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Design of a Pulse-Radiating Dipole Antenna as Related to High-Frequency and Low-Frequency Limits

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Abstract

In this note we consider some of the design parameters for a pulse-radiating dipole antenna as related to the high-frequency and low-frequency content of the radiated waveform. For the high-frequency portion the use of a biconical wave launcher is discussed. The low-frequency content depends on such parameters as generator voltage and capacitance and the capacitance and mean charge separation distance of the antenna. The mean charge separation distance of the antenna is shown to be the same as its equivalent height. Finally, a technique is discussed for achieving a large antenna capacitance by making the dipole structure have an electrically-large radius while still maintaining a sparse mechanical structure. This technique involves the use of wires (or strips, etc.) to approximate a conducting circular cylinder.

I. Introduction

In a recent note[1] we have discussed the low frequency behavior of a pulse-radiating dipole antenna and showed the importance of the late-time dipole moment to both the radiated waveform and the low-frequency portion of its Fourier transform. In another note[2] we have discussed the early-time characteristics of the radiated waveform from a conical structure which can be used as part of such a pulse-radiating antenna; these early-time characteristics are related to the high-frequency portion of the Fourier transform of the waveform. The coefficients of these high-frequency and low-frequency asymptotic forms are functions of certain parameters of the antenna geometry. These coefficients can be optimized by appropriate choice of these geometric parameters.

In the present note we discuss some of the antenna parameters related to the high-frequency and low-frequency asymptotes. Particular emphasis is given to those parameters related to the low-frequency asymptote; the low-frequency characteristics of a pulse-radiating antenna impose a significant limitation on its performance. The important low-frequency parameters are the mean charge separation distance and capacitance of the antenna. The mean charge separation distance is shown to be the same as the equivalent height of the antenna when used as an electric field sensor. Related to the antenna capacitance we discuss the characteristics of a cylindrical wire array which might be used in place of a continuous circular cylinder for part of the antenna structure.

II. High-Frequency Characteristics

For purposes of the present discussion of the high-frequency characteristics of dipole antennas we assume that there is a generator which can be represented as a single capacitor charged to a voltage V_o which is switched onto a symmetrical biconical wave launcher with angles θ_o and $\pi - \theta_o$ as illustrated in figure 1. The remainder of the antenna, beyond the two cones, is assumed to lie within the two regions $0 \leq \theta \leq \theta_o$ and $\pi - \theta_o \leq \theta \leq \pi$; the generator is assumed to be contained inside the antenna structure and driving the biconical structure at the common apex of the two cones. Spherical (r,θ,ϕ) and cylindrical (Ψ,ϕ,z) coordinates are also illustrated in figure 1.

For the present discussion we ignore the nonzero risetime of a real generator and assume that the capacitive generator places a pulse with a step rise of value V_o on the biconic structure introduced at the apex at $t = 0$ where t is time.[3] Then the voltage at the apex for early times (with the upper cone by convention positive) for $\theta_o < \theta < \pi - \theta_o$ is given by

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1. Capt Carl E. Baum, Sensor and Simulation Note 65, Some Limiting Low-Frequency Characteristics of a Pulse-Radiating Antenna, October 1968.
2. Capt Carl E. Baum, Sensor and Simulation Note 36, A Circular Conical Antenna Simulator, March 1967.
3. All units are rationalized MKSA.

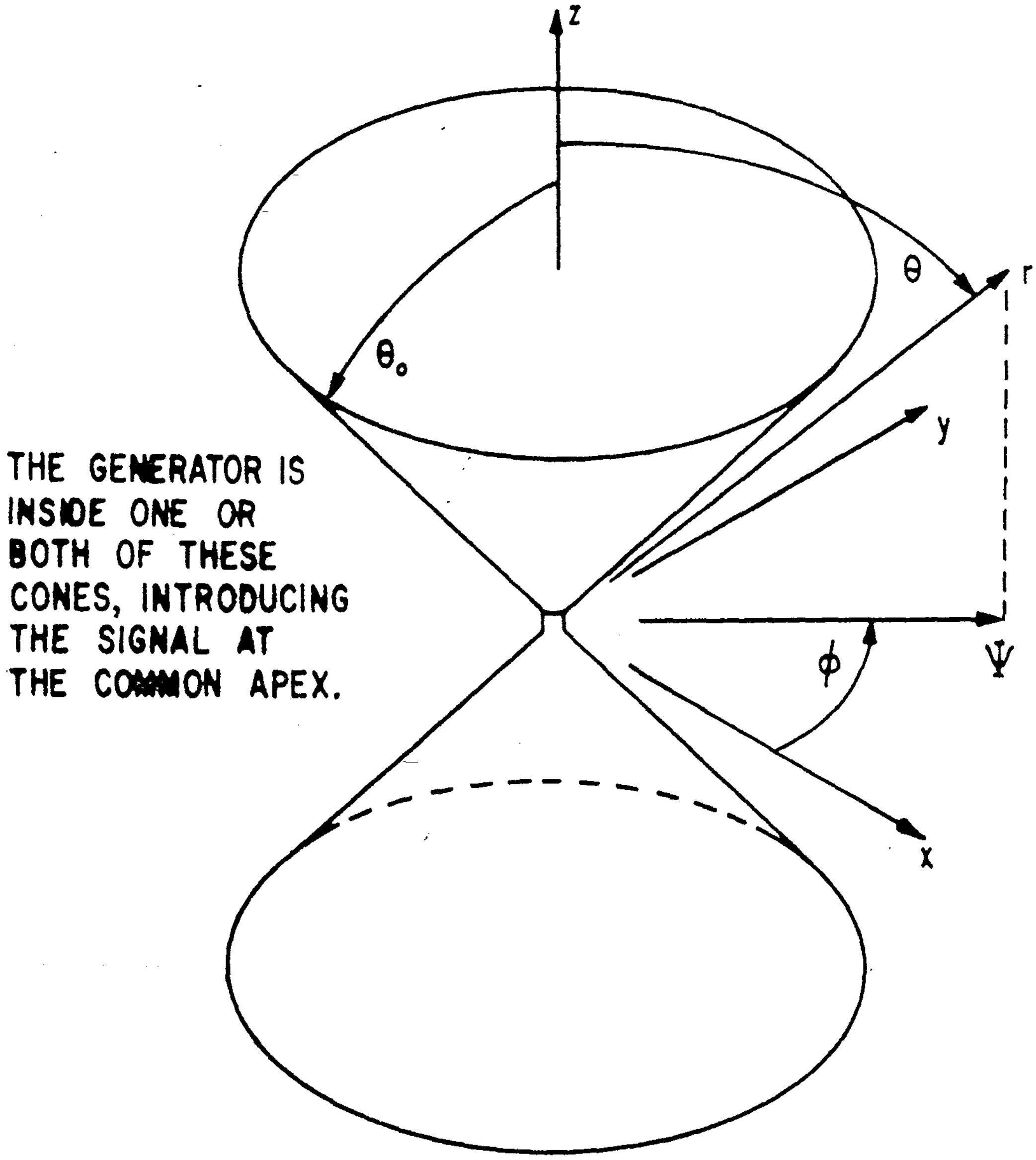

FIGURE 1. BICONICAL HIGH-FREQUENCY WAVE LAUNCHER

$$V_a(t) = V_o e^{-\frac{t}{Z_b C_g}} u(t) \tag{1}$$

where C_g is the generator capacitance and Z_b is the pulse impedance of the biconical section given by

