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The Distributed Source for Launching Spherical Waves

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Abstract

In designing an antenna for radiating a fast rising transient pulse one can think of radiating the high frequencies from some small region of space, such as near the apex of a biconical antenna. Launching all the high frequency energy from a small region of space, however, implies large fields there. In order to reduce the peak electric fields one can make the source region larger. In this note we discuss an approach which in principle allows one to make the source region arbitrarily large while still radiating a fast rising spherical wave. This approach relies on a uniqueness theorem for the solution of electromagnetic boundary value problems in which it is only required to specify the tangential electric (or magnetic) field over the boundary surfaces to determine the fields in the volume of interest. The tangential electric field on the boundary surfaces is specified to match any particular form of wave desired as long as the wave satisfied Maxwell's equations. One can approximately specify some forms of the tangential electric field on a surface with an array of capacitors, conductors, and switches.

I. Introduction

One type of simulator for the nuclear electromagnetic pulse consists of a pulse-radiating electric dipole antenna. One approach to establishing good antenna characteristics for radiating high frequencies is to make the central portion of the antenna as a biconical wave launcher.[1] The pulser puts an electrical pulse on the antenna by driving between the two cones at or near the common apex of the two cones. If the pulser output has a fast rise time and if the region (near the apex) where the conical geometry is distorted to allow the introduction of the pulser signal is sufficiently small, then the antenna can radiate a fast rising electromagnetic pulse. The initial part of this pulse has the form of a spherical wave with an angular distribution of its amplitude appropriate to the biconical wave launcher. The approach here has been to make the source region (where the wave is introduced onto the biconical wave launcher) sufficiently small such that some of the details of the source region are not critical for launching a desired fast rising wave. The source region is then considered from a quasi static viewpoint. One disadvantage, however, to a small source region is that for a given voltage put on the antenna (to get a certain amplitude for the radiated field at some particular distance away) the electric fields in the source region can be rather large, thereby leading to insulation problems.

In this note we consider another approach to this problem of radiating a fast rising pulse in the form of a spherical wave. This concept allows one to use large source regions. We call this concept the distributed source for launching spherical waves.

By the general concept of a distributed source we mean some electrical energy source which has, at least approximately, the property of specifying something about the electromagnetic fields at some surface which we call the source surface. In particular, we are concerned here with a distributed source which specifies the tangential components of the electric field at the source surface. There are various possible types of such distributed sources. One might use a planar source surface (or even other shapes) to launch a plane wave in free space or on a TEM transmission line. Another example, already discussed in the notes,[2,3] is the distributed

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1. Capt Carl E. Baum, Sensor and Simulation Note 69, Design of a Pulse-Radiating Dipole Antenna as Related to High-Frequency and Low-Frequency Limits, January 1969.

2. Capt Carl E. Baum, Sensor and Simulation Note 48, The Planar, Uniform Surface Transmission Line Driven from a Sheet Source, August 1967.

3. Capt Carl E. Baum, Sensor and Simulation Note 66, A Simplified Two-Dimensional Model for the Fields Above the Distributed-Source Surface Transmission Line, December 1968.

source for launching a wave into a conducting medium (like earth) using a planar source sheet with a particular form of propagating source amplitude along the source surface.

Conceptually, an important feature of a distributed source is that its design can be specified by first specifying the form of electromagnetic wave (satisfying Maxwell's equations). Then given appropriate boundary surfaces the tangential component of the electric field required of the sources on the boundary surfaces is precisely that tangential electric field specified by the desired electromagnetic wave. The present note gives another application of the concept of a distributed source, in this case to certain types of outward propagating spherical waves.

II. Basic Concept of the Distributed Source for Launching Spherical Waves

As illustrated in figure 1 consider some closed source surface designated S_s. Assume that S_s contains the coordinate origin on which we center cartesian (x, y, z), cylindrical (Ψ, ϕ, z), and spherical (r, θ, ϕ) coordinate systems. Coordinates referring to points on S_s are designated by adding a subscript s to the coordinates shown in figure 1 and listed above. The position vector of a point is $\vec{r}$ with $\vec{r} = \vec{0}$ as the coordinate origin. For the calculations in this note the medium external to S_s will be taken the same as free space with permittivity ε_o, permeability μ_o, and zero conductivity. The basic concepts, however, apply for even more general types of media.

Now consider some time-domain solution of Maxwell's equations applying to the volume outside of S_s. Assume that this solution is of a form such that the electromagnetic fields are zero before some particular time which we will typically take as $t = 0$ where t is time.[4] Let $\vec{E}(\vec{r}, t)$ be the electric field vector in the chosen solution of Maxwell's equations outside S_s. Let $\vec{n}$ be the outward pointing unit normal vector for S_s. The electric field has a tangential component on S_s which we express as $\vec{E}_s$ where $\vec{E}_s$ is parallel to S_s; this tangential field is given by

$$\vec{E}_s(\vec{r}_s, t) = -[\vec{E}(\vec{r}_s, t) \times \vec{n}(\vec{r}_s)] \times \vec{n}(\vec{r}_s) \qquad (1)$$

