Switching Notes
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OPERATING THEORY OF POINT-PLANE SPARK GAPS

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Abstract

The point-plane spark gap design proposed by J. C. Martin of AWRE has achieved wide use in recent pulsed power systems. Extremely precise breakdown time is one of this gap's most useful properties; an explanation of this behavior is developed using J. C. Martin's gas breakdown relations as a starting point. Martin's definition of the "effective time" of a switching voltage pulse is also derived.

I. Introduction

The point-plane pulse-overvolted spark gap design of J.C. Martin has achieved wide application in pulsed power systems. Its primary advantage is the extremely precise time of sparkover, or switch closure, which this switch exhibits. Mr. Martin has often explained this phenomenon, but the author knows of no written exposition on the general subject of point-plane spark gaps which includes such an explanation. Since this may be useful to both system designers and users in the pulsed-power field, the following is a very simple discussion of streamer propagation data, the use of streamer velocity relations and the application of these data to point-plane gap design.

II. Breakdown Data and Streamer Velocity

Suppose the following experiment is performed. Given a point-plane spark gap, and a source of voltage, we apply a rising voltage waveform to the gap and measure the streamer "formation time"; in this case defined as the time interval between initial application of the voltage and onset of heavy current conduction across the gap. Suppose further that the voltage source has a simple linear ramp time dependency, for simplicity in the present arguments. We will generalize this a bit later in the discussion.

Then the experimental set-up could look like Figure 1.

Now by design,

$$V(t) = \alpha t \tag{1}$$

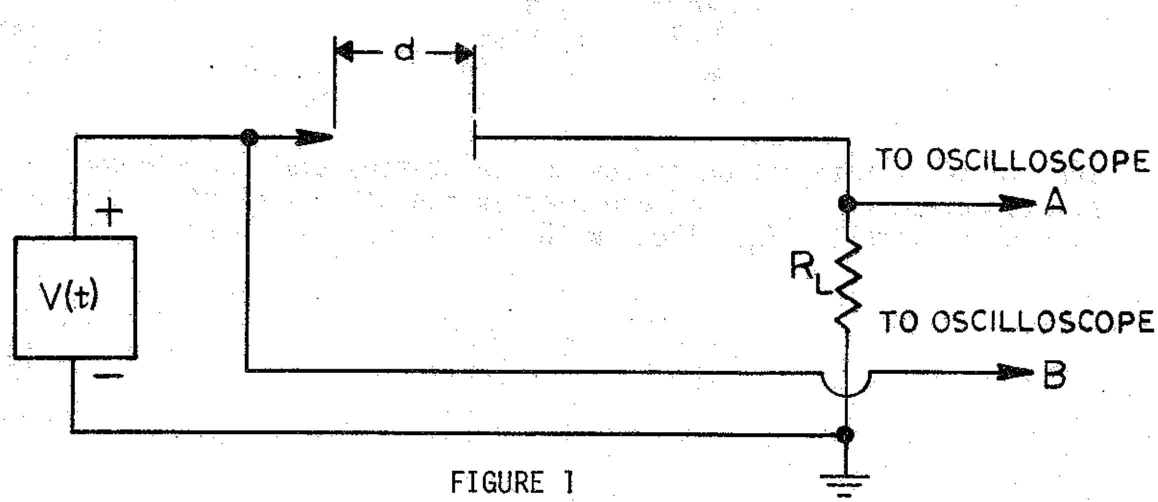

FIGURE 1

Then we vary the gap spacing, d, for a given rate of voltage increase, α. In the general case, assuming no other important parameters in the problem, we will get a relation between the breakdown voltage V_o, the gap spacing d and the time to breakdown T_o, of this form:

$$V_o^P T_o^Q d^M = x \text{ (a constant)} \tag{2}$$

This equation (2) is all we can write directly from our experimental data. If we now want to know the propagation velocity of the streamer across the gap, we can try to deduce an appropriate form of it from (2).

It is important to note that the velocity relation we are seeking cannot be uniquely determined from (2), and this will be shown by example. However, one can examine several possible velocity relations to see which most nearly fits the current theory of streamer propagation, and select that relation for further use. One of the chief uses for the streamer velocity relation will be to explain the small observed percentage jitter obtained from point-plane spark gaps, which will be done in section 3 of this discussion.

