TRANSIENT PULSE TRANSMISSION USING IMPEDANCE LOADED CYLINDRICAL ANTENNAS

Sensor and Simulation Note #70

Final Report

EE-149

by
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February 1968

This Final Report was prepared under the supervision and direction of R. H. Williams for the Sandia Corporation

UNM Contract 290-774
Sandia Purchase Order No. 57-1887
Buyer: H. C. Strauss, 4312
Requester: C. S. Williams, 1425

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ABSTRACT

Two aspects of the problem of pulse transmission using impedance loaded cylindrical antennas have been considered. First, an approximate solution is obtained for the current distribution on a thin cylindrical antenna driven at an arbitrary point along its length. This solution may be applied to an antenna of any length, and its simplicity makes the current distribution rapidly computable. This solution is ideal for the prediction of the transient electromagnetic field radiated when an arbitrary voltage transient is impressed across the input terminals of a long, thin, cylindrical antenna loaded with lumped impedances. Second, an antenna synthesis procedure was evolved. This synthesis procedure yields a selection of resistor pairs to be used to symmetrically load a cylindrical antenna so that the radiated electromagnetic field pulse approximates some prescribed waveshape. The length of the antenna required to obtain a useful approximation is also considered. Since a voltage step was the assumed input to the antenna, the waveshape that can be approximated is limited to fast rising, generally decaying functions of time.

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LIST OF SYMBOLS

$\vec{A}$	the monochromatic vector potential at point $\vec{p}$, $\vec{A}(\vec{p})$
A_i	the normalized voltage across R_i, Equations (3.20) and (3.31)
$A_{i,j}$	an amplitude constant related to A_i Equation (3.31)
A_z	the z-component of the monochromatic potential at point (r, ϕ, z), $A_z(r,z)$
a	the antenna radius, meters
$B_\phi(r,z)$	the ϕ-component of the monochromatic magnetic field at point (r, ϕ, z)
C,D	undefined voltage constants, Equation (2.16)
C',D'	undefined numerical constants, Equation (2.19)
c	the velocity of light in vacuo, 3×10^8 meter/sec
d	the location of the voltage source on an arbitrarily driven antenna; also the distance from the center of a cylindrical antenna to symmetric voltage sources
$\vec{E}$	the monochromatic electric field vector
$E_r(r,z)$	the r-component of the monochromatic electric field vector
E_s	the z-component of the monochromatic electric field vector at the antenna surface, $E_s(z)$
$E_{s;U_\ell}, E_{s;V_\ell}$	the z-components of the surface electric field; due to current components U_ℓ and V_ℓ
E_z	the z-component of the monochromatic electric field vector, $E_z(r,z)$
e(t)	a prescribed electric field transient (Fig. 3.8)
$e_a(t)$	a step approximation to $e_N(t)$

$e_N(t)$	a normalized electric field transient (Fig. 3.8)
$e_z(r,z)$	the time history of the z-component of the electric field
$F(X)$	an integral function, (A2.41)
$G(d)$	the z-component of the electric field at the point of observation, Fig. 1.1, due to a 1-volt monochromatic voltage source applied to the antenna at $z = d$ (1.9), $G(d,\omega)$,
$G(X)$	an integral function, (A2.46)
$G^S(d,\omega)$	the z-component of the electric field at the point of observation, Fig. 1.1, due to a 1-volt monochromatic voltage source, symmetrically applied to the antenna at $z = \pm d$, (3.44), $(\text{meters})^{-1}$
$G_T(\omega)$	the z-component of the electric field at the point of observation, Fig. 1.1, due to a 1-volt monochromatic voltage source applied to the center of a cylindrical antenna symmetrically loaded with lumped resistors, (3.48)
$g_T(t)$	the time history of the electric field at the point of observation, Fig. 1.1, when a unit impulsive voltage is applied to the center terminals of a cylindrical antenna symmetrically loaded with lumped resistors.
h	the antenna half-length, meters
$I(z)$	the axial component of the monochromatic current at point z on a center driven antenna
$I'(z,d)$	the axial component of the monochromatic current at point z on a cylindrical antenna driven at point $z = d$
$I^S(z,d)$	the axial component of the monochromatic current at point z on a cylindrical antenna, symmetrically driven by sources at $z = \pm d$

$I_T(z)$	the axial component of the total current observed on a multiply-driven or multiply-loaded antenna
$i(z)$	the time history of the current observed at point z on an unloaded center driven antenna
$i(z,d)$	the time history of the current at point z due to a transient voltage source at $z = d$
$i_T(z)$	the time history of the current observed at point z on a multiply-driven or multiply-loaded antenna
$J(p)$	the volume current density at point p, amperes/meter^3
$J_1(\ell,m), J_2(m)$	integral functions, Equations (A2.3) and (A2.4)
$K(z,z')$	the approximate kernel of the vector potential integral, Equation (2.7)
$K_1(\ell,m), K_2(m)$	integral functions, Equations (A2.18) and (A2.19)
k_o	the free space of propagation constant, $k_o = \omega/c = 2\Pi/\lambda$
$L(\ell,m)$	an integral function, Equation (A2.32)
ℓ,m,n	dummy coordinate variables $\ell,m,n \epsilon$ [h,-h,d]
P_1^ℓ, P_2^ℓ	integral Equations (2.50) and (2.52)
R_i	the value of the resistor located at d_i
S	dummy variable of integration, Appendix 2
$S(X)$	an integral function, Equation (A2.50)
T	dummy variable of integration, Appendix 2
T	experimental pulse duration, Equation (2.54)
U,V	current components, Equations (2.39) and (2.43)

V_d	the monochromatic source voltage impressed at $z = d$ on a cylindrical antenna
V_i	the monochromatic voltage measured across R_i.
V_o	source voltage (or the spectral density of a transient source voltage) applied to the center of a cylindrical antenna, $V_o(\omega)$
X	dummy variable, Equation (A2.41)
$Y(z,d)$	the axial component of a monochromatic current at point z on a cylindrical antenna due to a 1-volt source at $z = d$, Equation (1.2), mhos
$Y_p(z,\ell)$	the solution to the integral equation,(2.21)
$y(z,t)$	the time history of the axial current at point z on a symmetric, multiply-loaded antenna with a unit impulsive voltage source applied to the center of the antenna, Equation (3.39)
$y^s(z)$	the time history of the axial current at point z on the antenna, symmetrically driven by unit impulsive voltages applied at $z = \pm d$, Equation (3.39)
$Z(\omega)$	the input impedance of an infinite antenna
Z_a	the characteristic impedance of an antenna, Equation (3.18)
$Z_d(\omega)$	the input impedance of a cylindrical antenna
Z_j	the general impedance located at $z = \pm d$ on a symmetrically loaded cylindrical antenna
$\delta(x)$	Dirac delta function
ϵ_o	the permittivity of free space 8.85×10^{-12} farads/meter

λ	the free space wavelength of a monochromatic wave
μ_o	the permeability of free space, $4\pi \times 10^{-7}$ henrys/meter
ϕ	scaler potential, Equation (2.9) and (2.10)
$\Psi(z)$	the expansion parameter, Equation (3.7)
ψ	the near-constant value of $\Psi(z)$
ω	the radian frequency of an applied monochromatic voltage source or the Fourier transform variable

ACKNOWLEDGMENTS

The author expresses his sincere appreciation to Professor R. H. Williams, who supervised this research in its development, and to the University of New Mexico, who supported this research under a research contract funded by the Air Force Weapons Laboratory through the Sandia Corporation.

Sincere appreciation is expressed to Professors W. J. Byatt, M. D. Bradshaw, and A. Steger for their assistance, advice, and encouragement at the University of New Mexico. A special note of appreciation is also extended to Mr. C. S. Williams Jr., Mr. R. L. Parker, and Dr. C. W. Harrison Jr. of the Sandia Corporation for their continuing interest in this study. The author is also grateful to Mr. T. F. Ezell, Mr. E. A. Aronson, and Mrs. M. Houston for their assistance with the computer programming.

A special note of gratitude is extended by the author to his wife, Torksey, and to their children, Pamalee, Kimball, and Evan, for their patience and encouragement.

Finally, the patient efforts of Mrs. Sue Brannon in typing the report and to Mr. Bill Prather for drafting and printing the report are deeply appreciated.

CHAPTER 1

INTRODUCTION

Intense electromagnetic pulses are created by nuclear explosions and lightning flashes. Accurate knowledge of the field waveshape generated can often yield insight into the physical processes which create the field pulse. For this reason, research has been devoted to the analysis of transient-field generation, propagation, and reception.[1-4]

Since transient field pulses can be very intense (10^5v/m at 1 km from a lightning flash),[5] research has also been performed to determine the effects of intense field pulses on power distribution and communications equipment as well as on missile systems.[6] Associated with the study of the effects of intense electromagnetic pulses on electronic equipment is the problem of the generation of an intense electromagnetic field pulse for testing purposes. Since the vulnerability of an electronic system may depend upon the frequency content of the pulse as well as on the maximum intensity,[7] the waveshape of the field pulse to which the system may be exposed should be simulated. Very-high-voltage equipment is normally employed in the design of the field-pulse generating equipment; therefore, the system designer cannot easily shape the current into the antenna terminals by filtering and switching in the transmitting equipment. For this reason, attention is devoted to the problem of antenna synthesis, i.e., designing an antenna system which will take the conventional output of high-voltage equipment and produce the desired field waveshape incident upon the test object. This report considers the characteristics of one type of antenna that might be used in this application.

The antenna considered is the multiply-loaded dipole antenna shown in Figure 1.1. The origin of the assumed cylindrical coordinate system is at the center of the antenna. Two important aspects of the problem are considered here. In Chapter 2, an antenna theory is developed which will allow the calculation of the electric field pulse radiated when a fast-rising voltage step is applied to the input terminals of a long impedance loaded dipole antenna. In Chapter 3, using a simplified antenna theory, a synthesis procedure is evolved that will yield a selection of lumped resistor pairs to be used to symmetrically load the antenna. The values of the resistors are chosen so that when a voltage step is applied to the antenna the radiated electric field pulse approximates some prescribed, fast-rising, generally decaying function of time.

When the antenna is excited by a monochromatic voltage source (viz, $V_o e^{+j\omega t}$), the electric field at the point of observation has only a z-component.[8]

$$E_z(r,o) = - j\omega A_z(r,o) \doteq - \frac{j\omega\mu_o}{4\pi r} e^{-jk_o r} \int_{-h}^{h} I_T(z)\, dz . \quad (1.1)$$

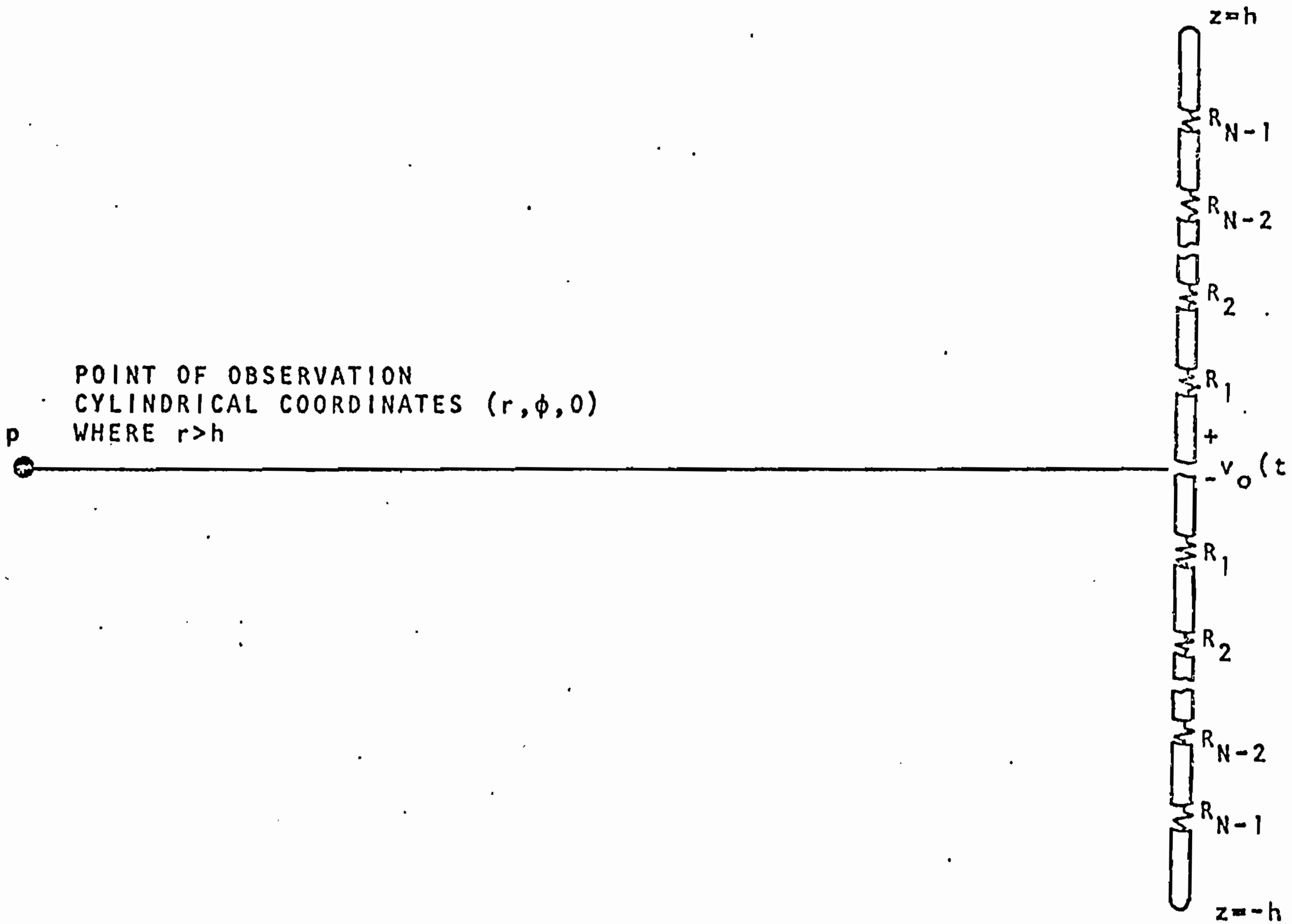


Figure 1.1. Multiply Loaded-Antenna

where:

$A_z(r,o)$ = the z-component of the magnetic vector potential evaluated at the point of observation

$I_T(z)$ = the total axial current flowing at point z on the antenna

2h = the total length of the dipole antenna

k_o = the free space propagation constant, $k_o = \omega/c$

ω = the radian frequency of the applied excitation, $\omega = 2\pi f$

c = the velocity of light in free space, 3×10^8 meters/second

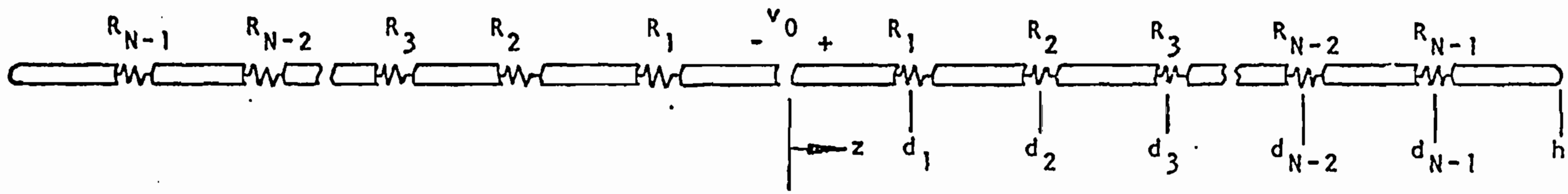
μ_o = the permeability of free space, $4\pi \times 10^{-7}$ henry/meter

ϵ_o = the permittivity of free space, 8.85×10^{-12} farads/meter

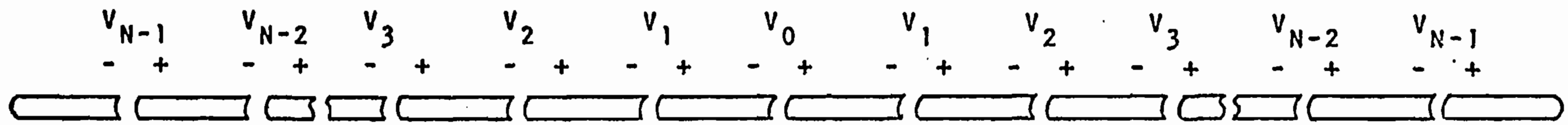
The relation of the total current to the resistive loading and to the applied source voltage is determined by applying the Compensation Theorem of network theory.[9]

By the Compensation Theorem, any load impedance, Z_L, can be replaced by an equivalent voltage source, $V_L = - I_L Z_L$, without disturbing the network. Therefore, the symmetric resistance loaded dipole of Figure 1.2a is equivalent to the multiply-driven structure shown in Figure 1.2b, where each equivalent voltage source has zero internal impedance.

The total current on the antenna shown in Figure 1.2b can be obtained by superposition.



a) Multiply-Loaded Dipole Antenna



b) Multiply-Driven Dipole Antenna

$$V_i = -I(d_i)R_i$$

Figure 1.2. Equivalent Multiply-Loaded and Multiply-Driven Dipole Antennas

$$I_T(z) = V_o Y(z,o) + \sum_{i=1}^{N-1} V_i \left(Y(z,d_i) + Y(z,-d_i) \right) \qquad (1.2)$$

where $I_T(z)$ is the total axial current on the antenna measured at point z,

$Y(z,d_i)$ is the current measured at the same point z when a unit voltage source is applied at point d_i.

The load voltage V_i is to be determined by the Compensation Theorem,

$$V_i = -I_T(d_i)Z_i. \qquad (1.3)$$

Combination of (1.2) and (1.3) yields a set of (N-1) simultaneous equations for determining the (N-1) unknown load voltages,

$$\sum_{j=1}^{N-1} V_j\, a_{ij} = -V_o Y(d_i,o) \qquad i = 1,\cdots,(N-1).$$

Here

$$a_{ij} = Y(d_i,d_j) + Y(d_i,-d_j), \qquad i \neq j$$

$$a_{ii} = Y(d_i,d_i) + Y(d_i,-d_i) + 1/Z_i \qquad . \qquad (1.4)$$

Substituting the solutions obtained from (1.4) back into (1.3) yields the complete distribution of current on the antenna needed to compute the electric field by (1.1).

$$E_z(r,o) = -\frac{j\omega\mu_o}{4\pi r} e^{-jk_o r} \left\{ V_o \int_{-h}^{h} Y(z,o)dz \right.$$

$$\left. + \sum_{i=1}^{N-1} V_i \int_{-h}^{h} \left[Y(z,d_i) + Y(z,-d_i) \right] dz \right\} . \quad (1.5)$$

But by symmetry

$$Y(z,-d_i) = Y(-z,d_i), \quad (1.6)$$

and

$$\int_{-h}^{h} Y(-z,d_i)dz = \int_{-h}^{h} Y(x,d_i)dx, \quad (1.7)$$

so that the total electric field at the point of observation is written

$$E_z(r,o) = V_o G(o) + 2 \sum_{i=1}^{N-1} V_i \; G(d_i) . \quad (1.8)$$

Here

$$G(d) \equiv -\frac{j\omega\mu_o e^{-jk_o r}}{4\pi r} \int_{-h}^{h} Y(z,d)dz . \quad (1.9)$$

To compute the transient response, V_o is interpreted as the spectral density of the applied voltage transient defined by

$$V_o(\omega) = \int_{-\infty}^{\infty} v_o(t)e^{-j\omega t}dt, \tag{1.10}$$

with the inverse transform

$$v_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V_o(\omega)e^{+j\omega t}d\omega. \tag{1.11}$$

The time history of the electric field transient is then obtained by taking the inverse Fourier transform, of (1.8).

CHAPTER 2

ANTENNA THEORY

In Chapter 1 it was determined that the transient electric field radiated by an impedance loaded antenna could be determined if the current distribution on the antenna driven at an arbitrary point along its length and the radiated field produced by this current distribution were known.

Theories have been developed that provide this information if the electrical length of the antenna is not too long. However, to consider fast-rising pulses in impedance loaded antennas, an approximate theory that has no fundamental frequency limitation is required. In this chapter, an approximate theory is developed that satisfies this requirement. The current distributions predicted by the theory agree reasonably well with the measured current distributions on both unloaded and impedance loaded electrically long antennas as reported by Altschuler.[10]

2.1 Formulation of the Antenna Problem

In recent work on antenna theory, it has been found convenient to determine the field maintained by a given distribution of electric currents from Maxwell's equations with the intermediary of the magnetic vector potential.

$$\vec{A}(\vec{p}) = \frac{\mu_o}{4\Pi} \int_v \vec{J}(\vec{p}\,') \frac{e^{-jk_o|\vec{p} - \vec{p}\,'|}}{|\vec{p} - \vec{p}\,'|} dV. \qquad (2.1)$$

$\vec{J}(\vec{p}')$ is the electric current density at the point p' located in V, and p is the point of observation located either inside or outside the volume V. To evaluate properly, $\vec{A}(\vec{p})$, the indicated integration must be taken over all currents flowing in the antenna, its feeding transmission line, and the exciting transmitter. One normally is interested in only that portion of the field which is due to currents distributed on the intended radiating element; therefore, most theories have been developed using an idealized model similar to that shown in Figure 2.1. The antenna is assumed to be constructed of an extremely thin-walled tube of infinite conductivity without endcaps. The length of the antenna is 2h and the radius is "a".[11] The antenna is driven by a monochromatic voltage generator, $V_d e^{+j\omega t}$, applied across a narrow circumferential gap located a distance, d, from the center of the antenna. The voltage across the gap is defined by

$$V_d = - \int_{gap} E_s dz, \qquad (2.2)$$

where E_s is the z-component of the electric field evaluated at the surface of the antenna. E_s is zero everywhere on the antenna surface except in the small gap, since the walls of the tube have been assumed to be perfectly conducting. If the gap width is decreased while the voltage V_d is held constant, E_s approaches the form

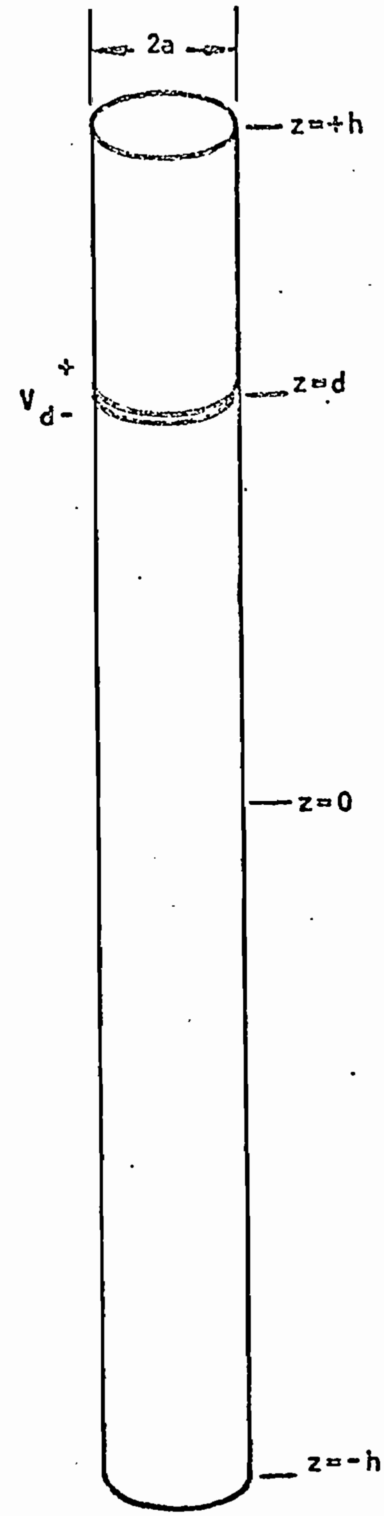


Figure 2.1. Idealized Arbitrarily Driven Antenna

$$E_s = -V_d\,\delta(z-d), \qquad \text{for } |z| \leq h, \tag{2.3}$$

where δ denotes the Dirac delta function.

