A Laboratory Method for the Measurement of the Dielectric Constant of Rock and Soil Samples in the Frequency Range $10^2 - 10^8$ Hertz

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ABSTRACT

A two-terminal bridge method for determining the dielectric constant, $\varepsilon$, of geologic materials over the frequency range $10^2 - 10^8$ Hz has been developed by the U.S. Geological Survey. The method is designed to minimize (1) errors associated with the presence of electrode polarization by using nonpolarizing electrodes, (2) errors associated with parasitic impedances present in the measuring system at high frequency by calibration with known standards, and (3) errors associated with sample inhomogeneity by using relatively large samples. Using this method, representative values of $\varepsilon$ may be obtained for rock and soil samples with errors of less than 10 percent.
NOTE 88

A LABORATORY METHOD FOR THE MEASUREMENT OF THE
DIELECTRIC CONSTANT OF ROCK AND SOIL SAMPLES
IN THE FREQUENCY RANGE $10^2 - 10^8$ Hertz

by

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SECTION I
INTRODUCTION

The propagation of an electromagnetic wave in any medium depends upon the resistivity, permittivity, and magnetic permeability of the medium. Generally the attenuation constant and the wave number appearing in the plane-wave equations are assumed to be described by values of permittivity, permeability, and resistivity which are relatively constant. In geologic materials propagation is generally through a three-phase system, consisting of air, water containing dissolved ionic species, and a solid matrix, and the values of permittivity and resistivity of such materials appear to be strongly frequency dependent, the dielectric constant exhibiting values as high as $2 \times 10^5$ at low frequency ($10^2$ Hz). Table I lists some of the dielectric constants that have been measured for representative geologic materials. Because EMP vulnerability criteria are dependent upon the earth impedance surrounding missile sites, it is of importance to attempt to explain the mechanisms which cause these phenomena and to examine their relationship to specific geologic materials.

Described herein is the laboratory method developed by the U.S. Geological Survey for determining the dielectric constant and resistivity of rock and soil samples using a two-terminal method over the frequency range $10^2$-$10^8$ Hz.

Previous work by the U.S. Geological Survey (Ref. 1) aimed at reducing the error caused by interfacial polarization in two-terminal measurements of the impedance of earth samples resulted in a workable
Table I

DIELECTRIC CONSTANT OF SEVERAL GEOLOGIC MATERIALS
AT $10^2$ Hz*

<table>
<thead>
<tr>
<th>Material</th>
<th>Water Content (Percent by Volume)</th>
<th>Dielectric Constant</th>
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</thead>
<tbody>
<tr>
<td>Limestone</td>
<td>0.45</td>
<td>$1.8 \times 10^2$</td>
</tr>
<tr>
<td>Quartz Monzonite</td>
<td>1.1</td>
<td>$1.1 \times 10^3$</td>
</tr>
<tr>
<td>Sandstone</td>
<td>10</td>
<td>$4.6 \times 10^3$</td>
</tr>
<tr>
<td>Shale</td>
<td>3.8</td>
<td>$1.3 \times 10^4$</td>
</tr>
<tr>
<td>Alluvium</td>
<td>13</td>
<td>$1.9 \times 10^4$</td>
</tr>
<tr>
<td>Topsoil</td>
<td>50</td>
<td>$1.9 \times 10^5$</td>
</tr>
</tbody>
</table>

* Adapted from Reference 1.
electrode system using platinized-platinum electrodes to minimize the effects of electrode impedance in the measurement. A disc of blotter paper containing colloidal silver and silver chloride is interposed between the platinized surface and the sample. This work resulted in a reliable method for determining resistivity and permittivity over the frequency range $10^2$-$10^6$ Hz. Subsequent work (Ref. 2) was directed toward developing a method for separating out the effect of electrode polarization in measurements of the sample impedance. Application of this method to the aforementioned electrode system enables estimation of the actual error induced by electrode polarization. It verifies that with proper precautions in electrode preparation, polarization has a relatively small effect on the measured impedance of the sample.