$$Z_b = \frac{1}{\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \ln\left[\cot\left(\frac{\theta_o}{2}\right)\right] \simeq 120\ \ln\left[\cot\left(\frac{\theta_o}{2}\right)\right] \tag{2}$$

where we have assumed that the parameters of the medium are the same as in free space. Note that the impedance and field amplitudes differ by factors of 2 from those in reference 2 because here we are concerned with a biconical structure instead of a single cone over a ground plane. The electric field for early retarded times for $\theta_o < \theta < \pi - \theta_o$ has only a θ component given by

$$E_\theta(\vec{r},t) = \frac{1}{r} V_a(t - \frac{r}{c}) \left\{ 2\sin(\theta)\ \ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]\right\}^{-1} \tag{3}$$

As in reference 1 we define a retarded time as

$$t^* \equiv t - \frac{r}{c} \tag{4}$$

and a normalized radiation field, at large r, as

$$\xi(\theta,t^*) \equiv \frac{r\,E_\theta(\vec{r},t)}{V_o} \tag{5}$$

For the present early-time results for $\theta_o < \theta < \pi - \theta_o$ this gives

$$\xi(\theta,t^*) = \frac{V_a(t^*)}{V_o}\ f_o(\theta) \tag{6}$$

where we have defined

$$f_o(\theta) \equiv \left\{ 2\sin(\theta)\ \ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]\right\}^{-1} \tag{7}$$

This is precisely 1/2 of the parameter f_E used in reference 2. Note for these early-time results in the limited range $\theta_o < \theta < \pi - \theta_o$ that the fields near the apex of the cones and the radiated fields far from the antenna have the same form and $\xi(\theta,t^*)$ applies to both. For each observation position in the range $\theta_o < \theta < \pi - \theta_o$ these results for ξ apply until such time that diffraction due to the end of the conical structure can reach the observer.

The initial discontinuity in the normalized radiated waveform for $\theta_o < \theta < \pi - \theta_o$ is given by

$$\xi(\theta,0+) = f_o \tag{8}$$

Assuming that there are no further step discontinuities in the radiated waveform then the transform behaves as $|s| \to \infty$ asymptotically like

$$\tilde{\xi} \approx \frac{f_o}{s} \tag{9}$$

where the tilde is used to denote the Laplace, two-sided Laplace, or Fourier transform as appropriate and where s is the transform variable (taken as $j\omega$ in the Fourier case). Then on a frequency domain plot as $\omega \to \infty$ we have asymptotically

$$|\tilde{\xi}| \approx \frac{f_o}{\omega} \tag{10}$$

Thus, f_o is the coefficient for the high-frequency asymptote for $|\tilde{\xi}|$.

Now the high-frequency or early-time content of the radiated waveform at a fixed r is proportional to $V_o f_o$. Thus, one obvious way to increase the high-frequency content is to increase V_o, the initial voltage on the capacitor. A second way to increase the high-frequency content is to increase f_o by increasing θ_o toward $\pi/2$. However, θ_o should perhaps not be made so large that the change in geometry as each cone connects to the rest of the antenna introduces (at the appropriate time) a drastic decrease in ξ. Thus, θ_o might be better chosen such that the early-time form of ξ transitions smoothly into the later parts of the waveform which are influenced by the rest of the antenna including its geometry, distribution of resistive loading, etc. Another point of interest is that the generator capacitance should be large for reasons related to the low-frequency content of the radiated waveform (which will be discussed in the next section). Then with a sufficiently large C_g the early-time portion of the radiated waveform occurs in retarded times significantly less than $Z_b C_g$ so that in the early-time region the exponential decay of the antenna voltage (as in equation 1) and of the radiated waveform (as in equation 6) can be neglected. The waveform will later decay as time goes on due to the antenna characteristics beyond the cones.

III. Low-Frequency Characteristics

From reference 1 (equations 21 and 35) the low-frequency form of the radiation electric field or far electric field as $s \to 0$ is given asymptotically by

$$\vec{E}_f \approx \frac{\mu_o}{4\pi} \frac{s}{r} (\vec{p}(\infty) \times \vec{e}_r) \times \vec{e}_r \tag{11}$$

where $\vec{e}_r$ is the unit vector in the r direction (and similarly for other unit vectors) and where $\vec{p}(\infty)$ is the late-time dipole moment. As discussed in reference 1 one can let the dipole moment $\vec{p}(t)$ start from zero at $t = 0$ and go to some nonzero late-time value $\vec{p}(\infty)$ at the later times of interest. Eventually it must go to zero as the antenna is discharged but this time can be much larger than times of interest. Then $\vec{p}(\infty)$ is the interesting parameter for the low-frequency portion of the radiated waveform.

Referring to a dipole (such as illustrated in figure 2) which has two pieces joined through a single generator, we can define Q_∞ as the late-time charge on one section, say the upper section (z>0) for convenience. Then we can define a mean charge separation distance as

$$\vec{h}_a \equiv \frac{\vec{p}(\infty)}{Q_\infty} \tag{12}$$

This mean charge separation distance is a property of the antenna geometry including distribution of conductors, dielectrics, etc. Such a dipole also has a capacitance C_a, which combined with the generator capacitance C_g gives a late-time antenna voltage V_∞ as

$$V_\infty = V_o \frac{C_g}{C_a+C_g} \tag{13}$$

The late-time antenna charge Q_∞ is given by

$$Q_\infty = C_a V_\infty \tag{14}$$

Note that we have assumed that the generator can be represented as a simple capacitor initially charged to V_o which is switched into the dipole with the voltage settling to its nonzero late-time value V_∞. This ignores such elements as resistors which might be used to discharge the antenna and generator at times much larger than times of interest. Using the terms in equations 12 through 14 the far electric field as $s \to 0$ has the asymptotic form

$$\vec{E}_f \approx \frac{\mu_o}{4\pi} \frac{s}{r} \frac{V_o C_a C_g}{C_a+C_g} (\vec{h}_a \times \vec{e}_r) \times \vec{e}_r \tag{15}$$