Now suppose $\vec{E}_s$ is specified on S_s as this particular function of $\vec{r}_s$ and t associated with our originally assumed $\vec{E}(\vec{r}, t)$ which solves Maxwell's equations outside S_s by hypothesis. Clearly then $\vec{E}$ satisfies Maxwell's equations and the boundary condition on S_s; $\vec{E}$ and the other associated electromagnetic fields are also zero for $t < 0$ by hypothesis. Furthermore these conditions are sufficient

4. All units are rationalized MKSA.

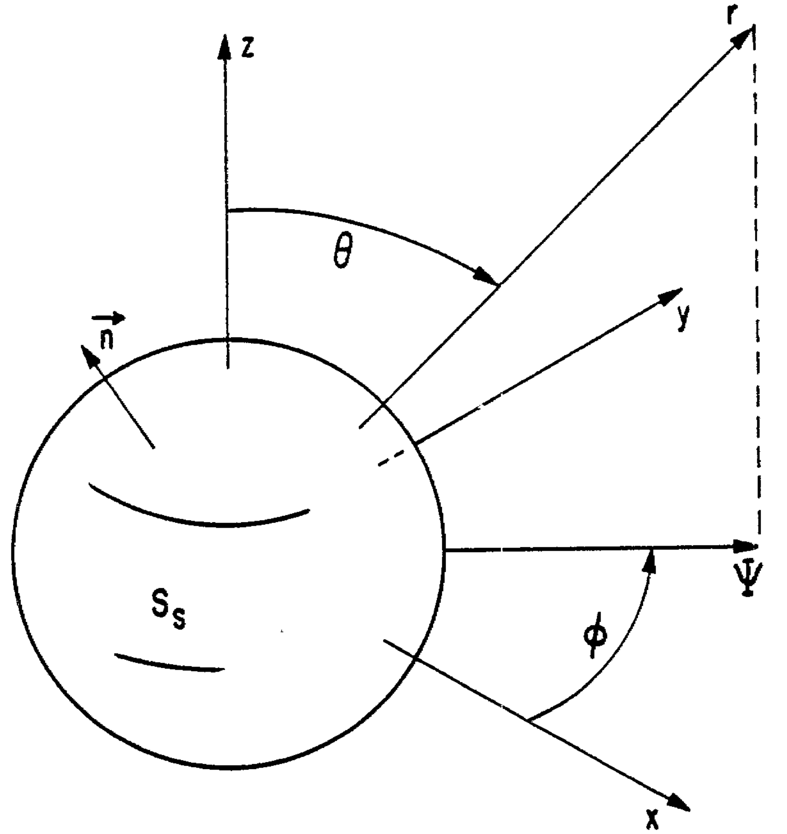

FIGURE 1. SOURCE SURFACE WITH COORDINATES

to determine a unique solution.[5] Thus if we specify $\vec{E}_s(\vec{r}_s, t)$ then the associated $\vec{E}(\vec{r}, t)$ is uniquely determined. The basic concept of this distributed source is then to specify some outward propagating electromagnetic field outside S_s satisfying Maxwell's equations, use the $\vec{E}$ field so specified to calculate $\vec{E}_s$ and take this $\vec{E}_s$ and impose it on S_s; the fields produced outside S_s will just be $\vec{E}$ and the other associated electromagnetic fields which were specified at the start of the problem.

There are various sizes and shapes one might choose for S_s including spheres, ellipsoids, cylinders with end caps, etc. This choice could depend on various considerations such as the peak magnitude of $\vec{E}$ on S_s; mechanical convenience, etc. Now the peak magnitude of $\vec{E}(\vec{r}, t)$ for each $\vec{r}$ will generally decrease with increasing r, depending on the exact form of $\vec{E}$ used. An approximate 1/r decrease of the peak $|\vec{E}|$ is typical of many forms of $\vec{E}(\vec{r}, t)$ of interest for this type of simulator (a pulse-radiating electric dipole antenna). In these cases one can choose the typical value of r_s large enough that the peak $|\vec{E}|$ is not too large over S_s. The typical r_s for a particular case might be chosen as a compromise between various electrical and mechanical factors.

If $\vec{e}_r$ is the unit vector in the r direction (and similarly for other unit vectors) then we call S_s an outward pointing surface if for all $\vec{r}_s$ we have $\vec{e}_r \cdot \vec{n}(\vec{r}_s) \geq 0$. Note that the location of $\vec{r} = \vec{0}$ relative to S_s then is a part of this definition. The examples in this note have S_s as an outward pointing surface. If the Poynting vector has only a positive r component then the energy nowhere flows into an outward pointing surface. In the high-frequency or geometrical-optics limit for a wave propagating only in the $\vec{e}_r$ direction (a spherically expanding wave) an outward pointing S_s will nowhere absorb the wave.

Note that not only are there fields generated outside S_s; there are also fields generated inside. We do not consider the fields inside S_s in this note. The shape of S_s, however, can be influenced by the internal fields. For example, one might not want to generate an inwardly propagating spherical wave inside S_s focusing at $\vec{r} = \vec{0}$ because of the large local fields which could be produced.