The electric field may also be obtained from the magnetic vector potential, (2.1). Because the model is symmetric about the z-axis, there is only a z-component of the magnetic vector potential;

$$A_z(r,z) = \frac{\mu_o}{4\Pi}\int_{-h}^{h} I(z',d)\,K(z,z')\,dz' , \tag{2.4}$$

where $I(z,d)$ is the axial distribution of current on the antenna due to voltage V_d applied at $z = d$. The kernel, $K(z,z')$, is defined by

$$K(z,z') = \frac{1}{2\Pi}\int_{o}^{2\Pi} \frac{e^{-jk_oR}}{R}\,d\emptyset', \tag{2.5}$$

where

$$R = \sqrt{r^2 + a^2 - 2r\,a\cos\emptyset' + (z-z')^2}. \tag{2.6}$$

For thin antennas, for which $a \ll h$ and $k_o a \ll 1$, a sufficiently accurate approximation of the kernel is

$$K(z,z') = \frac{e^{-jk_oR}}{R}, \tag{2.7}$$

where

$$R \doteq \sqrt{r^2 + (z-z')^2} \qquad (2.8)$$

This approximation yields accurate results even when the field near the surface of the antenna is to be evaluated. The electric field at all points in space is given by[12]

$$\vec{E} = -\nabla\phi - j\omega\vec{A}, \qquad (2.9)$$

where ϕ is the scaler potential, related to the divergence of $\vec{A}$ by the Lorentz condition,

$$\nabla\cdot\vec{A} + \frac{jk_o^2}{\omega}\phi = 0. \qquad (2.10)$$

Equations (2.9) and (2.10) may be combined to give the non-zero components of the electromagnetic field in cylindrical coordinates:[12]

$$E_z(r,z) = -\frac{j\omega}{k_o^2}\left(\frac{\partial^2}{\partial z^2} + k_o^2\right) A_z(r,z), \qquad (2.11)$$

$$E_r(r,z) = -\frac{j\omega}{k_o^2}\frac{\partial^2 A_z(r,z)}{\partial r\partial z}, \qquad (2.12)$$

$$B_\phi(r,z) = -\frac{\partial A_z(r,z)}{\partial r}. \qquad (2.13)$$

Knowledge of the distribution of current on the antenna yields the entire description of the electromagnetic field.

If (2.11) is evaluated at the surface of the antenna where (2.3) must be satisfied, then,

$$\left(\frac{\partial^2}{\partial z^2} + k_o^2\right)A_z(a,z) = -j\,\frac{k_o^2}{\omega}\,V_d\,\delta(z-d), \qquad (2.14)$$

where $|z| \leq h$. Appropriate solutions to the homogeneous equation are e^{+jk_oz} and e^{-jk_oz}. These may be combined to yield a solution to (2.14) in the form

$$A_z(a,z) = \frac{1}{c}\left[C \cos k_c z + D \sin k_o z + \frac{V_d}{2}\, e^{-jk_o|z-d|}\right]. \qquad (2.15)$$

An integral equation for I(z,d) can be obtained by substituting for the left side of the equation the defining integral, (2.4).

$$\frac{\mu_o}{4\pi}\int_{-h}^{h} I(z',d)K(z,z')dz' =$$

$$\frac{1}{c}\left[C \cos k_o z + D \sin k_o z + \frac{V_d}{2}\, e^{-jk_o|z-d|}\right], \qquad (2.16)$$

where the kernel K(z,z') is given by (2.5), or approximately by (2.7) evaluated at r=a. Henceforth it will be assumed that the radius of the antenna is small so that the use of the approximate kernel is appropriate.

Equation (2.16) is Hallén's integral equation for the arbitrarily driven cylindrical antenna. Other forms of Hallén's integral equation are the starting point for most theoretical treatments of the antenna problem. C and D, the unknown constants appearing in the right side of (2.16), may be eliminated by applying the boundary condition that the current at each end of the antenna must vanish.

2.2 Previous Solutions of the Antenna Problem

Studies of the current distribution on the antenna have primarily been limited to the important special case of the center driven antenna for which d = 0 and D = 0 in (2.16). Many analytical procedures have been applied to this problem:[13] iteration (Hallén),[14] (King and Middleton),[15] Fourier Series (Duncan and Hinchey),[16] Numerical Integration (Mei),[17] and an application of the Wiener-Hopf Technique (Wu).[18] These theories all yield the approximate current distribution on the antenna. However, the complexity of the solutions has limited their utility to the determination of the input impedance of the isolated antenna. In 1959, King showed that the current distribution on a center driven antenna could be represented approximately by the linear combination of two terms,[19]

$$I(z) = A \sin k_o(h-|z|) + B [\cos k_o z - \cos k_o h], \quad (2.17)$$

provided the half length of the antenna did not exceed $5\lambda/8$.

The simplicity of this function has led to its use in the solution of many complicated problems ranging from the Yagi-Uda array theory[20] to the leakage of RF energy into the electronics of a missile in flight.[21]

King used direct substitution in Hallen's integral equation to obtain the coefficients A and B.[19] Storer showed that the calculus of variations could be employed to optimize the choice of these coefficients.[22] His results, however, do not differ significantly from King's. Tai also applied the calculus of variations to the antenna problem, using one different trial function;[23]

$$I(z)=A \sin k_o(h-|z|) + B\, k_o(h-|z|) \cos k_o(h-|z|). \quad (2.18)$$

This form has the advantage that there are no frequencies at which the input current is zero, a defect suffered by King's assumed current. Tai's results may be applied to longer antennas, $h > 5\lambda/8$; however, the resulting current distribution is generally not similar to the measured data. In the frequency range where comparison is appropriate, $h < 5\lambda/8$, Tai's results support the results of both Storer and King.[13]

King added a third function to his assumed solution in 1966 which further increases the accuracy of the approximation at the expense of additional complexity.[24] In 1967, King and Wu extended the theory to the antenna driven at an arbitrary point along its length,[25] thus allowing the study of multiply-driven or impedance loaded antennas which satisfy the requirement that $h < 5\lambda/8$.

For longer antennas, the results have not been as far reaching; Wu's theory[18] could be used to find an accurate current distribution for the arbitrarily-driven dipole. However, these results would not be in a simple form, as are those which have been found to be so useful in working with shorter antennas.

King and Saunders were able to find a trigonometric expansion for the current distribution on the center drive resonant antenna.[26] They concluded that no simple trigonometric form could be obtained for currents on long anti-resonant antennas.

In the treatment to follow, a simple expansion of the current distribution on an arbitrarily driven cylindrical antenna is developed. Unlike the previous approximate distributions developed for short center driven antennas, trigonometric current components are not employed in this development; instead, attenuated traveling waves of current are assumed to emanate from the driving point and from the ends of the antenna. Conversion of this form of solution to that of attenuated trigonometric components is possible. However, a significant reduction in mathematical complexity is obtained by retaining the traveling wave form of solution. Only linearly attenuated waves are considered in the numerical analysis given, but the addition of higher order terms required for greater accuracy is straightforward.

2.3 Traveling Wave Antenna Theory

The development begins with (2.19), which is the equivalent of Hallén's integral, (2.16).

$$\frac{\mu_o}{4\pi}\int_{-h}^{h} I(z',d)K(z,z')dz' =$$

$$\frac{V_d}{2c}\left[C' e^{-jk_o|z-h|} + D' e^{-jk_o|z+h|} + e^{-jk_o|z-d|}\right]. \quad (2.19)$$

$$C' = \left(\frac{C-jD}{V_d}\right)e^{+jk_o h} \quad \text{and} \quad D' = \left(\frac{C+jD}{V_d}\right)e^{+jk_o h}.$$

In this form it is evident that the total current can be considered to be the sum of three components.

$$I(z,d) = V_d\left[C'Y_p(z,h) + D'Y_p(z,-h) + Y_p(z,d)\right], \quad (2.20)$$

where the components of the form $Y_p(z,\ell)$ are solutions of the integral equation.

$$\frac{\mu_o}{4\pi}\int_{-h}^{h} Y_p(z',\ell)K(z,z')dz' = \frac{1}{2c}\, e^{-jk_o|z-\ell|}, \quad (2.21)$$

where $|z| \leq h$ and $|\ell| \leq h$.

The current distribution on an infinite antenna driven at an arbitrary point along its length by a unit voltage source is the solution of (2.21) with $h=\infty$. It has been previously determined that an approximate solution for this case is[27]

$$Y_p(z,\ell) = \frac{2\pi}{Z_o} \times \frac{e^{-jk_o|z-\ell|}}{\ell n\left[\frac{-2j|z-\ell|}{\Gamma^2 k_o a^2}\right]}, \qquad (2.22)$$

where $k_o a \ll 1$, $a \ll |z-\ell|$, and $\ell n\,\Gamma = 0.577216$, Euler's constant. The value of the integral in (2.21) at a point, z, is determined primarily by the current within a small distance of that point. This is due to a very sharp peak in the kernel at the point $z' = z$. Accordingly, (2.22) is also an approximate solution of (2.21) except near the ends of the antenna.

Since the inverse logarithmic attenuation of the traveling wave in (2.22) is gradual, the solution of (2.21) may also be approximated over the finite length of the antenna by a more tractable linearly attenuated traveling wave.

$$Y_p(z,\ell) \doteq [A + B\,k_o|z-\ell|]\,e^{-jk_o|z-\ell|}, \qquad (2.23)$$

where A and B are suitably selected complex coefficients. When (2.23) is substituted into (2.20), the total current on the antenna is expressed in the form

$$I(z,d) = V_d \Big[(A_h + B_h k_o |z-h|)\, e^{-jk_o|z-h|}$$

$$+ (A_{-h} + B_{-h} k_o |z+h|)\, e^{-jk_o|z+h|}$$

$$+ (A_d + B_d k_o |z-d|)\, e^{-jk_o|z-d|} \Big] . \qquad (2.24)$$

Here, C' and D' have been incorporated into the unknown A's and B's. The total current on the antenna is expressed as the sum of attenuated traveling waves emanating from the driving source and from each end of the structure.

Two of the constants could be eliminated by enforcing the boundary condition that the current is zero on the ends of the antenna; however, a more general class of functions is allowed if this smooth approximation is not required to vanish at the ends of the antenna. The appropriate boundary condition is that current exists only on the antenna, where a discontinuity is allowed at the end of the structure.

Clearly, an increase in the accuracy of the distribution could be obtained by increasing the number of terms in the indicated series expansion of the attenuation function. For the problem at hand, the analysis of transients in impedance loaded structures, many frequencies and several drive points must be considered. For this type of problem, simplicity is often more important than extreme accuracy.

The reaction concept was employed to determine appropriate values for the six constants required. This technique is equivalent[28] to Galerkin's method or to the variational approach employed by Storer[22] to optimize the choice of coefficients for the current components previously selected by King.[19]

The reaction concept was invented by V. H. Rumsey in 1954,[29] and while it is equivalent to the other techniques, it is conceptually easier to apply. The reaction between a field, a, and a source, b, is defined as

$$\langle a,b \rangle = \int_{vol} \vec{E}^a \cdot \vec{J}^b \, dv. \qquad (2.25)$$

The reciprocity theorem in Rumsey's notation is

$$\langle a,b \rangle = \langle b,a \rangle . \qquad (2.26)$$

One may also define the self-reaction as the reaction of a field on its own source.

$$\langle a,a \rangle = \int_{vol} \vec{E}^a \cdot \vec{J}^a \, dv. \qquad (2.27)$$

For the arbitrarily driven antenna, the current exists only on the surface of the antenna, so the integration reduces to

$$\langle a,a \rangle = \int_{-h}^{h} E_s(z)\ I(z,d)dz, \tag{2.28}$$

where $E_s(z)$ is the z-component of the electric field at the surface of the antenna. Since the antenna model is that of a perfectly conducting tube, the electric field is nonzero only in the gap where the antenna is driven, as can be seen from (2.3). Therefore, the value of the reaction is given by

$$\langle a,a \rangle = -V_d I(d,d). \tag{2.29}$$

$V_d = I(d,d)Z_d$, where Z_d is the input impedance of the antenna, and may be written

$$Z_d = \frac{-\langle a,a \rangle}{I(d,d)^2}. \tag{2.30}$$

The reaction between any two approximate sources is stationary if it is subjected to the constraint:[30]

$$\langle a,b \rangle = \langle c_a,b \rangle = \langle a,c_b \rangle, \tag{2.31}$$

where c_a and c_b represent the "correct" sources and fields. The application of this constraint yields a stationary approximation to the reaction and is hence equivalent to the variational approach used by Storer.[22]

The method of determining the appropriate current distribution coefficients is straightforward. Suppose that the total current is to be represented by two trial components: $I^a = V_d [A\,U + B\,V]$. Set the reaction between the approximate field and each trial current equal to the reaction between the true field and each trial current.[30]

$$\langle a,U\rangle = \langle c,U\rangle \tag{2.32}$$

$$\langle a,V\rangle = \langle c,V\rangle \tag{2.33}$$

The reaction between the correct field and each trial current is known, since $E_z = -V_d\,\delta(z-d)$. Therefore,

$$A\,\langle U,U\rangle + B\langle V,U\rangle = -U(d), \tag{2.34}$$

$$A\,\langle U,V\rangle + B\langle V,V\rangle = -V(d). \tag{2.35}$$

By reciprocity, $\langle U,V\rangle = \langle V,U\rangle$, and there are three reactions in the coefficient matrix to be determined. The extension to six variables is straightforward.

Let

$$U_\ell = e^{-jk_o|z-\ell|} \tag{2.36}$$

and

$$V_\ell = k_o|z-\ell|e^{-jk_o|z-\ell|} \; . \tag{2.37}$$

The following matrix equation results

$$
\begin{bmatrix}
\langle U_{-h},U_{-h}\rangle & \langle V_{-h},U_{-h}\rangle & \langle U_{d},U_{-h}\rangle & \langle V_{d},U_{-h}\rangle & \langle V_{h},U_{-h}\rangle & \langle U_{h},U_{-h}\rangle \\
\langle U_{-h},V_{-h}\rangle & \cdot & & & & \\
\langle U_{-h},U_{d}\rangle & & \cdot & & & \\
\langle U_{-h},V_{d}\rangle & & & \cdot & & \\
\langle U_{-h},V_{h}\rangle & & & & \cdot & \\
\langle U_{-h},U_{h}\rangle & & & & & \langle U_{h},U_{h}\rangle
\end{bmatrix}
\begin{bmatrix}
A_{-h} \\ B_{-h} \\ A_{d} \\ B_{d} \\ B_{h} \\ A_{h}
\end{bmatrix}
= -
\begin{bmatrix}
e^{-jk_o(h+d)} \\
k_o(h+d)e^{-jk_o(h+d)} \\
1. \\
0 \\
k_o(h-d)e^{-jk_o(h-d)} \\
e^{-jk_o(h-d)}
\end{bmatrix}
\tag{2.38}
$$

All the elements of the reaction matrix are of the form $\langle U_\ell, U_m \rangle$, $\langle U_\ell, V_m \rangle$, $\langle V_\ell, U_m \rangle$, or $\langle V_\ell, V_m \rangle$. By reciprocity, $\langle U_\ell, V_m \rangle = \langle V_m, U_\ell \rangle$, so that only three general formulas are required to define all 36 matrix elements. Therefore, the analytical part of this solution, using a 6-component trial current, is obtained as easily as when using a general 2-component solution.

It is this significant reduction in computational effort which makes the traveling wave form of an assumed solution superior to the trigonometric forms extensively used in antenna theory.

In Appendix 1, the electric field at the surface of the antenna due to each of the current component forms is derived. These fields are then used to determine the three required reaction formulas in Appendix 2.

It was found that each of these reactions could be reduced to expressions involving the tabulated sine and cosine integrals, so that no numerical integration was necessary.

In the remaining sections of this chapter, the accuracy of this theory is evaluated by comparison with existing experimental data and theoretical evidence.

2.4 Current Distributions

Very accurate measurement tecniques have been devised for determining both the real and the imaginary components of the current distribution on a center driven dipole antenna.[11] In Figure 2.2, both the measured current distribution and the current distribution predicted by the traveling wave theory are given for center driven antennas of four different lengths. The agreement between the measured and the predicted distributions is quite good except for the long antiresonant antenna. Although this discrepancy is unimportant for transient analysis because the total antenna current is quite small at antiresonant frequencies, it is apparent that a linear attenuated current model will not adequately describe the rapid variations which occur near the end of the antiresonant antenna.

No measurement exists of the current distribution on an antenna not driven at its center; however, Altschuler[10] has reported the current distribution on a center driven dipole symmetrically loaded with a pair of resistors located one quarter of a wavelength from the ends of the structure. The total current flowing on this structure can be obtained from (1.2) and (1.3).

$$I_T(z) = V_o\, Y(z,o) + V_d\, [Y(z,d) + Y(z,-d)], \quad (2.39)$$

where

$$V_d = -I_T(d)\, R_d. \quad (2.40)$$

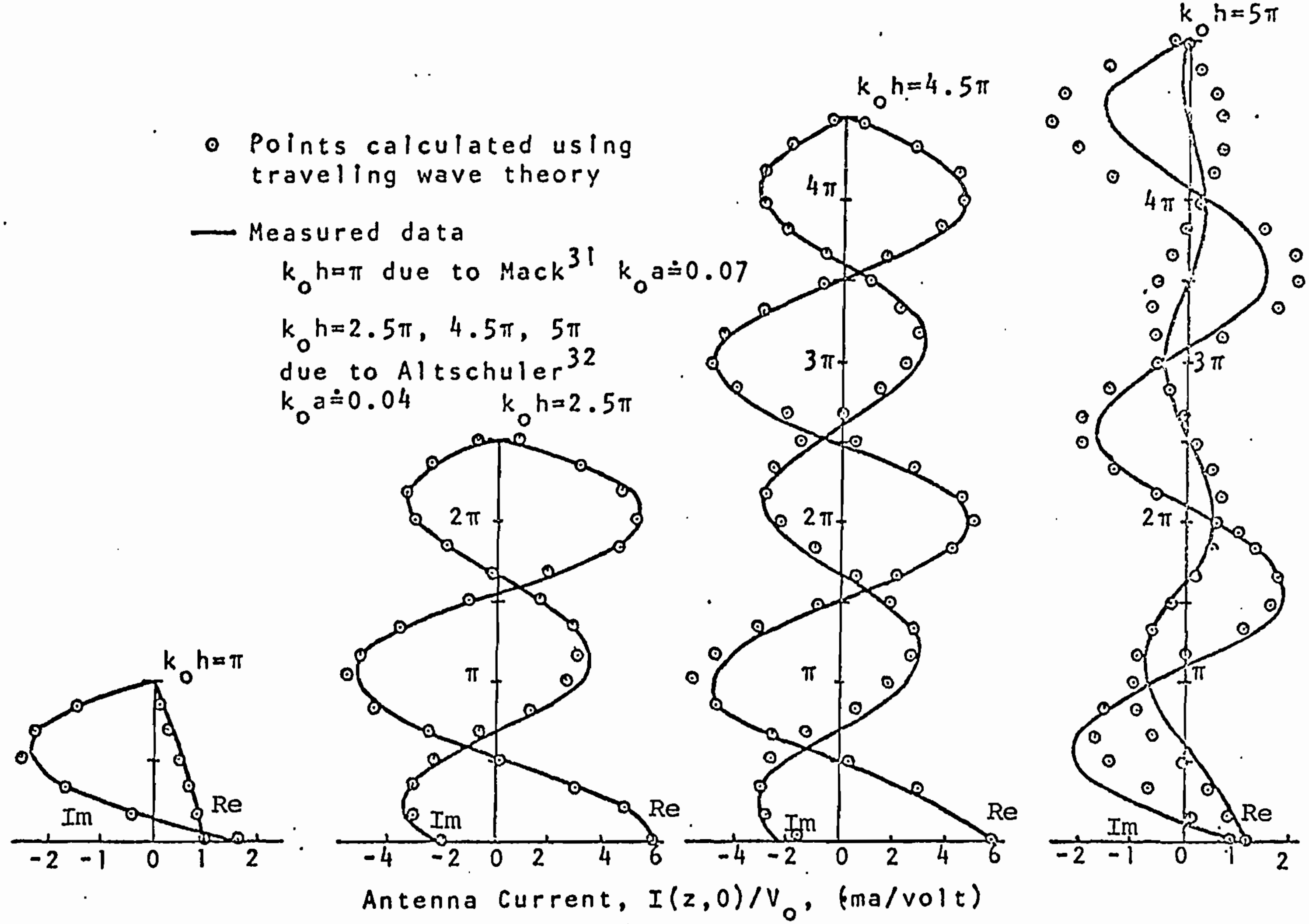


Figure 2.2. Comparison of Current Distributions on Standing Wave Antennas

Since $Y(z,-d) = Y(-z,d)$, (1.6), only the symmetrical component of the current distribution on an arbitrarily driven antenna can be evaluated by comparing the calculated results to Altschuler's measured data.[10,32]

In Figure 2.3, both the predicted and the measured current distributions are plotted for seven different antenna lengths. The value of the resistor was 240 ohms. Experimentally, Altschuler found that this value yielded a traveling wave distribution of current between the driving terminals and the resistor. Since much of the rapid variation of the current distribution is removed when these resistors are placed in the antenna, the predicted current distributions are acceptable for both resonant and anti-resonant lengths.