The developmental work chiefly reported here was undertaken by the U.S. Geological Survey at the request of AFWL to extend the range of measurement to $10^8$ Hz, in order to determine a high frequency value of dielectric constant. To accomplish this required the design of a test cell to minimize lead inductance and stray capacitance at high frequencies. To calibrate the system, several materials of precisely known electrical characteristics which exhibited a wide range in dielectric constant were used. In addition, a platinized-platinum electrode comprised of a platinum layer on a brass substrate was developed. This electrode has the same electrical characteristics as one of solid platinum; however, this composite electrode is more mechanically durable and less expensive than the solid platinum electrode.
SECTION II
EQUIPMENT

1. TEST CELL

The test cell containing the sample to be measured was designed with the following criteria in mind: The cell should

a. Be suitable for the frequency range $10^2-10^8$ Hz.

b. Be large enough to contain a representative amount of the geologic sample so as to minimize the effects of sample inhomogeneity.

c. Have a length-to-area ratio and perimeter small enough to minimize fringing effects.

d. Be sealed from the atmosphere to prevent moisture loss during measurement.

The cell, shown schematically on Figure 1 and pictorially on Figure 2, consists of a lucite sample holder 4 inches in diameter and 1 inch long with a 2-1/8-inch diameter hole through the center to accommodate the sample. Around the edge of the center hole is an L-shaped groove to hold a teflon O-ring which, when the sample and electrodes are in place, seals the cell against moisture loss. Twelve equally spaced holes are tapped around a 3-1/8-inch diameter circle to receive the nylon screws used to secure the end plates.

The end plates are 4-inch-diameter, 1-3/8-inch-thick lucite pieces which contain 2-inch-diameter, 1-inch-long gold-plated copper contact blocks. These blocks make contact with the backs of the electrodes used in the measurements. The purpose of the blocks is to minimize the radial resistance in the connection to the electrodes. The end
Figure 1.—Schematic Diagram of the Test Cell.
Figure 2. Assembled Test Cell (Background) and Opened Test Cell (Foreground).
plates also have 12 holes bored on a 3-1/8-inch-diameter circle; the nylon screws pass through these holes and attach to the sample holder. Electrical connection is made to the contact blocks via banana plugs.

2. IMPEDANCE MEASURING EQUIPMENT

For measurements in the frequency range of $10^2$ to $2 \times 10^4$ Hz, a Wayne Kerr Universal Bridge (Type B221) is used, driven by a Hewlett Packard audio oscillator (Type 201CR). For the frequency range of $2 \times 10^4$ to $10^6$ Hz, a Wayne Kerr Radio Frequency Bridge (Type B601) is used, driven by a Hewlett Packard test oscillator (Type 650A). A Rohde and Schwarz Tunable Indicating Amplifier (Type UBM) feeding a Tektronix oscilloscope (Type RS16) is used as a null detector for both bridges. The frequency of the oscillators is monitored on a Beckmann EPUT and Timer (Type 7360R). A block diagram is shown on Figure 3. This is essentially the same equipment used in previous work by the USGS (Ref. 1). The oscilloscope was added because it gives a sharper null, and consequently, a more sensitive indication of the quadrature component in the output of the bridge. The radio receiver used previously (Ref. 1) as a frequency converter for frequencies above $6 \times 10^5$ Hz (the upper frequency limit of the amplifier in its tuned mode) was eliminated since the oscilloscope provides an accurate null indication using the amplifier in its untuned mode. Connections to these bridges are made by means of a pair of shielded cables.

For the frequency range $10^6$ to $10^8$ Hz a Hewlett Packard RX meter (Type 250B) is used. This instrument is a self-contained impedance
Figure 3.—Block Diagram of the Impedance Measuring Equipment for the Frequency Range $10^2$ to $10^6$ Hz.
bridge, oscillator, and null detector. Connection is made by means of a banana plug mounted on the HI-terminal and a band of phosphor-bronze connected to the LO-terminal and terminated with another banana plug. Figure 4 shows the 250B with and without the test cell in place.

At a frequency of $10^8$ Hz, a correction must be made for the series inductance of the phosphor-bronze connecting band. This is accomplished by converting the measured parallel $R$ and $C$ of the cell and band combination to the equivalent series values of $R$ and $X$ (reactance at $10^8$ Hz), subtracting the band inductance term, and converting the result back to parallel equivalents which are then used with the formulas in Section III to compute $\kappa$ and $\rho$. At a certain value of cell capacitance (that is, dielectric constant of the sample) the band and cell become series resonant (that is, $X$ becomes 0 and $C$ parallel $\rightarrow + \infty$); thus, a certain range of dielectric constant (in this case $23 < \kappa < 30$) is out of the range of the bridge. Therefore, a second connecting band of phosphor-bronze having a different value of inductance must be used to cover this range of dielectric constants.

