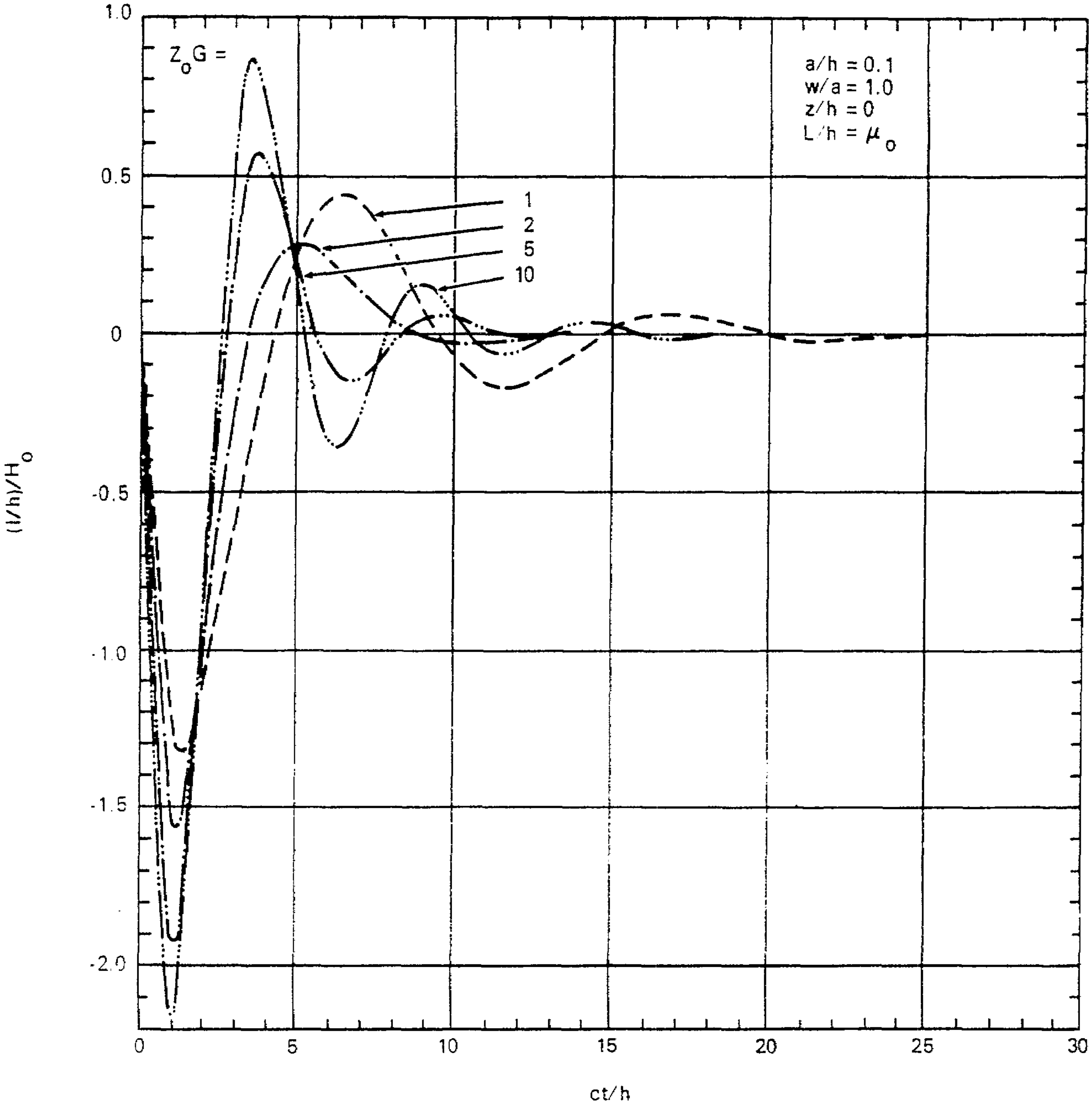

Figure 15. Current on a Loaded Cylinder versus ct/h

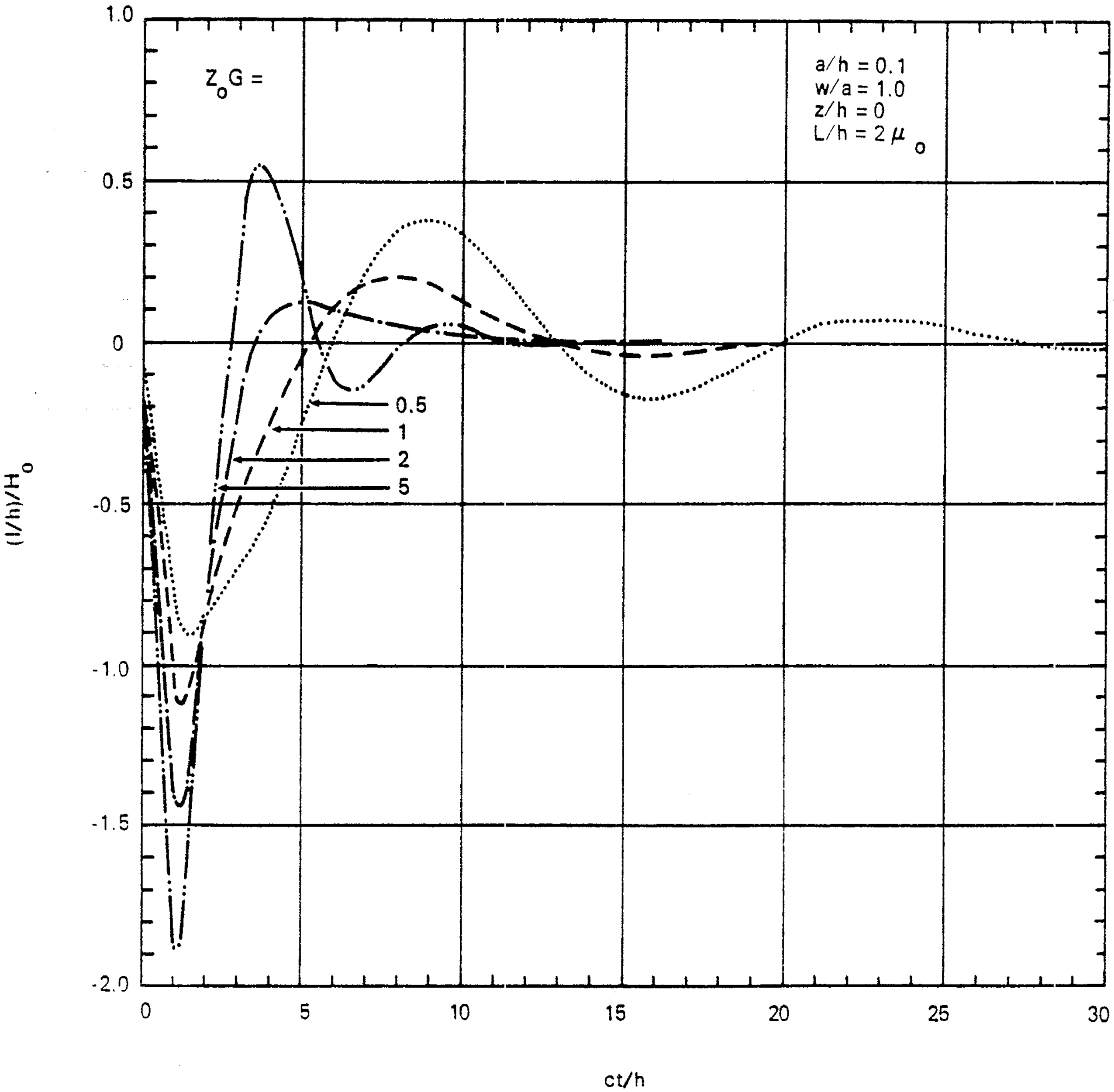