2.5 Radiated Fields

The normalized radiated electromagnetic field at the point of observation, when the antenna is driven at an arbitrary point along its length, (1.9) is required to complete the analysis of the pulses radiated by a symmetrically loaded structure. The desired component is

$$G(d) = -\frac{j\omega\mu_o e^{-jk_o r}}{4\pi r} \int_{-h}^{-h} Y(z,d)dz, \qquad (2.41)$$

where $Y(z,d)$ is obtained from (2.24). Substitution yields

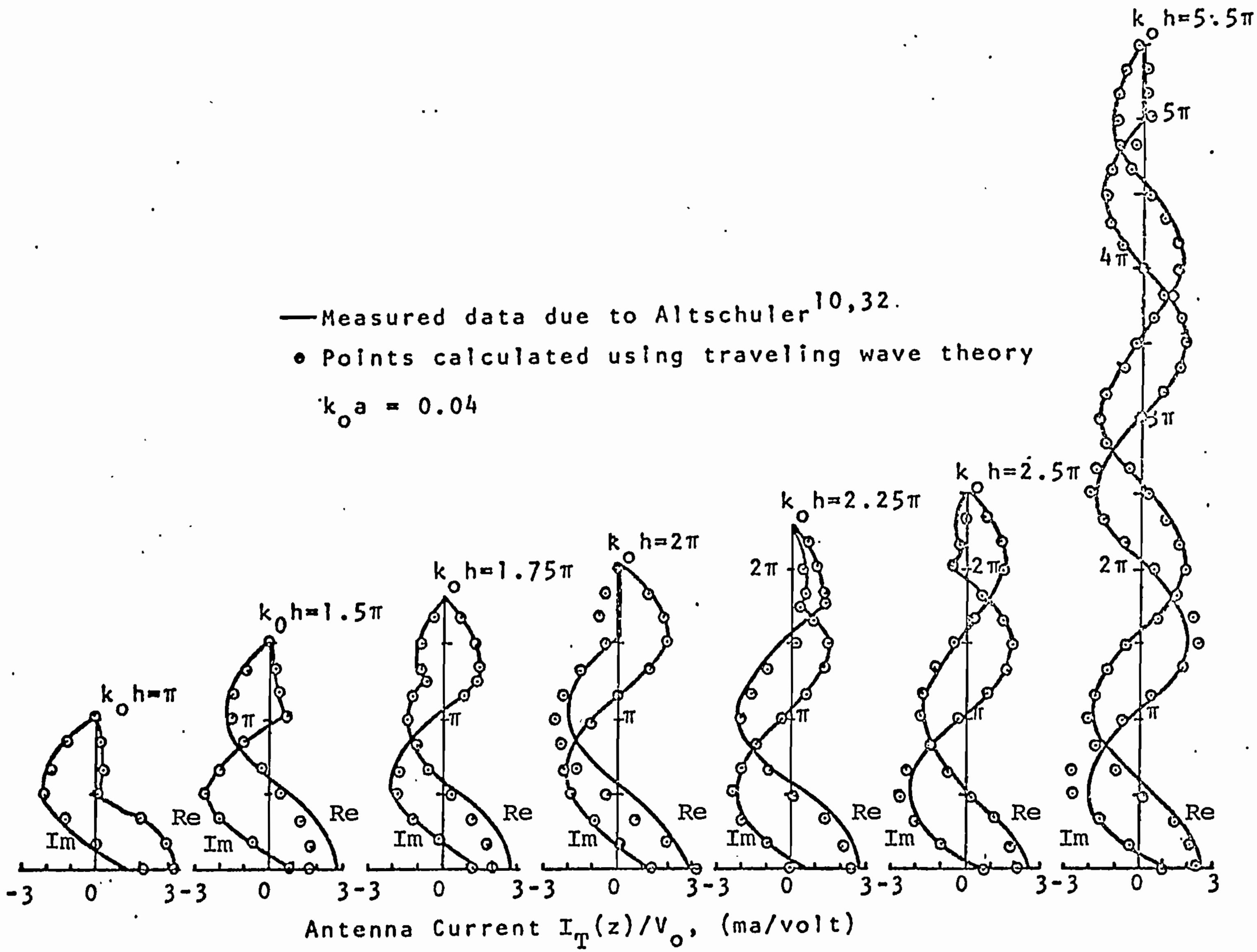


Figure 2.3. Comparison of Current Distributions of Traveling Wave Antennas

$$G(d) = A_h P_1^h + B_h P_2^h$$

$$+ A_{-h} P_1^{-h} + B_{-h} P_2^{-h}$$

$$+ A_d P_1^d + B_d P_2^d, \qquad (2.42)$$

where

$$P_1^\ell = \frac{-j\omega\mu_o e^{-jk_o r}}{4\pi r} \int_{-h}^{h} e^{-jk_o|z-\ell|} dz, \qquad (2.43)$$

$$P_1^\ell = \frac{-Z_o}{2\pi r} e^{-jk_o r} [1 - e^{-jk_o h} \cos k_o \ell], \qquad (2.44)$$

and

$$P_2^\ell = \frac{-j\omega\mu_o e^{-jk_o r}}{4\pi r} \int_{-h}^{h} k_o|z-\ell| e^{-jk_o|z-\ell|} dz, \qquad (2.45)$$

$$P_2^\ell = -\frac{jZ_o}{4\pi r} e^{-jk_o r} [(1 + jk_o(h-\ell)) e^{-jk_o(h-\ell)}$$

$$+ (1 + jk_o(h+\ell)) e^{-jk_o(h+\ell)} - 2] . \qquad (2.46)$$

Here, $Z_o = 120\pi$ ohms.

Another interesting check on the traveling wave theory is to compare the radiated electric field transients predicted by this theory with those both predicted and measured by

Schmitt, Harrison, and Williams.[3] Schmitt, et al used King-Middleton antenna theory[15] for frequencies where $h < 5\lambda/8$ and the theory developed by Wu[18] for frequencies where $h \geq 5\lambda/8$ to predict the radiated electric field transient.

The configuration considered is shown in Figure 2.4a, a cylindrical monopole fed by a coaxial line with a characteristic impedance of 50 ohms. The input voltage pulse is the type of square pulse that can be generated in the laboratory. This pulse is well approximated by the analytical expression:

$$v_g(t) = V_B\,[f(t)u(t) - f(t-T)u(t-T)], \qquad (2.47)$$

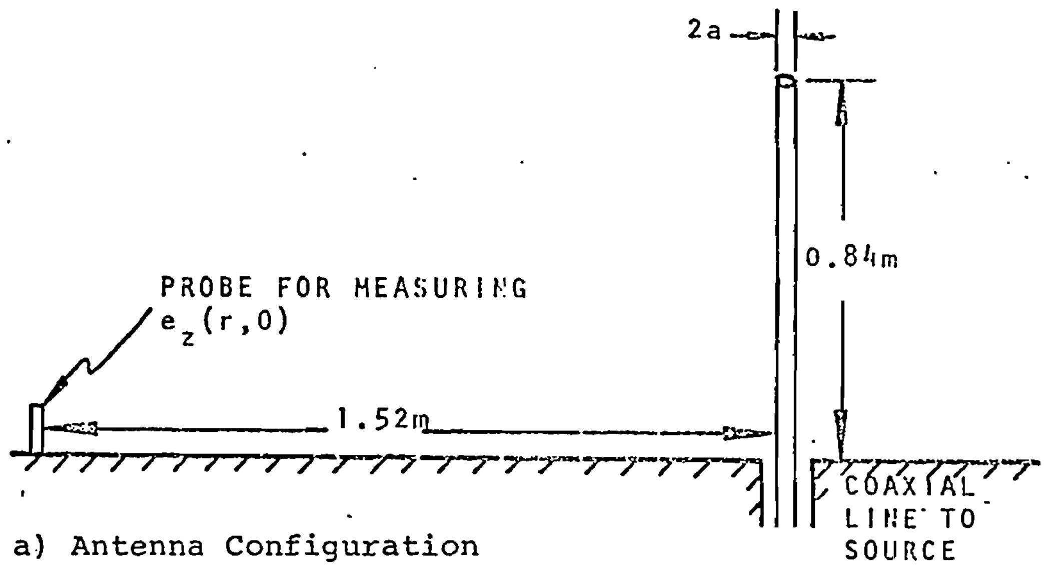
where

$$f(t) = [1 - (1+t/t_1)\,e^{-t/t_1}], \qquad (2.48)$$

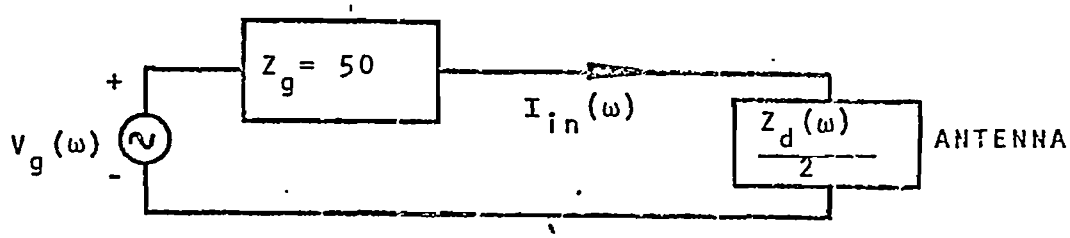
$$u(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases} \qquad (2.49)$$

t_1 is a parameter related to the rise time of the pulse (rise time $\cong 5t_1$), and T is the pulse duration in seconds. The Fourier transform of this voltage wave is

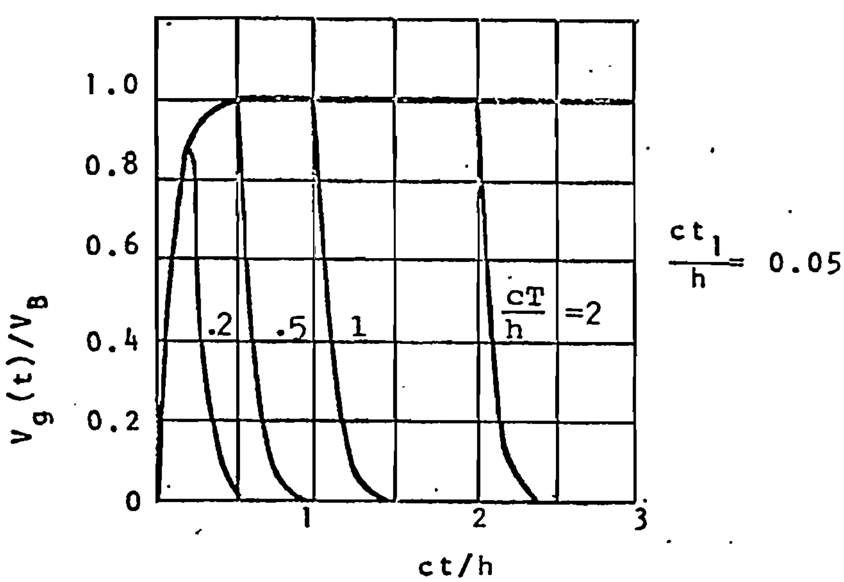
$$V_g(\omega) = \frac{V_B(1-e^{-j\omega T})}{j\omega(1+j\omega t_1)^2} \text{ Volts/Hz}. \qquad (2.50)$$



a) Antenna Configuration



b) Equivalent Circuit



c) Excitation Pulse Shape

Figure 2.4. Experimental Configuration

The time history of this voltage pulse is shown in Figure 2.4c. All times have been normalized by the factor h/c, the one-way travel time from the input of the antenna to the end of the structure. The time history of the radiated electric field pulse is obtained by taking the inverse Fourier transform of the frequency description of the pulse. Using traveling wave theory, the frequency description of the pulse is

$$E_z(r,o) = 2\,\frac{V_g(\omega)\; Z_d(\omega)}{Z_d(\omega) + 2Z_g}\, G(o,\omega). \qquad (2.51)$$

Here, $Z_d(\omega)$ is the input impedance of the equivalent dipole:

$$Z_d(\omega) \equiv 1/Y(o,o). \qquad (2.52)$$

The factors of 2 are required to apply the dipole antenna data to the monopole configuration used in the experiment. In Figure 2.5, the calculated radiation zone field along the ground plane is presented for one value of ct_1/h, 0.05, and four values of relative pulse width cT/h. These data were obtained by numerically calculating the inverse Fourier transform of (2.51).

In Figures 2.6 and 2.7, the calculated and measured data obtained by Schmitt, Harrison, and Williams[3] for the same antenna and exciting sources are presented. The

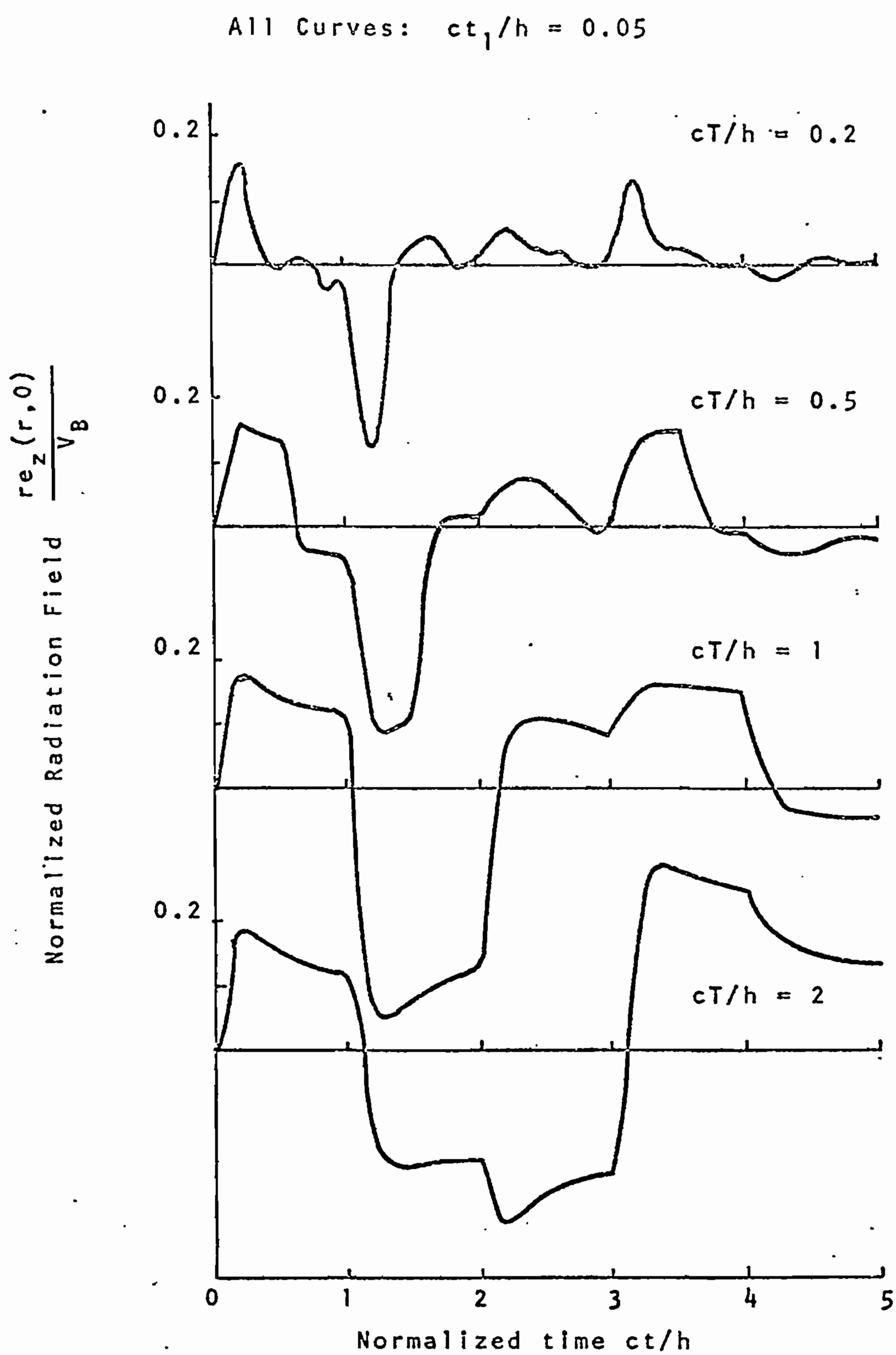


Figure 2.5. Radiated Pulses Predicted by Traveling Wave Theory

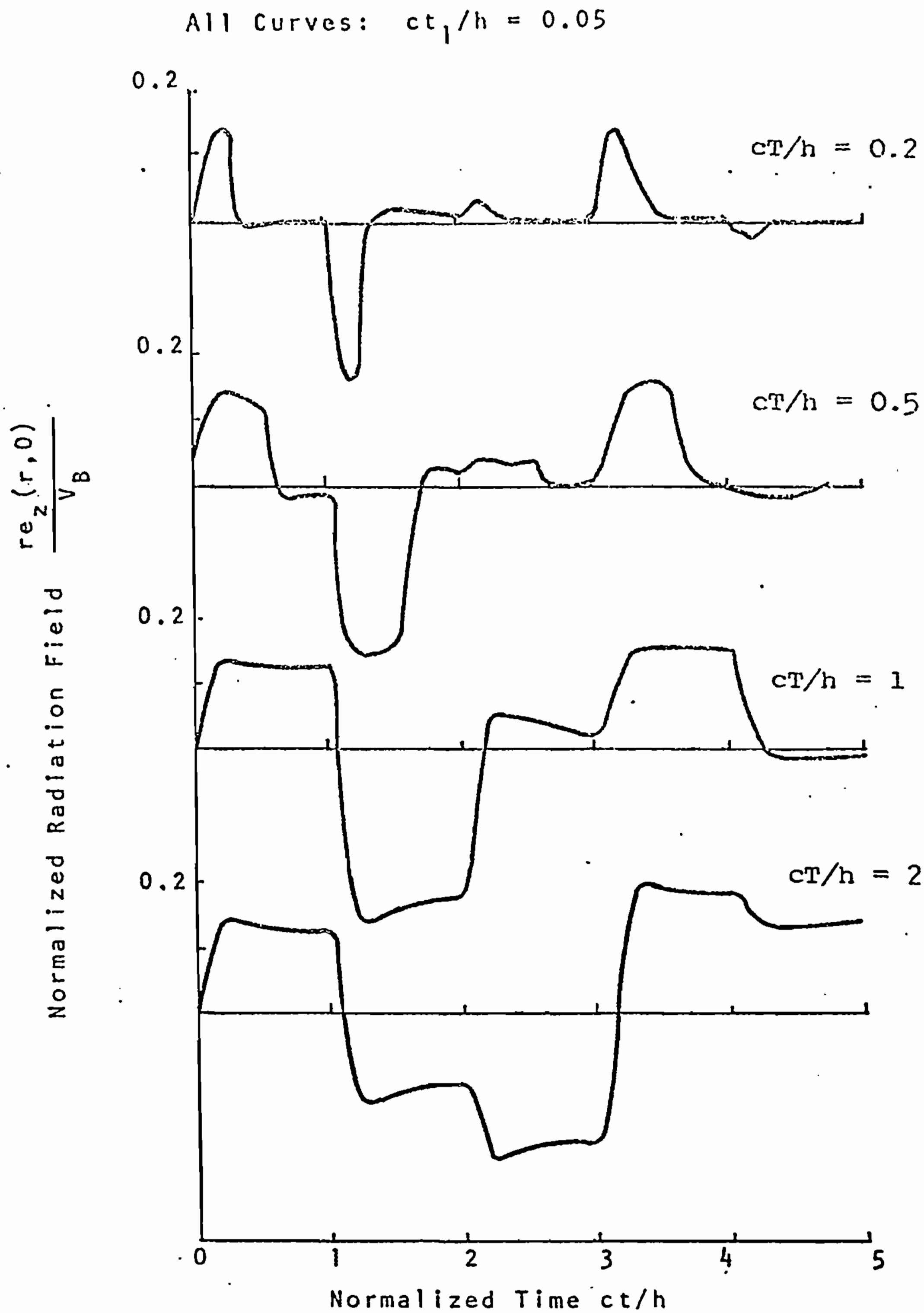


Figure 2.6. Radiated Pulsed Predicted by Schmitt, Harrison, and Williams[3]

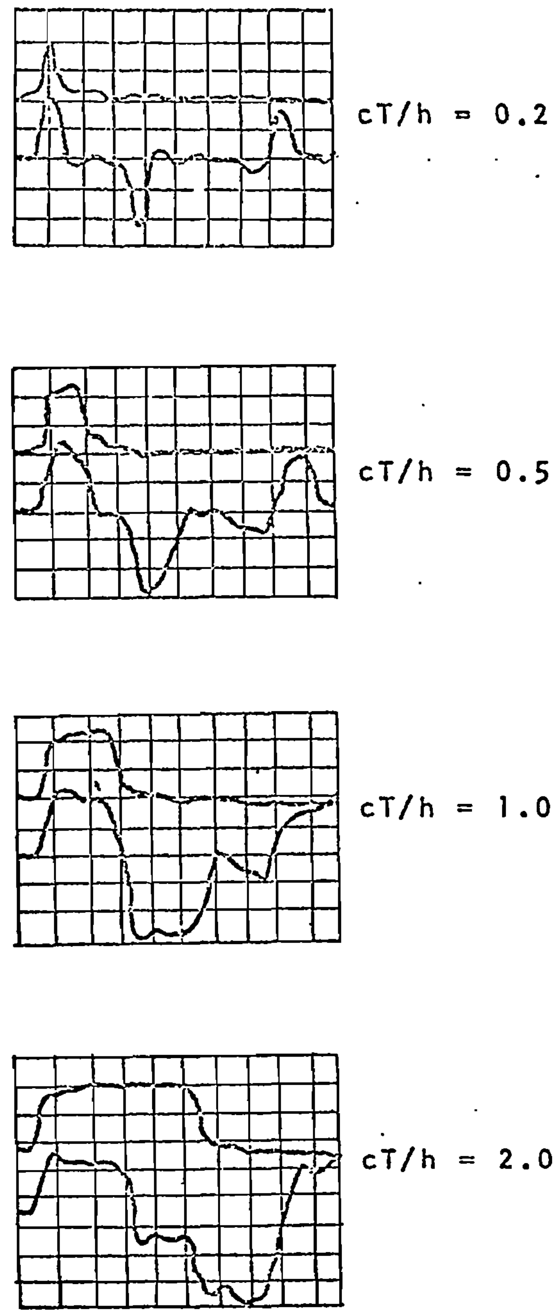


Figure 2.7. Transient Electric Fields Measured by Schmitt, Harrison, and Williams[3] $h=0.84$ meters, $h/a=904$, $r=1.52$ meters. For all sweeps, $ct_1/h=0.05$ and the time scale is 1.25 nsec/div. $V_g(t)$ is shown in the upper traces.

experimental configuration did not well satisfy the far field radiation requirement; thus, precise agreement between the measured data and the calculated data cannot be expected. In their publication, Schmitt, Harrison, and Williams[3] showed that the discrepancies between the measured data and the calculated data could be attributed to the difference in configuration at least until ct/h=2.