For convenience we define

$$h_a \equiv |\vec{h}_a| \tag{16}$$

Then specializing the dipole to one with axial and vertical symmetry (as illustrated in figure 2) we have

$$\vec{h}_a = h_a \vec{e}_z \tag{17}$$

The far electric field has only a θ component given asymptotically as $s \to 0$ by

$$\vec{E}_{f_\theta} \approx -\frac{\mu_o}{4\pi} \frac{s}{r} \vec{p}(\infty) \cdot \vec{e}_\theta = \frac{\mu_o}{4\pi} \frac{s}{r} h_a Q_\infty \cos(\theta)$$

$$= \frac{\mu_o}{4\pi} \frac{s}{r} \frac{V_o C_a C_g}{C_a+C_g} h_a \cos(\theta) \tag{18}$$

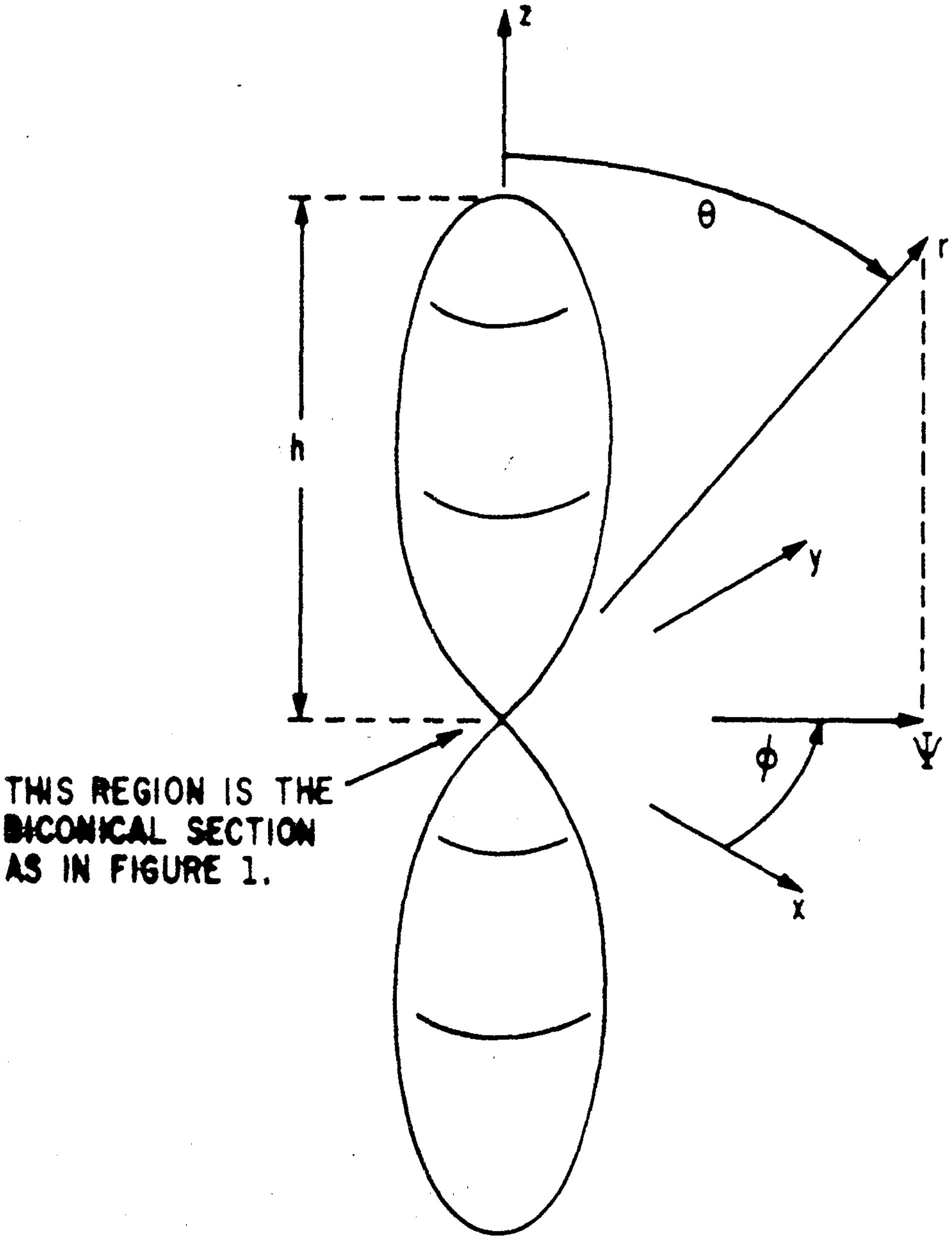

FIGURE 2. AXIALLY AND LENGTHWISE SYMMETRIC PULSED DIPOLE ANTENNA

The normalized radiated waveform is given asymptotically as $s \to 0$ by

$$\tilde{\xi} \equiv \frac{r}{V_o} \tilde{E}_{f_\theta} \approx \frac{\mu_o}{4\pi} \left[C_a^{-1}+C_g^{-1}\right]^{-1} h_a \cos(\theta)\ s$$

$$= \left(\frac{h}{c}\right)^2 f_\infty\ s = t_h^2 f_\infty\ s \qquad (19)$$

where the speed of light in vacuum is given by

$$c \equiv \frac{1}{\sqrt{\mu_o \varepsilon_o}} \qquad (20)$$

and where h is the half length of the dipole structure as illustrated in figure 2. We have defined a convenient time based on the antenna half length as

$$t_h \equiv \frac{h}{c} \qquad (21)$$

and we have defined a dimensionless parameter as

$$f_\infty(\theta) \equiv \frac{\mu_o}{4\pi} \left(\frac{c}{h}\right)^2 h_a \left[C_a^{-1}+C_g^{-1}\right]^{-1} \cos(\theta)$$

$$= \frac{1}{4\pi\varepsilon_o} \frac{h_a}{h^2} \left[C_a^{-1}+C_g^{-1}\right]^{-1} \cos(\theta)$$

$$= \frac{1}{4\pi} \frac{h_a}{h} \left[\frac{\varepsilon_o h}{C_a} + \frac{\varepsilon_o h}{C_g}\right]^{-1} \cos(\theta) \qquad (22)$$