Now we assume a form for the streamer velocity, and then integrate it to obtain (2). As one example, we assume

$$\frac{dx}{dt} = A\frac{V^N}{x^S} \tag{3}$$

where x is the length of the streamer at a given instant, and A is a constant. Then, with such a "separated-variable" formula, we can integrate quite easily.

$$\int_o^d x^S dx = A\int_o^{T_o} V^N dt \tag{4}$$

The limits of integration correspond to stating that the streamer starts out at t = o, x = o, and reaches the other side of the gap (x = d) at time t = T_o. Then, with the use of (1), we get

$$\left.\frac{x^{S+1}}{S+1}\right]_o^d = A\alpha^N \left.\frac{t^{N+1}}{N+1}\right]_o^{T_o} \tag{5}$$

so

$$\frac{d^{s+1}}{s+1} = \frac{A\alpha^N T_o^{N+1}}{N+1} \tag{6}$$

Now, from (1) we see that the voltage at breakdown, V_o, is given by

$$V_o = \alpha T_o \tag{7}$$

so (6) can be rewritten as

$$\frac{d^{s+1}}{s+1} = \frac{AV_o^N T_o}{N+1} \tag{8}$$

Then (8) can be rearranged to give the assumed experimental data equation format of (2).

$$\left(\frac{s+1}{N+1}\right) V_o^N T_o d^{-(s+1)} = A^{-1} \tag{9}$$

Mr. Martin normally writes equation (2) in terms of F (the mean breakdown field, defined as V_o/d), T_o and d. So we follow this convention, rearranging (9) in accordance with the definition of F to read

$$\left(\frac{s+1}{N+1}\right) F^N T_o d^{N-s-1} = A^{-1} \tag{10}$$

A further convention of Martin's is to set the exponent on F equal to 1. This can be done by writing:

$$F\left[\frac{T_o(s+1)}{(N+1)}\right]^{\frac{1}{N}} d^{\frac{N-s-1}{N}} = A^{\frac{-1}{N}} \tag{11}$$

Now we can do the same thing to (2) finding:

$$FT_o^{Q/P} d^{\frac{M+P}{P}} = X^{\frac{1}{P}} \equiv K \text{ (a constant)} \qquad (12)$$

Comparing (11) and (12), we can identify

$$\frac{1}{N} = \frac{Q}{P}$$

$$\frac{N-s-1}{N} = \frac{M+P}{P} \qquad (13)$$

and $$A^{-\frac{1}{N}} = K$$

So the streamer velocity relation in (3) can be related to the experimental data expressed in (12) by the relations (13).

As a concrete example, consider J.C. Martin's breakdown equation for point-plane gaps in a gas:

$$FT_e^{1/6} d^{1/6} = K' \qquad (14)$$

(F is in kV/cm, T_e is in μs, d in cm. $K' = 23$ for air at STP; the relation between K' and our constant K will be given later).

Here T_e is defined as the "effective time," or the interval between the 88%-height points of the voltage pulse. The reasons for this definition will be touched on later. For now, we note that the identifications (13), when applied to (12) and (14), give the following results for the streamer velocity constants:

$$\frac{1}{N} = \frac{Q}{P} = \frac{1}{6}$$

$$\therefore \quad N = 6$$

$$\frac{N-s-1}{N} = \frac{5-s}{6} = \frac{M+P}{P} = \frac{1}{6} \qquad (15)$$

$$\therefore \quad S=4$$

and $$A = K^{-6}$$

So the streamer velocity, in the same unit system (MV, μS, cm) as (14), is given by

$$\frac{dx}{dt} = \frac{V^6}{K^6} \cdot \frac{1}{x^4} \tag{16}$$

which is the relation also given by Martin as the one he prefers to use[1], apart from a factor 1/5 which will be explained further on. This factor will appear when we relate Martin's constant, K′, to the constant K used in (12).

In regard to the physical "reasonableness" of (16), note that as the streamer starts its journey (x = 0) it would <u>seem</u> to have infinite velocity. Of course, at this time V is also very near zero, so this is misleading. The main argument in favor of (16) is that in a point-plane geometry, the streamer moves out into a diverging (decreasing) field pattern as it leaves the pointed electrode, and so could be expected to slow down, since the local field in the vicinity of the streamer tip stores progressively less energy. This, of course, ignores the probable effect that the streamer itself has a strong influence on the local field, but the arguments are only approximate at best. At least, (16) seems to have some basis in the expected physical behavior.

It was remarked earlier that (16) is not a unique solution to the problem; we can show this by finding yet another streamer velocity relation which leads to the identical "data" equations (12-14) yet has markedly different properties. Simply change the distance variable in the velocity relation from x to d-x. Then (16) becomes

$$\frac{dx}{dt} = \frac{V^6}{K^6} \cdot \frac{1}{(d-x)^4} \tag{17}$$

Now integrating this over the same ranges of x and t as before, and continuing to use the linear time-voltage relation (1), we get

$$K^6 \int_0^d (d-x)^4 \, dx = \alpha^6 \int_0^{T_0} t^6 dt \tag{18}$$

Then

$$- \frac{K^6 (d-x)^5}{5} \Bigg]_0^d = \frac{\alpha^6 T_0{}^7}{7} \tag{19}$$

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[1]J. C. Martin, AWRE, Private Communication

or

$$\frac{K^6 d^5}{5} = \frac{V_o^{\,6} T_o}{7} \tag{20}$$

since $\alpha T_o = V_o$ from (1).