The comparison of the transients predicted by the traveling wave theory (Figure 2.5) with Figures 2.6 and 2.7 reveals good general agreement among all three sets of curves. The two theoretical predictions agree more closely with each other than with the measured data, although the traveling wave theory predicts somewhat higher levels and a larger pulse at the time that the current pulse, reflected by the end of the antenna, returns to the driving source.

Based upon this comparison and the previous comparisons of current distribution, it is expected that the traveling wave theory with linear attenuation is sufficiently accurate to describe the electromagnetic pulse radiated when a transient voltage is impressed across the input terminals of an impedance loaded dipole antenna.

CHAPTER 3

TRANSIENT ELECTRIC FIELD SYNTHESIS

The traveling wave antenna theory developed in Chapter 2 can be employed to predict accurately the time history of the electromagnetic pulse radiated when a fast-rising transient voltage is applied to a long impedance loaded dipole. However, due to the complexity of the equations involved, the theory is not well suited for application to the synthesis problem--the problem of determining the set of resistors with which to load the antenna to obtain a prescribed electromagnetic field pulse.

In this chapter, a simplified theory is employed to determine the current distribution on the symmetrically driven cylindrical antenna. The synthesis problem is then solved, using this simplified current distribution model.

In Section 3.4, examples of the pulses radiated when a transient voltage is applied to the input terminals of an antenna loaded with resistors are considered. The pulses predicted, using the simplified model, are compared to those predicted by the application of the more accurate traveling wave theory. This comparison indicates that the error expected when employing the synthesis procedure is not very severe.

3.1 The Symmetrically Driven Cylindrical Antenna --Simplified Theory--

The idealized antenna model to be considered is shown in Figure 3.1. It is a thin tubular model of infinite conductivity without endcaps. It has a total length of 2h and a radius of "a". It is assumed to be symmetrically driven across two narrow circumferential gaps located a distance, d, from the center of the antenna.

While the model selected is not as general as that used in the development of the traveling wave theory, it is of sufficient generality to be employed in the solution of the problem at hand.

The electric field on the surface of the idealized antenna is given by

$$E_s = -V_d\,[\delta(z-d) + \delta(z+d)], \qquad |z| \leq h. \tag{3.1}$$

Accordingly, Hallén's integral equation for the symmetrically driven antenna is

$$A_z^s(a,z) = \frac{\mu_o}{4\pi}\int_{-h}^{h} I^s(z')\,K\,(z,z')\,dz'$$

$$= \frac{1}{c}\left[\frac{V_d}{2}\left(e^{-jk_o|z-d|} + e^{-jk_o|z+d|}\right)\right.,$$

$$\left. + C\cos k_o z\right], \tag{3.2}$$

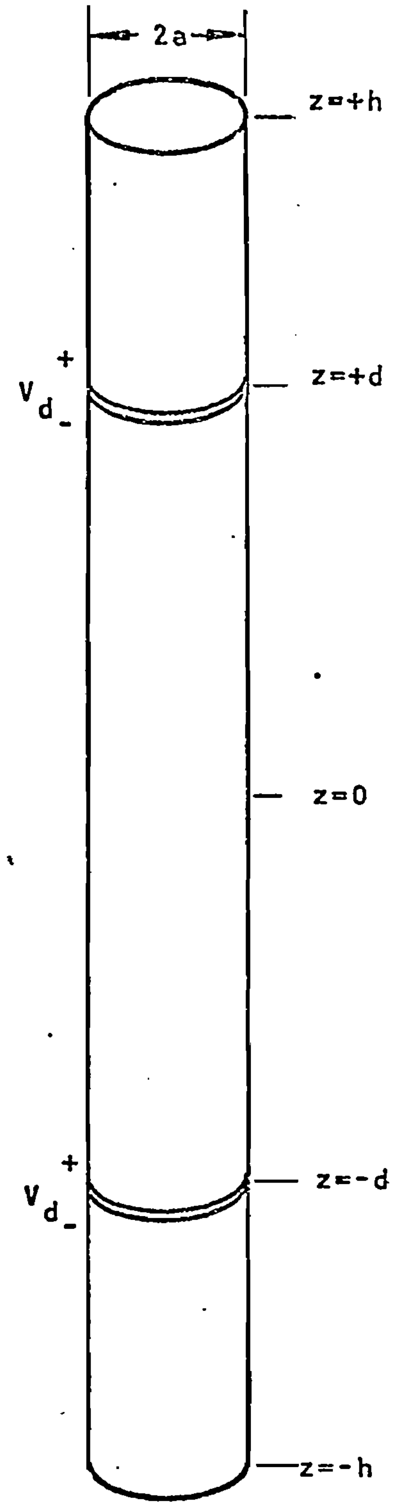


Figure 3.1. Idealized Symmetrically-Driven Antenna

where the kernel $K(z,z')$ is approximately

$$K(z,z') = \frac{e^{-jk_o R}}{R} , \tag{3.3}$$

and

$$R = \sqrt{a^2 + (z-z')^2} . \tag{3.4}$$

Some insight into the expected current distribution can be gained by noting that the real part of the kernel,

$$K_R(z,z') = \frac{\cos k_o R}{R} , \tag{3.5}$$

is a decaying oscillatory function with a very large peak at the point of $z = z'$. The imaginary part of the kernel,

$$K_I(z,z') = \frac{-\sin k_o R}{R} , \tag{3.6}$$

is also a decaying oscillatory function, but it does not have a very large peak at $z = z'$. Due to this peaking property of the kernel, the "expansion parameter" defined by

$$\Psi(z) \equiv \frac{4\pi A_z(a,z)}{\mu_o I(z)} = \frac{1}{I(z)} \int_{-h}^{h} I(z')K(z,z')dz' , \tag{3.7}$$

is almost a constant real value, ψ, independent of z, except near the ends of the antenna. The properties of the expansion parameter have been extensively studied by King.[33] He has shown that the value of ψ is nearly independent of the choice of current distributions used in (3.7). Physically, the implication is that the magnetic vector potential evaluated at a given point on the surface of the antenna is primarily determined by the current very close to that point.[34] If the approximation,

$$A_z^s(a,z) = \frac{\mu_o}{4\pi} \psi \, I^s(z), \tag{3.8}$$

is employed, (3.2) defines the current distribution on the antenna.

$$I^s(z) = \frac{4\pi}{\psi Z_o} \left[C \cos k_o z + \frac{V_d}{2} \left(e^{-jk_o|z-d|} + e^{-jk_o|z+d|}\right)\right]. \tag{3.9}$$

The constant C can be determined by employing the boundary condition that the current at the end of the antenna must vanish, I(h) = 0. The resulting value is

$$C = -V_d \,[\cos k_o d/\cos k_o h]\, e^{-jk_o h} \tag{3.10}$$

When (3.10) is substituted into (3.9), a simple approximation to the current distribution is obtained.

$$I^s(z) = \frac{j2\Pi V_d}{\psi Z_o \cos k_o h} [\sin k_o(h-|z-d|) + \sin k_o(h-|z+d|)]. \tag{3.11}$$

The result is simple enough to apply to the synthesis problem. Note that when d is allowed to approach zero, this current is twice the zeroth order solution given by King for the center driven antenna.[35] This results because the voltage applied to the center of the antenna is then $2V_d$, (3.1).

Before proceeding to the consideration of the multiply-loaded antenna, it is instructive to consider the transient response of the symmetrically driven antenna. Dividing (3.11) by V_d yields a transfer function relating the Fourier transform of the current on the antenna to the voltage applied symmetrically to the two pairs of input terminals on the antenna.

$$\frac{I^s(z)}{V_d} = \frac{j2\Pi}{\psi Z_o \cos k_o h} [\sin k_o(h-|z-d|) + \sin k_o(h-|z+d|)]. \tag{3.12}$$

Taking the inverse Fourier transform of (3.12) results in the "impulse response" of the antenna. This is the time history of the current which would be observed at a point z on the antenna if an impulsive voltage were applied to the terminals of the antenna.

In exponential notation, (3.12) is written

$$\frac{I^s(z)}{V_d} = \frac{2\pi}{\psi Z_o}\left[\frac{e^{+jk_o(h-|z-d|)} - e^{-jk_o(h-|z-d|)}}{e^{+jk_oh} - e^{-jk_oh}}\right.$$

$$\left. + \frac{e^{+jk_o(h-|z+d|)} - e^{-jk_o(h-|z+d|)}}{e^{+jk_oh} + e^{-jk_oh}}\right] . \qquad (3.13)$$

Multiply the numerator and the denominator of each term by $e^{-jk_oh} - e^{-jk_o3h}$.

$$\frac{I^s(z)}{V_d} = \frac{2\pi}{\psi Z_o}\left[\frac{e^{-jk_o|z-d|} - e^{-jk_o(2h - |z-d|)}}{1 - e^{-j4k_oh}}\right.$$

$$- \frac{e^{-jk_o(2h + |z-d|)} - e^{-jk_o(4h - |z-d|)}}{1 - e^{-j4k_oh}}$$

$$+ \frac{e^{-jk_o|z+d|} \quad e^{-jk_o(2h-|z+d|)}}{1 - e^{-j4k_oh}}$$

$$\left. - \frac{e^{-jk_o(2h + |z+d|)} - e^{-jk_o(4h - |z+d|)}}{1 - e^{-j4k_oh}}\right] . \qquad (3.14)$$

If ψ may be considered to be a frequency independent real constant, the inverse Fourier transform of (3.14) is easily obtained. With the expansion,

$$\frac{1}{1 - e^{-j4k_o h}} = 1 + e^{-j\omega 4h/c} + e^{-j\omega 8h/c} + \text{----}, \quad (3.15)$$

it is apparent that the denominator of each fractional term in (3.14) serves to make the time function represented by the numerator periodic after t = o, with a period $T = 4h/c$.

Employing (3.15), the inverse Fourier transform of (3.14) is written

$$\begin{aligned} y^s(z) = \frac{2\Pi}{\psi' Z_o} [& \delta(t - \frac{|z+d|}{c}) + \delta(t - \frac{|z-d|}{c}) \\ & - \delta(t - \frac{2h - |z+d|}{c}) - \delta(t - \frac{2h - |z-d|}{c}) \\ & - \delta(t - \frac{2h + |z+d|}{c}) - \delta(t - \frac{2h + |z-d|}{c}) \\ & + \delta(t - \frac{4h - |z+d|}{c}) + \delta(t - \frac{4h - |z-d|}{c})] \end{aligned} \quad (3.16)$$

for $t \le 4h/c$, and $y^s(z)$ is periodic in time with period $T = 4h/c$.

A bounce diagram[36] is employed in Figure 3.2 to illustrate the physical phenomena described by (3.16). At the time that the voltage is applied to each pair of driving terminals, a

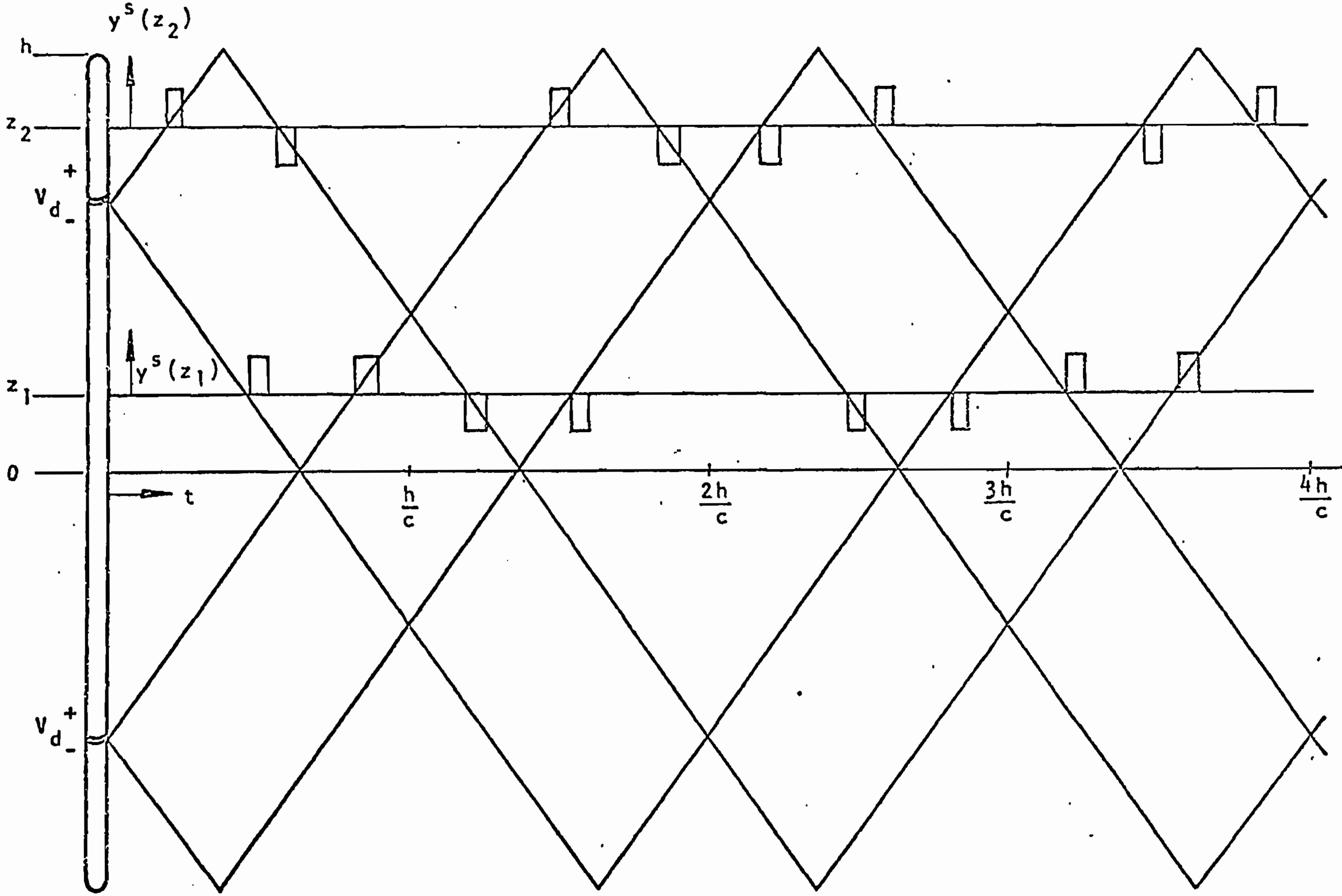


Figure 3.2. Current Pulses on a Symmetrically-Driven Antenna

pulse of current leaves each pair of terminals traveling in both directions. Each of the four current pulses impressed upon the antenna proceeds along the antenna until it is reflected by the end, when the polarity and the direction of the pulse are reversed and the pulse travels back toward the source. Each of the four current pulses bounces back and forth from one end of the antenna to the other. Due to the several simplifying assumptions which were required to obtain these results, the radiation of energy by the antenna has been neglected. Consequently, this simplified theory predicts that these pulses bounce back and forth forever, undistorted and unattenuated. In effect, the assumptions required have reduced the symmetrically driven cylindrical antenna to the symmetrically driven open ended transmission line shown in Figure 3.3. By examination of (3.16) it is apparent that the characteristic impedance of the transmission line must be

$$Z_{ch} = \psi Z_o / 4\Pi \qquad (3.17)$$

The quantity

$$Z_a = 2Z_{ch} = \psi Z_o / 2\Pi \qquad (3.18)$$

can also be defined as the ratio of the amplitude of the impulsive voltage applied at the input terminals of the antenna to the amplitude of the resultant input current

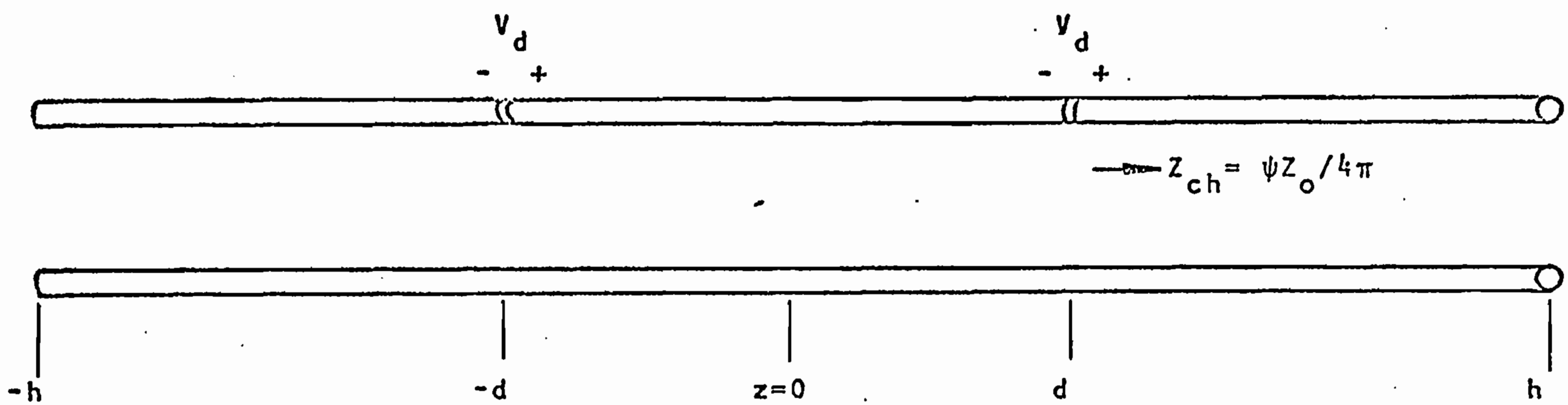


Figure 3.3. Transmission Line Model

impulse. Equation (3.18) can also be used to define the expansion parameter ψ in terms of a measurable quantity, Z_a. In Appendix 3, approximate solutions for Z_a and ψ are obtained by consideration of the infinite antenna theory.

One may question the validity of employing an antenna model which allows no radiation of energy, to determine the radiated electric field produced by the antenna. However, it is well known that the current distribution on a radiating cylindrical antenna is quite similar to the current distribution on an open-ended transmission line;[37] hence, a reasonable approximation to the radiated field should be obtained. Moreover, if the resistive loading of the antenna is significant, the energy dissipated in the antenna will far exceed the radiated energy. The neglected radiated energy will then be less important.

3.2 The Symmetric Resistance Loaded Dipole Antenna

The approximate current distribution on a dipole antenna symmetrically loaded with resistors can be obtained by application of the superposition and compensation theorems, as described in Chapter 1. The resultant equation for the total current distributed on the antenna when excited by a monochromatic voltage source is

$$I_T(z) = \frac{j2\pi V_o}{\psi Z_o \cos k_o h} \Big\{ \sin k_o(h - |z|) + \sum_{i=1}^{N-1} A_i [\sin k_o(h - |z-d_i|) + \sin k_o(h - |z+d_i|)] \Big\}, \qquad (3.19)$$

where:

V_o = the voltage applied to the antenna's center terminals

$(N-1)$ = the total number of resistor pairs,

A_i = the ratio of the voltage developed across the resistor R_i to the input voltage, determined by the compensation theorem,

$$A_i = -I_T(d_i)\, R_i/V_o, \quad i = 1, \text{---} (N-1), \qquad (3.20)$$

$d_i = ih/N$.

Only periodic loading need be considered, since any realizable aperiodic symmetric loading of a finite antenna can be described in some periodic system.

When (3.19) is substituted into (3.20), the following system of N-1 simultaneous equations results.

$$\sum_{j=1}^{N-1} A_j a_{ij} = - \sin k_o(h-d_i), \; i = 1, \text{---}, (N-1). \qquad (3.21)$$

Here

$$a_{ij} = \sin k_o(h - |d_i - d_j|) + \sin k_o(h - |d_i + d_j|), \; i \neq j \quad (3.22)$$

$$a_{ii} = \sin k_o h + \sin k_o(h - 2d_i) + \frac{1}{YR_i}, \quad (3.23)$$

and

$$Y = \frac{j2\Pi}{\psi Z_o \cos k_o h} . \quad (3.24)$$

The total approximate current distribution is obtained by substituting the solution to the (N-1) simultaneous equations back into (3.19). This required matrix inversion prevents the calculation of the transient response of the antenna directly from (3.19). However, another approach is possible. It has been shown that the approximations which have been made have reduced the antenna to an equivalent transmission line; transmission line concepts can therefore be used to find the approximate currents and voltages which will be observed on the antenna.