Now the low-frequency content of the radiated waveform at a fixed r is proportional to $V_o f_\infty h^2$. Keeping f_∞ fixed for the moment note that if we increase V_o and h each by, say, a factor of two then the low-frequency content is increased by a factor of eight, almost an order of magnitude. Note that h is particularly important, appearing as h^2. Now consider f_∞ which is an efficiency factor for any particular antenna-generator design for fixed V_o and h. First, f_∞ can be increased by increasing the generator capacitance C_g, but as C_g is increased to the condition $C_g >> C_a$ there is little more gained in increasing f_∞ by further increase in C_g. Second, f_∞ can be increased by increasing the mean charge separation distance h_a. Now we have $h_a \leq 2h$ and typically h_a may be of the order of h or a little less. Thus, h_a/h is one indicator of the efficiency of use of the antenna half length. Third, one can try to increase the antenna capacitance C_a; $C_a/(\epsilon_o h)$ is another indicator of the efficiency of use of the antenna half length. However, for fixed h, increasing C_a may decrease h_a and conversely, depending on the technique(s) used to increase one of these parameters. For the case that $C_g >> C_a$ we define another parameter as

$$f'_\infty \equiv \frac{1}{4\pi} \frac{h_a}{h} \frac{C_a}{\epsilon_o h} \tag{23}$$

so that

$$f_\infty \simeq f'_\infty \sin(\theta) \tag{24}$$

In f'_∞ we have the product $h_a C_a$ which one would like to maximize for a fixed h. One way to do this is to give the dipole a larger average radius (Ψ in figure 2) for its conductors. Of course, the electromagnetic advantages of large radii will have to be compared with the mechanical difficulties of achieving large radii for long antennas. There is also the trade-off problem concerned with how large C_g should be as compared to C_a because of the various other electrical and mechanical problems associated with increasing the generator capacitance at a fixed V_o.

As one can see there are several electromagnetic parameters of the generator and antenna which determine the low-frequency content of the radiated waveform. In general one should try to maximize $V_o f_\infty h^2$ while taking account of the various other important problems such as those related to the high-voltage generator and the mechanical (structural and weight) problems related to the entire pulsed-antenna system and considering such things as means of support or lift and antenna orientation.

IV. Mean Charge Separation Distance and Equivalent Height

In section III we introduced a mean charge separation distance $\vec{h}_a$ as one of the important low-frequency parameters for the design of a pulsed dipole antenna. Another parameter used in connection with dipole antennas is $\vec{h}_{eq}$, the equivalent height. Instead of driving the antenna with a generator, consider the open-circuit voltage V_{oc} at the antenna terminals which is produced by an incident plane wave with electric field $\vec{E}_{inc}$. For frequencies low enough that wavelengths are much larger than the antenna dimensions such that the problem becomes an electrostatic one then the equivalent height relates the voltage and electric field as $s \to 0$ asymptotically by

$$\tilde{V}_{oc} \approx - \tilde{\vec{E}}_{inc} \cdot \vec{h}_{eq} \tag{25}$$

This can be considered the defining relation for $\vec{h}_{eq}$. Note that, for generality, $\vec{h}_{eq}$ is a vector because the dipole is sensitive at low frequencies to the component of the electric field in a particular direction.

As will be shown $\vec{h}_{eq}$ and $\vec{h}_a$ are equal. This is a particular result related to the more general question of the reciprocity of an antenna in transmission and reception[4]. In order to do this consider two dipole antennas as illustrated in figure 3 which we call antenna 1 and antenna 2. The unit vector $\vec{e}_r$ is taken to point from antenna 1 to antenna 2 and the distance r between the antennas is assumed to be very large compared to the dimensions of the antennas. For later use we construct two unit vectors, $\vec{e}_1$ and $\vec{e}_2$, at antenna 1, both perpendicular to $\vec{e}_r$ and to each other and having a relative orientation such that $\vec{e}_1 \times \vec{e}_2 = \vec{e}_r$. All the asymptotic expressions are for $s \to 0$ and $r \to \infty$.

The reciprocity relation which we use is that if a given current I is driven into the terminals of antenna 1 and an open circuit voltage V_{oc} appears at the terminals of antenna 2, then if I is driven into the terminals of antenna 2 the same open circuit voltage V_{oc} appears at the terminals of antenna 1. Now the electric field incident on antenna 2 when antenna 1 is driven is given, using the results of reference 1, by

$$\tilde{\vec{E}}_{inc_2} \approx \frac{\mu_o}{4\pi} \frac{s^2}{r} (\tilde{\vec{p}}_1 \times \vec{e}_r) \times \vec{e}_r \tag{26}$$

where we have the dipole moment of antenna 1 given by

$$\tilde{\vec{p}}_1 \approx \vec{h}_{a_1} \tilde{Q}_{a_1} = \vec{h}_{a_1} \frac{\tilde{I}}{s} \tag{27}$$

Since we have $r \to \infty$ then $\vec{E}_{inc_2}$ is a plane wave in the vicinity of antenna 2 so that the open circuit voltage at antenna 2 is

$$\tilde{V}_{oc} \approx - \tilde{\vec{E}}_{inc_2} \cdot \vec{h}_{eq} \approx - \frac{\mu_o}{4\pi} \frac{s\tilde{I}}{r} [(\vec{h}_{a_1} \times \vec{e}_r) \times \vec{e}_r] \cdot \vec{h}_{eq_2} \tag{28}$$

Similarly driving antenna 2 with $\tilde{I}$ gives the open circuit voltage at antenna 1 as

$$\tilde{V}_{oc} \approx - \tilde{\vec{E}}_{inc_1} \cdot \vec{h}_{eq_1} \approx - \frac{\mu_o}{4\pi} \frac{s\tilde{I}}{r} [(\vec{h}_{a_2} \times (-\vec{e}_r)) \times (-\vec{e}_r)] \cdot \vec{h}_{eq_1} \tag{29}$$