Figure 16. Current on a Loaded Cylinder versus ct/h

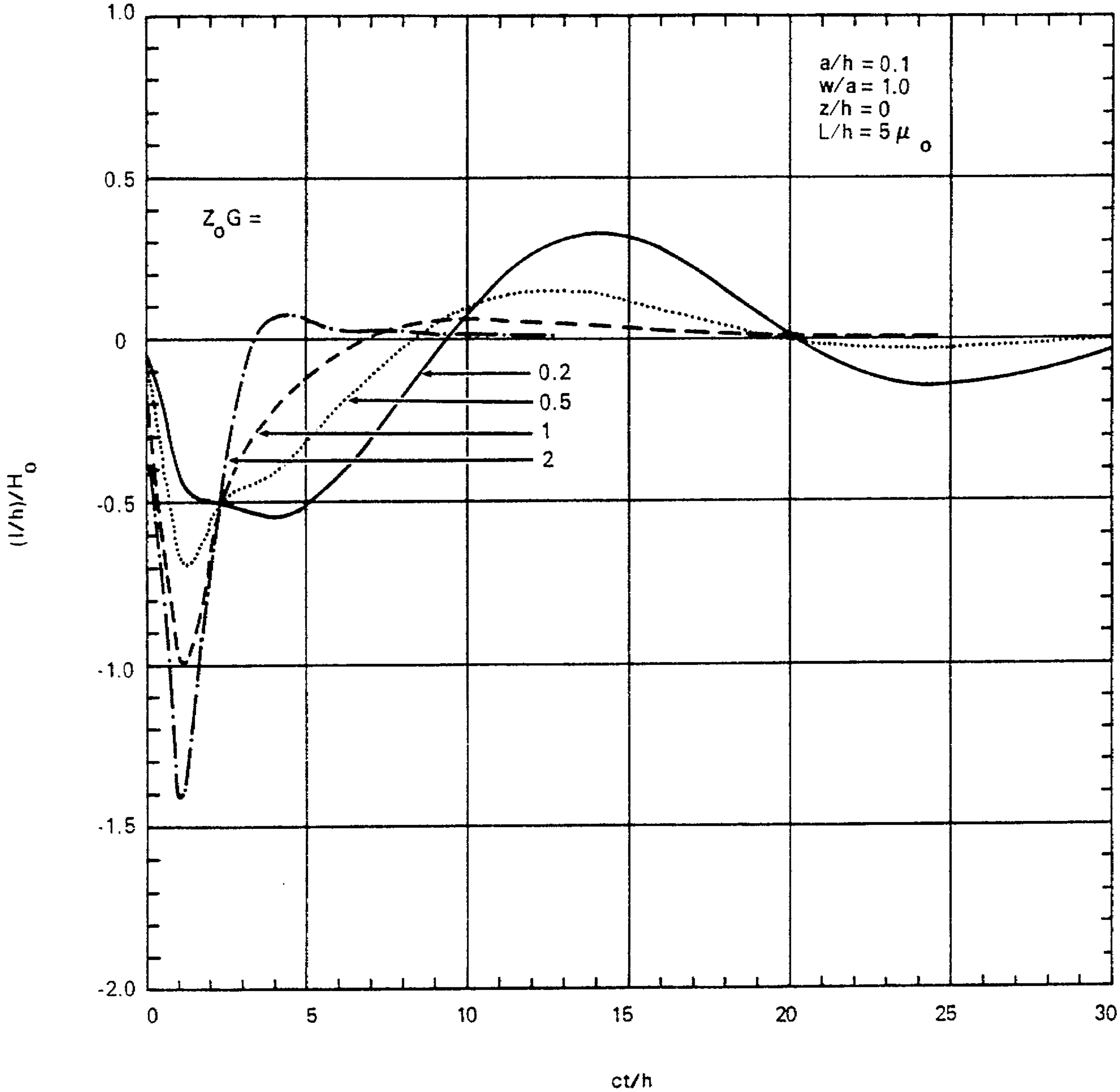

Figure 17. Current on a Loaded Cylinder versus ct/h