3. PREPARATION OF ELECTRODES

The electrodes are similar to the platinized-platinum foil and silver-silver chloride blotters reported previously (Ref. 1), with the difference that a platinum-plated brass disc is used as the base instead of the platinum foil. The advantages of the platinum-plated brass are its mechanical strength, rigidity, and economy. The pure platinum foil has a tendency to crease and dent even under small stress, and such imperfections prevent uniform platinization. The
Figure 4. 250B RX Meter and Connections to the Test Cell. Photograph (a) shows the Test Cell in Place, and (b) shows the Cell Removed.
plated electrodes have shown no tendency toward such mechanical imperfection. It is also much less expensive to produce a plated electrode than a pure one. No apparent electrical difference (within the limits of sample and electrode reproducibility) was noted when the two types of electrodes were compared using the procedures and cell type described previously (Ref. 1).

For the preparation of the electrodes, a stock solution of chloroplatanic acid (H₂PtCl₆) (0.3 percent PtCl₄) to which has been added 0.025 percent lead acetate is used both for platinum plating and platinizing. The electrode disc is cut 2-3/8 inches in diameter from 0.005-inch-thick shim brass, cleaned with trichloroethylene, and placed flat on the bottom of a 250 ml beaker. It is covered to a depth of 1 centimeter with a solution consisting of one part stock solution and five parts distilled water. The zinc in the brass reacts with the platinum in solution:

\[ 2\text{Zn} + \text{Pt}^{4+} \rightarrow 2\text{Zn}^{2+} + \text{Pt} \]

The reduced platinum deposits on the surface of the disc.

After about 15 minutes the disc has a uniform coat of platinum metal. The disc is removed from the solution, rinsed, dried, buffed on a wire buffing wheel, and cleaned. It is then returned to the solution to repeat the process three to four times until a bright platinum surface is obtained.

For platinizing, the stock solution is used at full strength. The negative lead of a controlled constant-current DC power supply is connected to the electrode to be platinized and the positive lead is connected to a strip of platinum metal. The two electrodes are immersed in
the solution and a current of 0.5 ampere is passed through it until a uniform coat of platinum black is obtained. The platinized electrode is then rinsed and placed in a beaker of distilled water until ready for use.

The electrolyte blotters are prepared by first cutting discs of blotter paper to the same diameter as the sample, and then preparing 0.1 N solutions of silver nitrate, hydroxamine hydrochloride, and sodium chloride. The disc is dipped in the silver nitrate and then blotted between layers of lab wipers. Next it is dipped in the hydroxamine hydrochloride and blotted. Finally it is dipped in the sodium chloride and blotted. For best results the cell is assembled and measured immediately after the blotters are prepared.

4. PREPARATION OF SAMPLES

a. Rocks

A right circular cylinder of rock, 2-1/8 inches in diameter and 1 inch long, is cut. If the natural state water is not preserved, the rock is placed in a dessicator and the vessel evacuated. Enough water of the proper salinity is introduced into the vessel under a vacuum to cover the sample. After the saturation process is completed, (36 to 48 hours) the sample is removed from the vacuum; its sides are wiped and wrapped with pressure bonding tape and it is inserted into the sample holder.

b. Soils

If the natural state water content of the sample is preserved, it is packed into the sample holder to its natural state density.
If the sample is dry, then water having the required salinity is added to restore it to its natural state water content and it is packed into the cell to its natural state density.
SECTI0N 111
THEORY OF MEASUREMENT

1. ELECTRICAL EQUIVALENT

In a two-terminal system the combination of sample, electrodes, and holder may be described electrically as follows:

At any frequency the sample is considered to be a parallel combination of capacitance, \( C_s \) and resistance, \( R_s \). In series with the sample is the electrode impedance, \( Z_e \), which is essentially due to a boundary-layer formation at the sample-electrode interfaces (Ref. 3). In parallel with this combination are two additional terms: (1) the electrical loss (represented by a resistance, \( R_f \)) of the cell, and (2) the fringing capacitance, \( C_f \), of the cell. This circuit is depicted schematically in Figure 5a.