Consider the infinite transmission line circuit containing an impulsive voltage source and a resistor R_d, shown in Figure 3.4. The superposition and compensation theorems may again be employed to find all the current pulses on the line and their location at any time. The total current, $i_T(z)$, may be separated into two components,

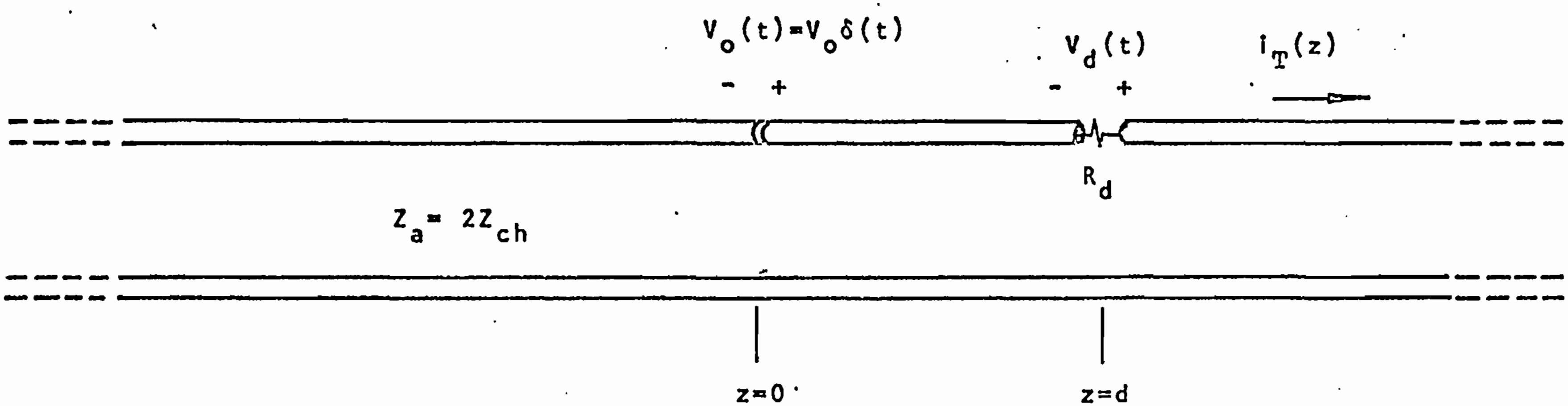


Figure 3.4. Infinite Transmission Line Circuit

$$i_T(z) = i(z,o) + i(z,d). \tag{3.25}$$

$i(z,o)$ is the impulsive current emanating from the impulsive voltage source applied at $t = 0$, and located at $z = 0$.

$$i(z,o) = \frac{V_B}{Z_a}\,\delta\left(t - \frac{|z|}{c}\right). \tag{3.26}$$

$i(z,d)$ is the impulsive current emanating from the equivalent voltage source $v_d(t)$ located at $z = d$.

$$i(z,d) = v_d\left(t - \frac{|z-d|}{c}\right)/Z_d. \tag{3.27}$$

$v_d(t)$ may be determined by use of the Compensation Theorem.

$$v_d(t) = -i_T(d)\,R_d. \tag{3.28}$$

Substitution of (3.26) and (3.27) into (3.28) yields

$$v_d(t) = V_{R_d}\,\delta\left(t - \frac{|d|}{c}\right). \tag{3.29}$$

Here,

$$V_{R_d} = -\frac{R_d}{R_d + Z_a}\,V_B\,, \tag{3.30}$$

With (3.29) and (3.30), a complete description of currents on the infinite transmission line has been obtained.

The approximate voltages and currents observed on a multiply-loaded antenna may be obtained in the same manner. A bounce diagram, as shown in Figure 3.5, helps to clarify the time dependence of the voltages and currents observed on the antenna. An impulsive voltage source of unit amplitude shall be assumed to be applied to the center terminals of the antenna at t = 0. Due to the periodicity of the loading, the voltage across each of the resistors is a collection of delta functions. The first impulse occurs at the time that the current wave arrives from the source, and additional impulses occur at time intervals of $2\Delta/c$ thereafter, where Δ is the separation distance between two resistors.

For an infinite antenna periodically loaded with resistors, the voltage observed across any resistor R_i may be written,

$$A_i(t) = \sum_{j=1}^{\infty} A_{i,j}\ \delta[t-(i+2(j-1))\ \Delta/c)], \qquad (3.31)$$

for $i = 1,2, -----,\infty$.

The amplitude of each impulse, $A_{i,j}$, can be ascertained by inspection of Figure 3.5.

The amplitude of the first impulse across any resistor is proportional to the sum of the input impulse amplitude and the contributions from other resistors located between the source and the resistor in question.

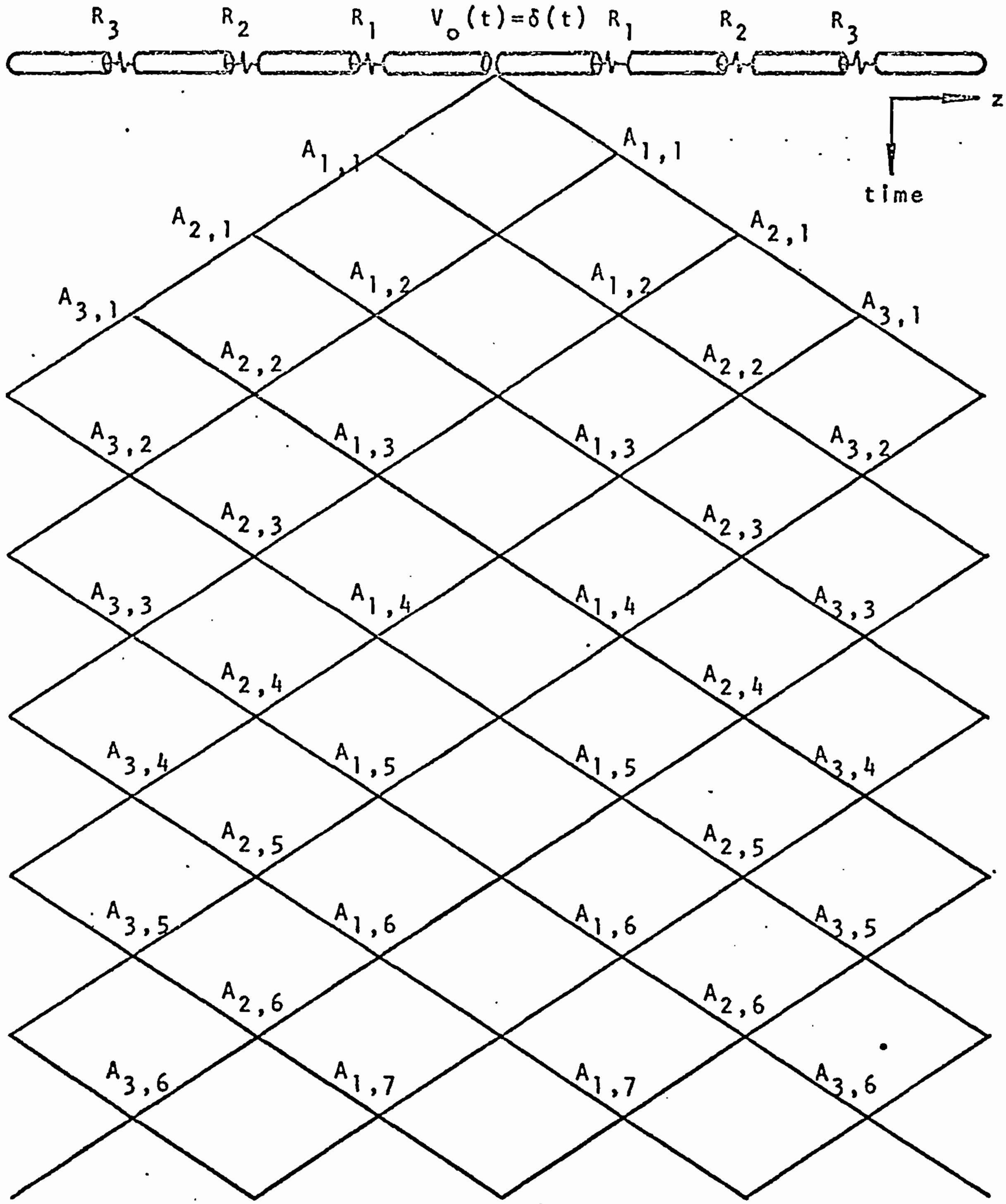


Figure 3.5. Bounce Diagram for a Multiply-Loaded Antenna

$$A_{i,1} = -\frac{R_i}{R_i+Z_a}\left(1 + \sum_{k=1}^{i-1} A_{k,1}\right). \qquad (3.32)$$

On the bounce diagram, the amplitude is proportional to the sum of all amplitudes recorded on the upper half of the diagonal passing through $A_{i,1}$. The amplitudes of the succeeding impulses, $A_{i,j}$, where j is greater than 2, can also be determined from Figure 3.5. $A_{i,j}$ is proportional to the sum of all the amplitudes recorded on the upper half of the two diagonals passing through $A_{i,j}$.

$$A_{i,j} = -\frac{R_i}{R_i+Z_a}\left(\sum_{k=i+1}^{i+j-1} A_{k,j+i-k} + \sum_{k=1}^{i-1} A_{k,j} + \sum_{k=1}^{j-1} A_{k,j-k}\right), \qquad (3.33)$$

where i = 1,2, -------; ∞ , and j=2,3,-----∞ .

Having solved the case of the symmetrically loaded infinite structure, the truncation to a finite structure with N-1 symmetric resistor pairs is easily accomplished. If R_N is allowed to approach infinity, voltages across resistors R_i, $i > N$, approach zero and voltages across resistors R_i, $i < N$, approach those observed on the finite structure. Equations (3.32) and (3.33) become:

$$A_{i,1} = -\frac{R_i}{R_i+Z_a}\left(1 + \sum_{k=1}^{i-1} A_{k,1}\right), \qquad (3.34)$$

for i = 1,2,------, (N-1) and,

$$A_{N,1} = -\left(1 + \sum_{k=1}^{N-1} A_{k,1}\right), \tag{3.35}$$

$$A_{i,j} = -\frac{R_i}{R_i+Z_a}\left(\sum_{k=i+1}^{M1} A_{k,i+j-k} + \sum_{k=1}^{i-1} A_{k,j} + \sum_{k=1}^{M2} A_{k,j-k}\right), \tag{3.36}$$

for i = 1,2------, N, and j = 2,3,----∞,

where

$$M1 = \mathrm{Min}(N,i+j-1), \tag{3.37}$$

and

$$M2 = \mathrm{Min}(N,j-1). \tag{3.38}$$

The notation L = Min(J,K) means that L should be set equal to the smaller of the two integers J or K.

The current anywhere on the structure, driven by a delta function source of unit amplitude, can be obtained by superposition.

$$y(z,t) = \frac{1}{Z_a} \left\{ \delta\left(t - \frac{|z|}{c}\right) + \sum_{i=1}^{N} \left[A_i\left(t - \frac{|z-i\Delta|}{c}\right) + A_i\left(t - \frac{|z+i\Delta|}{c}\right) \right] \right\}. \quad (3.39)$$

Substituting (3.31) into (3.39) yields

$$y(z,t) = \frac{1}{Z_a} \left\{ \delta\left(t - \frac{|z|}{c}\right) + \sum_{i=1}^{N} \sum_{j=1}^{\infty} A_{i,j} \left[\delta\left(t - \frac{|z-i\Delta|}{c} - \frac{(i+2(j-1))\Delta}{c}\right) + \delta\left(t - \frac{|z+i\Delta|}{c} - \frac{(i+2(j-1))\Delta}{c}\right) \right] \right\}. \quad (3.40)$$

Equation (3.40) completes the description of the currents and voltages observed on the antenna when excited by a unit impulse. The response of the antenna to an arbitrary time function can be obtained by use of the convolution theorem. For example, the current at point z on the antenna when a causal voltage, $v_o(t)$, is applied to the input terminals is

$$i_T(z) = \int_0^t v_o(t-\tau) y(z,\tau) d\tau. \quad (3.41)$$

Since y(z,t) is a collection of delta functions, the integration is easily performed.

$$i_T(z) = \frac{1}{Z_a}\left\{v_o\left(t - \frac{|z|}{c}\right)\right.$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{\infty} A_{i,j} \left[v_o\left(t - \frac{|z-i\Delta|}{c} - \frac{(i+2(j-1))\Delta}{c}\right)\right.$$

$$\left.\left. + v_o\left(t - \frac{|z+i\Delta|}{c} - \frac{(i+2(j-1))\Delta}{c}\right)\right]\right\} . \qquad (3.42)$$

3.3 Radiation Field

The approximate current distribution resulting from a single pair of symmetric sources is given in (3.11). The electric field at the point of observation (as shown in Figure 1.1) resulting from this distribution can be obtained by substituting (3.11) into (1.1).

$$E_z(r,o) = -\frac{j\omega\mu_o}{4\pi r} e^{-jk_or} \int_{-h}^{h} \left[\frac{j2\pi V_d}{\psi Z_o \cos k_oh}\right]$$

$$[\sin k_o(h - |z+d|) + \sin k_o(h - |z-d|)]dz. \qquad (3.43)$$

Carrying out the integration,

$$G^s(d,\omega) \equiv \frac{E_z(r,o)}{V_d} = -\frac{2\,e^{-jk_or}}{\psi r}\left[1 - \frac{\cos k_od}{\cos k_oh}\right]. \qquad (3.44)$$

$G^s(d,\omega)$, defined by (3.44), is the Fourier transform of the "impulse response" of the network made up of symmetrically driven antenna and the transmission path to the point of observation. Taking the inverse transform of (3.44) will yield the electric field observed when a unit voltage impulse is applied to the input terminals of the antenna. Multiplying the numerator and the denominator of the latter term by

$$e^{-j\omega h/c} - e^{-j\omega 3h/c}, \tag{3.45}$$

The following form results

$$G^{s}(d,\omega) = -\frac{2e^{-j\omega r/c}}{\psi r}\left[1 - \frac{e^{-j\omega(h+d)/c} - e^{j\omega(3h+d)/c}}{1 - e^{-j\omega 4h/c}}\right.$$

$$\left. - \frac{e^{-j\omega(h-d)/c} - e^{-j\omega(3h-d)/c}}{1 - e^{-j\omega 4h/c}}\right] . \tag{3.46}$$

When ψ may be considered to be a frequency independent real constant, the inverse Fourier transform of (3.46) is a collection of delta functions.

The factor,

$$e^{-j\omega r/c}, \tag{3.47}$$

accounts for the expected delay between the application of the signal to the input terminals and the observable effect at the point of observation. If signals at the point of observation are measured in retarded time, this factor needs no further consideration. The "1" within the brackets accounts for the first pulse to reach the point of observation. It corresponds to the time that the current pulses emerge from the driving sources onto the antenna. The fractional terms represent components periodic after t = 0, with period T = 4h/c. These components represent successive radiations from the ends of the antenna as each of the four current pulses placed on the antenna by the two impulse voltage sources bounce back and forth from one end of the antenna to the other. This type of behavior has been verified by both measurement and calculation, using much more accurate theories.[2,3]

The radiation field, when a monochromatic voltage source is applied to the center of a symmetric multiply-loaded antenna, may be obtained by superposition.

$$G_T(\omega) = -\frac{1}{\psi}\frac{1}{r} e^{-jk_o r} \left\{ \left[1 - \frac{1}{\cos k_o h}\right] + 2 \sum_{i=1}^{N-1} A_i \left[1 - \frac{\cos k_o d_i}{\cos k_o h}\right] \right\} . \qquad (3.48)$$

A_i has been defined (3.20). Equation (3.48) can also be written in the form

$$G_T(\omega) = -\frac{1}{\psi r} e^{-j\omega r/c} \Bigg\{ 1 + 2 \sum_{i=1}^{N-1} A_i$$

$$- \frac{e^{-j\omega h/c} - e^{-j\omega 3h/c}}{1 - e^{-j\omega 4h/c}}$$

$$-2 \sum_{i=1}^{N-1} A_i \frac{e^{-j\omega(h-d_i)/c} - e^{-j\omega(3h-d_i)/c}}{1 - e^{-j\omega 4h/c}}$$

$$-2 \sum_{i=1}^{N-1} A_i \frac{e^{-j\omega(h+d_i)/c} - e^{-j\omega(3h+d_i)/c}}{1 - e^{-j\omega 4h/c}} \Bigg\} . \quad (3.49)$$

In (3.49), the fractional terms again represent successive reflection from the ends of the antenna. The form of (3.49) can be simplified.

In solving for the voltage across each resistor in a finite antenna periodically loaded with (N-1) resistor pairs, it was convenient to consider the infinite structure loaded with N resistor pairs, where the last resistor was so large that total reflection occurs. This same principle can be applied here. If total reflection occurs at R_N, no current exists above R_N and (3.49) becomes,

$$G_T(\omega) = -\frac{1}{\psi r} e^{-j\omega r/c} \left\{ 1 + 2 \sum_{i=1}^{N} A_i \right\} . \quad (3.50)$$

The inverse Fourier transform of (3.50) is apparent. In retarded time, when the antenna is driven by a unit impulse, the radiated field is a collection of impulses. The first occurs when the current pulse emerges from the driving source; the second, of opposite polarity, occurs when the current pulse reaches the first resistor. Further pulses occur as the primary wave from the center of the antenna passes each successive periodically spaced resistor.

$$g_T(t) = -\frac{1}{\psi r}\left\{\delta(t)+2\sum_{i=1}^{N}\sum_{j=1}^{\infty} A_{ij}\,\delta\!\left(t-(i+2(j-1))\frac{\Delta}{c}\right)\right\}. \tag{3.51}$$

The radiated field which occurs when an arbitrary causal voltage pulse is applied to the input terminals of the antenna can be obtained by use of the convolution theorem

$$e_z(r,o) = \int_o^t v_o(t-\tau)\, g_T(\tau)\,d\tau \,. \tag{3.52}$$

Substitution of (3.51) into (3.52) yields

$$e_z(r,o) = -\frac{1}{\psi r}\left\{v_o(t)\right.$$

$$\left. + 2\sum_{i=1}^{N}\sum_{j=1}^{\infty} A_{ij}\, v_o\!\left(t-(i+2(j-1))\frac{\Delta}{c}\right)\right\} . \tag{3.53}$$

With (3.53), the approximate radiated field at the point of observation has been completely evaluated.

3.4 Accuracy of the Simplified Theory

In Chapter 2, an antenna theory was developed which would permit the prediction, with reasonable accuracy, of the radiated electromagnetic pulse observed when a transient voltage is impressed across the input terminals of a dipole antenna symmetrically loaded with resistors. In this chapter accuracy of the antenna theory was sacrificed to obtain a simple description of the current on the antenna and the radiated electromagnetic pulse that can be applied to the antenna synthesis problem. It was reasoned that the radiation of energy, neglected by the simplified theory, would become less important as the energy dissipated within the antenna is increased.

In this section, the predicted response of three resistively loaded antennas is computed, using both theories, and the results are compared. The voltage impressed across the input terminals of the antenna was the type of square pulse that can be generated in the laboratory, previously used by Schmitt and described in Section 2.5,[3]

$$v_o(t) = V_B[f(t)u(t) - f(t-T)u(t-T)], \qquad (3.54)$$

where

$$f(t) = 1 - (1+t/t_1)\, e^{-t/t_1}, \qquad (3.55)$$

and u(t) is the unit step function,

$$u(t) = 0, \qquad t < 0$$

$$u(t) = 1, \qquad t \geq 0 \tag{3.56}$$

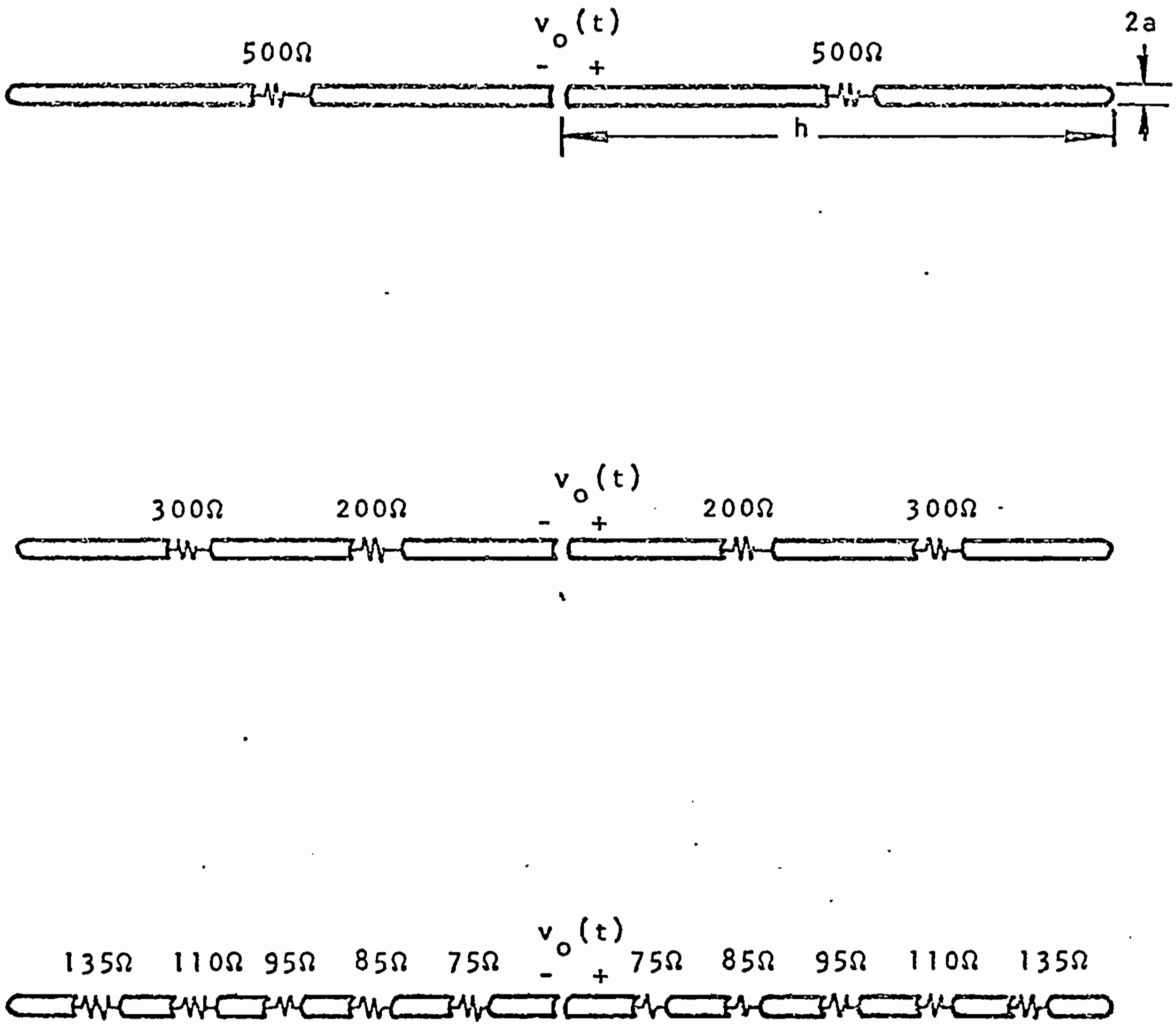
The time history of the input voltage is shown in Figure 2.4c. For the calculation made in this section, $V_B = 1$, and the pulse duration related to T was made much larger than the time interval of observation. The waveform then approximates the type of unit voltage step which can be generated in the laboratory.