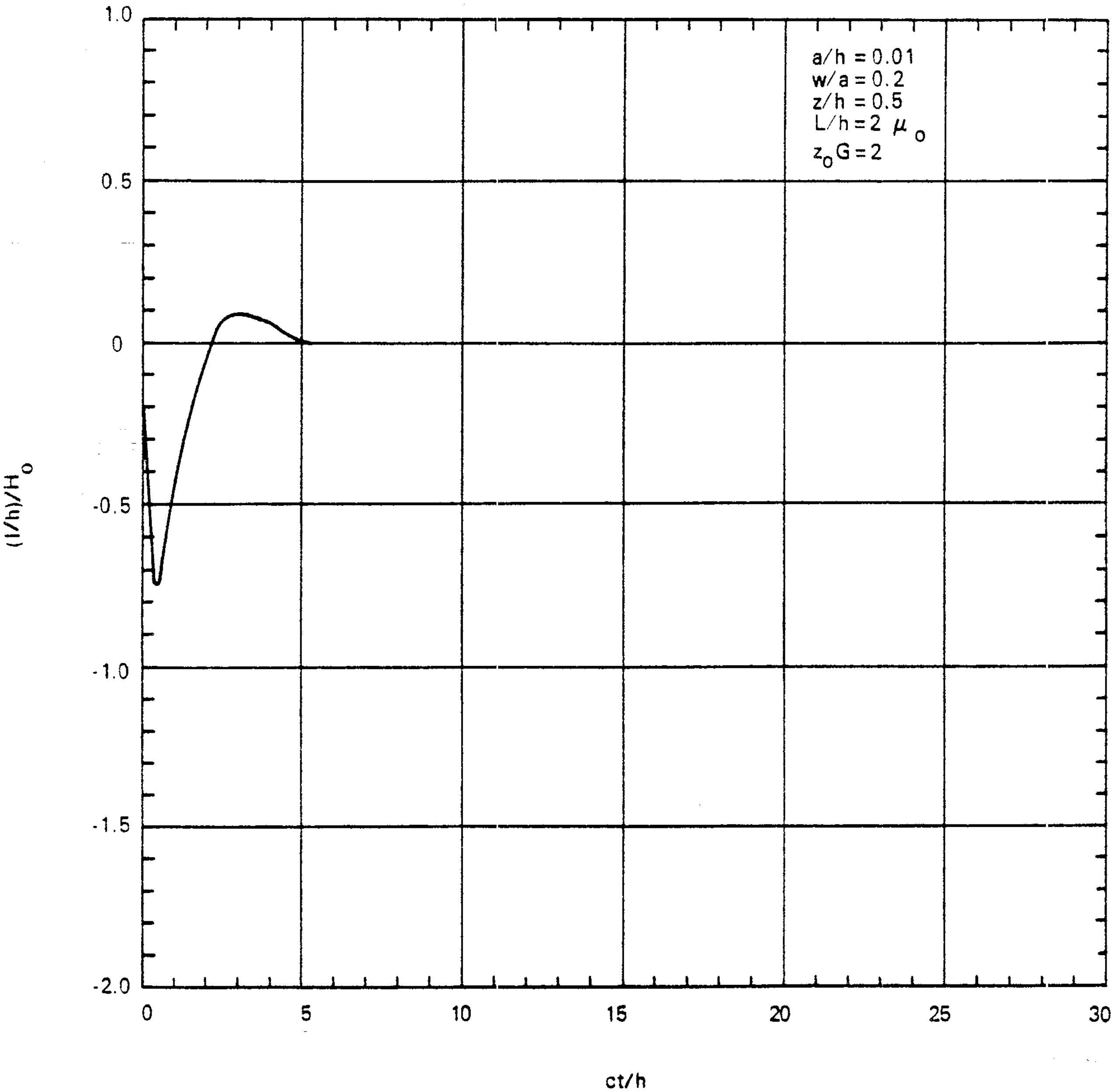

Figure 18. Current on a Loaded Cylinder versus ct/h

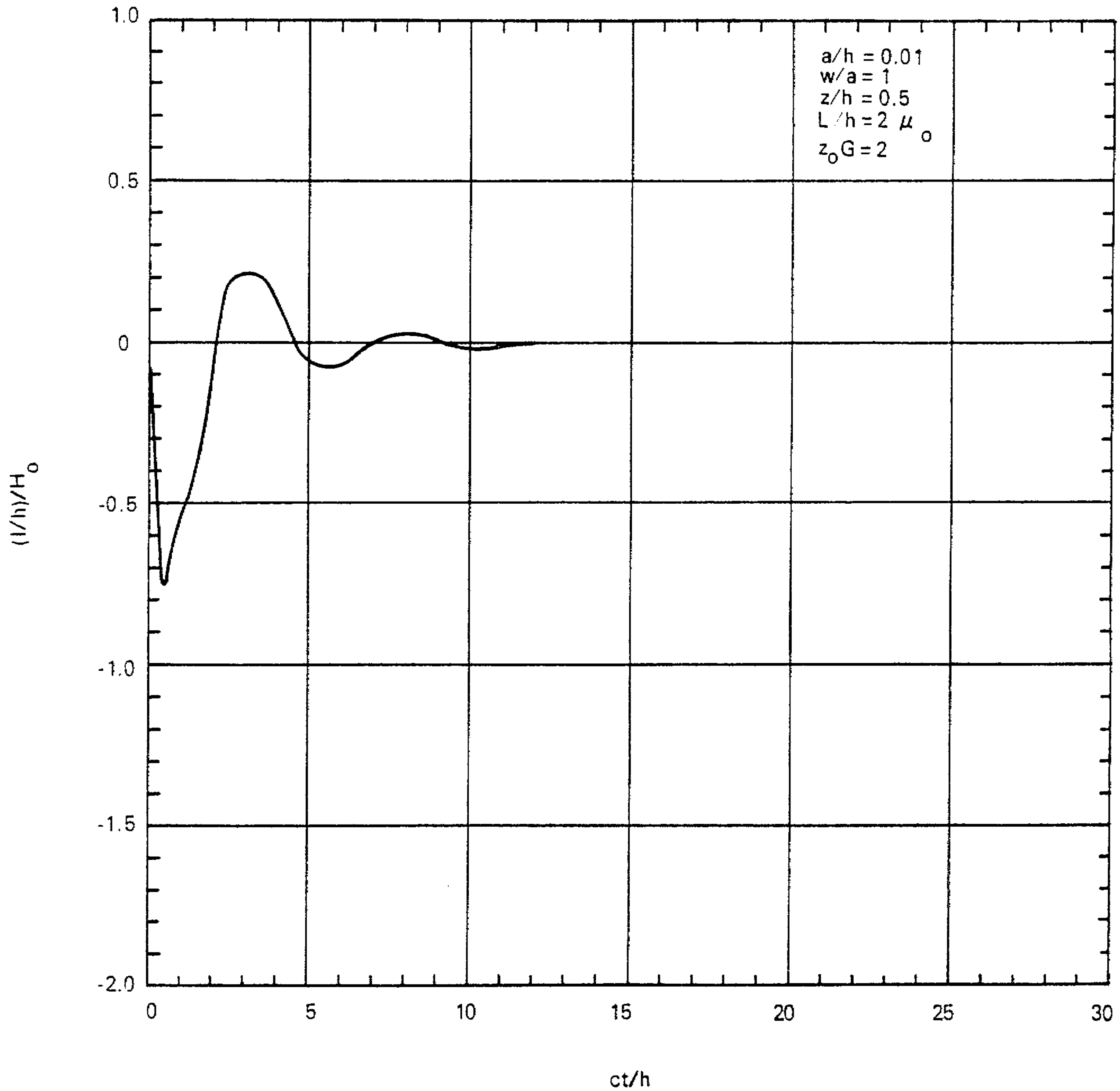