If \( Z_e \) is much smaller than the impedance of the sample, it can be neglected and the total parallel equivalent capacitance, \( C \), and resistance, \( R \), can be expressed simply as:

\[
C = C_s + C_f
\]  
\[
\frac{1}{R} = \frac{1}{R_s} + \frac{1}{R_f}
\]

or by substituting \( C_s = \kappa \varepsilon_0 \frac{A}{L} \) and \( \frac{1}{R_s} = \frac{1}{\rho} \frac{A}{L} \) where \( \varepsilon_0 \) is the dielectric permittivity of free space, \( \kappa \) is the dielectric constant of the sample, \( \rho \) is the resistivity of the sample, \( A \) is the cross-sectional area of the sample, and \( L \) is the length of the sample, then:

\[
C = \kappa \varepsilon_0 \frac{A}{L} + C_f
\]  
\[
\frac{1}{R} = \frac{1}{\rho} \frac{A}{L} + \frac{1}{R_f}
\]
Figure 5.---Electrical Equivalent Circuits of The Test Cell, (a) Containing the Sample and (b) Containing Air.
If the sample is removed from the test cell, the equivalent circuit of Figure 5b applies, where $C_a$ is the capacitance of the sample chamber when it is empty (that is, contains air only). If $\epsilon_o \frac{A}{L}$ is substituted for $C_a$, the parallel equivalent capacitance, $C'$, and resistance, $R'$, of the empty cell are:

$$C' = \epsilon_o \frac{A}{L} + C_f$$

(5)

$$\frac{1}{R'} = \frac{1}{R_f}$$

(6)

where $C_f$ and $R_f$ are the same as those for the cell containing the sample.

The values of $C$, $C'$, $R$, and $R'$ are obtained directly from the instruments used to make the impedance measurements.

Subtracting (5) from (3) and rearranging gives

$$\kappa = \frac{L}{A \epsilon_o} (C - C') + 1$$

(7)

and subtracting (6) from (4) and rearranging gives

$$\frac{1}{\rho} = \frac{L}{A} \left(\frac{1}{R} - \frac{1}{R'}\right)$$

(8)

These equations apply only if (1) $R_f$ and $C_f$ are constant, that is, independent of the material within the test cell; and (2) $Z_e$ is negligible compared to the series impedance of the sample.

In practice, $R_f$ and $C_f$ are not constant but will vary slightly due to changes in the shape of the electric field which is a function of the dielectric constant and resistivity of the material within the cell. $Z_e$ is also affected implicitly by the values of $\kappa$ and $\rho$, as these and $Z_e$ are functions of the sample, as well as its water content and salinity.
2. SOURCES OF ERROR AND ACCURACY

One may consider the following as sources of error in the techniques described in this report:

(1) The assumption that the electrode impedance is negligible compared with the sample impedance.

(2) Errors due to accuracy limitations in the components of the measuring system.

(3) The assumptions that the sample cell resistance, $R_f$, and the fringing capacitance, $C_f$, are independent of the material in the cell.

With regard to the electrode impedance, $Z_e$, previous work (Ref. 1) was directed toward reducing the error due to polarization at the electrode sample interface in two-terminal systems. Several electrode systems were studied. This was done by measuring the parallel $R$ and $C$ of the sample and electrode system and calculating the apparent values of $\rho$ and $\kappa$. Then the length of the sample was halved and the measurement repeated and the apparent values of $\rho$ and $\kappa$ again were calculated. If the two values agreed the effects of polarization were considered negligible. The results of this work indicated that of the electrode types tested, the platinized-platinum electrode with silver-silver chloride electrolyte blotters produced a minimal amount of polarization error. Electrode polarization was indicated to be negligible, but its effect was not quantitized.

Further work (Ref. 2) demonstrated that it is possible to separate the electrode polarization impedance from that of the sample in a two-terminal system if various lengths of the sample material are available.
The total series impedance, $Z$, of the sample electrode system is given by:

$$Z = Z_e + Z_m$$

where $Z_e$ is the electrode polarization impedance, which is localized at the sample electrode interface and hence is independent of the length of the sample, and $Z_m$ is the impedance of the sample material. $Z_e$ can be broken down into its real and imaginary parts designated by $R_e$ and $-\omega I_e$, respectively. Taking the electrical equivalent of the sample to be a parallel combination of capacitance and resistance, $C_S$ and $R_S$, equation (9) becomes

$$Z = \left[ R_e + \frac{R_S}{1 + \omega^2 R_S^2 C_S^2} \right] - j\omega \left[ I_e + \frac{R_S^2 C_S}{1 + \omega^2 R_S^2 C_S^2} \right]$$