4. S.A. Schelkunoff and H. T. Friis, Antennas: Theory and Practice, Wiley, 1952, chapter 9.

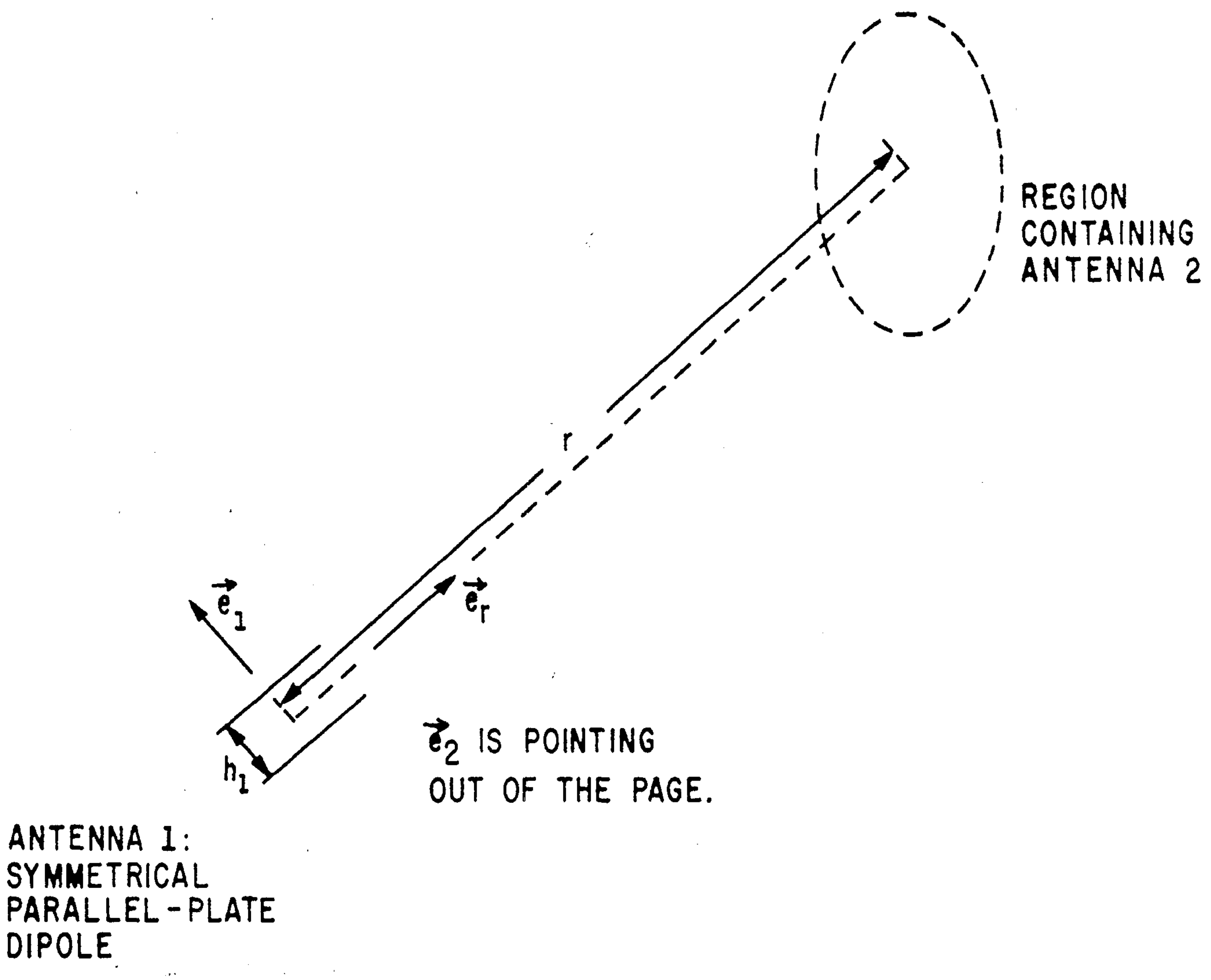

FIGURE 3. TWO DIPOLE ANTENNAS WITH COORDINATES

where we have used $-\vec{e}_r$ as the unit vector pointing from antenna 2 to antenna 1. Equating the results of equations 28 and 29 we have the interesting result

$$[(\vec{h}_{a_1} \times \vec{e}_r) \times \vec{e}_r] \cdot \vec{h}_{eq_2} = [(\vec{h}_{a_2} \times \vec{e}_r) \times \vec{e}_r] \cdot \vec{h}_{eq_1} \tag{30}$$

Using the scalar triple product relationships we have

$$[(\vec{h}_{a_2} \times \vec{e}_r) \times \vec{e}_r] \cdot \vec{h}_{eq_1} = (\vec{h}_{a_2} \times \vec{e}_r) \cdot (\vec{e}_r \times \vec{h}_{eq_1})$$

$$= (\vec{h}_{eq_1} \times \vec{e}_r) \cdot (\vec{e}_r \times \vec{h}_{a_2})$$

$$= [(\vec{h}_{eq_1} \times \vec{e}_r) \times \vec{e}_r] \cdot \vec{h}_{a_2} \tag{31}$$

which we substitute in equation 30 to give

$$[(\vec{h}_{a_1} \times \vec{e}_r) \times \vec{e}_r] \cdot \vec{h}_{eq_2} = [(\vec{h}_{eq_1} \times \vec{e}_r) \times \vec{e}_r] \cdot \vec{h}_{a_2} \tag{32}$$

Now we specialize antenna 1 while we keep antenna 2 as a general dipole antenna. As illustrated in figure 3 make antenna 1 a symmetrical parallel-plate dipole with plate spacing h_1. Antenna 1 is also made axially symmetric with respect to an axis parallel to $\vec{e}_1$. Thus $\vec{h}_{a_1}$ and $\vec{h}_{eq_1}$ are both parallel to this axis and only have components in the $\vec{e}_1$ direction. Now we have chosen antenna 1 as a parallel-plate dipole with plate separation h_1 so that we can exactly calculate both the mean charge separation distance and the equivalent height. Specifically we have

$$\vec{h}_{a_1} = h_1 \vec{e}_1 \tag{33}$$

because all the charge is on the plates with spacing h_1 in the $\vec{e}_1$ direction over the entire plate areas. By convention we take I positive into the plate farthest in the $\vec{e}_1$ direction and measure V_{oc} with a positive convention for this same plate. We also have

$$\vec{h}_{eq_1} = h_1 \vec{e}_1 \tag{34}$$

because the component of the electric field in the $\vec{e}_1$ direction (incident on antenna 1) is not distorted by the antenna conductors, none of which are extended parallel to this $\vec{e}_1$ direction. Thus, we have for the ideal parallel-plate dipole the result

$$\vec{h}_{a_1} = \vec{h}_{eq_1} \tag{35}$$

One should note that this ideal parallel-plate dipole can be approached arbitrarily closely, even including the terminals for source and measurement connections, by making the ratio of h_1 to the plate radius arbitrarily small.