Figure 19. Current on a Loaded Cylinder versus ct/h

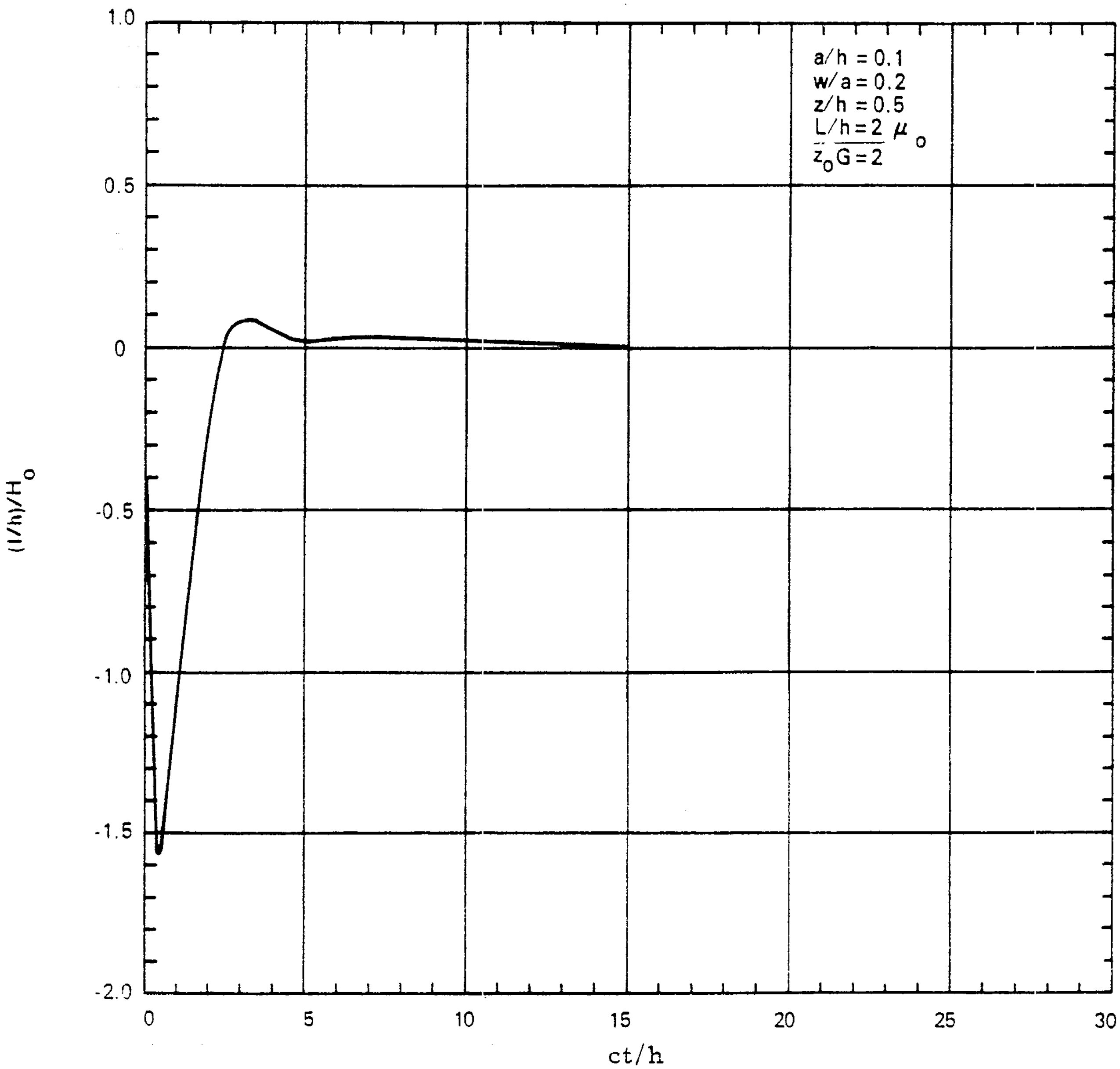

Figure 20. Current on a Loaded Cylinder versus ct/h