The values of $R_S$ and $C_S$ may be represented in terms of the sample dimensions and electrical parameters as follows:

$$R_S = \frac{\rho L}{A} \quad \quad C_S = \frac{\kappa \varepsilon_0 A}{L}$$

substituting these into equation (10) gives:

$$Z = \left[ R_e + \left( \frac{\rho A}{1 + \omega^2 \rho^2 \kappa^2 \varepsilon_0^2} \right) L \right] - j\omega \left[ I_e + \left( \frac{\rho^2 \kappa \varepsilon_0 A}{1 + \omega^2 \rho^2 \kappa^2 \varepsilon_0^2} \right) L \right]$$

in which the impedance due to the sample is a linear function of $L$.

If the equivalent parallel resistance, $R$, and capacitance, $C$, of the system are measured then:

$$Z = \frac{R}{1 + \omega^2 C^2 R^2} - j\omega \left( \frac{R^2 C}{1 + \omega^2 C^2 R^2} \right)$$
equating the real and imaginary parts of equations (11) and (12) gives

\[ \frac{R}{1 + \omega^2 C^2 R^2} = \text{Re} + \frac{\rho/A}{1 + \omega^2 \rho^2 \kappa^2 \epsilon_0^2} L \]  

(13)

and

\[ \frac{R^2 \epsilon_0}{1 + \omega^2 C^2 R^2} = \text{Im} + \frac{\rho^2 \kappa \epsilon_0^2 / \mu_0}{1 + \omega^2 \rho^2 \kappa^2 \epsilon_0^2} L \]  

(14)

If \( R \) and \( C \) are measured for several lengths of the material and the left-hand sides of equations (13) and (14) are computed for each length and plotted against \( L \), two linear plots should result whose slopes give numerical values for the coefficients of \( L \) in equations (13) and (14). Since \( \omega \), \( \epsilon_0 \), and \( A \) are known, the result is two equations in two unknowns (\( \rho \) and \( \kappa \)) which can be solved simultaneously. This process results in values of \( \rho \) and \( \kappa \) which are not affected by the electrode polarization impedance. Values determined by this method are therefore true values of these parameters.

An apparent dielectric constant, \( \kappa_a \), and an apparent resistivity, \( \rho_a \), can be calculated from the measured values of \( C \) and \( R \) for the system at each length. If the apparent values equal the true values as determined above, then the electrode polarization impedance is negligible. The departure of the apparent values from the true values is then a measure of the polarization error induced by the electrode system.

This method has been applied to two electrode systems with the results shown in Table II. The table gives, for selected earth materials, the percentage difference between the apparent and true
values of $\rho$ and $\kappa$ for the platinized-platinum, silver-silver chloride blotter system electrode, and for a system using the platinized platinum electrode without the blotter system. The use of the blotter system decreases the polarization error by providing an electrochemically reversible electrode. The average error for the blotter system computed from the table is 3.8 percent. The data for the blotter-electrode system, listed in Table II, indicates that polarization impedance is effectively negligible when using this electrode system.

The second element affecting the accuracy of the measurement is the accuracy of the impedance measuring equipment. Table III indicates the accuracies of the equipment used to measure impedance. The accuracy of the 250B impedance bridge is a function of the frequency and read values of $R$ and $C$. The values given in the table represent "worst case" errors in this application.

The final source of error is due to variations in the parasitic impedances $R_f$ and $C_f$. In order to minimize the error from these sources several liquids, whose values of $\rho$ and $\kappa$ are well known experimentally, were selected for use as standards to calibrate the test cell over the entire frequency range from $10^2$ to $10^8$ Hz.
Table II
PERCENTAGE DEVIATION OF THE APPARENT VALUES OF $\rho$ AND $\kappa$ FROM THE TRUE VALUES AS DETERMINED FROM MULTIPLE LENGTH SAMPLES FOR PLATINIZED-PLATINUM ELECTRODES