Then

$$\frac{V_o^{\,6}}{d^6} \cdot d \cdot \frac{5T_o}{7} = K^6 \tag{21}$$

which leads to

$$F\, d^{1/6} \left(\frac{5}{7} T_o \right)^{1/6} = K \tag{22}$$

This is identical to (11), with S=4, N=6 as derived before. But the form of the streamer velocity equation (17) is markedly different from (16), since in (17) the streamer velocity becomes truly infinite at the end of the pulse, where V>0, rather than quasi-infinite (indefinite) at the beginning of the pulse, where V∼0. The real case probably combines these effects, with the streamer moving rapidly away from the pointed electrode, then slowing down somewhat as the electric field decreases, then gathering speed again as it nears the planar electrode where the field between streamer tip and electrode increases again with x. This kind of relation can be represented by a velocity equation of the form:

$$\frac{dx}{dt} = \frac{V^6}{K^6 x^2 \, (d-x)^2} \tag{23}$$

where the reader can show that essentially the same "data" equation (14) will result upon integration, with a modified constant term.

So much for the basic uncertainty in the streamer velocity relation. In summary, we have shown that given only the basic data of breakdown voltage, breakdown time and gap spacing, no unique form of the velocity relation can be derived. However, one can fall back on other data, such as time-resolved photography of streamer propagation, to uphold a particular choice of the velocity relation which is consistent with the time-integrated data. Such a choice is given for gases, by (16) according to J.C. Martin (we will multiply (16) by 1/5 later on to give Martin's result) and we will use this relation for our further discussion of streamer velocity and breakdown data.

We now return to an important factor which has been "glossed over" up to this point. Note that in (14), the time T_e is defined as an "effective time" equal to the width of the voltage pulse at the 88% amplitude points. However, in (22), the time factor is 5/7 T_o, which, for a linear ramp voltage pulse, is surely not the indicated width at 88% amplitude. Why not, and why choose the 88% figure in the first place?

The reasons behind these factors can be seen by manipulating the streamer velocity relation (16), using it to perform a "thought experiment."

Suppose we do our original experiment with several different pulse shapes, rather than the linear ramp voltage assumed in (1). Then, for each new pulse shape, a new set of breakdown data will result. How can we unify these data under one roof?

To be quantitative, we consider the family of pulse shapes given by

$$V(t) = \alpha t^N \tag{24}$$

where N has any value $\geq$ zero that we wish. The family of voltage pulses represented by (24) is quite large, and has the common property of being monotone increasing in time (the curve and its first derivative are always ≥ 0 while the second derivative has a constant algebraic sign, either (+) or (-)). Some curves, such as the popular 1-cos(wt) function, do not have this property and cannot be well approximated by (24). However, for point-plane gaps we usually want the gap to break over during the rising portion of the pulse (as in peaking-capacitor generators) so this part of the voltage waveform can be closely approximated by correctly choosing α and N in (24). A sketch of the rather wide family of curves represented by (24) is shown in Figure 2.