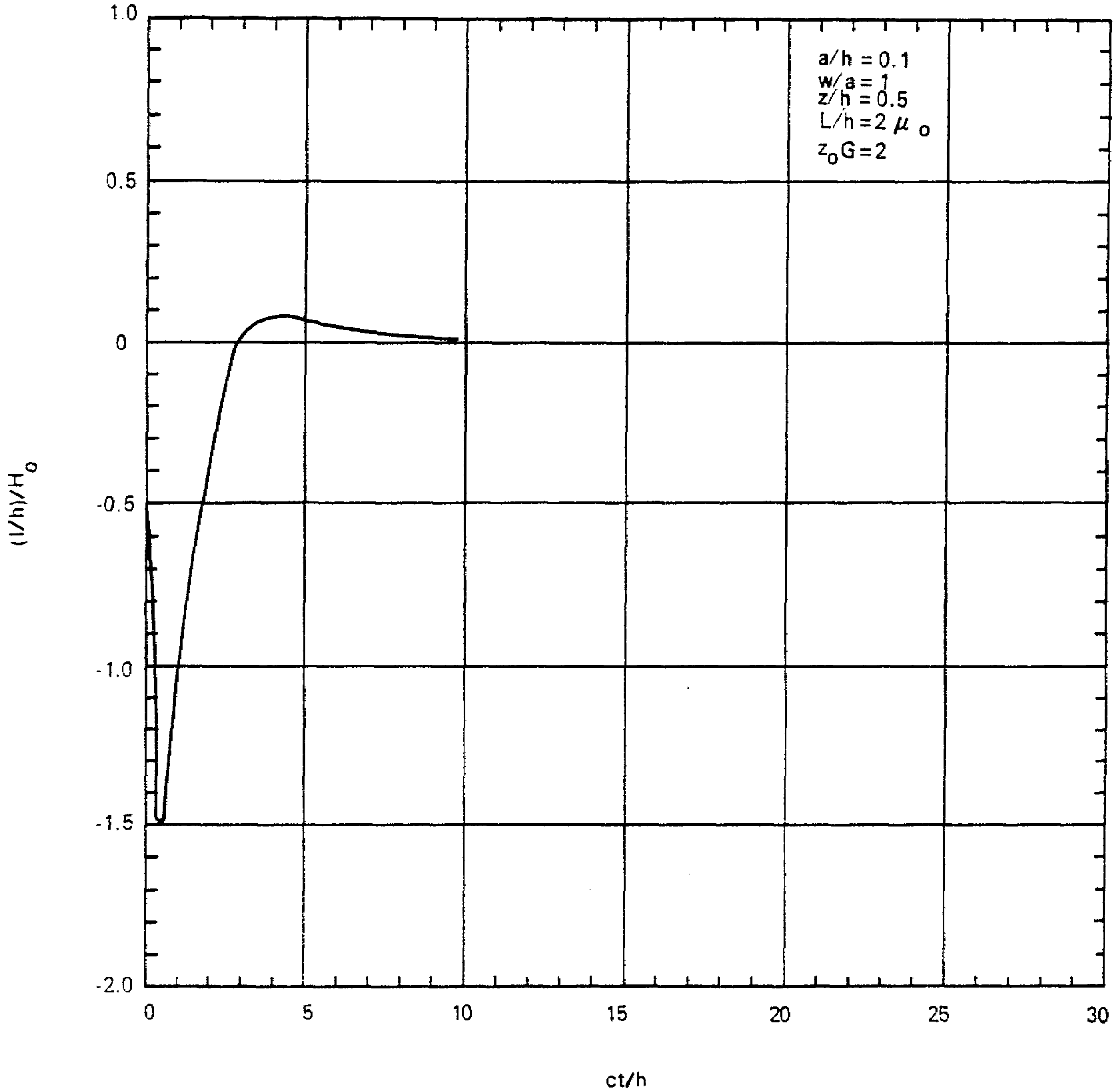

Figure 21. Current on a Loaded Cylinder versus ct/h

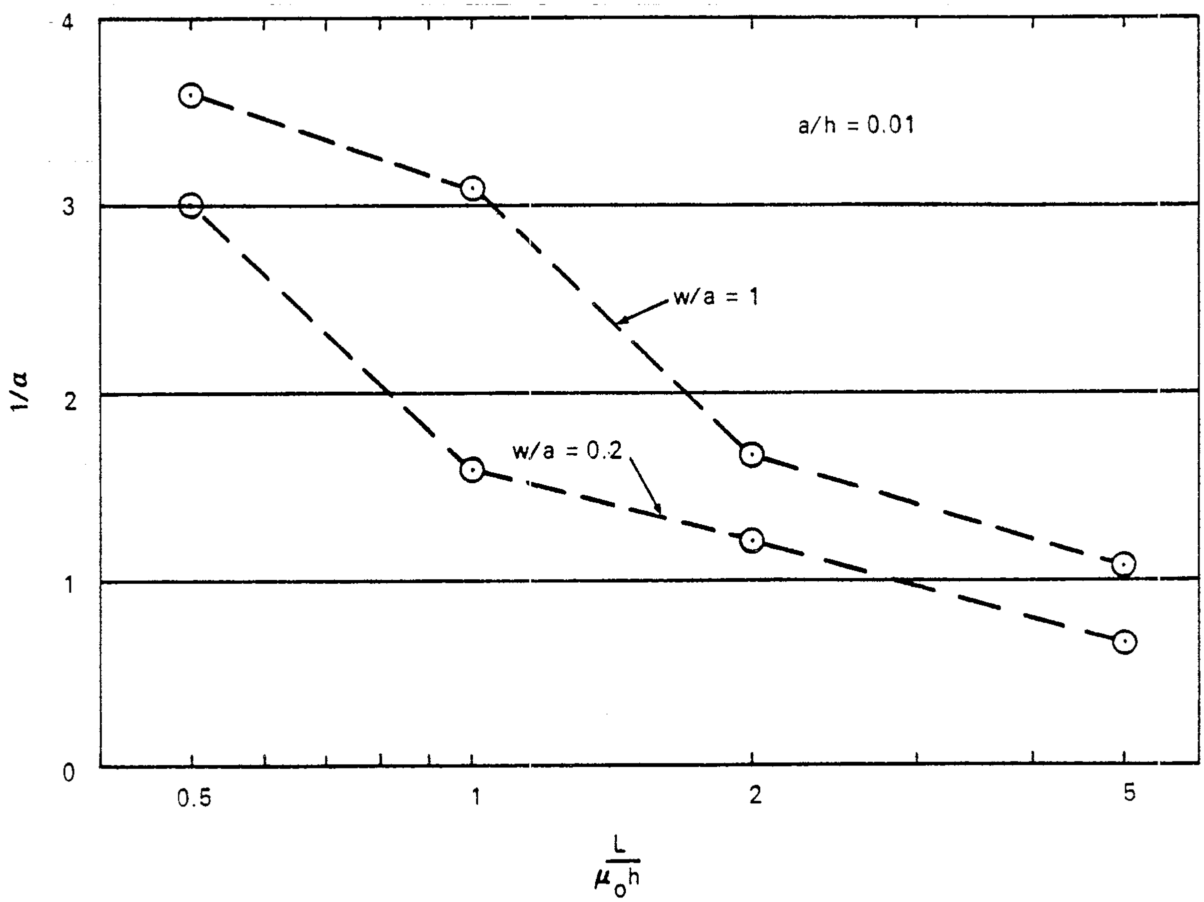