One way to approximate a given $\vec{E}_s$, depending on the shape of the waveform desired, is to distribute capacitors with switches on or near S_s and trigger the switches in an appropriate sequence. Of course, capacitors and switches do not give a smooth distribution for $\vec{E}_s$ but can approximate $\vec{E}_s$ in a macroscopic view, i.e. over dimensions larger than the spacings of capacitors and switches. This

5. J. A. Stratton, Electromagnetic Theory, McGraw-Hill, 1941, pp. 486-488.

non smooth characteristic of such a distributed source can limit its performance in producing a desired radiated field. Perhaps such problems can be considered in future notes.

III. Distributed Source for Launching a Spherical TEM Wave on a Symmetrical Bicone

Now we consider some of the features of a distributed source for launching a particular type of spherical wave. Specifically, consider the outward propagating spherical TEM wave which can propagate on a perfectly conducting biconical structure with axial symmetry. In a free-space medium this wave has the form

$$\vec{E} = E_\theta \vec{e}_\theta \ , \qquad \vec{H} = H_\phi \vec{e}_\phi \ , \qquad E_\theta = Z_o H_\phi \tag{2}$$

The wave impedance and speed of light are

$$Z_o \equiv \sqrt{\frac{\mu_o}{\varepsilon_o}} \ , \qquad c \equiv \frac{1}{\sqrt{\mu_o \varepsilon_o}} \tag{3}$$

where μ_o and ε_o are respectively the permeability and permittivity of free space. E_θ has the form

$$E_\theta = \frac{V' f(t^*)}{r \sin(\theta)} \tag{4}$$

where the retarded time is

$$t^* \equiv t - \frac{r}{c} \tag{5}$$

V' is some convenient constant with dimension volts and f is some function of t^*, as yet unspecified.

As illustrated in figure 2 choose a perfectly conducting symmetrical bicone; let the bicone have both axial and lengthwise symmetry by specifying the two cones by $\theta = \theta_o$ and $\theta = \pi - \theta_o$ where $0 < \theta_o < \pi/2$. The spherical TEM wave (equations 2) has the electric field perpendicular to the two cones at the conical surfaces. Thus we only have the fields for $\theta_o < \theta < \pi - \theta_o$ with the boundary condition satisfied on the perfectly conducting cones. For the present calculations the cones are assumed to extend from the source surface (S_s) to infinity. In a real application the cones will have finite length. The present results then apply for times before reflections can propagate from discontinuities in the conical geometry to each position of interest.