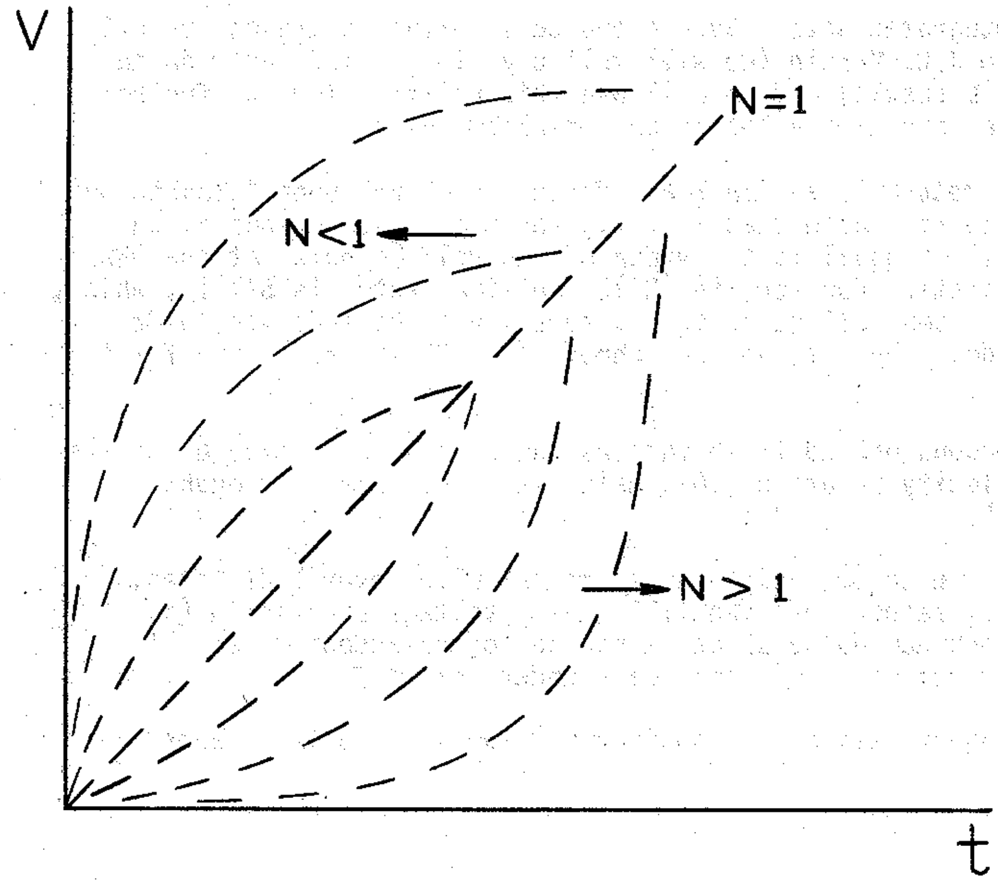

Figure 2

Sketch of the possible curves obtainable by correct choice of α and N in (24).

Now if we take (16) to adequately represent the form of the streamer velocity in a point-plane gap for given V(t), then we can substitute the voltage pulse (24) into (26) and derive the resulting "breakdown data" results.

Proceeding:

$$\frac{dx}{dt} = \frac{V^6}{K^6 x^4} = \frac{\alpha^6 t^{6N}}{K^6 x^4} \tag{25}$$

$$K^6 \int_0^d x^4\, dx = \alpha^6 \int_0^{T_0} t^{6N} dt \tag{26}$$

or

$$\frac{K^6 d^5}{5} = \frac{\alpha^6 T_o^{(6N+1)}}{6N+1} \qquad (27)$$

Now since T_o is the observed breakdown time of the gap, (16) gives us

$$V_o = \alpha T_o^N \qquad (28)$$

where V_o is the observed breakdown voltage.

Then (27) becomes

$$\frac{K^6 d^5}{5} = \frac{V_o^6 T_o}{6N+1} \qquad (29)$$

Then, as before, let $F = \frac{V_o}{d}$. So

$$5\left(\frac{F^6 d\ T_o}{6N+1}\right) = K^6 \qquad (30)$$

Then

$$F\left(\frac{5d\ T_o}{6N+1}\right)^{1/6} = K \qquad (31)$$

is the resulting breakdown data relation.

Now, for a given gap spacing (d) and total breakdown time T_o, we have a wide variety of breakdown field levels F (hence wide variation in voltage V_o) depending on the shape of the voltage curve (or value of N) chosen. What, if anything, do all these data have in common? Suppose we try to relate them by defining an "effective time" T_e such that for a given gap spacing and T_e, the breakdown voltage is independent of the exact shape of the pulse used. This would be a useful artifice.

How is T_e to be defined?

We follow J.C. Martin's lead and try using the time interval between a certain pulse amplitude and the peak time T_o. Here, we assume that the pulse falls very steeply just at T_o, which amounts to saying that the gap closes very rapidly compared to T_e (resistive and inductive times are small). An extension of this is to define T_e as the interval during which the pulse exceeds a given fraction of its observed peak amplitude, which is Martin's definition, and includes the effect of finite pulse fall-time. The difference between these definitions is generally small and will not be considered further.

Then, define T_e as the time interval between the amplitudes βV_o and V_o, where $0<\beta<1$ is to be correctly chosen.

Using (24) we have the definitions:

$$V_o = \alpha T_o^N \tag{32}$$

and

$$\beta V_o = \alpha T_\beta^N \tag{33}$$

whence

$$T_o = \left(\frac{V_o}{\alpha}\right)^{1/N} \tag{34}$$

and

$$T_\beta = \left(\frac{\beta\, V_o}{\alpha}\right)^{1/N} \tag{35}$$

Then our trial definition of "effective time" becomes

$$T_e = T_o - T_\beta = \left(\frac{V_o}{\alpha}\right)^{1/N}\left(1-\beta^{1/N}\right) \tag{36}$$

Now from (32) again,

$$\left(\frac{V_o}{\alpha}\right)^{1/N} = T_o \tag{37}$$

so

$$T_e = T_o\left(1-\beta^{1/N}\right) \tag{38}$$

If we want to write this kind of "effective time" function (38) into (31) to eliminate all dependence on N, we see that the problem is to find β, and some constant multiplier which we will call δ, such that

$$T_e = T_o\left(1-\beta^{1/N}\right) \cong \delta\left[\frac{5T_o}{6N+1}\right] \tag{39}$$

Thus,

$$1-\beta^{1/N} \cong \frac{5\delta}{6N+1}\ ; \quad 0<\beta<1 \tag{40}$$

A little time spent sketching curves will convince the reader that the basic form of the functions on either side of (40) is remarkably similar over the range of $N\geq0$. In fact, the congruence of shapes is excellent. So one can be sure that choosing β and δ properly will indeed provide a proper solution to the problem.