Again, following Schmitt, all times were normalized to the one-way travel time between the input at the center and the end of the antenna.[3] The parameter t_1, related to the rise time of the pulse, was taken to be 0.05h/c, as used in Section 2.5.

The three antenna configurations considered are shown in Figure 3.6. The length-to-diameter ratio of the antenna, h/a, was taken to be 904, for which the "thickness parameter,"[11]

$$\Omega \equiv 2 \ln \frac{2h}{a} = 15.0 \ . \tag{3.57}$$

The predicted electromagnetic field pulses are shown in Figure 3.7. The top curve for each antenna configuration represents the radiated electromagnetic field predicted by the traveling wave theory given in Chapter 2 and computed by the procudre given in Chapter 1. Numerical integration



For all antennas: $2h/a = 904$, $\Omega = 15.0$

Figure 3.6. Impedance-Loaded Antennas

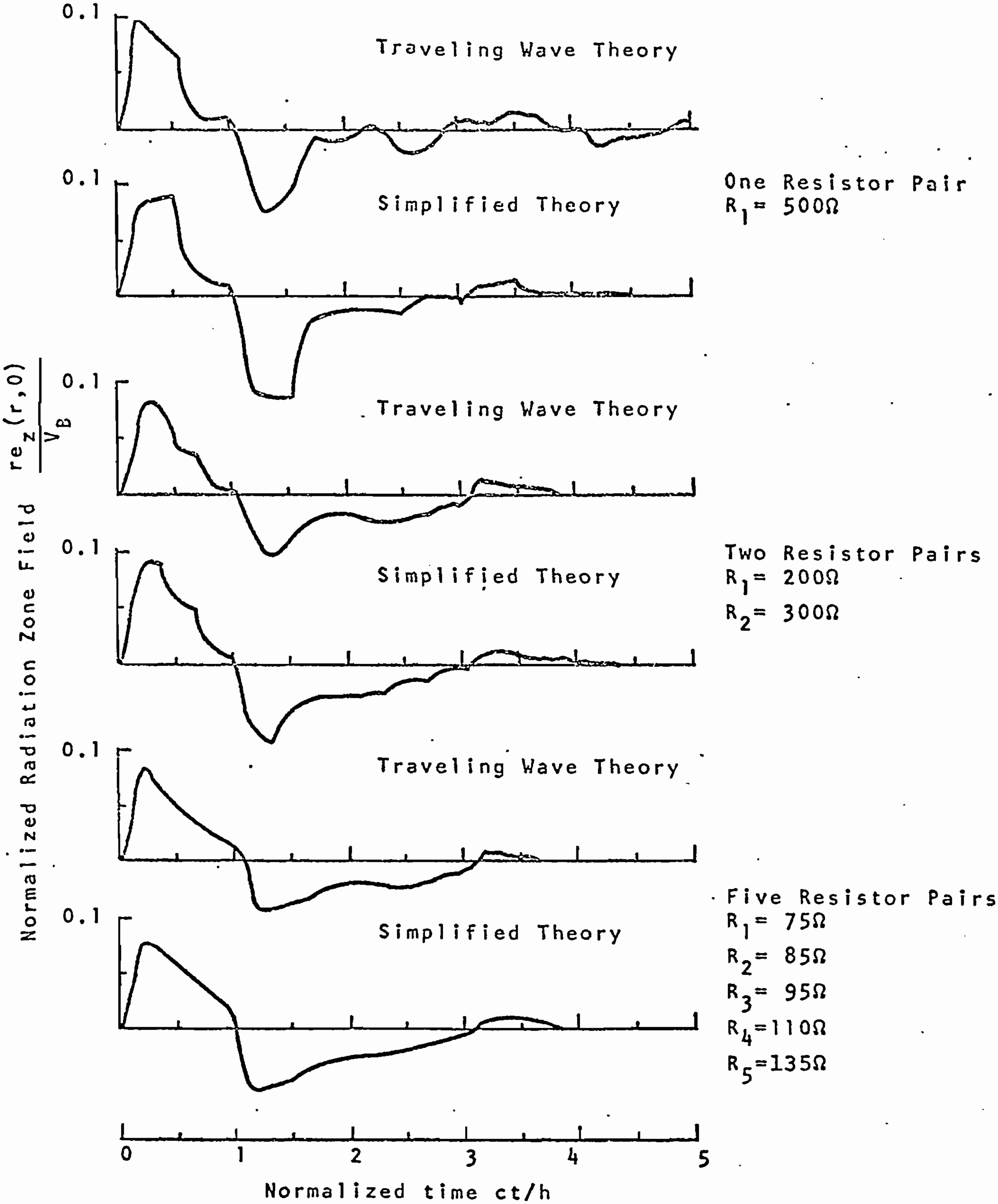


Figure 3.7. Predicted Radiated Electric Field Transients

was used to compute the inverse Fourier transform. The voltage across each resistor was obtained by inverting the matrix given by (1.3) at each of the frequencies used in the integration. It should be noted that the traveling wave theory yielded current distributions which only approximately satisfied the reciprocity condition,

$$Y(d_i, d_j) = Y(d_j, d_i) \quad . \tag{3.58}$$

Although the differences were not great, the average value of the two numbers was used in calculating the matrix coefficients required in solving (1.4).

The lower curve for each antenna configuration in Figure 3.7 is the radiated electromagnetic field predicted by the simplified theory. Equation (3.53) was used to obtain this result. The characteristic antenna impedance, Z_a, used in this calculation was 667 ohms, as determined from the considerations given in Appendix 3.

In each of the configurations considered, the values of the resistors that loaded the antenna were such that the amplitude of the electric field pulse had become quite small when the current wave reached the end of the structure. Reflection of the current wave from the end of the structure is apparent in the negative swing of the pulse at t=h/c.

As anticipated, the agreement between the two predicted field pulses becomes more acceptable as the resistive loading of the antenna becomes more significant. While the agreement is never perfect, it is evident that the simplified theory will yield a good engineering approximation to the pulse produced. The simplified theory can therefore be employed in the desired synthesis procedure to yield at least an initial selection of resistors.

An interesting point is that although the discrete nature of the loading is apparent in the predicted electric field transient when the antenna is loaded with either one or two resistor pairs, loading the antenna with five resistor pairs resulted in a smooth predicted electric field transient. This results because the travel time between resistors is smaller than the rise time of the pulse.

3.5 Synthesis of Radiated Electromagnetic Field Transients

In preceding sections of this chapter, an approximate formula has been developed for the electric field transient radiated when an arbitrary voltage pulse is applied to the input terminals of a dipole antenna symmetrically loaded with resistors. It has also been shown that this simplified approximation does yield a good engineering approximation to the radiated pulse expected.

From this point, the procedure may be inverted to obtain a selection of resistors with which to load the antenna to approximate a prescribed electric field transient. A voltage step will be assumed to be the input voltage to the antenna.

The voltage step has been found to be a waveform which can be generated with high-voltage equipment and therefore is a typical choice for a voltage input. This choice limits the type of functions which can be simulated to fast-rising pulses that decay with time.

Another limitation on the type of pulse that can be generated by the antenna is not apparent in the formulas given. This limitation is that the average value of the radiated field must be zero. It is essential to the physical nature of radiation that the signal be time-varying, no static (DC) fields can be radiated by a meaningful source of finite dimensions. This does not limit the use of the impedance loaded dipole however, since, presumably, any radiated field pulse which one would desire to approximate would be subject to the same requirement.

From (3.53), it is apparent that the rise time of the radiated field is equal to the rise time of the input voltage pulse, provided the spacing between resistors is no smaller than

$$\Delta_{min} = ct_r , \qquad (3.59)$$

where t_r is the rise time of the desired field pulse. If the rise time of the high-voltage step is made equal to the rise time of the desired electric field transient, and the spacing between resistors is no smaller than Δ_{min}, the rise time of both the input voltage and the desired electric field transient can be ignored in the process of selecting resistors with which to load the antenna.

The development begins with the normalized radiated field pulse observed when a voltage step is applied to the input terminals of the antenna. From (3.53),

$$-\frac{\psi \, r e_z(r,o)}{V_B} = \Big\{u(t)$$

$$+ 2 \sum_{i=1}^{N} \sum_{j=1}^{\infty} A_{ij} \, u[t-(i+2(j-1)) \, \Delta/c]\Big\}, \qquad (3.60)$$

where V_B is the amplitude of the voltage step and $u(t)$ is the unit step function previously defined.

Consider the transient electric field illustrated in Figure 3.8a. In Figure 3.8b, a normalized pulse is shown. The normalization is somewhat unusual; the average of the values of pulse at $t=0$ and at $t=t_1$ was selected as the normalizing factor, where $[t_1, t_2 \text{----}]$ is a periodic time sequence which will adequately describe the character of the pulse. In Figure 3.8c, a step function approximation of the normalized pulse is illustrated. If the time interval between steps is equated to the travel time between resistors on the antenna, the step approximation to e_N is written

$$e_a(t) = [u(t) + \sum_{k=1}^{\infty} M_k u(t-k\Delta/c)] \, , \qquad (3.61)$$

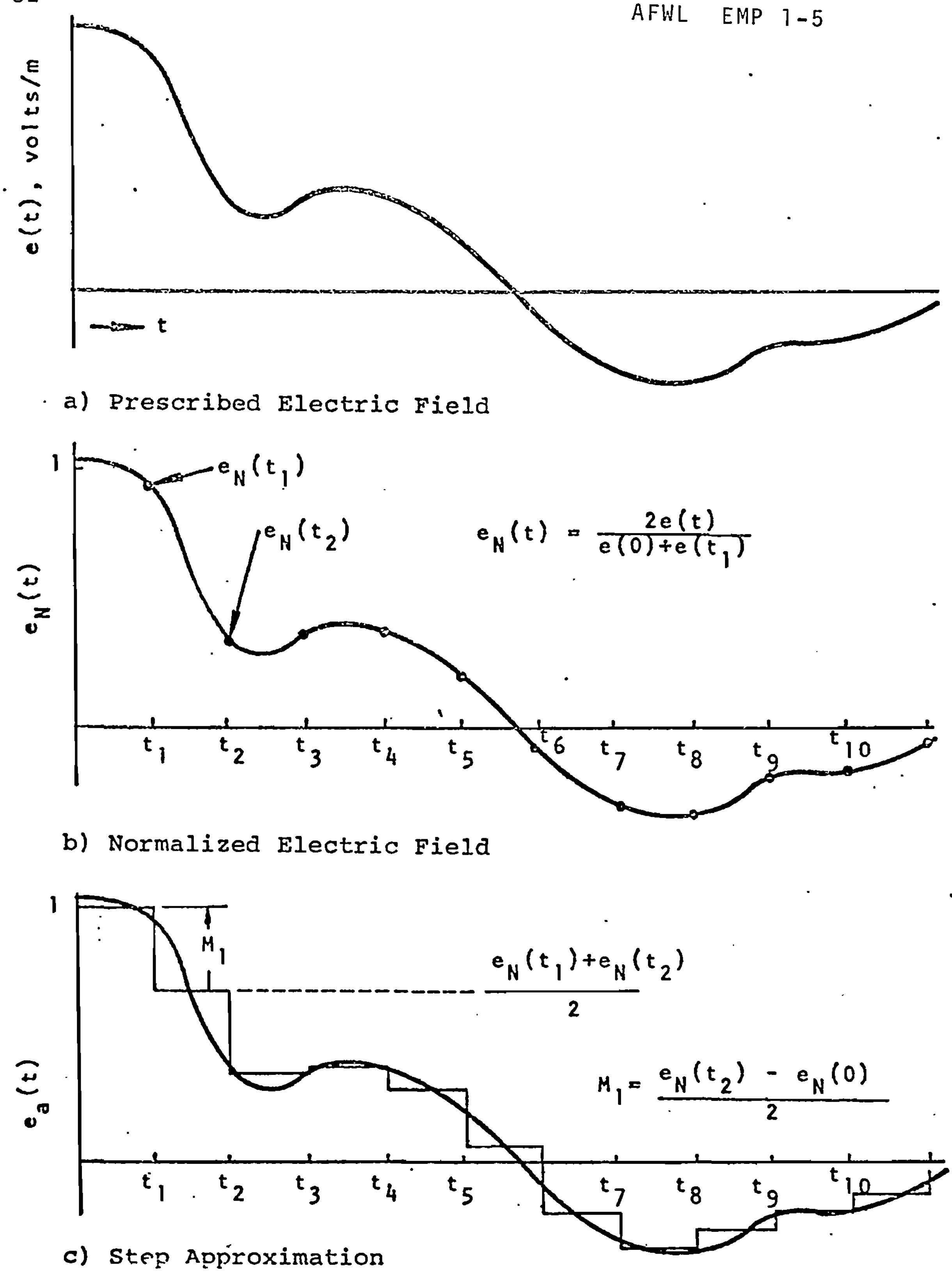


Figure 3.8. Development of a Step Approximation to an Electric Field Transient

where decrements, M_k, have been selected to yield an average fit to the normalized pulse,

$$M_k = \frac{e_N(t_{k+1})-e_N(t_{k-1})}{2} \ , \ k=1,2,----\infty \ . \qquad (3.62)$$

It is evident that the step size may be reduced by adding more resistors to the antenna to increase the accuracy of the approximation within the limitation imposed by (3.59).

An equation implicitly involving the selection of resistors on the antenna is obtained by equating the right sides of (3.60) and (3.61), which yields

$$M_k = 2 \sum_{j=1}^{JM} A_{k-2(j-1),j} \qquad k \leq N' \qquad (3.63)$$

where $k \leq (N-1)$ and $JM=(k+1)/2$ if k is odd, and $JM=k/2$ if k is even. $A_{k,1}$ is the voltage step developed across the kth resistor by the primary wave of current traveling away from the driving terminals toward the end of the antenna, while the other terms in the summation are terms due to second and higher order bounces between the previously chosen resistors. Each resistor may be chosen to yield the desired decrement in the electric field at the time that the primary wave reaches that resistor. The following equation for $A_{k,1}$ results:

$$A_{k,1} = M_k/2 - \sum_{j=2}^{JM} A_{k-2(j-1),j} \ . \qquad (3.64)$$

An apparent limitation on the type of waveform that can be generated is that the quantity $A_{k,1}$, determined by (3.64), must be negative. If the desired waveform is a smooth decaying function, the indicated summation is normally positive and the requirement is easily satisfied. Having determined the required value of $A_{k,1}$, the needed value of R_k can be obtained from (3.34).

Using the procedure given above, all the resistor values have been chosen when the primary wave reaches the end of the antenna. The waveform after t = h/c is outside the control of the designer. Accordingly, the length of the antenna should be chosen so that the significant part of the desired transient will be obtained when the wave reaches the end of the antenna. As an example of the effect of truncating the antenna on the radiated waveshape, the pulse

$$e(t) = 2e^{-1.386\, t/t_c} - e^{-0.693t/t_c} \qquad (3.65)$$

was considered. This pulse has a maximum value of one at t=0, the signal passes through zero at $t=t_c$, and the average value of the pulse is zero.

Since the transient has no clearly defined end, it serves as a good example of the effect of truncating the antenna on the pulse shape. The spacing between resistors was chosen to be

$$\Delta = ct_c/20. \tag{3.66}$$

The values of the resistors required to approximate the pulse were determined by use of (3.64) and (3.34). In Figure 3.9, the normalized field transient $e_N(t)$ is compared to the step function approximation of the pulses generated by four antennas ranging in length from $0.75c\ t_c$ to $1.50c\ t_c$. Of interest here is the deviation of the approximation from the desired function after the wave reaches the end of the antenna, $t=h/c$. This is the portion of the approximation over which the designer has no control. For shorter antennas, the deviation is quite severe, but as the length of the antenna is increased, more of the waveform has been accurately described and less current approaches the end of the structure. This reduces the amplitude of the reflection from the end of the antenna and results in an overall improvement of the quality of the approximation.

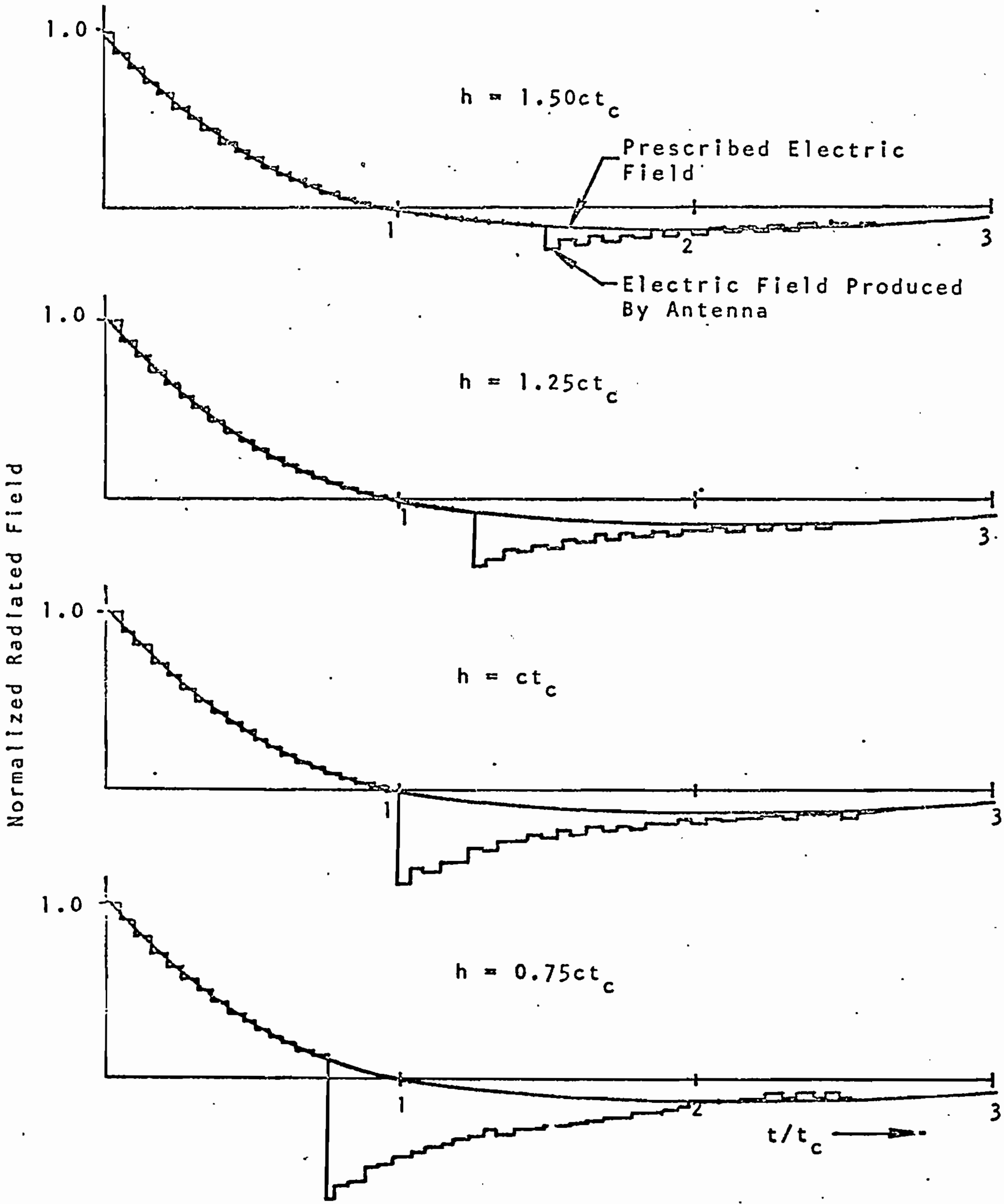


Figure 3.9. Effect of Finite Antenna Length on Pulse Synthesis

CHAPTER 4

SUMMARY

The primary use of dipole antennas multiply-loaded with resistors has been as an electromagnetic pulse generator. In this report, two important problems associated with this application have been considered.

In Chapter 2, an approximate solution for the current distribution on an arbitrarily driven cylindrical antenna was obtained. The problem of determining the current distribution on a cylindrical antenna has been treated many times in the past. However, no solution was available which could be employed to calculate the electromagnetic field transient radiated when an arbitrary voltage source is applied to the input terminals of a long impedance loaded dipole antenna.

The success of the theory developed in Chapter 2 results from expressing the current distribution as the summation of attenuated traveling waves emanating from the driving point and from the ends of the antenna. This formulation overcomes several of the disadvantages found in previous solutions. First, there is no difference in the form of the solution when the antenna is center driven or arbitrarily driven. Second, there is no fundamental limitation on the electrical length of the antenna which may be considered. Third, the accuracy of the solution may be increased by adding self-evident additional terms to the attenuation function. And fourth, the simplicity of the solution makes it ideal for those problems where a large number of calculations must be made.

In Chapter 3, using a simplified antenna theory, a synthesis procedure was evolved that would yield the selection of resistors with which to load a dipole antenna so that the radiated electric field transient approximates some prescribed waveshape. A voltage step was assumed to be impressed across the antenna terminals, limiting the type of wave form which can be simulated to fast-rising transients which generally decay with time. The accuracy of this synthesis procedure was investigated by comparing the pulses predicted by this simplified theory with the more accurate predictions obtained by employing the traveling wave theory developed in Chapter 2. These results indicate that, for resistively loaded antennas, the simplified antenna theory will yield a reasonably good approximation to the radiated pulse. The synthesis procedure can therefore be used to obtain a good initial selection of resistors with which to load the antenna.