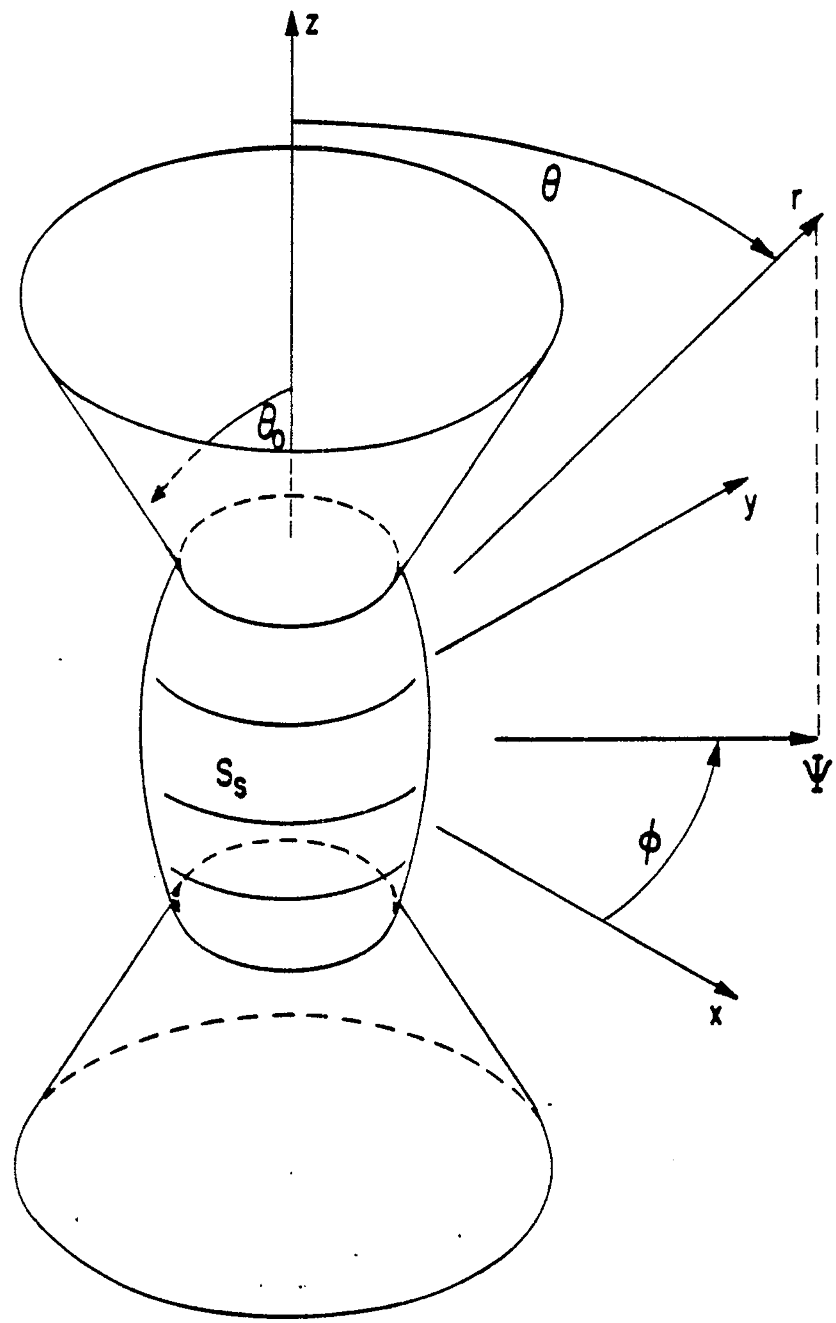

FIGURE 2. SOURCE SURFACE WITH SYMMETRICAL BICONE

As discussed in reference 1 the outward propagating wave on such a symmetrical bicone can be written as

$$E_\theta = \frac{1}{r} V_a(t^*) f_o(\theta) \tag{6}$$

where

$$f_o(\theta) = 2\left\{\sin(\theta)\ \ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]\right\}^{-1} \tag{7}$$

with

$$\int_{\theta_o}^{\pi-\theta_o} f_o(\theta)d\theta = 1 \tag{8}$$

Here V_a can be considered as the voltage on the bicone provided the appropriate line integral of the electric field is restricted to a path with constant r. For convenience we define

$$V_a(t^*) \equiv V_o f(t^*) \tag{9}$$

so that we have

$$E_\theta = \frac{V_o}{r} f_o(\theta) f(t^*) \tag{10}$$

where V_o has dimension volts and $f(t^*)$ is the same waveform function as in equation 4. For convenience one can take V_o as the peak of the antenna voltage (V_a) so that $f(t^*)$ has a peak value of unity.

With this special wave (equations 2 through 10) we have an outward propagating wave for $\theta_o < \theta < \pi - \theta_o$ with the boundary condition of zero tangential electric field satisfied on $\theta = \theta_o$ and $\theta = \pi - \theta_o$. This leaves the source surface as the remaining boundary surface to consider. As shown in figure 2 the source surface S_s connects the two perfectly conducting cones in a manner so as to form a continuous surface which divides space into two separate regions which we can call outside and inside. The outside is where we have the wave of interest as discussed above; the inside contains the origin ($\vec{r} = \vec{0}$). Note that the two perfectly conducting cones are only needed as part of the boundary of the outside region and do not have to extend into the inside region after connecting with S_s. It is now only necessary to specify $\vec{E}_s$ on S_s by equation 1 in order to obtain the desired form of $\vec{E}$ in the outside region. Note that we must restrict S_s to not intersect the z axis where $\vec{E}$ would be singular; the z axis must be entirely in the inside region. The inclusion of the perfectly conducting bicone with S_s in order to support this spherical TEM wave is an extension of the concept discussed in section II where the only boundary surface was S_s and it was finite in extent.

Since this spherical TEM wave has only a θ component equation 1 for the tangential electric field on S_s becomes

$$\vec{E}_s(\vec{r}_s, t) = -E_\theta [\vec{e}_\theta \times \vec{n}(\vec{r}_s)] \times \vec{n}(\vec{r}_s)$$

$$= -\frac{V_o}{r_s} f_o(\theta_s) f(t_s^*) [\vec{e}_\theta \times \vec{n}(\vec{r}_s)] \times \vec{n}(\vec{r}_s) \qquad (11)$$

where t_s^* is the retarded time referred to points on S_s so that

$$t_s^* \equiv t - \frac{r_s}{c} \qquad (12)$$

Now for convenience let S_s have axial symmetry so that the shape of S_s is independent of ϕ. This makes $\vec{n}$ have no ϕ component. Then $\vec{e}_\phi$ is a unit tangent vector for S_s. Define another unit tangent vector for S_s as

$$\vec{e}_s \equiv \vec{e}_\phi \times \vec{n} \qquad (13)$$

$\vec{n}$, $\vec{e}_s$, $\vec{e}_\phi$ (in this cyclic order) form a right handed set of orthogonal unit vectors referenced to S_s with the restriction of axial symmetry. Since we have

$$\vec{n} = \vec{e}_s \times \vec{e}_\phi \qquad (14)$$

then from equation 11 we find

$$\vec{E}_s(\vec{r}_s, t) = E_\theta \vec{n} \times [\vec{e}_\theta \times \vec{n}]$$

$$= E_\theta (\vec{e}_s \times \vec{e}_\phi) \times [\vec{e}_\theta \times (\vec{e}_s \times \vec{e}_\phi)] \qquad (15)$$