Perhaps the simplest way to force the approximate equality in (40) is by trial-and-error. We will use a semi-systematic method for this. First, we set the N=o values of (40) exactly equal, since this value does not depend on β. Then

$$(1-\beta^{\infty}) = 1 = 5\delta \tag{41}$$

or

$$\delta = \frac{1}{5}$$

This is the "missing" factor of 1/5 in equation (16), and we will come back here later and use it.

Now we set the two sides of (40) exactly equal for several other values of N and observe the results: Try N = 2.

$$(1-\beta^{\frac{1}{2}}) = \frac{1}{13} \qquad (42)$$

$$\beta^{1/2} = \frac{12}{13} \qquad (43)$$

$$\beta = 0.852 \qquad (44)$$

Similarly, for any N,

$$1-\beta^{1/N} = \frac{1}{6N+1} \qquad (45)$$

$$\beta = \left(\frac{6N}{6N+1}\right)^N \qquad (46)$$

We inquire about the limit of (46) as N grows arbitrarily large. Noting

$$\beta = \left(\frac{1}{1+\frac{1}{6N}}\right)^N \qquad (47)$$

we write

$$\ln\beta = N\left[\ln(1)-\ln(1+\frac{1}{6N})\right] \qquad (48)$$

and since

$$\ln(1+x) \approx x \quad \text{as} \quad x \to 0\pm \tag{49}$$

we get

$$\lim_{N \to \infty} \beta = e^{-\frac{1}{6}} = 0.846 \tag{50}$$

So β varies almost not at all over the range $1<N<\infty$. We can be sure this variation is smooth and non-oscillatory (so we haven't just checked a pair of "nice" values of N) by our prior curve-tracing experience, or by noting that $\frac{d\beta}{dN}$ is negative over the entire range of positive N.

The limit of (47) as N vanishes can also be evaluated, and turns out to be

$$\lim_{N \to 0} \beta = 1 \tag{51}$$

Hence, we have very little variation in β from 1 to ∞ in N, but somewhat more variation from 0 to 1. If we insist on covering the entire range of possible voltage waveforms with a single value of β, this value should be some sort of average over the possible values of N.

It is apparent, from the sketch of Figure B-2, that the family of curves represented by (24) divides evenly at N = 1, with curves of lesser N having negative curvature, and those for greater N showing positive curvature. Also, given a curve with a certain N value, a "mirror-image" curve exists with exponent value 1/N.

That is, if the normalized form of (24), with V = 1 at t = 1, is

$$V = t^N \tag{52}$$

then exchanging N for 1/N gives

$$V = t^{1/N} \tag{53}$$

or

$$t = V^N \tag{54}$$

which is the same curve shape with the V and t axes interchanged.

All of these things suggest that an exponent of value N is equally likely to occur as an exponent of value 1/N, so the appropriate "average" value of β would be an average over several decades of N, giving each β value equal weight.

A numerical table, with β calculated from (46), is shown as Table 1, with the appropriate average value given as 0.884. This explains Martin's choice of the effective time described earlier, as the interval between 88% - amplitude points of the pulse.

TABLE I

Values of β For Four Decades of N

N	β
0.01	0.971692
0.03	0.945152
0.1	0.906574
0.3	0.875859
1	0.857143
3	0.85027
10	0.847645
30	0.846872
100	0.846599
Total	7.94781
Average	0.88309

All of this discussion leads us to replace the time dependence relation

$$f(t) = \left(\frac{5T_o}{6N+1}\right)^{\frac{1}{6}} \qquad (55)$$

appearing in (31), with a simpler "effective time" T_e which includes the explicit dependence on pulse shape (the exponent N) in its verbal definition. Then, the resulting breakdown equation will be that given by Martin, equation (14).

We saw in (39) that

$$\frac{1}{\delta} T_e = \frac{5T_0}{6N+1} \tag{56}$$

and from (41) that $\delta = 1/5$. So, making the direct substitution (56) in equation (31) leads to

$$F\,(dT_e)^{1/6} = K\delta^{1/6} \tag{57}$$

or

$$Fd^{1/6}\,T_e{}^{1/6} = \frac{K}{(5)^{1/6}} = K' \tag{58}$$

Values of K' are used by Martin, (such as 23 for air). These are related to the constant K used in deriving (16), by (58). Using (58), we have a modified version of (16):

$$\frac{dx}{dt} = \frac{1}{5} \cdot \frac{V^6}{(K')^6} \cdot \frac{1}{x^4} \tag{59}$$

in which K' is Martin's constant. This is the exact form of the streamer velocity relation given by Martin. It makes the correction factor due to the "effective time" definition (39) explicit. This factor is, by contrast, "hidden" in our equation (16). In most of the following discussions, Martin's equation (59) will be used.