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Parameter</th>
<th>Alluvium</th>
<th>Topsoil</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Without Blotters (Percent)</td>
<td>With Blotters (Percent)</td>
</tr>
<tr>
<td>$10^2$</td>
<td>$\kappa$</td>
<td>5.7</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>3.6</td>
<td>3.5</td>
</tr>
<tr>
<td>$10^3$</td>
<td>$\kappa$</td>
<td>9.8</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>3.4</td>
<td>3.1</td>
</tr>
<tr>
<td>$10^4$</td>
<td>$\kappa$</td>
<td>10.4</td>
<td>6.0</td>
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<tr>
<td></td>
<td>$\rho$</td>
<td>3.2</td>
<td>3.1</td>
</tr>
<tr>
<td>$10^5$</td>
<td>$\kappa$</td>
<td>6.3</td>
<td>.5</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>1.9</td>
<td>.5</td>
</tr>
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<td>$10^6$</td>
<td>$\kappa$</td>
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<td>.8</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>.3</td>
<td>.1</td>
</tr>
</tbody>
</table>

Table III
ACCURACIES OF VARIOUS INSTRUMENTS USED TO MEASURE IMPEDANCE

<table>
<thead>
<tr>
<th>Bridge</th>
<th>Frequency (Hz)</th>
<th>Accuracy of C</th>
<th>Accuracy of R</th>
</tr>
</thead>
<tbody>
<tr>
<td>B221</td>
<td>$10^2 - 10^4$</td>
<td>$\pm0.25%$</td>
<td>$\pm0.2%$</td>
</tr>
<tr>
<td>B601</td>
<td>$10^5 - 10^6$</td>
<td>$\pm1.0%$</td>
<td>$\pm1.0%$</td>
</tr>
<tr>
<td>250B</td>
<td>$10^7$</td>
<td>$\pm0.55% \pm 0.15 \text{ pf}$</td>
<td>$\pm7.2% \pm 0.2\Omega$</td>
</tr>
<tr>
<td>250B</td>
<td>$10^8$</td>
<td>$\pm5.5% \pm 0.15 \text{ pf}$</td>
<td>$\pm5.8% \pm 0.2\Omega$</td>
</tr>
</tbody>
</table>
SECTION IV

METHOD OF MEASUREMENT

1. CALIBRATION OF THE TEST CELL

The test cell is calibrated in the following manner. First, the values $R'$ and $C'$ are measured for the empty cell at each frequency of interest. Second, the cell is filled with a liquid standard and $R$ and $C$ are again measured. The differences between $C$, $C'$ and $R$, $R'$ are then used to calculate the apparent values of $\rho$ and $\kappa$ for the liquid using equations (7) and (8). This process is repeated for several standards with a range of values of $\rho$ and $\kappa$. Calibration curves are then obtained for the cell relating the apparent values of $\rho$ and $\kappa$ to the known values at each frequency.

Table IV lists the standards used for calibration along with the true values of $\rho$ and $\kappa$. Methanol-water mixtures were used to cover the range of $\kappa$ from 31 to 78 and Figure 6 shows typical curves relating the true values of $\rho$ and $\kappa$ as a function of the weight percent of methanol for these mixtures.

2. MEASUREMENT OF THE SAMPLE

Using the calibrated cell, the equivalent parallel capacitance and resistance, $C$ and $R$, of the cell containing the sample are measured, and using $C'$ and $R'$ for air, the apparent values of $\rho$ and $\kappa$ are calculated. The calibration data are then used to find the true values.
### Table IV