Using vector identities together with $\vec{e}_\theta \cdot \vec{e}_\phi = 0$ and $\vec{e}_s \cdot \vec{e}_\phi = 0$ this equation reduces to

$$\vec{E}_s(\vec{r}_s, t) = E_\theta(\vec{e}_s \cdot \vec{e}_\theta)\vec{e}_s$$

$$= \frac{V_o}{r_s} (\vec{e}_s \cdot \vec{e}_\theta) f_o(\theta_s) f(t_s^*) \vec{e}_s \qquad (16)$$

Thus $\vec{E}_s$ is parallel to $\vec{e}_s$ and has a distribution in θ_s (at a fixed retarded time t_s^*) proportional to $(\vec{e}_s \cdot \vec{e}_\theta) f_o(\theta_s)/r_s$.

Let S_s be an outward pointing surface, as discussed in section II so that no energy is flowing into some portion of S_s. Also let S_s have axial and lengthwise symmetry for convenience. Figure 3 illustrates such a case. In order to describe this surface we can consider r_s as a function of θ_s. If S_s is also restricted such that Ψ_s is a single-valued function of z_s then we can also use this kind of a description for S_s. For convenience we define

$$\psi = \frac{\pi}{2} - \theta \qquad (17)$$

so that ψ is the angle from the x, y plane and r_s is now an even function of ψ_s because of the assumed lengthwise symmetry of S_s.

For purposes of constructing a distributed generator on S_s one can divide S_s into many small regions, each small region having its own generator which might consist of one or more charged capacitors and a switch. For the form of $\vec{E}_s$ in equation 16 it is convenient to first divide S_s on circles of constant θ which are also circles of constant ψ. Note that $\vec{E}_s$ in equation 16 has no ϕ component. Thus one could put conducting strips along circles of constant ψ and connect generators so as to put transient voltage between these strips. The generators would be uniformly distributed in ϕ to approximate the required ϕ independent source. The conducting strips and associated generators are distributed with respect to θ_s in a manner to approximate the required distribution of $\vec{E}_s$ with respect to θ_s (or ψ_s).

Divide S_s with respect to θ_s into M source regions which we call bands. For convenience make this division symmetrical with respect to ψ_s. There are two cases to consider.

<u>Case 1</u>: M = 2N (M even)