With the result of equations 33 and 34 we can look at the implications for the arbitrary dipole antenna, namely antenna 2. Using the relationships among the orthogonal unit vectors equation 32 becomes

$$[-h_1\vec{e}_2 \times \vec{e}_r] \cdot \vec{h}_{eq_2} = [-h_1\vec{e}_2 \times \vec{e}_r] \cdot \vec{h}_{a_2} \qquad (36)$$

or

$$-h_1\vec{e}_1 \cdot \vec{h}_{eq_2} = -h_1\vec{e}_1 \cdot \vec{h}_{a_2} \qquad (37)$$

or

$$\vec{e}_1 \cdot \vec{h}_{eq_2} = \vec{e}_1 \cdot \vec{h}_{a_2} \qquad (38)$$

Now keeping antenna 2 fixed in space note that the direction to antenna 1 from antenna 2 can be arbitrarily chosen. Thus, $\vec{e}_r$ can take on any direction; this allows $\vec{e}_1$ to take on any direction. Since $\vec{e}_1$ is then arbitrary we conclude

$$\vec{h}_{eq_2} = \vec{h}_{a_2} \qquad (39)$$

Since antenna 2 is an arbitrary antenna then we have for all such dipole antennas

$$\vec{h}_{eq} = \vec{h}_a \qquad (40)$$

so that the mean charge separation distance and equivalent height are the same for all dipole antennas, whether axially symmetric or not and whether lengthwise symmetric or not.

For convenience we define

$$h_a \equiv |\vec{h}_a| \quad , \quad h_{eq} \equiv |\vec{h}_{eq}| \qquad (41)$$

Now, since $h_a = h_{eq}$, we can calculate either one and have the other. This result can then be used sometimes to simplify the calculation of h_a or h_{eq}, whichever one is being considered for a particular problem.

V. Increasing Antenna Capacitance

One way to improve the low-frequency performance of a pulse-radiating dipole is to increase the antenna capacitance for a given antenna half length while maintaining the mean charge separation distance roughly constant. As illustrated in figure 2 one might try to make C_a large by making the average cylindrical radius (with coordinate Ψ) of the antenna large.

Now for very long antennas, making the conductor surface have a large cylindrical radius may present significant mechanical problems including such things as structural, weight, and wind-loading problems. However, it is not necessary that the conductors (and/or resistors) at each position (z) along the antenna length be completely continuous around the antenna. Conductors can be sparsely distributed around the antenna and give an equivalent radius (as a conducting cylinder) not too much smaller than the actual antenna radius. Thus, we try to approximate a conducting cylinder by an array of wires approximately parallel to the antenna axis and uniformly spaced around the geometrical antenna surface which is roughly a circular cylinder. This approach may be particularly useful for a pulsed dipole for the case that the antenna is suspended vertically and the wires can be used in tension for vertical support with perhaps hoops to space the wires apart. Note that this technique is similar to the one in which a conducting plate is approximated by a wire grid.[5]

Assume that we have N equally charged wires, each of radius r_o, centered on the circumference of a circular cylinder of radius Ψ_1, and uniformly spaced around the circumference an angle of $2\pi/N$ apart. For this problem we use cartesian (x,y,z) and cylindrical (Ψ,ϕ,z) coordinates. We assume that the geometry and fields are independent of z. Define complex variables as

$$\zeta \equiv x + jy = \Psi e^{j\phi} \quad , \qquad w \equiv u + jv \tag{42}$$

Consider the conformal transformation

$$\zeta = \Psi_1 \left(e^{Nw} + 1\right)^{1/N}$$
$$w = \frac{1}{N} \ln\left[\left(\frac{\zeta}{\Psi_1}\right)^N - 1\right] \tag{43}$$

Now w is singular at the N points given by

$$\frac{\zeta}{\Psi_1} = 1^{1/N} = e^{j\frac{2\pi n}{N}} \quad \text{for } n = 0,1,2,\ldots, N-1 \tag{44}$$

These N points are taken as the wire centers. Expanding equations 43 in terms of u, v, Ψ, and ϕ we have

5. Lt Carl E. Baum, Sensor and Simulation Note 21, Impedances and Field Distributions for Parallel Plate Transmission Line Simulators, June 1966.

$$u = \frac{1}{2N} \ln \left\{ \left[\left(\frac{\Psi}{\Psi_1} \right)^N \cos(N\phi) - 1 \right]^2 + \left(\frac{\Psi}{\Psi_1} \right)^{2N} \sin^2(N\phi) \right\}$$

$$= \frac{1}{2N} \ln \left\{ \left(\frac{\Psi}{\Psi_1} \right)^{2N} - 2\cos(N\phi) \left(\frac{\Psi}{\Psi_1} \right)^N + 1 \right\} \qquad (45)$$

$$v = \frac{1}{N} \left\{ \arctan \left[\frac{\left(\frac{\Psi}{\Psi_1} \right)^N \sin(N\phi)}{\left(\frac{\Psi}{\Psi_1} \right)^N \cos(N\phi) - 1} \right] + \pi k \right\}$$

$$k = 0,1,2,\ldots,2N-1 \qquad (46)$$

$$\frac{\Psi}{\Psi_1} = \left[(e^{Nu} \cos(Nv) + 1)^2 + e^{2Nu} \sin^2(Nv) \right]^{\frac{1}{2N}}$$

$$= \left[e^{2Nu} + 2\cos(N\phi)\, e^{Nu} + 1 \right]^{\frac{1}{2N}} \qquad (47)$$

$$\phi = \frac{1}{N} \left\{ \arctan \left[\frac{e^{Nu} \sin(Nv)}{e^{Nu} \cos(Nv) + 1} \right] + \pi k \right\}$$

$$k = 0,1,2,\ldots,2N-1 \qquad (48)$$

To convert to cartesian coordinates we have

$$x = \Psi \cos(\phi) \quad , \quad y = \Psi \sin(\phi) \qquad (49)$$

The equipotentials are given in the x,y plane by lines of constant u. As an example, lines of constant u and lines of constant v are plotted in figure 4 for the case of N = 4.