Figure 22. Decay Time Constant versus Load Inductance

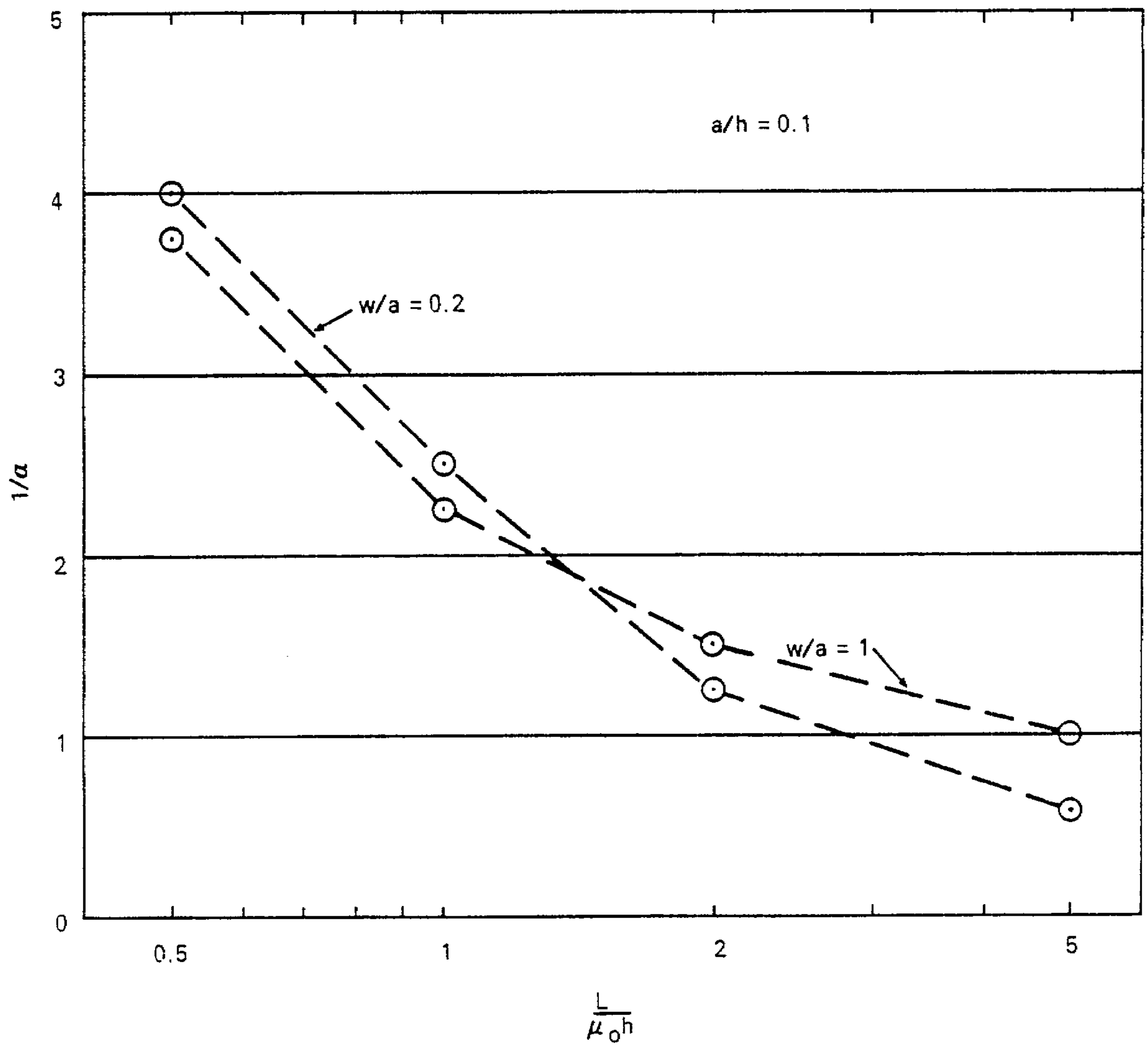

Figure 23. Decay Time Constant versus Load Inductance