Define M + 1 = 2N + 1 angles by

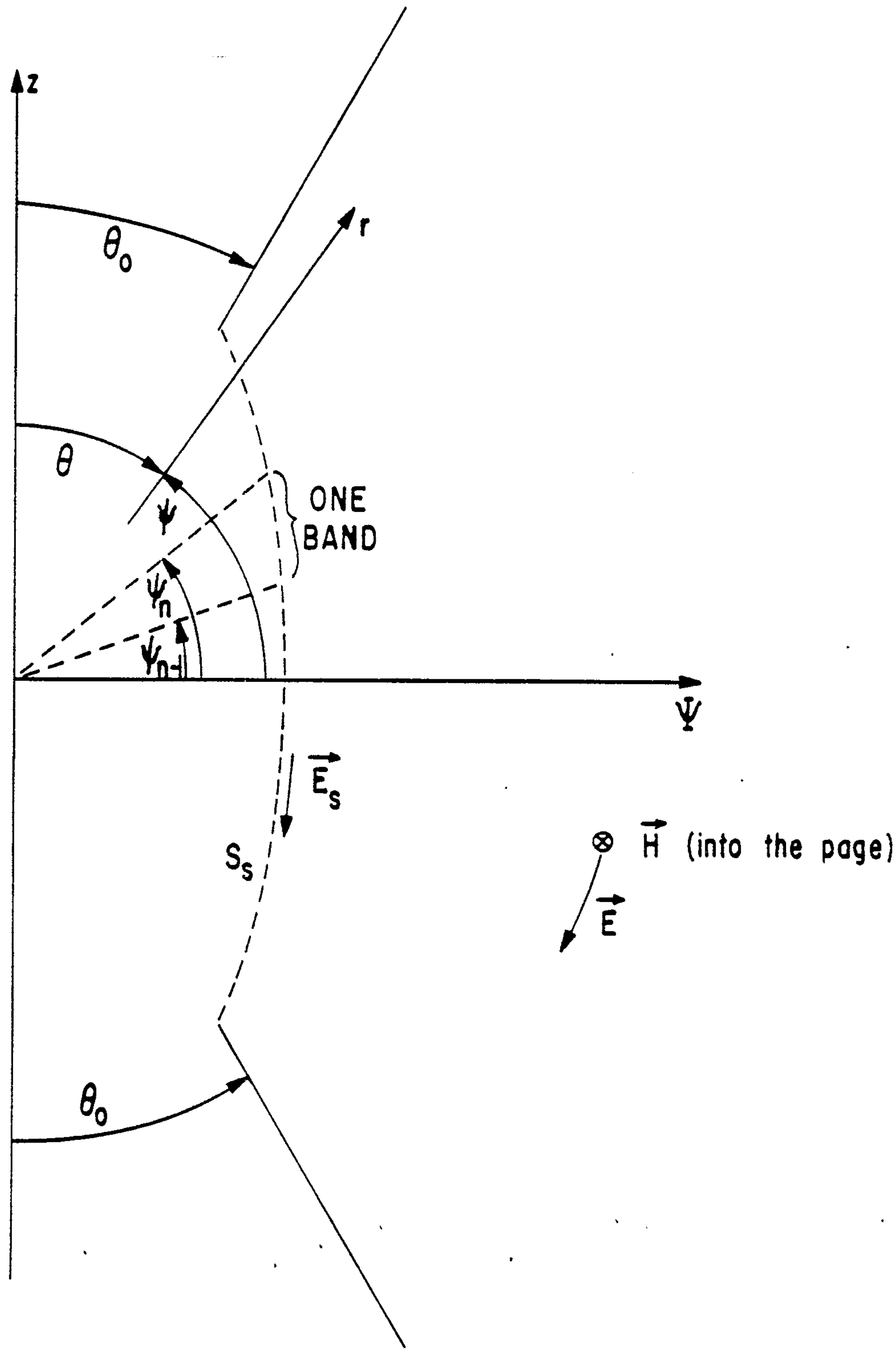

FIGURE 3. SOURCE SURFACE AND BICONE WITH AXIAL AND LENGTHWISE SYMMETRY: PLANE OF CONSTANT ϕ

$$\psi_{-N} < \ldots < \psi_{-2} < \psi_{-1} < \psi_o < \psi_1 < \psi_2 < \ldots < \psi_N \quad (18)$$

where

$$\psi_N \equiv \frac{\pi}{2} - \theta_o \equiv -\psi_{-N}$$

$$\psi_o \equiv 0 \quad (19)$$

$$\psi_{-n} \equiv -\psi_n \qquad \text{for } n = -N, \ldots, -2, -1, 0, 1, 2, \ldots, N$$

Note that n = 0 is used to number one of the angles. Setting $\psi_s = \psi_n$ defines M + 1 circles symmetrically placed on S_s. These circles are taken as the borders of the bands. The bands are numbered from -N to N excluding 0. Band number n is defined for $n \geq 1$ by

$$\psi_{n-1} < \psi_s < \psi_n \quad (20)$$

and for $n \leq -1$ by

$$\psi_n < \psi_s < \psi_{n+1} \quad (21)$$

Note that n = 0 is not used to number one of the bands.

Case 2: M = 2N - 1 (M odd)

Define M + 1 = 2N angles by

$$\psi_{-N} < \ldots < \psi_{-2} < \psi_{-1} < \psi_1 < \psi_2 < \ldots < \psi_N \quad (22)$$

where

$$\psi_N \equiv \frac{\pi}{2} - \theta_o \equiv -\psi_{-N}$$
$$(23)$$
$$\psi_{-n} \equiv -\psi_n \qquad \text{for } n = -N, \ldots, -2, -1, 1, 2, \ldots, N$$

Note that n = 0 is not used to number one of the angles. The bands are numbered from -N + 1 to N - 1 including zero. Band number n is defined for $n \geq 1$ by

$$\psi_n < \psi_s < \psi_{n+1} \tag{24}$$

for $n \leq -1$ by

$$\psi_{n-1} < \psi_s < \psi_n \tag{25}$$

and for $n = 0$ by

$$\psi_{-1} < \psi_s < \psi_1 \tag{26}$$

The spherical TEM wave being considered here has a potential function or voltage distribution of the form[6]

$$V = \frac{V_o}{2} f(t^*) f_V(\theta) \tag{27}$$

where

$$f_V(\theta) = \frac{\ln\left[\tan\left(\frac{\theta}{2}\right)\right]}{\ln\left[\tan\left(\frac{\theta_o}{2}\right)\right]} = \frac{\ln\left[\cot\left(\frac{\theta}{2}\right)\right]}{\ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]} \tag{28}$$

with special values

$$f_V(\theta_o) = 1 , \qquad f_V\left(\frac{\pi}{2}\right) = 0 , \qquad f_V(\pi - \theta_o) = -1 \tag{29}$$

In terms of ψ we have

$$f_V = \frac{\ln\left[\cot\left(\frac{\pi}{4} - \frac{\psi}{2}\right)\right]}{\ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]} \tag{30}$$

One can calculate E_θ (as in equation 10) from

6. Capt Carl E. Baum, Sensor and Simulation Note 36, A Circular Conical Antenna Simulator, March 1967.

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} \tag{31}$$

The two distribution functions in θ for E_θ and V are related as

$$f_o(\theta) = -\frac{1}{2}\frac{df_V(\theta)}{d\theta} \tag{32}$$

It is convenient to use V to define the source distribution on S_s. V implies a certain $\vec{E}$ and the corresponding $\vec{E}_s$.

Now choose the M bands of S_s such that the source voltages driving them are all the same as a function of t_s^*. Call this voltage on each band $V_s(t_s^*)$. This requires that the ψ_n be chosen such that the change in f_V across a band be the same for all M bands; we express this as

$$\Delta f_V \equiv \frac{2}{M} \tag{33}$$

since f_V goes between -1 and +1 over the range $\pi - \theta_o \geq \theta \geq \theta_o$. Again consider the two cases.