Now all N wires are at the same potential which we call u_o. Consider the wire centered on $\zeta = \Psi_1$, i.e., on $(x,y) = (\Psi_1, 0)$. Then define a new complex variable ν by

$$\zeta \equiv \Psi_1 (1 + \nu) \qquad (50)$$

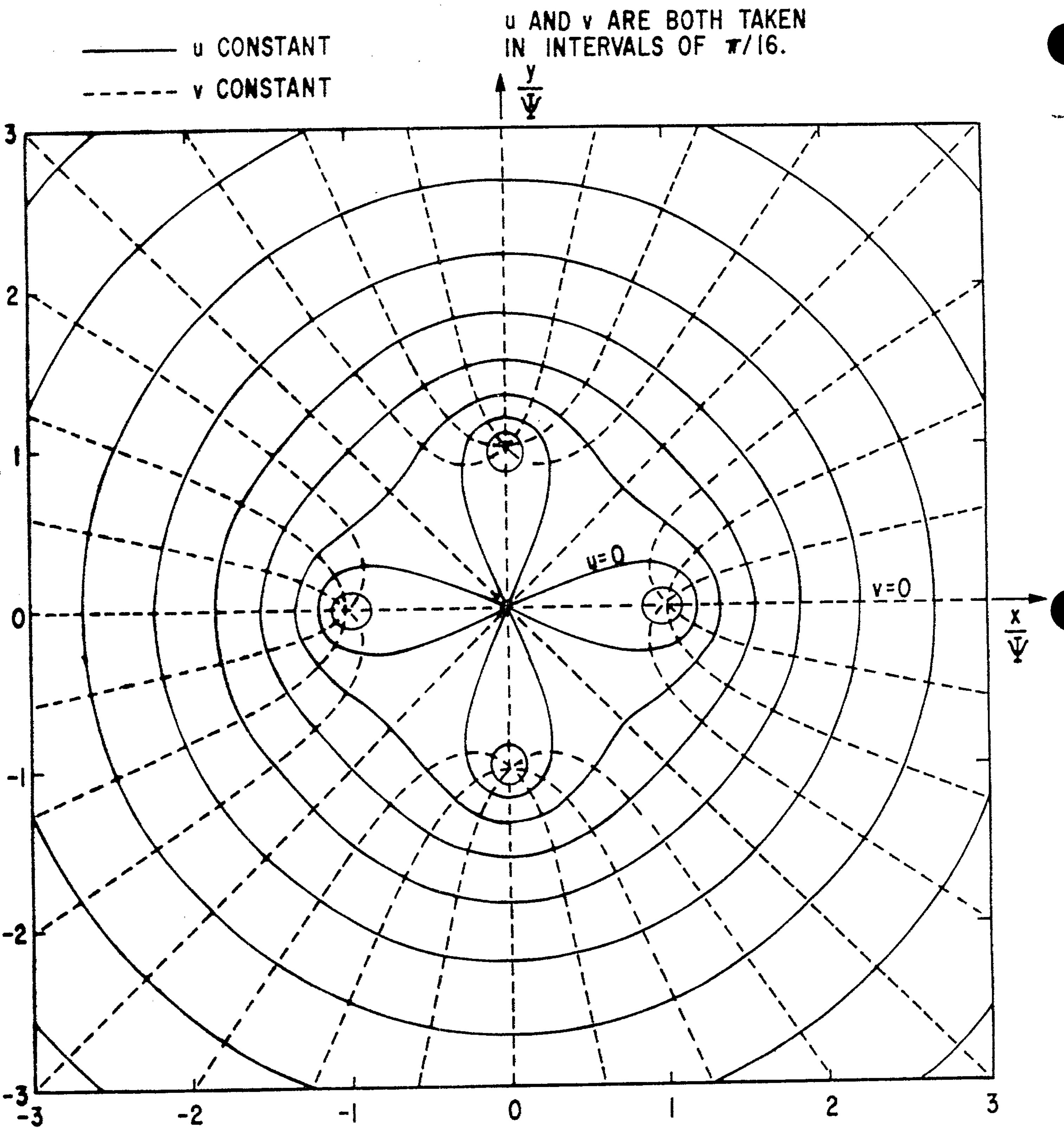

FIGURE 4. POTENTIAL PLOT FOR THE CASE OF FOUR WIRES

Now from the second of equations 43 as $\nu \to 0$ we have

$$w = \frac{1}{N} \ln[(1+\nu)^N - 1] = \frac{1}{N} \ln [1+N\nu+0(\nu^2)-1]$$

$$= \frac{1}{N} \ln[N\nu(1+0(\nu))] = \frac{1}{N} \ln(N\nu) + 0(\nu) \qquad (51)$$

Thus, the potential function u as $\nu \to 0$ has the asymptotic form

$$u \approx \frac{1}{N} \ln(N|\nu|) \qquad (52)$$

Setting $u = u_o$ and $|\nu| = r_o/\Psi_1$ we have an approximation for u_o as

$$u_o \approx \frac{1}{N} \ln \left(\frac{Nr_o}{\Psi_1} \right) \qquad (53)$$

At large distances from the structure so that $|\zeta| \to \infty$ the second of equations 43 becomes

$$w = \frac{1}{N} \ln\left[\left(\frac{\zeta}{\Psi_1}\right)^N\right] + \frac{1}{N} \ln \left[1- \left(\frac{\Psi_1}{\zeta}\right)^N\right]$$

$$= \ln \left(\frac{\zeta}{\Psi_1}\right) + 0(\zeta^{-N}) \qquad (54)$$

Note that we define w such that at large $|\zeta|$ it assumes this form and let $0 \leq \arg(w) < 2\pi$. This removes some of the ambiguity in the multiple-valued expressions in equations 43. The potential function u at large Ψ is then

$$u = \ln \left(\frac{\Psi}{\Psi_1}\right) + 0 \ (\Psi^{-N}) \qquad (55)$$

Now a conducting cylinder with a surface defined by $\Psi = \Psi_{eq}$ has a corresponding potential distribution u' for the same charge on the cylinder as on all N wires of the form

$$u' = \ln \left(\frac{\Psi}{\Psi_1}\right) \qquad (56)$$

The potential from the equivalent cylinder is chosen to match that of the N wires at large Ψ. Setting $\Psi = \Psi_{eq}$ and $u' = u_o$ on the equivalent cylinder gives

$$u_o = \ln \left(\frac{\Psi_{eq}}{\Psi_1}\right) \qquad (57)$$

Combining equations 53 and 57 gives an approximation for the radius of the equivalent cylinder as

$$\frac{\Psi_{eq}}{\Psi_1} \simeq \left(\frac{Nr_o}{\Psi_1}\right)^{\frac{1}{N}} \tag{58}$$

This approximation is accurate as long as the wire radius is small compared to the distance to the nearest wire so that the equipotential u_o accurately fits the circular wire of radius r_o centered on $\zeta = \Psi_1$. Thus, we need $Nr_o/\Psi_1 << 1$. Figure 5 plots the results of equation 58 for various values of N.