APPENDIX 1

COMPUTATION OF SURFACE FIELDS DUE TO CURRENT COMPONENTS

To compute the reactions required in the traveling wave theory, the axial electric field along the surface of the antenna produced by each current component must be evaluated. Throughout this appendix and Appendix 2, extensive use is made of the fact that although there are six components of current assumed on the antenna, there are only two forms:

$$U_\ell = e^{-jk_o|z-\ell|} , \tag{A1.1}$$

and

$$V_\ell = k_o|z-\ell|e^{-jk_o|z-\ell|} . \tag{A1.2}$$

The calculation of the electric field is made by first determining the magnetic vector potential along the antenna; then the value of the surface electric field is determined from (2.11)

$$E_s = -j \frac{\omega}{k_o^2} \left\{\frac{\partial^2 A_z(a;z)}{\partial z^2} + k_o^2 A_z(a,z)\right\} \tag{A1.3}$$

The magnetic vector potential evaluated at the antenna surface is given by

$$A_z(a,z) = \frac{\mu_o}{4\pi}\int_{-h}^{h} I(z')K(z,z')\, dz \qquad (A1.4)$$

It will be assumed that the antenna is thin enough, $k_o a \leq 0.1$, that the kernel is well approximated by

$$K(z,z') = \frac{e^{-jk_o R}}{R} \qquad (A1.5)$$

where

$$R = \sqrt{a^2 + (z-z')^2} \qquad (A1.6)$$

Tangential Axial Field due to Current Component $U_\ell = e^{-jk_o|z-\ell|}$

Evaluate the magnetic vector potential at the surface of the antenna,

$$A_z(a,z) = \frac{\mu_o}{4\pi}\int_{-h}^{h} \frac{e^{-jk_o(R + |z'-\ell|)}}{R}\, dz' \,, \qquad (A1.7)$$

where

$$R = \sqrt{a^2 + (z-z')^2} \,. \qquad (A1.8)$$

Removing the absolute value notation requires separating the integral into two parts.

$$A_z(a,z) = \frac{\mu_o}{4\pi}\left\{I_1 + I_2\right\} , \qquad (A1.9)$$

where

I_1 and I_2 are given by

$$I_1 = \int_{\ell}^{h} \frac{e^{-jk_o(R + (z'-\ell))}}{R}\, dz' , \qquad (A1.10)$$

and

$$I_2 = \int_{-h}^{\ell} \frac{e^{-jk_o(R - (z'-\ell))}}{R}\, dz' . \qquad (A1.11)$$

With the substitution, $\sigma = z-z'$,

$$I_1 = e^{-jk_o(z-\ell)} \int_{z-h}^{z-\ell} \frac{e^{-jk_o(R_\sigma - \sigma)}}{R_\sigma} d\sigma, \qquad (A1.12)$$

and

$$I_2 = e^{+jk_o(z-\ell)} \int_{z-\ell}^{z+h} \frac{e^{-jk_o(R_\sigma + \sigma)}}{R_\sigma} d\sigma . \qquad (A1.13)$$

The differentiation required to determine $E_s(z)$ can be carried out quite easily, using Leibniz's rule.

$$\frac{\partial I_1}{\partial z} = - jk_o I_1 + \frac{e^{-jk_o R_{1\ell}}}{R_{1\ell}} - \frac{e^{-jk_o(R_{1h}+(h-\ell))}}{R_{1h}} , \qquad (A1.14)$$

and

$$\frac{\partial I_2}{\partial z} = + jk_o I_2 - \frac{e^{-jk_o R_{1\ell}}}{R_{1\ell}} + \frac{e^{-jk_o(R_{2h}+(h+\ell))}}{R_{2h}} \qquad (A1.15)$$

where

$$R_{1\ell} = \sqrt{a^2+(z-\ell)^2} \qquad (A1.16)$$

$$R_{1h} = \sqrt{a^2+(z-h)^2} \qquad (A1.17)$$

$$R_{2h} = \sqrt{a^2+(z+h)^2} . \qquad (A1.18)$$

Accordingly,

$$\frac{\partial^2 I_1}{\partial z^2} = -k_o^2 I_1 - e^{-jk_o R_{1\ell}}\left[\frac{jk_o}{R_{1\ell}} + \frac{jk_o(z-\ell)}{R_{1\ell}^2} + \frac{(z-\ell)}{R_{1\ell}^3}\right]$$

$$+ e^{-jk_o(R_{1h}+(h-\ell))}\left[\frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^2} + \frac{(z-h)}{R_{1h}^3}\right], \tag{A1.19}$$

and

$$\frac{\partial^2 I_2}{\partial z^2} = -k_o^2 I_2 - e^{-jk_o R_{1\ell}}\left[\frac{jk_o}{R_{1\ell}} - \frac{jk_o(z-\ell)}{R_{1\ell}^2} - \frac{(z-\ell)}{R_{1\ell}^3}\right]$$

$$+ e^{-jk_o(R_{2h}+(h+\ell))}\left[\frac{jk_o}{R_{2h}} - \frac{jk_o(z+h)}{R_{2h}^2} - \frac{(z+h)}{R_{2h}^3}\right]. \tag{A1.20}$$

E_{s,U_ℓ}, the surface field due to current component U_ℓ, may be determined from (A1.3).

$$E_{s,U_\ell} = -j \frac{\omega}{k_o^2} \left\{ \frac{\partial^2 A_z(a,z)}{\partial z^2} + k_o^2 A_z(a,z) \right\}$$

$$= - j \frac{Z_o}{4\pi k_o} \left\{ \frac{\partial^2}{\partial z^2} + k_o^2 \right\} \left\{ I_1 + I_2 \right\} . \qquad \text{(A1.21)}$$

$$E_{s,U_\ell} = -j \frac{Z_o}{4\pi k_o} \left\{ - j2k_o \frac{e^{-jk_o R_{1\ell}}}{R_{1\ell}} \right.$$

$$+ e^{-jk_o(R_{1h}+(h-\ell))} \left[\frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^2} + \frac{(z-h)}{R_{1h}^3} \right]$$

$$\left. + e^{-jk_o(R_{2h}+(h+\ell))} \left[\frac{jk_o}{R_{2h}} - \frac{jk_o(z+h)}{R_{2h}^2} - \frac{(z+h)}{R_{2h}^3} \right] \right\} .$$

(A1.22)

<u>Tangential Axial Field due to Current Component</u>

$$\underline{V_\ell = k_o|z-\ell|e^{-jk_o|z-\ell|}}$$

Evaluate the magnetic vector potential at the surface of the antenna,

$$A_z(a,z) = \frac{\mu_o k_o}{4\Pi}\left\{I_3 - I_4\right\} ; \qquad \text{(A1.23)}$$

where

$$I_3 \equiv \int_\ell^h \frac{(z'-\ell)\, e^{-jk_o(R+(z'-\ell))}}{R}\, dz', \qquad \text{(A1.24)}$$

and

$$I_4 \equiv \int_{-h}^\ell \frac{(z'-\ell)\, e^{-jk_o(R-(z'-\ell))}}{R}\, dz. \qquad \text{(A1.25)}$$

With the substitution, $\sigma = z-z'$, I_3 and I_4 are written

$$I_3 = e^{-jk_o(z-\ell)}\left\{(z-\ell)\int_{z-h}^{z-\ell}\frac{e^{-jk_o(R_\sigma-\sigma)}}{R_\sigma}d\sigma\right.$$

$$\left. - \int_{z-h}^{z-\ell}\frac{\sigma e^{-jk_o(R_\sigma-\sigma)}}{R_\sigma}\,d\sigma\right\}, \qquad \text{(A1.26)}$$

and

$$I_4 = e^{+jk_o(z-\ell)}\left\{(z-\ell)\int_{z-\ell}^{z+h}\frac{e^{-jk_o(R_\sigma+\sigma)}}{R_\sigma}d\sigma\right.$$

$$\left. - \int_{z-\ell}^{z+h}\frac{\sigma e^{-jk_o(R_\sigma+\sigma)}}{R_\sigma}\,d\sigma\right\} . \qquad \text{(A1.27)}$$

Derivatives are obtained, using Leibniz's rule,

$$\frac{\partial I_3}{\partial z} = -jk_oI_3 + e^{-jk_o(z-\ell)}\int_{z-h}^{z-\ell}\frac{e^{-jk_o(R_\sigma-\sigma)}}{R_\sigma}\,d\sigma$$

$$- (h-\ell)\,\frac{e^{-jk_o(R_{1h}+(h-\ell))}}{R_{1h}} , \qquad \text{(A1.28)}$$

$$\frac{\partial^2 I_4}{\partial z^2} = -k_o^2 I_4 + j2k_o e^{+jk_o(z-\ell)} \int_{z-\ell}^{z+h} \frac{e^{-jk_o(R_\sigma+\sigma)}}{R_\sigma} d\sigma$$

$$- \frac{e^{-jk_o R_{1\ell}}}{R_{1\ell}} + \frac{e^{-jk_o(R_{2h}+(h+\ell))}}{R_{2h}}$$

$$- (h+\ell)\, e^{-jk_o(R_{2h}+(h+\ell))} \left[\frac{jk_o}{R_{2h}} - \frac{jk_o(z+h)}{R_{2h}^2} - \frac{(z+h)}{R_{2h}^3}\right], \qquad \text{(A1.31)}$$

and E_{s,V_ℓ}, the tangential component of surface field due to current component V_ℓ is given by (A1.3),

$$E_{s,V_\ell} = -j \frac{\omega}{k_o^2} \left\{\frac{\partial^2 A_z}{\partial z^2} + k_o^2 A_z\right\}$$

$$= -j \frac{Z_o}{4\Pi} \left\{\frac{\partial^2}{\partial z^2} + k_o^2\right\} \left\{I_3 - I_4\right\}. \qquad \text{(A1.32)}$$

Hence,

$$E_{s,V_\ell} = -j \frac{Z_o}{4\pi} \Big\{ - j2k_o \, e^{-jk_o(z-\ell)} \int_{z-h}^{z-\ell} \frac{e^{-jk_o(R_\sigma - \sigma)}}{R_\sigma} d\sigma$$

$$-j2k_o \, e^{+jk_o(z-\ell)} \int_{z-\ell}^{z+h} \frac{e^{-jk_o(R_\sigma + \sigma)}}{R_\sigma} d\sigma$$

$$+ 2 \frac{e^{-jk_o R_{1\ell}}}{R_{1\ell}} - \frac{e^{-jk_o(R_{1h} + (h-\ell))}}{R_{1h}} - \frac{e^{-jk_o(R_{2h} + (h+\ell))}}{R_{2h}}$$

$$+ (h-\ell) \, e^{-jk_o(R_{1h} + (h-\ell))} \left[\frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^2} + \frac{(z-h)}{R_{1h}^3} \right]$$

$$+ (h+\ell) \, e^{-jk_o(R_{2h} + (h+\ell))} \left[\frac{jk_o}{R_{2h}} - \frac{jk_o(z+h)}{R_{2h}^2} - \frac{(z+h)}{R_{2h}^3} \right] \Big\} . \tag{A1.33}$$

Equations (A1.22) and (A1.33) are used in Appendix 2 to compute the reactions required to determine the current coefficients of the traveling wave antenna theory.

$$\frac{\partial I_4}{\partial z} = + jk_o I_4 + e^{+jk_o(z-\ell)} \int_{z-\ell}^{z+h} \frac{e^{-jk_o(R_\sigma+\sigma)}}{R_\sigma} d\sigma$$

$$- (h+\ell) \frac{e^{-jk_o(R_{2h}+(h+\ell))}}{R_{2h}} \quad ; \qquad \text{(A1.29)}$$

where

$$R_{1h} = \sqrt{a^2+(z-h)^2} \text{ and } R_{2h} = \sqrt{a^2+(z+h)^2} .$$

$$\frac{\partial^2 I_3}{\partial z^2} = - k_o^2 I_3 - j2k_o e^{-jk_o(z-\ell)} \int_{z-h}^{z-\ell} \frac{e^{-jk_o(R_\sigma-\sigma)}}{R_\sigma} d\sigma$$

$$+ \frac{e^{-jk_o R_{1\ell}}}{R_{1\ell}} - \frac{e^{-jk_o(R_{1h}+(h-\ell))}}{R_{1h}}$$

$$+ (h-\ell) e^{-jk_o(R_{1h}+(h-\ell))} \left[\frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^2} + \frac{(z-h)}{R_{1h}^3}\right],$$

(A1.30)

APPENDIX 2

COMPUTATION OF REACTIONS

Although there are six current components employed in expanding the current distribution on the antenna, only two forms are assumed:

$$U_\ell = e^{-jk_o|z-\ell|}$$

and

$$V_\ell = k_o|z-\ell|e^{-jk_o|z-\ell|} .$$

Having only two assumed forms greatly reduces the computational effort required to compute the coefficients in the reaction matrix; three algebraic equations are sufficient to define all 36 elements of the matrix.

In this appendix, three equations for the reactions, $\langle U_\ell, U_m\rangle$, $\langle U_\ell, V_m\rangle$, and $\langle V_\ell, V_m\rangle$ are determined. No additional approximations are introduced at this point. The reactions are computed exactly, within the limitation imposed by the use of the electric field, which is computed under the assumption that the antenna is thin, $k_o a \ll 1$.

Evaluation of $\langle U_\ell, U_m \rangle$

$$\langle U_\ell, U_m \rangle = \int_{-h}^{h} E_{s}\{U_\ell\} U_m \, dz \quad . \tag{A2.1}$$

Substitution of (A1.22) into (A2.1) yields

$$\langle U_\ell, U_m \rangle = -j \frac{Z_o}{4\pi} \left\{ J_1(\ell,m) + e^{-jk_o(h-\ell)} J_2(m) + e^{-jk_o(h+\ell)} J_2(-m) \right\}, \tag{A2.2}$$

where

$$J_1(\ell,m) = -2j \int_{-h}^{h} e^{-jk_o|z-m|} \left\{ \frac{e^{-jk_o R_{1\ell}}}{R_{1\ell}} \right\} dz, \tag{A2.3}$$

$$J_2(m) = \frac{1}{k_o} \int_{-h}^{h} e^{-jk_o|z-m|} e^{-jk_o R_{1h}}$$

$$\cdot \left\{ \frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^{2}} + \frac{(z-h)}{R_{1h}^{3}} \right\} dz, \tag{A2.4}$$

and

$$R_{1h} = \sqrt{a^2 + (z-h)^2} \tag{A2.5}$$

$$R_{1\ell} = \sqrt{a^2 + (z-\ell)^2}. \tag{A2.6}$$

Each of the integrals will be evaluated separately;

$$J_1(\ell,m) = -2j \left\{ \int_m^h \frac{e^{-jk_o(R_{1\ell}+(z-m))}}{R_{1\ell}} dz \right.$$

$$\left. + \int_{-h}^m \frac{e^{-jk_o(R_{1\ell}-(z-m))}}{R_{1\ell}} dz \right\} . \qquad \text{(A2.7)}$$

With the substitution, $S = k_o(z-\ell)$ and $T = k_o(\ell-z)$, $J_1(\ell,m)$ is written as

$$J_1(L,M) = -2j \left\{ e^{-j(L-M)} \int_{M-L}^{H-L} \frac{e^{-j(R_S+S)}}{R_S} dS \right.$$

$$\left. + e^{+j(L-M)} \int_{L-M}^{H+L} \frac{e^{-j(R_T+T)}}{R_T} dT \right\} , \qquad \text{(A2.8)}$$

where $L = k_o\ell$, $H = k_oh$, $A = k_oa$, etc,

and

$$R_S = \sqrt{A^2+S^2},$$

$$R_T = \sqrt{A^2+T^2} .$$

Define the commonly occurring integral,

$$F(X) \equiv \int_0^X \frac{e^{-j(R_S+S)}}{R_S} dS. \qquad \text{(A2.9)}$$

Then,

$$J_1(L,M) = -2j\left\{e^{-j(L-M)}[F(H-L)-F(M-L)] + e^{+j(L-M)}[F(H+L)-F(L-M)]\right\} . \qquad (A2.10)$$

So, $J_1(L,M)$ has been completely determined. The function $F(X)$ occurs in all the reactions, and will be further discussed in the last section of this appendix.

$J_2(M)$ remains to be evaluated.

$$J_2(M) = \frac{1}{k_o}\left\{\int_m^h e^{-jk_o(R_{1h}+(z-m))} \cdot \left[\frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^2} + \frac{(z-h)}{R_{1h}^3}\right] dz + \int_{-h}^m e^{-jk_o(R_{1h}-(z-m))} \cdot \left[\frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^2} + \frac{(z-h)}{R_{1h}^3}\right] dz\right\} . \qquad (A2.11)$$

Substitute $S = k_o(z-h)$ and $T = k_o(h-z)$,

$$J_2(M) = \left\{ - e^{-j(H-M)} \int_0^{M-H} e^{-j(R_S+S)} \left[\frac{j}{R_S} + \frac{jS}{R_S^2} + \frac{S}{R_S^3} \right] dS \right.$$

$$\left. + e^{+j(H-M)} \int_{H-M}^{2H} e^{-j(R_T+T)} \left[\frac{j}{R_T} - \frac{jT}{R_T^2} - \frac{T}{R_T^3} \right] dT \right\} . \quad (A2.12)$$

The integrand of the first integral is a perfect differential. The second integral may be integrated by parts.

Taking

$$U = e^{-j2T} \text{ and } dV = e^{-j(R_T-T)} \left[\frac{j}{R_T} - \frac{jT}{R_T^2} - \frac{T}{R_T^3} \right] dT,$$

J_2 can be evaluated as:

$$J_2(M) = e^{-j(H-M)} \left. \frac{e^{-j(R_S+S)}}{R_S} \right|_0^{M-H}$$

$$+ e^{+j(H-M)} \left\{ \left. \frac{e^{-j(R_T+T)}}{R_T} \right|_{H-M}^{2H} \right.$$

$$\left. + 2j \int_{H-M}^{2H} \frac{e^{-j(R_T+T)}}{R_T} dT \right\} . \quad (A2.13)$$

So that

$$J_2(M) = \frac{e^{-j(H+M+\sqrt{A^2+4H^2})}}{\sqrt{A^2+4H^2}} - \frac{e^{-j(H-M+A)}}{A}$$

$$+ 2je^{+j(H-M)}[F(2H)-F(H-M)] \quad . \qquad (A2.14)$$

Substituting (A2.10) and (A2.14) into (A2.2) yields

$$\langle U_\ell, U_m\rangle = \frac{Z_o}{2\Pi}\Big\{j\,\frac{e^{-j(2H+A)}\cos(L+M)}{A} - j\,\frac{e^{-j(2H+P)}\cos(L-M)}{P}$$

$$+ e^{+j(L-M)}[F(2H)-F(H-M)-F(H+L)+F(L-M)]$$

$$+ e^{-j(L-M)}[F(2H)-F(H+M)-F(H-L)+F(M-L)]\Big\} \; , \qquad (A2.15)$$

where

$$P = \sqrt{A^2+4H^2} .$$

<u>Evaluation of $\langle U_\ell, V_m\rangle$</u>

$$\langle U_\ell, V_m\rangle = \int_{-h}^{H} E_{s,U_\ell}\cdot V_m \, dz . \qquad (A2.16)$$

Substituting (A1.22) into (A2.16) yields

$$\langle U_\ell, V_m \rangle = -j \frac{Z_o}{4\Pi} \Big\{ K_1(\ell,m) + e^{-jk_o(h-\ell)} K_2(m)$$

$$+ e^{-jk_o(h+\ell)} K_2(-m) \Big\} , \qquad (A2.17)$$

where

$$K_1(\ell,m) = -2j \int_{-h}^{h} k_o|z-m| e^{-jk_o|z-m|} \left\{ \frac{e^{-jk_o R_{1\ell}}}{R_{1\ell}} \right\} dz , \qquad (A2.18)$$

$$K_2(m) = \frac{1}{k_o} \int_{-h}^{h} k_o|z-m| e^{-jk_o(R_{1h}+|z-m|)}$$

$$\cdot \left\{ \frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^{\;2}} + \frac{(z-h)}{R_{1h}^{\;3}} \right\} dz , \qquad (A2.19)$$

and

$$R_{1h} = \sqrt{A^2+(z-h)^2} , \qquad (A2.20)$$

$$R_{1\ell} = \sqrt{A^2+(z-\ell)^2} . \qquad (A2.21)$$

Each of the integrals will be evaluated separately;

$$K_1(\ell,m) = -2j\left\{ \int_m^h k_o(z-m)\, \frac{e^{-jk_o(R_{1\ell}+(z-m))}}{R_{1\ell}}\, dz - \int_{-h}^{m} k_o(z-m)\, \frac{e^{-jk_o(R_{1\ell}-(z-m))}}{R_{1\ell}}\, dz \right\}. \quad \text{(A2.22)}$$

With the substitutions, $S = k_o(z-\ell)$ and $T = k_o(\ell-z)$, (A2.22) is written

$$K_1(L,M) = e^{-j(L-M)}\left\{-2j \int_{M-L}^{H-L} S\, \frac{e^{-j(R_S+S)}}{R_S}\, dS - 2j\,(L-M) \int_{M-L}^{H-L} \frac{e^{-j(R_S+S)}}{R_S}\, dS\right\}$$

$$+\, e^{+j(L-M)}\left\{-2j \int_{L-M}^{H+L} T\, \frac{e^{-j(R_T+T)}}{R_T}\, dT - 2j\,(M-L) \int_{L-M}^{H+L} \frac{e^{-j(R_T+T)}}{R_T}\, dT\right\}, \quad \text{(A2.23)}$$

where

$L = k_o\ell$, $H = k_o h$, $A = k_o a$, etc.

Define the commonly occurring function $G(X)$ by

$$G(X) = 2j \int_0^X S\, \frac{e^{-j(R_S+S)}}{R_S}\, dS. \quad \text{(A2.24)}$$

The properties of $G(X)$ are discussed in the last section of this appendix.

In terms of the defined auxiliary functions, $K_1(L,M)$ is

$$K_1(L,M) = e^{-j(L-M)}\Big\{G(M-L)-G(H-L)-2j(L-M)[F(H-L)-F(M-L)]\Big\}$$

$$+ e^{+j(L-M)}\Big\{G(L-M)-G(H+L)-2j(M-L)[F(H+L)-F(L-M)]\Big\} . \quad \text{(A2.25)}$$

$K_2(m)$ remains to be evaluated.

$$K_2(m) = \frac{1}{k_o}\Big\{\int_m^h k_o(z-m)e^{-jk_o(R_{1h}+(z-m))}$$

$$\cdot \left[\frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^2} + \frac{(z-h)}{R_{1h}^3}\right] dz$$

$$- \int_{-h}^m k_o(z-m)\, e^{-jk_o(R_{1h}-(z-m))}$$

$$\cdot \left[\frac{jk_o}{R_{1h}} + \frac{jk_o(z-h)}{R_{1h}^2} + \frac{(z-h)}{R_{1h}^3}\right] dz\Big\} . \quad \text{(A2.26)}$$

With the substitutions, $S = k_o(z-h)$ and $T = k_o(h-z)$, $K_2(m)$ becomes

$$K_2(M) = e^{-j(H-M)}\Big\{ - \int_0^{M-H} S\, e^{-j(R_S+S)}\left[\frac{j}{R_S} + \frac{jS}{R_S^{\,2}} + \frac{S}{R_S^{\,3}}\right] dS$$

$$- (H-M) \int_0^{M-H} e^{-j(R_S+S)}\left[\frac{j}{R_S} + \frac{jS}{R_S^{\,2}} + \frac{S}{R_S^{\,3}}\right] dS\Big\}$$

$$+ e^{+j(H-M)}\Big\{ \int_{H-M}^{2H} T\, e^{-j(R_T+T)}\left[\frac{j}{R_T} - \frac{jT}{R_T^{\,2}} - \frac{T}{R_T^{\,3}}\right] dT$$

$$- (H-M) \int_{H-M}^{2H} e^{-j(R_T+T)}\left[\frac{j}{R_T} - \frac{jT}{R_T^{\,2}} - \frac{T}{R_T^{\,3}}\right] dT\Big\}. \quad (A2.27)$$

The second and fourth integrals were encountered in determining $J_2(M)$. (A2.12).