Case 1: M = 2N (M even)

Define ψ_n for M + 1 angles by setting

$$f_V \equiv \frac{n}{N} \qquad \text{for } n = -N, \ldots, -2, -1, 0, 1, 2, \ldots, N \tag{34}$$

Case 2: M = 2N - 1 (M odd)

Define ψ_n for M + 1 angles by setting

$$f_V \equiv \begin{cases} \dfrac{2n+1}{2N-1} & \text{for } n = -N, \ldots, -2, -1 \\ \\ \dfrac{2n-1}{2N-1} & \text{for } n = 1, 2, \ldots, N \end{cases} \tag{35}$$

The source current associated with the magnetic field just outside S_s is

$$I_s(t_s^*) = \int_0^{2\pi} H_\phi(\vec{r}_s, t_s^*)\Psi_s d\phi$$

$$= 2\pi\Psi_s H_\phi(\vec{r}_s, t_s^*) \tag{36}$$

since H_ϕ is independent of ϕ and where some ψ_s in a band of interest is chosen for the path of integration. From equations 2 we replace H_ϕ in terms of E_θ giving

$$I_s(t_s^*) = \frac{2\pi}{Z_o}\Psi_s E_\theta(\vec{r}_s, t_s^*) \tag{37}$$

Replacing E_θ from equation 6 gives

$$I_s(t_s^*) = \frac{2\pi}{Z_o}\frac{\Psi_s}{r_s} V_a(t_s^*) f_o(\theta_s)$$

$$= \frac{2\pi}{Z_o}\sin(\theta_s) f_o(\theta_s) V_a(t_s^*)$$

$$= \frac{\pi}{Z_o}\frac{V_a(t_s^*)}{\ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]} \tag{38}$$

Now from reference 1 the impedance of the bicone is

$$Z_a = \frac{Z_o}{\pi}\ln\left[\cot\left(\frac{\theta_o}{2}\right)\right] \tag{39}$$

Thus

$$I_s(t_s^*) = \frac{V_a(t_s^*)}{Z_a} \tag{40}$$

showing that the source current on S_s through a conical surface of constant θ_s is independent of θ_s for a fixed retarded time and that I_s and V_a have the same history in retarded time. The voltage across each band in S_s is just

$$V_s(t_s^*) = \frac{1}{M} V_a(t_s^*) \tag{41}$$

Since the current through each band is $I_s(t_s^*)$ we can then define an impedance driven by each band as

$$Z_1 \equiv \frac{Z_a}{M} \tag{42}$$

so that

$$V_s(t_s^*) = Z_1 I_s(t_s^*) \tag{43}$$

Note that if r_s varies over a band then t_s^* similarly varies for fixed t. Ideally the size of the bands and spacing between the generators is small enough that variations over the area covered by one unit of the distributed source can be ignored. Since, however, the size of the bands and generator spacing are greater than zero then equation 43 can only be approximately applied to a band in a real distributed source. Also there are source currents associated with the magnetic field inside S_s which are not included in I_s.

Consider an example of a waveform defined by

$$f(t^*) \equiv e^{-t^*/t_o} u(t^*) \tag{44}$$

where u is the unit step function and $t_o > 0$ is some time constant. Suppose we have a capacitive generator on S_s with the same capacitance C_1 per band and total generator capacitance C_g related as

$$C_g = \frac{C_1}{M} \tag{45}$$

Let each band be switched on at $t_s^* = 0$. Since the resistive impedance driven by each band is Z_1 then we have the same time constant for the discharge of each band given by

$$t_o = Z_1 C_1 = \frac{Z_a}{M} (MC_g) = Z_a C_g \tag{46}$$

The capacitors in each band are charged to give the same initial voltage on each band given by

$$V_1 = \frac{V_o}{M} \tag{47}$$

so that

$$V_s(t_s^*) = \frac{V_o}{M} e^{-t_s^*/t_o} u(t_s^*) \tag{48}$$

Depending on the design of the equipment inside S_s it should be possible to have the source currents associated with the fields inside S_s decay in times of the order of some typical transit time across the inside of S_s. If this transit time is much smaller than t_o then the internal loading should not be significant enough to speed the pulse decay. The internal loading may, however, increase the pulse rise time, depending on various features of the generator design. Note that our idealized $f(t^*)$ does not include the non zero rise time of a real generator. Also a real bicone will not extend indefinitely so that the later portions of the waveform will be altered by the reflections introduced by changes in the antenna geometry at the ends of the bicone.

IV. Source Surface Shaped as a Circular Cylinder

Continuing with the particular spherical TEM wave used in section III we consider some additional aspects of the shape of S_s. Still keeping axial and lengthwise symmetry one can still specify the form of r_s as a function of ψ_s, or Ψ_s as a function of z_s. One way to specify this shape is to relate it to the magnitude of the electric field or some component of the electric field at all points of S_s. For our present calculations consider an example defined by setting the maximum magnitude of $\vec{E}$ at each point on S_s equal to some constant E_o. Note that outside S_s for $\theta_o < \theta < \pi - \theta_o$ the maximum magnitude of $\vec{E}$ decreases with increasing r. The region inside S_s including the distributed source and extending to (or even partly into) the two conducting cones might be filled with some gas of higher dielectric strength than air to minimize electrical breakdown problems.