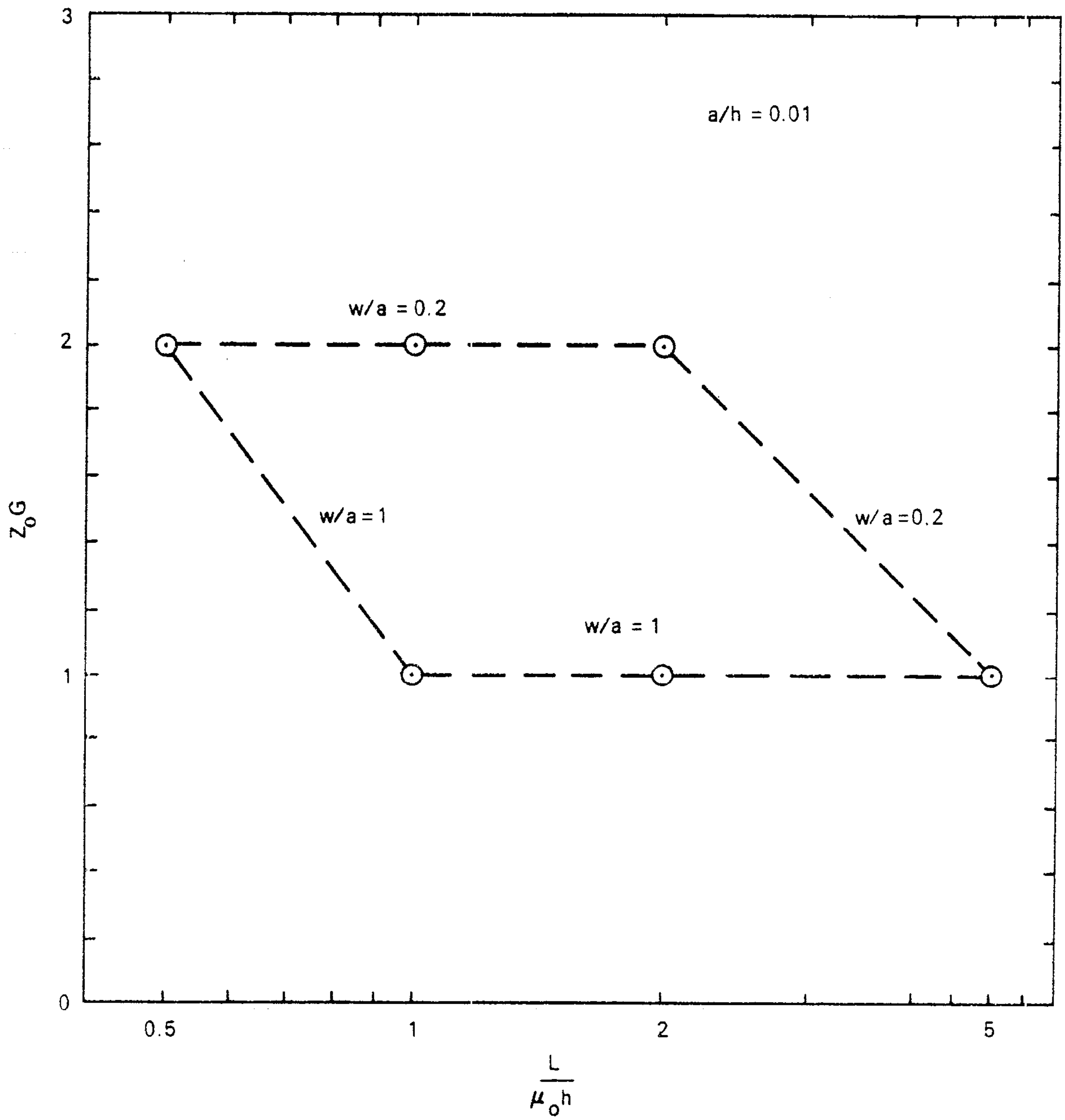

Figure 24. Optimum Load Conductance versus Load Inductance

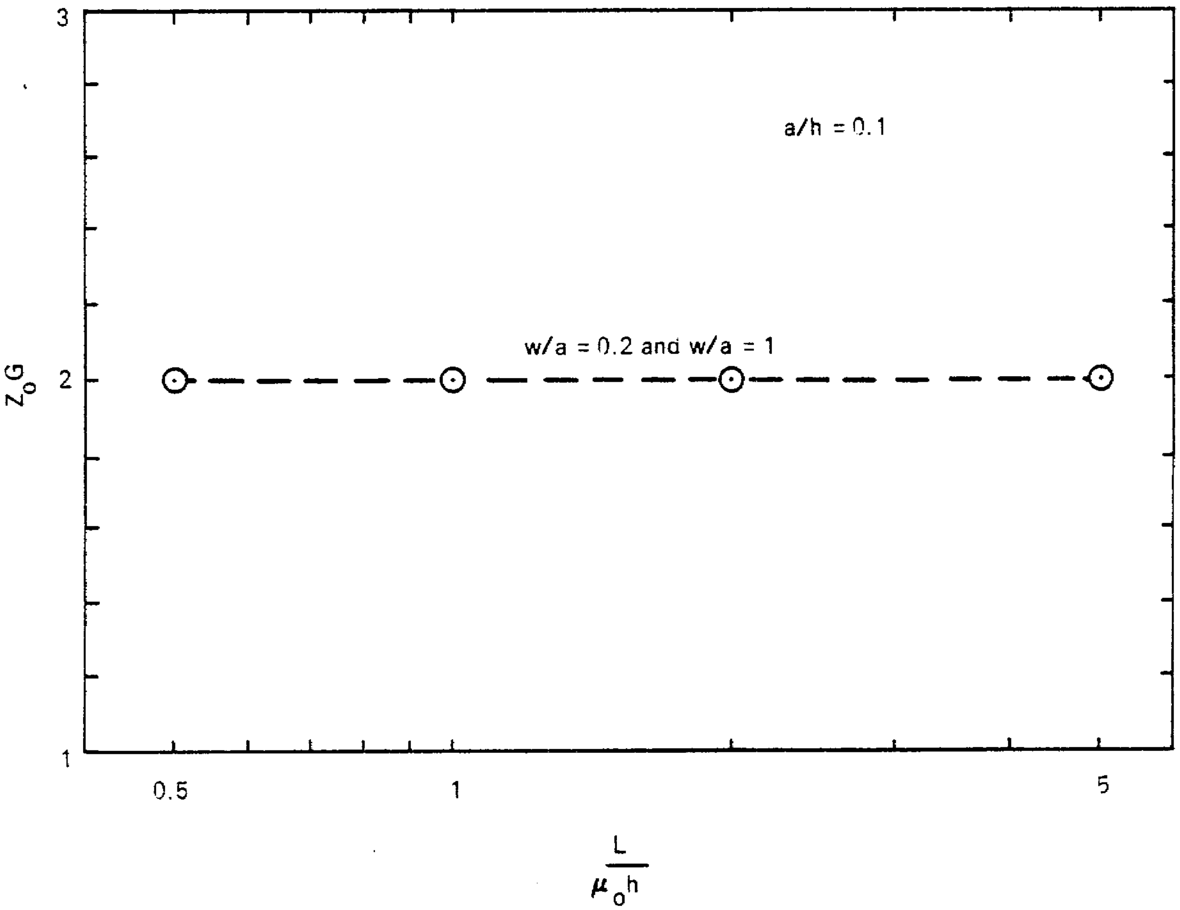

Figure 25. Optimum Load Conductance versus Load Inductance

References

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