III. Breakdown on a Falling Waveform

All of the preceding deals with gap breakdowns which occur on the "leading edge", or rising portion, of the applied voltage waveform. This is the usual case in point-plane switch design, and the results of section 2 will be useful in that regard. However, in the design of gaseous insulation systems for high voltage pulsers, it is important to know how J.C. Martin's data equation (59) applies. To get an approximate, but useful, appreciation for this problem we first idealize the situation as follows.

A. Assume the gaseous insulation system is weak at one or more points, due to imperfections of various kinds, and that these points (presumably located on high-voltage electrodes) do launch streamers into the gas at the peak of the applied voltage pulse.

B. Assume that the rise time of the applied pulse is much less than its fall time, so that most of the streamer's path will be traversed on the falling portion of the pulse.

We could repeat a fairly general analysis for this case, using a family of voltage pulses similar in form to (24). However, in most cases of interest the falling portion of the voltage pulse has exponential shape, so this case will be the only one considered here.

Once again, we follow J.C. Martin and assume that the streamer velocity relation has the form (59). Then, if

$$V(t) = V_o e^{-t/\tau} \tag{60}$$

we want to know whether (59) will give the same breakdown law (14) as before, in this new case.

Repeating the integration of (59) as before, using the voltage pulse (60) this time, we find

$$\frac{(K')}{d} = \frac{\tau}{6} F^6 \left[1-e^{-6T_0/\tau}\right] \tag{61}$$

or

$$F\left(\frac{\tau}{6}\left[1-e^{-6T_0/\tau}\right]\right)^{1/6} d^{1/6} = K' \tag{62}$$

where T_0 is the point on the wavetail where the streamer bridges the gap d.

Now how does the time dependence

$$\frac{\tau\left[1-e^{-6T_0/\tau}\right]}{6} = f(t_0) \tag{63}$$

compare with the prior definition of effective time? If we use the 88% - width of this exponential pulse, we find that

$$0.88 = e^{-T_e/\tau}$$

or

$$T_e = -\tau \ln(0.88) = 0.128\tau \tag{64}$$

Then for what closure time T_o does (63) equal (64)? We find

$$1-e^{-6T_o/\tau} = 0.768 \tag{65}$$

or

$$T_o/\tau = 0.244 \tag{66}$$

Thus, if one uses the breakdown relation (14) for the case of a falling exponential pulse, and uses T_e as defined by Martin, one will find the spacing d and field F for which the gap (d) closes in about one-quarter time constant. This is an interesting set of F and d values, but is an unsafe set, since it assures flashover of the gap. In the usual case, one wants to know how wide the gap must be made to avoid flashover. To answer this question, let the closure time T_o become infinite in (62) and we have

$$F \left(\frac{\tau}{6}\right)^{1/6} d^{1/6} = K' \tag{67}$$

Equation (67) gives the F and d values, for a given fall-time constant τ, such that the gap closes at $t=\infty$, when no energy is left in the system. Note here that the time one uses is not the usual 88% effective time but is instead one-sixth of the time constant.

Of course, this conclusion rests entirely on the question of how well the streamer velocity relation (59) describes the true state of affairs. One can conclude that Martin's breakdown relation (14), streamer velocity relation (59) and definition of effective time T_e all form a self-consistent set of very useful tools, but that care must be taken in using these tools to predict breakdown behavior on a falling voltage waveform.

IV. Time Precision of Point-Plane Gaps

Martin's streamer velocity relation (59) will now be used to explain the low-jitter performance of point-plane spark gaps. Suppose there is non-negligible jitter in the first event, namely launching of the streamer into the gap. The geometry produces a highly enhanced field, so this event occurs early on a rising voltage waveform, but statistical delay in emission of the initiatory election in the avalanche process can cause this event to occur within a band of time intervals, measured over a large number of pulses. For an assumed "bandwidth" or "jitter" for this first event, what is the corresponding "bandwidth" for the final event -- namely closure of the gap?

To analyze this situation, let the rising voltage waveform be that of (24).

$$V(t) = \alpha t^N \tag{68}$$

In addition, let the streamer starting time be T_1 and the closure time be T_2. The sharp-pointed electrode assures that T_1 will be considerably smaller than T_2.