Outside S_s for $\theta_o < \theta < \pi - \theta_o$ the electric field is given from equations 2, 7, and 10 as

$$\vec{E}(\vec{r}, t^*) = \frac{V_o}{r} f(t^*) \left\{ 2 \sin(\theta) \ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]\right\}^{-1} \vec{e}_\theta \tag{49}$$

Let the maximum value of f equal 1 and occur at $t^* = 0$. Let the maximum magnitude of $\vec{E}$ on S_s be called E_o giving

$$E_o = |\vec{E}(\vec{r}_s, 0)| = \frac{V_o}{r_s}\left\{2 \sin(\theta_s) \ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]\right\}^{-1}$$

$$= \frac{V_o}{2\Psi_s \ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]} \tag{50}$$

which implies

$$\Psi_s = \frac{V_o}{2E_o \ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]} \tag{51}$$

Now E_o is chosen as a constant, applying to all of S_s. Since V_o and θ_o are also constants then Ψ_s is a constant. Therefore S_s is a circular cylinder of radius Ψ_s, a rather simple geometrical shape. Since Ψ_s is a constant we can define another constant as

$$h \equiv \Psi_s \cot(\theta_o) = \frac{V_o}{2E_o} \frac{\cot(\theta_o)}{\ln\left[\cot\left(\frac{\theta_o}{2}\right)\right]} \tag{52}$$

The source surface S_s has radius Ψ_s with extension in the z direction given by $-h < z_s < h$. At $z_s = \pm h$ the biconical perfectly conducting surface begins.

In equations 27 through 35 in the previous section we split up S_s into bands of equal potentials (in retarded time). To do this certain values of $\psi = \psi_n$ were defined. For the case that S_s is a circular cylinder (as in this section) it is convenient to use z_s to define the bands. Since we have

$$\cot\left(\frac{\theta_s}{2}\right) = \frac{1 + \cos(\theta_s)}{\sin(\theta_s)} = \frac{r_s}{\Psi_s} + \frac{z_s}{\Psi_s}$$

$$= \frac{z_s}{\Psi_s} + \left[1 + \left(\frac{z_s}{\Psi_s}\right)^2\right]^{1/2} \tag{53}$$

then we can write the potential distribution function for S_s from equation (28) as

$$f_V = \frac{\ln\left[\frac{z_s}{\Psi_s} + \left[1 + \left(\frac{z_s}{\Psi_s}\right)^2\right]^{1/2}\right]}{\ln\left[\frac{h}{\Psi_s} + \left[1 + \left(\frac{h}{\Psi_s}\right)^2\right]^{1/2}\right]} = \frac{\operatorname{arcsinh}\left(\frac{z_s}{\Psi_s}\right)}{\operatorname{arcsinh}\left(\frac{h}{\Psi_s}\right)} \tag{54}$$

This can be rewritten as

$$\frac{z_s}{\Psi_s} = \sinh\left[f_V \operatorname{arcsinh}\left(\frac{h}{\Psi_s}\right)\right] \tag{55}$$

Define the boundaries of the bands by

$$z_n \equiv \Psi_s \tan(\psi_n) \tag{56}$$

Replacing z_s in equation 55 by z_n and f_V by the discrete values in equation 34 or 35, as appropriate, the boundaries of the M bands are then defined.

V. Summary

It is then not necessary to launch a fast rising spherical wave from some small region of space which one might think of as a point source. An alternate approach (discussed in this note) is to use an array of sources distributed over an appropriate surface with amplitudes and firing sequence arranged in such a manner that the desired fast rising spherical wave is produced outside the source surface. Conceptually one can define the source distribution over the source surface by defining a desired outward propagating field distribution which solves Maxwell's equations in a region outside the source surface (plus any other appropriate perfectly conducting surfaces which one might include). This desired external field distribution implies a certain field distribution (including the tangential electric field) on the source surface. From the uniqueness theorem we only need to specify this tangential electric field on the source surface to give the desired fields outside S_s. Within some limitations one might specify the tangential electric field on the source surface with an appropriate array of capacitors, conductors, and switches.

By using a distributed-source approach for launching spherical waves one can avoid having very large electric fields at something like a point source such as near the apex of a biconical wave launcher. The distributed source launches a spherical wave outside the source; the wave appears to come from a point source. There are also fields inside the distributed source which will be determined by the source fields and the materials inside the source surface. Part of the design problem for a distributed source is the minimization of any adverse effects associated with the internal fields. Perhaps some design considerations for the internal fields can be considered in future notes.

In this note we have given particular attention to some of the design considerations for a distributed source for launching a spherical TEM wave as propagates on a biconical structure. There are various other types of fast rising spherical waves which one might also consider.