**CALIBRATION STANDARDS**

<table>
<thead>
<tr>
<th>Material</th>
<th>Temperature (°C)</th>
<th>Frequency (Hz)</th>
<th>$\rho$(Ω-m)</th>
<th>$\kappa$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>25</td>
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<td>$5.75 \times 10^3$</td>
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<td>4</td>
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<tr>
<td></td>
<td></td>
<td>$10^6$</td>
<td>$5.75 \times 10^3$</td>
<td>78.2</td>
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<td>$10^7$</td>
<td>$5.00 \times 10^3$</td>
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<tr>
<td></td>
<td></td>
<td>$10^8$</td>
<td>$4.61 \times 10^2$</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>Methanol</td>
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<td>$10^6$</td>
<td>$2.90 \times 10^3$</td>
<td>31</td>
<td>4</td>
</tr>
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<td></td>
<td></td>
<td>$10^7$</td>
<td>$2.23 \times 10^3$</td>
<td>31.0</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>$10^8$</td>
<td>$1.54 \times 10^2$</td>
<td>31.0</td>
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</tr>
<tr>
<td>1-Propanol</td>
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<td>$10^6$</td>
<td>$4.97 \times 10^4$</td>
<td>20.1</td>
<td>4</td>
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<td></td>
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<td>$10^7$</td>
<td>$5.26 \times 10^3$</td>
<td>20.1</td>
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<td></td>
<td></td>
<td>$10^8$</td>
<td>$4.76 \times 10^1$</td>
<td>19.0</td>
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<tr>
<td>1-Butanol</td>
<td>25</td>
<td>$10^5$</td>
<td>$8.61 \times 10^4$</td>
<td>17.4</td>
<td>4</td>
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<td></td>
<td></td>
<td>$10^6$</td>
<td>$1.03 \times 10^5$</td>
<td>17.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^7$</td>
<td>$1.09 \times 10^6$</td>
<td>17.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^8$</td>
<td>$4.30 \times 10^3$</td>
<td>17.4</td>
<td></td>
</tr>
<tr>
<td>2-Methyl-1-Propanol</td>
<td>30</td>
<td>$3.16 \times 10^7$</td>
<td>$5.69 \times 10^2$</td>
<td>17</td>
<td>5</td>
</tr>
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<td></td>
<td></td>
<td>$9.74 \times 10^7$</td>
<td>$4.64 \times 10^1$</td>
<td>16.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1.65 \times 10^8$</td>
<td>$1.96 \times 10^1$</td>
<td>13.6</td>
<td></td>
</tr>
<tr>
<td>1-Heptanol</td>
<td>25</td>
<td>$1.00 \times 10^7$</td>
<td>---------</td>
<td>11.7</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1.26 \times 10^7$</td>
<td>$1.52 \times 10^3$</td>
<td>11.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$9.59 \times 10^7$</td>
<td>$3.86 \times 10^1$</td>
<td>7.86</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1.30 \times 10^8$</td>
<td>$3.29 \times 10^1$</td>
<td>7.05</td>
<td></td>
</tr>
<tr>
<td>Benzene</td>
<td>25</td>
<td>0</td>
<td>---------</td>
<td>2.28</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$9.00 \times 10^9$</td>
<td>---------</td>
<td>2.28</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6. \( \kappa \) and \( \rho \) of Methanol Aqueous Solutions as a Function of the Weight Percent of CH\(_3\)OH Compensated for Temperature at 25°C (Adapted from Ref. 6).
SECTION V
RESULTS OF CALIBRATION

The appendix contains the calibration curves for the dielectric constant at each frequency decade from $10^2$ to $10^8$ Hz inclusive for the cell described in Section II. The values of apparent dielectric constant, $\kappa_a$, as calculated from equation (7), are plotted horizontally against the true value, $\kappa_T$, from Table IV or Figure 6, vertically. In most cases the points fall close to the ideal line ($\kappa_a = \kappa_T$). This indicates that the variations in the $R_f$ and $C_f$ terms are minimal. A linear curve was fitted to the points by the method of least squares. The standard error of $\kappa_T$ on $\kappa_a$ obtained from the fit is taken as the error introduced by the fringing terms and experimental scatter in taking the calibration data. The average standard error for all the calibration curves is 1.8 with a range of 1.4 to 2.1. Exceptions occurred (4 points at $10^2$ Hz and 2 points at $10^3$ Hz) but were not included in the least squares fitting because they were attributed to electrode failure. Such failure is occasionally experienced and is due mainly to dirt and grease that remained on the brass disc during the platinum plating and/or platinizing process and prevented proper deposition of the metal. The equation for the linear least-squares fit to the data is given on the graph, and this equation is used to convert the apparent value of dielectric constant to the true value for an unknown.

Measured values of resistivity were consistently lower than the expected values for the materials used, and showed no correlation.
In some cases they were off by as much as an order of magnitude. This is apparently due to the fact that the materials used for calibration (purposely selected for their dielectric constants) tend to pick up contaminants, including water, in minute quantities, yet sufficient to cause a marked decrease in resistivity. However, these results indicate that although the values of resistivity of some of the materials had been greatly altered, the dielectric constants of these same materials showed no tendency to deviate from the expected values, indicating (1) that the parallel R-C equivalent circuit is accurate at any one frequency, and (2) that it is therefore possible to calibrate the cell for resistivity independently of the dielectric constant by using aqueous solutions of sodium chloride.

Table V shows the results of a series of measurements on a single geologic sample, in this case a sandy soil. Included are the apparent values of dielectric constant, $\kappa_a$, and resistivity, $\rho_a$, and the true dielectric constant, $\kappa_T$, calculated from the curve equations given in the appendix.