Now we put this information into (59) and integrate to relate T_1 and T_2.

$$5(K')^6 \int_0^d x^4 dx = \alpha^6 \int_{T_1}^{T_2} t^{6N} dt \tag{69}$$

$$(K')^6 d^5 = \frac{\alpha^6}{6N+T} \left[T_2^{\,6N+1} - T_1^{\,6N+1} \right] \tag{70}$$

Then let

$$\frac{(K')^6 d^5 (6N+1)}{\alpha^6} = K'' \text{ (a new constant)} \tag{71}$$

so

$$T_2^{6N+1} = T_1^{6N+1} + K'' \tag{72}$$

Now, differentiate (72) with respect to T_1, and find

$$\frac{dT_2}{dT_1} = \left(\frac{T_1}{T_2}\right)^{6N} \tag{73}$$

If the jitter in T_1 is ΔT_1, then an approximation to the jitter in T_2 will be:

$$\Delta T_2 \cong \left(\frac{dT_2}{dT_1}\right)\Delta T_1 = \left(\frac{T_1}{T_2}\right)^{6N} \Delta T_1 \tag{74}$$

Expressing these as relative fractional changes, let

$$\frac{\Delta T_2}{T_2} = A \tag{75}$$

the fractional change in T_2, and

$$\frac{\Delta T_1}{T_2} = B \tag{76}$$

the change in T_1 expressed as a fraction of T_2.

Then these combine to show

$$\frac{A}{B} = \left(\frac{T_1}{T_2}\right)^{6N} \tag{77}$$

and (77) is useful for examining the relative jitter-reducing effects of point-plane gaps.

Consider a typical example: suppose a point-plane gap is designed to close in $T_2 = 30$ nS, and further suppose T_1 varies from 0 to 20 nS, with a mean value of 10 nS. This situation is considerably worse than any we are likely to encounter in practice. Then (77), using the largest (not the mean) value of T_1, gives

$$\frac{A}{B} = \left(\frac{2}{3}\right)^{6N} \quad (78)$$

For a linear voltage pulse (N=1), (78) shows

$$\frac{A}{B} = 0.096 \quad (79)$$

So the closure time jitter is less than ten percent of the starting time jitter, or 6.6 percent closure time jitter (the starting time was assumed to have the gross relative jitter of 66.7%).

In addition, a 1-cos(wt) waveform is more closely approximated by a parabola (N=2) than by the above linear function (N=1). And for this case, we see that the closure time would have about 0.6 percent jitter. This kind of performance is often experienced with point-plane gaps.

Equation (78) also points out that it is important to have dV/dt as large as possible (N>>1) at the time of closure, for the best jitter performance.

We conclude this section with two short tables of J.C. Martin's data, taken from his equation (59) for atmospheric air. These, combined with the graphical renderings in figure 3 and 4, should prove useful in air gap and insulator design problems.

Table II, figure 3, gives breakdown distance vs. mean electric field (at breakdown) with the usual (88%) "effective" time as a parameter.

Table III, figure 4, gives the minimum safe holdoff distance vs. mean electric field (at peak) with the pulse decay-time constant (not the "effective" time) as parameter.

TABLE II

FIELD VS. DISTANCE AS A FUNCTIØN ØF EFFECTIVE TIME

DISTANCE (CM)	TIME (μSECØNDS)	ELECT. FIELD (KV/CM)
1.0	.01	48.47
1.0	.10	33.03
1.0	1.00	22.50
1.0	10.00	15.33
1.0	100.00	10.44
1.0	1000.00	7.12
3.2	.01	40.01
3.2	.10	27.26
3.2	1.00	18.57
3.2	10.00	12.65
3.2	100.00	8.62
3.2	1000.00	5.87
10.0	.01	33.03
10.0	.10	22.50
10.0	1.00	15.33
10.0	10.00	10.44
10.0	100.00	7.12
10.0	1000.00	4.85
31.6	.01	27.26
31.6	.10	18.57
31.6	1.00	12.65
31.6	10.00	8.62
31.6	100.00	5.87
31.6	1000.00	4.00
100.0	.01	22.50
100.0	.10	15.33
100.0	1.00	10.44
100.0	10.00	7.12
100.0	100.00	4.85
100.0	1000.00	3.30
316.2	.01	18.57
316.2	.10	12.65
316.2	1.00	8.62
316.2	10.00	5.87
316.2	100.00	4.00
316.2	1000.00	2.73
1000.0	.01	15.33
1000.0	.10	10.44
1000.0	1.00	7.12
1000.0	10.00	4.85
1000.0	100.00	3.30
1000.0	1000.00	2.25