Another interesting result is to consider what happens for large N keeping r_o small enough that $Nr_o/\Psi_1 << 1$. Then from equation 58 we see that Ψ_{eq} is approaching Ψ_1 so that we define

$$\frac{\Delta\Psi}{\Psi_1} \equiv 1 - \frac{\Psi_{eq}}{\Psi_1} \tag{59}$$

From equations 53 and 57 for $\Delta\Psi << \Psi_1$ we have

$$\ln\left(1 - \frac{\Delta\Psi}{\Psi_1}\right) \simeq \frac{1}{N} \ln\left(\frac{Nr_o}{\Psi_1}\right) \tag{60}$$

or

$$\frac{\Delta\Psi}{\Psi_1} \simeq \frac{1}{N} \ln\left(\frac{\Psi_1}{Nr_o}\right) \tag{61}$$

Note for large N that one half the spacing between adjacent wires is approximately $\pi\Psi_1/N$. This is consistent with the results for a planar wire grid terminating a uniform field on one side as discussed in reference 5.

VI. Summary

There are many facets to the design of a pulse-radiating dipole antenna. In the present note we have considered some of the design features related to the high-frequency and low-frequency content of the radiated waveform. For the high-frequency portion we can use a biconical wave launcher as the central section of the antenna. This allows one to radiate a fast rising signal from the pulser with little distortion of the waveform at sufficiently early times.

Assuming one desires a significant low-frequency content in the radiated waveform, then the late-time dipole moment of the antenna is important. If the pulse generator can be approximated as a charged capacitor which is switched into the load then several features of the antenna and generator are significant. To maximize the late-time dipole moment one should maximize the generator charge voltage, generator capacitance, antenna length, and antenna capacitance and mean charge separation distance for a given antenna length. Of course, these

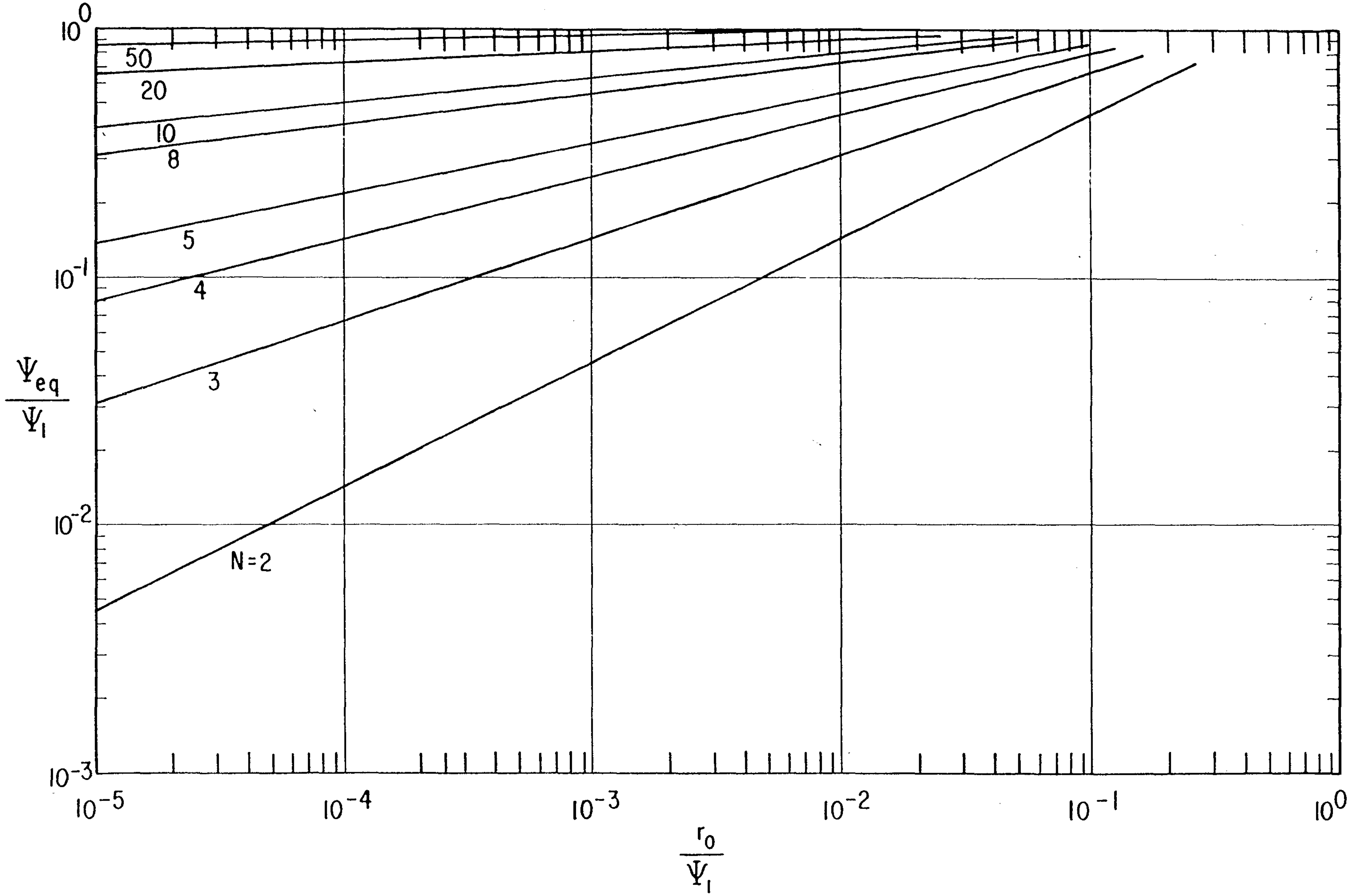

FIGURE 5. EQUIVALENT RADIUS OF WIRE ARRAY FOR VARIOUS N

parameters have different degrees of importance in maximizing the late-time dipole moment. Finally, the antenna and generator should be kept charged (to the extent possible) to times significantly beyond late times of interest.

One of the significant low-frequency parameters of the dipole, the mean charge separation distance, is the same as the equivalent height which applies to the dipole viewed as an electric field sensor. The mean charge separation distance may then also be calculated by calculating the equivalent height. In order to obtain a large antenna capacitance for a given antenna length (while also maintaining a large mean charge separation distance) one might increase its cross section dimensions, i.e., make it fat. One way to make it electrically fat while still retaining a light-weight and sparse mechanical structure is to space conducting wires (or strips or something similar) around the antenna structure with spacing between the wires large compared to the wire radius.

In this note we have considered some of the parameters of a pulsed dipole antenna as related to the high-frequency and low-frequency content of the radiated waveform. Using such parameters one can try to optimize the corresponding waveform characteristics. Of course, there are various other waveform characteristics which are not considered here. These other waveform characteristics might be related to what might be termed the intermediate-frequency content of the waveform. Related to the intermediate frequencies one might, for example, include resistive loading located at breaks in the antenna conductors in order to dampen resonances and achieve a smooth time-domain waveform after the fast initial rise.

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