The high dielectric constant at low frequency and the abrupt decrease in resistivity at $10^8$ Hz are notable. Both of these effects are considered real, the former may be attributed to the presence of polarizable material in the sample itself and the latter due possibly to permanent dipole loss effects at high frequency.
### Table V

**ELECTRICAL PARAMETERS OF A SANDY SOIL OVER THE FREQUENCY RANGE 10^2-10^8 Hz**

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$\rho_a$ (Ω-m)</th>
<th>$\kappa_a$</th>
<th>$\kappa_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^2$</td>
<td>749</td>
<td>3,110</td>
<td>2,960</td>
</tr>
<tr>
<td>$10^3$</td>
<td>734</td>
<td>361</td>
<td>369</td>
</tr>
<tr>
<td>$10^4$</td>
<td>719</td>
<td>59.9</td>
<td>58.5</td>
</tr>
<tr>
<td>$10^5$</td>
<td>698</td>
<td>21.9</td>
<td>22.0</td>
</tr>
<tr>
<td>$10^6$</td>
<td>606</td>
<td>8.97</td>
<td>9.56</td>
</tr>
<tr>
<td>$10^7$</td>
<td>402</td>
<td>4.65</td>
<td>6.87</td>
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<tr>
<td>$10^8$</td>
<td>105</td>
<td>5.51</td>
<td>3.89</td>
</tr>
</tbody>
</table>

This investigation chiefly represents the results of the development of the laboratory method and to date only a limited number of geologic materials have been measured. It is proposed to look further into these effects and their relationship to the composition of geologic materials.
SECTION VI
CONCLUSIONS AND RECOMMENDATIONS

The determination of the dielectric constant of rock and soil samples to an accuracy of 5 percent ±1.8 is possible in the frequency range $10^2$-$10^8$ Hz, provided that precautions are taken to minimize the effects of polarization errors at the electrode sample interface and parasitic impedances. This has been accomplished by using (1) an electrolytically reversible electrode system and (2) standards to calibrate the test cell.

Accuracies better than this may be obtained by a more detailed calibration using additional standards especially in the range of dielectric constants less than 20, which may be critical for earth materials at the high frequencies.

A detailed study of selected geologic materials is needed to determine the general variation of dielectric constant as a function of rock and soil type. It is also needed to determine to what extent the frequency dispersion of $\varepsilon$ and $\rho$ can be quantized, both in terms of an electrical analogue and in predicting the behavior with frequency of a particular rock given a minimum of electrical and/or chemical parameters of the rock.
APPENDIX

CALIBRATION CURVES FOR DIELECTRIC CONSTANT

Each graph (Figs. 7-10) is a plot of the true dielectric constant, \( \kappa_t \), against the apparent dielectric constant, \( \kappa_a \), at a discrete frequency. The line is the linear least-squares fit to the plotted data. The equation of this line is given on the plot. The symbols used for the various standards are shown below.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Benzene</td>
</tr>
<tr>
<td>○</td>
<td>1-Heptenol</td>
</tr>
<tr>
<td>△</td>
<td>2-Methyl-1-Propanol</td>
</tr>
<tr>
<td>▽</td>
<td>1-Butanol</td>
</tr>
<tr>
<td>□</td>
<td>1-Propanol</td>
</tr>
<tr>
<td>♦</td>
<td>Methanol</td>
</tr>
<tr>
<td>●</td>
<td>80% Methanol</td>
</tr>
<tr>
<td>▲</td>
<td>60% Methanol</td>
</tr>
<tr>
<td>▼</td>
<td>40% Methanol</td>
</tr>
<tr>
<td>■</td>
<td>20% Methanol</td>
</tr>
</tbody>
</table>
Figure 7.—Calibration Curves for the Test Cell for Determining Dielectric Constant at the Frequencies $10^2$ and $10^3$ Hz.

$k_f = 0.950 k_0 + 1.97$

$k_f = 1.20 k_0 + 0.905$
Figure 8.—Calibration Curves for the Test Cell for Determining Dielectric Constant at the Frequencies $10^4$ and $10^5$ Hz.
Figure 9.—Calibration Curves for the Test Cell for Determining Dielectric Constant at the Frequencies $10^6$ and $10^7$ Hz.
Figure 10.—Calibration Curve for the Test Cell for Determining Dielectric Constant at the Frequency $10^8$ Hz.
REFERENCES