The first and third integrals are evaluated by parts, with

$$U_1 = S \quad , dV_1 = -e^{-j(R_S+S)}\left[\frac{j}{R_S} + \frac{jS}{R_S^{\,2}} + \frac{S}{R_S^{\,3}}\right] dS,$$

$$U_2 = Te^{-j2T}, dV_2 = e^{-j(R_T-T)}\left[\frac{j}{R_T} - \frac{jT}{R_T^{\,2}} - \frac{T}{R_T^{\,3}}\right] dT.$$

Then,

$K_2(M)$ is given by

$$K_2(M) = e^{-j(H-M)}\left\{S \frac{e^{-j(R_S+S)}}{R_S}\Bigg|_0^{M-H} - \int_0^{M-H} \frac{e^{-j(R_S+S)}}{R_S}\,dS\right.$$

$$+ (H-M)\left.\frac{e^{-j(R_S+S)}}{R_S}\Bigg|_0^{M-H}\right\} + e^{+j(H-M)}\left\{T\frac{e^{-j(R_T+T)}}{R_T}\Bigg|_{H-M}^{2H}\right.$$

$$- \int_{H-M}^{2H}(1-j2T)\frac{e^{-j(R_T+T)}}{R_T}\,dT - (H-M)\frac{e^{-j(R_T+T)}}{R_T}\Bigg|_{H-M}^{2H}$$

$$\left. - j2(H-M)\int_{H-M}^{2H}\frac{e^{-j(R_T+T)}}{R_T}\,dT\right\}. \qquad \text{(A2.28)}$$

In terms of the auxiliary functions, F and G,

$$K_2(M) = \frac{(H+M)}{P}e^{-j(P+H+M)} - \frac{(H-M)}{A}e^{-j(H-M+A)} - e^{-j(H-M)}F(M-H)$$

$$- e^{+j(H-M)}\Big\{[1+j2(H-M)][F(2H)-F(H-M)]-G(2H)+G(H-M)\Big\},$$

$$\text{(A2.29)}$$

where

$$P = \sqrt{A^2+4H^2}.$$

Evaluation of $\langle V_\ell, V_m \rangle$

$$\langle V_\ell, V_m \rangle = \int_{-h}^{h} E_s \{ V_\ell \cdot V_m \, dz \ . \qquad \text{(A2.30)}$$

Substituting the equations for the electric field (A1.33) and the current component into (A2.30) yields

$$\begin{aligned} \langle V_\ell, V_m \rangle = -\frac{Z_o}{8\Pi} \Big\{ & e^{-jk_o(h-\ell)} K_1(h,m) + e^{-jk_o(h+\ell)} K_1(h,-m) \\ & - 2K_1(\ell,m) \\ & + 2jk_o(h-\ell) e^{-jk_o(h-\ell)} \cdot K_2(m) \\ & + 2jk_o(h+\ell) e^{-jk_o(h+\ell)} K_2(-m) \\ & + L(\ell,m) \Big\} \ . \end{aligned} \qquad \text{(A2.31)}$$

All the factors have been defined and evaluated except L, which is defined as

$$\begin{aligned} L(\ell,m) \equiv 4k_o \int_{-h}^{h} k_o |z-m| e^{-jk_o|z-m|} \Big\{ & e^{-jk_o(z-\ell)} \int_{\ell-z}^{h-z} \frac{e^{-jk_o(R_\sigma+\sigma)}}{R_\sigma} \\ & + e^{+jk_o(z-\ell)} \int_{z-\ell}^{h+z} \frac{e^{-jk_o(R_\sigma+\sigma)}}{R_\sigma} d\sigma \Big\} dz . \end{aligned} \qquad \text{(A2.32)}$$

With the change of variables, $Z = k_o z$, etc,

(A2.32) is written as

$$L(L,M) = 4 \int_{-H}^{H} |Z-M| e^{-j|Z-M|} \Big\{ e^{-j(Z-L)} [F(H-Z)-F(L-Z)]$$

$$+ e^{+j(Z-L)} [F(H+Z)-F(Z-L)] \Big\} dZ. \quad \text{(A2.33)}$$

Define

$$L_p(L,M,N) \equiv 4 \int_{-H}^{H} |Z-M| e^{-j|Z-M|} e^{-j(Z-L)} F(N-Z)\, dZ, \quad \text{(A2.34)}$$

Then, in terms of L_p, L is written

$$L(L,M) = L_p(L,M,H)-L_p(L,M,L) + L_p(-L,-M,H)-L_p(-L,-M,-L). \quad \text{(A2.35)}$$

L_p can be evaluated:

$$L_p(L,M,N) = 4e^{+j(M+L)} \int_{M}^{H} (Z-M) e^{-j2Z} F(N-Z)\, dZ$$

$$- 4e^{-j(M-L)} \int_{-H}^{M} (Z-M) F(N-Z)\, dZ. \quad \text{(A2.35)}$$

Integrate both integrals by parts with

$$V = F(N-Z),\ dU_1 = (Z-M)e^{-j2Z}\,dZ,\ dU_2 = (Z-M)\,dZ.$$

Then,

$$L_p(L,M,N) = 4e^{+j(M+L)}\left\{\left(\frac{1}{4} + j\,\frac{(Z-M)}{2}\right)e^{-j2Z}F(N-Z)\Bigg|_M^H\right.$$

$$+ \int_M^H \left(\frac{1}{4} + j\,\frac{(Z-M)}{2}\right)e^{-j2Z}\,\frac{e^{-j\left(\sqrt{A^2+(N-Z)^2} + (N-Z)\right)}}{\sqrt{A^2+(N-Z)^2}}\,dZ$$

$$- 4e^{-j(M-L)}\left\{\left(\frac{Z^2}{2} - MZ\right)F(N-Z)\Bigg|_{-H}^M\right.$$

$$\left. + \int_{-H}^M \left(\frac{Z^2}{2} - MZ\right)\frac{e^{-j\left(\sqrt{A^2+(N-Z)^2} + (N-Z)\right)}}{\sqrt{A^2+(N-Z)^2}}\,dZ\right\}. \quad \text{(A2.37)}$$

With the substitution, $S = Z-N$, in the first integral and $T = N-Z$ in the second integral,

L_p is written

$$L_p(L,M,N) = e^{-j(2H-M-L)}(1+j2(H-M))F(N-H)$$

$$- e^{-j(M-L)}(1-2M^2)F(N-M)$$

$$+ e^{-j(M-L)}(2H^2+4MH)F(N+H)$$

$$+ e^{-j(2N-M-L)}\left\{[1+j2(N-M)] \int_{M-N}^{H-N} \frac{e^{-j(R_S+S)}}{R_S} dS\right.$$

$$\left. + 2j \int_{M-N}^{H-N} S \frac{e^{-j(R_S+S)}}{R_S} dS\right\}$$

$$+ e^{-j(M-L)}\left\{(2N^2-4MN) \int_{H+N}^{N-M} \frac{e^{-j(R_T+T)}}{R_T} dT\right.$$

$$+ 4(M-N) \int_{H+N}^{N-M} T \frac{e^{-j(R_T+T)}}{R_T} dT$$

$$\left. + 2 \int_{H+N}^{N-M} T^2 \frac{e^{-j(R_T+T)}}{R_T} dT\right\} . \qquad \text{(A2.38)}$$

Define

$$S(X) = \int_0^X S^2 \frac{e^{-j(R_S+S)}}{R_S} dS. \qquad \text{(A2.39)}$$

Then,

$$L_p(L,M,N) = e^{-j(2H-M-L)}(1+j2(H-M))\,F(N-H)$$

$$+\, e^{-j(2N-M-L)}\Big\{(1+j2(N-M))[F(H-N)-F(M-N)]$$

$$+\, G(H-N) - G(M-N)\Big\}$$

$$+\, e^{-j(M-L)}\Big\{(2M^2+2N^2-4MN-1)F(N-M)$$

$$+\,(2H^2-2N^2+4MH+4MN)F(H+N)$$

$$-\,2j(M-N)\,[G(N-M) - G(H+N)]$$

$$+\,2[S(N-M) - S(H+N)]\Big\}. \qquad \text{(A2.40)}$$

Equations (2.40), (2.35) and (2.31) completely define $\langle V_\ell, V_m\rangle$ in terms of the auxiliary functions F(X), G(X), and S(X).

Auxiliary Functions F, G, and S

The function F is defined by

$$F(X) = \int_0^X \frac{e^{-j(R_S+S)}}{R_S}\, ds\ , \qquad \text{(A2.41)}$$

where

$$R_S = \sqrt{A^2+S^2} \;.$$

With the substitution, $Y = R_S+S$,

so that

$$\frac{dY}{Y} = \frac{dS}{R_S} \;; \qquad (A2.42)$$

F(X) is written

$$F(X) = \int_A^Q \frac{e^{-jY}}{Y}\, dY. \qquad (A2.43)$$

Here,

$$Q = X + \sqrt{A^2+X^2}. \qquad (A2.44)$$

F(X) is a combination of sine and cosine integrals

$$F(X) = Ci\ (Q) - Ci\ (A) - j \left\{Si\ (Q) - Si\ (A)\right\} . \qquad (A2.45)$$

The function G is defined by

$$G(X) = 2j \int_0^X S\, \frac{e^{-j(R_S+S)}}{R_S}\, dS. \qquad (A2.46)$$

With the substitution, $Y = R_S+S$,

G(X) is written

$$G(X) = j \int_A^Q \frac{Y^2-A^2}{Y^2} e^{-jY} dY. \qquad (A2.47)$$

$$G(X) = j\left\{\frac{e^{-jY}}{-j}\Bigg|_A^Q + A^2 \frac{e^{-jY}}{Y}\Bigg|_A^Q + A^2 j \int_A^Q \frac{e^{-jY}}{Y} dY\right\} \qquad (A2.48)$$

In terms of F(X),

G is written

$$G(X) = e^{-jA}[1-jA]$$

$$- e^{-jQ}\left[1-j \frac{A^2}{Q}\right]$$

$$- A^2F(X) . \qquad (A2.49)$$

The function S is defined by

$$S(X) = \int_0^X \frac{s^2 e^{-j(R_S+s)}}{R_S} ds. \qquad (A2.50)$$

With the substitution, $Y = R_S+S$,

S(X) is written

$$S(X) = \int_A^Q \left(\frac{Y}{4} + \frac{A^4}{4Y^3} - \frac{A^2}{2Y}\right) e^{-jY} dY. \qquad (A2.51)$$

$$S(X) = \frac{1}{4}\,(1+jY)e^{-jY}\Bigg|_{A}^{Q}$$

$$+ \frac{A^4}{8}\left(\frac{j}{Y} - \frac{1}{Y^2}\right)e^{-jY}\Bigg|_{A}^{Q}$$

$$- \left(\frac{A^4}{8} + \frac{A^2}{2}\right) F(X) \quad . \qquad \text{(A2.52)}$$

Or, finally,

$$S(X) = \frac{1}{8}\Big\{e^{-jQ}\Big[2+j2Q+A^4\left(\frac{j}{Q} - \frac{1}{Q^2}\right)\Big]$$

$$- e^{-jA}\Big[2+j2A+jA^3-A^2\Big]$$

$$- (A^4+4A^2)F(X)\Big\} \quad . \qquad \text{(A2.53)}$$

APPENDIX 3

THE CHARACTERISTIC IMPEDANCE OF AN ANTENNA

The characteristic impedance of an antenna, Z_a, and the expansion parameter, ψ, related by (3.18), have been used as parameters in the development of the antenna synthesis procedure. In this appendix, these quantities are related to the physical dimensions of the antenna.

When a short voltage pulse is impressed across the input terminals of an unloaded antenna, the initial response of the antenna is that of an infinitely long antenna of the same radius. The approximate input current predicted by the simplified theory, (3.42), is a pulse of similar shape and of amplitude $\frac{V_B}{Z_a}$, followed by subsequent reflections from the ends of the structure. In an experimental study, King and Schmitt found this to be true, and moreover, they demonstrated that a value of Z_a, suitable for computing the reflection coefficient of the antenna, could be expressed as an average of the input impedance of the infinite antenna of the same radius over the significant frequency range of the pulse.[38]

$$Z_a = \frac{1}{\omega_c} \int_0^{\omega_c} Z(\omega) d\omega. \qquad \text{(A3.1)}$$

$Z(\omega)$ is the input impedance of an infinite cylindrical antenna of the same radius. $Z(\omega)$, as determined by Papas, is[39]

$$Z(\omega) = \frac{Z_o}{\Pi} [\ln \frac{1}{k_o a} - 0.5772 - j \frac{\Pi}{2}]. \qquad (A3.2)$$

The antenna must be thin enough that $k_o a$ is small, $k_o a \leq 0.1$, at all frequencies of interest.

The integration is easily performed, yielding

$$Z_a = Z(\omega_c) + \frac{Z_o}{\Pi} . \qquad (A3.3)$$

The reflection coefficient is computed from

$$R = \frac{Z_a - R_c}{Z_a + R_c} \qquad (A3.4)$$

R_c is the characteristic impedance of the feeding transmission line. The reflection coefficient computed by (A3.4) is complex, but it was found that the magnitude of this coefficient agreed quite well with the real reflection coefficient measured in the laboratory.[37]

For long square pulses, the definition given by (A3.1) is not satisfactory. The spectrum envelope of the input voltage decreases in direct proportion to the frequency; therefore, the selection of a highest significant frequency becomes arbitrary. For these cases, King and Schmitt suggest a weighted average of the input impedance where the weighting function is the spectrum function of the input voltage pulse.[37]

The weighting procedure that seemed most appropriate to to the writer was to compute the input current pulse flowing into an infinite antenna when a voltage pulse is impressed across the input terminals. The characteristic impedance is then defined by the ratio of the amplitude of the input voltage pulse to the input current pulse.

Consider the input voltage pulse previously employed in Chapter 2.

$$v_o(t) = [f(t)u(t) - f(t-T)u(t-T)] \;, \qquad (A3.5)$$

where

$$f(t) = 1-(1+t/t_1)\, e^{-t/t_1} \quad . \qquad (A3.6)$$

This pulse was chosen because the parameter, t_1, which is related to the rise time of the pulse, can be selected to assure that the requirement, $k_o a \leq 0.1$, for the highest significant frequency, can be satisfied. Second, the type of square pulse most often measured in the laboratory is well approximated by this input. The Fourier transform of this input is

$$V_o(\omega) = \frac{1-e^{-j\omega T}}{j\omega(1+j\omega t_1)^2} \quad . \qquad (A3.7)$$

The time history of the current flowing into the antenna terminals is given by

$$i(o) = \frac{1}{2\Pi} \int_{-\infty}^{\infty} \left[\frac{1-e^{-j\omega T}}{j\omega(1+j\omega t_1)^2}\right]$$

$$\left[\frac{e^{+j\omega t}}{\frac{Z_o}{\Pi}\left(\ln \left|\frac{c}{\omega a}\right| - .5772 - j\frac{\Pi}{2}\frac{\omega}{|\omega|}\right)}\right] d\omega. \qquad (A3.8)$$

With the change of variables, $p=\omega a/c$,

i(o) is written

$$i(o) = \frac{1}{2\Pi} \int_{-\infty}^{\infty} \left[\frac{1-e^{-jp\frac{cT}{a}}}{jp(1+jp\frac{ct_1}{a})^2}\right]$$

$$\left[\frac{e^{+jp\frac{ct}{a}}}{\frac{Z_o}{\Pi}\left(\ln \frac{1}{|p|} - .5772 - j\frac{\Pi}{2}\frac{p}{|p|}\right)}\right] dp. \qquad (A3.9)$$

Since the parameter, T, accounts for the delay in the terminating step, the instantaneous current at a time, $t < T$, is determined by only one parameter, ct_1/a. The function given by (A3.9) can be integrated with the aid of a digital computer to yield a set of universal antenna current curves dependent upon only one parameter, ct_1/a. In Figure A3.1, this set of curves is presented. There is sufficient data presented to describe accurately the pulse of current flowing into the antenna when the length of the input voltage pulse,

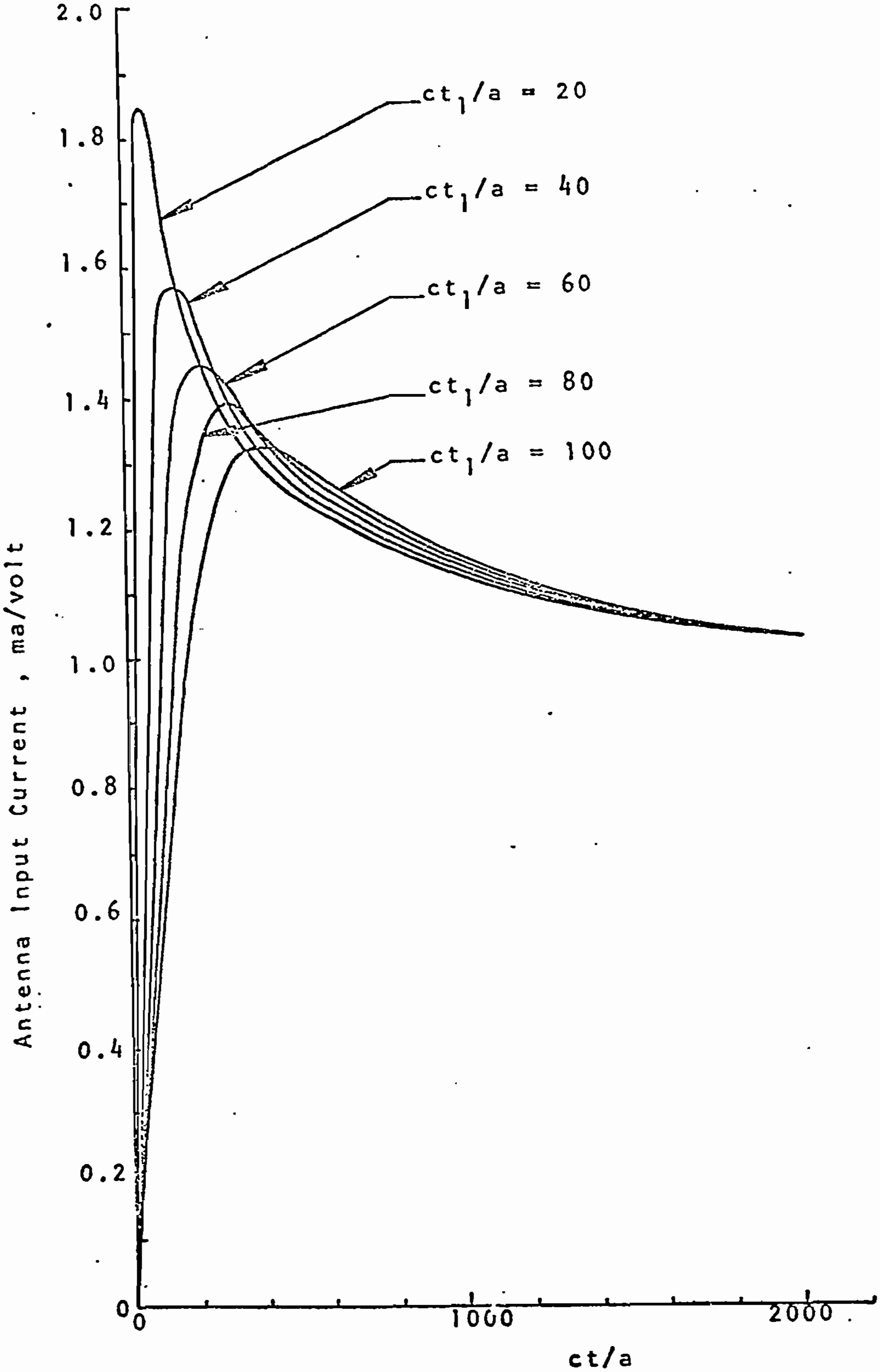


Figure A3.1. Infinite Antenna Response to a Laboratory Square Pulse

,T, is less than 2,000 a/c. The input current pulse is not exactly square. Current rapidly rushes onto the antenna to charge up the capacity in the immediate vicinity of the driving point. This results in a sharp peak on the front of the current pulse that is particularly evident for thicker antennas.

Since the current pulse is not exactly square, a value must be chosen for the characteristic impedance. For the examples given in Chapter 3, $ct_1/a = 45.2$, the peak value of the current was used to determine the characteristic impedance yielding $Z_a \cong 667$ ohms. The expansion parameter ψ was obtained from (3.18). As a consequence of this selection, the peak fields obtained by the simplified theory and the traveling wave theory agree more closely than the average values do, as indicated in Figure 3.6. Use of the average value of the current in the time interval, $0 < t < h/c$, to determine the characteristic impedance would result in a little better agreement between average values, but this difference is not significant.

In passing, we may note that the gradual decay of the field, before the current reaches the first resistor on the antenna, follows quite closely the decay of the input current observed on the antenna as indicated by (A3.1). It is a simple matter to substantiate this relationship. From the simplified theory, (3.53), the electric field during this time interval is

$$e_z(r,o) = -\frac{1}{\psi\, r}\, v_o(t). \tag{A3.10}$$

$\psi = 2\pi\, Z_a/Z_o$,

and the input voltage can be written in terms of the input current:

$$v_o(t) = i(o)\, Z_a\ . \tag{A3.11}$$

Substitution yields

$$e_z(r,o) = \frac{Z_o}{2\pi}\,\frac{i(o)}{r} = 60\frac{i(o)}{r}\ . \tag{A3.12}$$

This approximate expression is more accurate than (A3.10) because, with the substitutions, cancellation of the approximation, Z_a, is obtained.

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