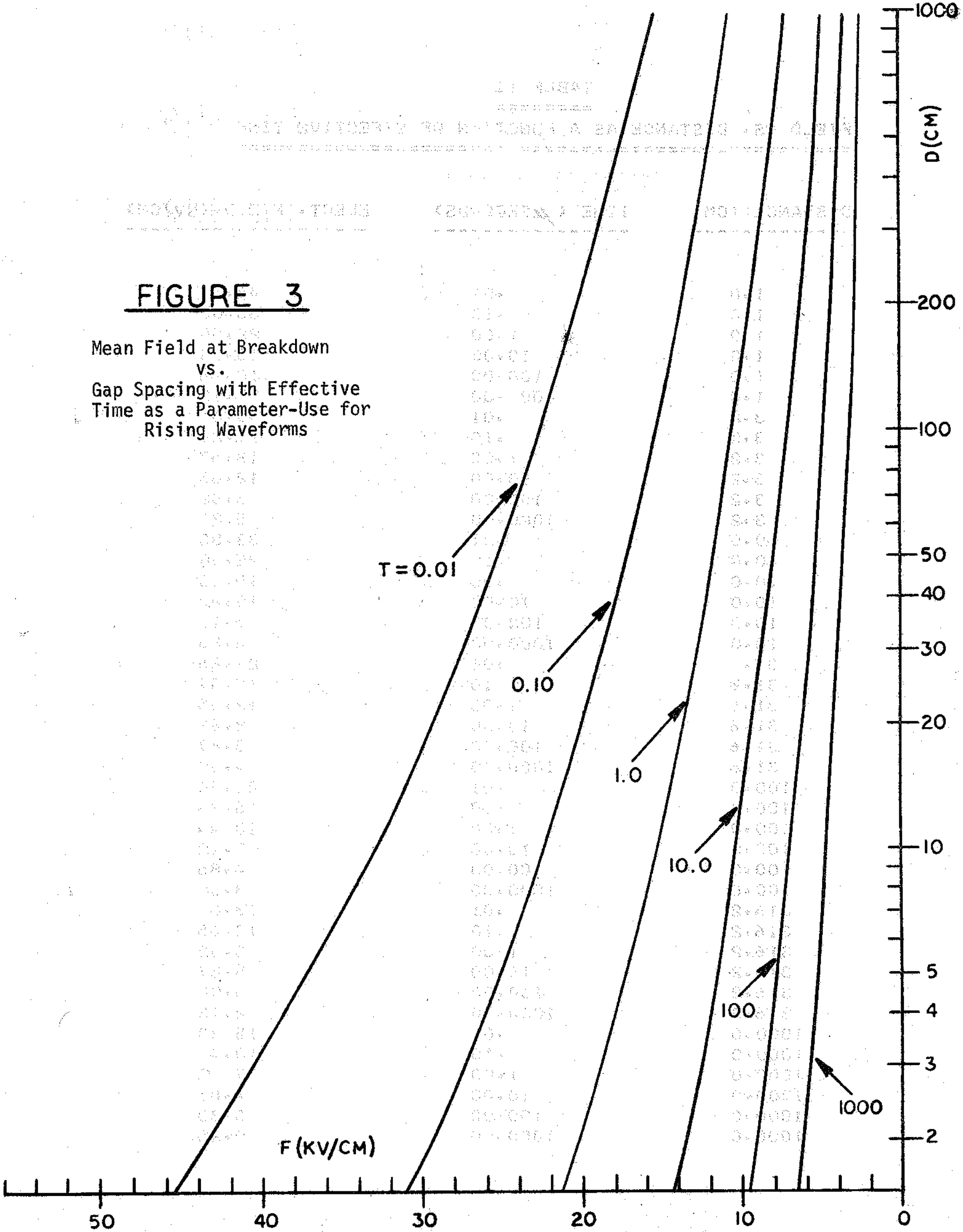

FIGURE 3

Mean Field at Breakdown
vs.
Gap Spacing with Effective
Time as a Parameter-Use for
Rising Waveforms

TABLE III

FIELD VS. DISTANCE AS A FUNCTIØN ØF TIME CØNSTANT

DISTANCE (CM)	TIME (μSECØNDS)	ELECT. FIELD (KV/CM)
1.0	.01	65.34
1.0	.10	44.52
1.0	1.00	30.33
1.0	10.00	20.66
1.0	100.00	14.08
1.0	1000.00	9.59
3.2	.01	53.94
3.2	.10	36.75
3.2	1.00	25.03
3.2	10.00	17.06
3.2	100.00	11.62
3.2	1000.00	7.92
10.0	.01	44.52
10.0	.10	30.33
10.0	1.00	20.66
10.0	10.00	14.08
10.0	100.00	9.59
10.0	1000.00	6.53
31.6	.01	36.75
31.6	.10	25.03
31.6	1.00	17.06
31.6	10.00	11.62
31.6	100.00	7.92
31.6	1000.00	5.39
100.0	.01	30.33
100.0	.10	20.66
100.0	1.00	14.08
100.0	10.00	9.59
100.0	100.00	6.53
100.0	1000.00	4.45
316.2	.01	25.03
316.2	.10	17.06
316.2	1.00	11.62
316.2	10.00	7.92
316.2	100.00	5.39
316.2	1000.00	3.67
1000.0	.01	20.66
1000.0	.10	14.08
1000.0	1.00	9.59
1000.0	10.00	6.53
1000.0	100.00	4.45
1000.0	1000.00	3.03

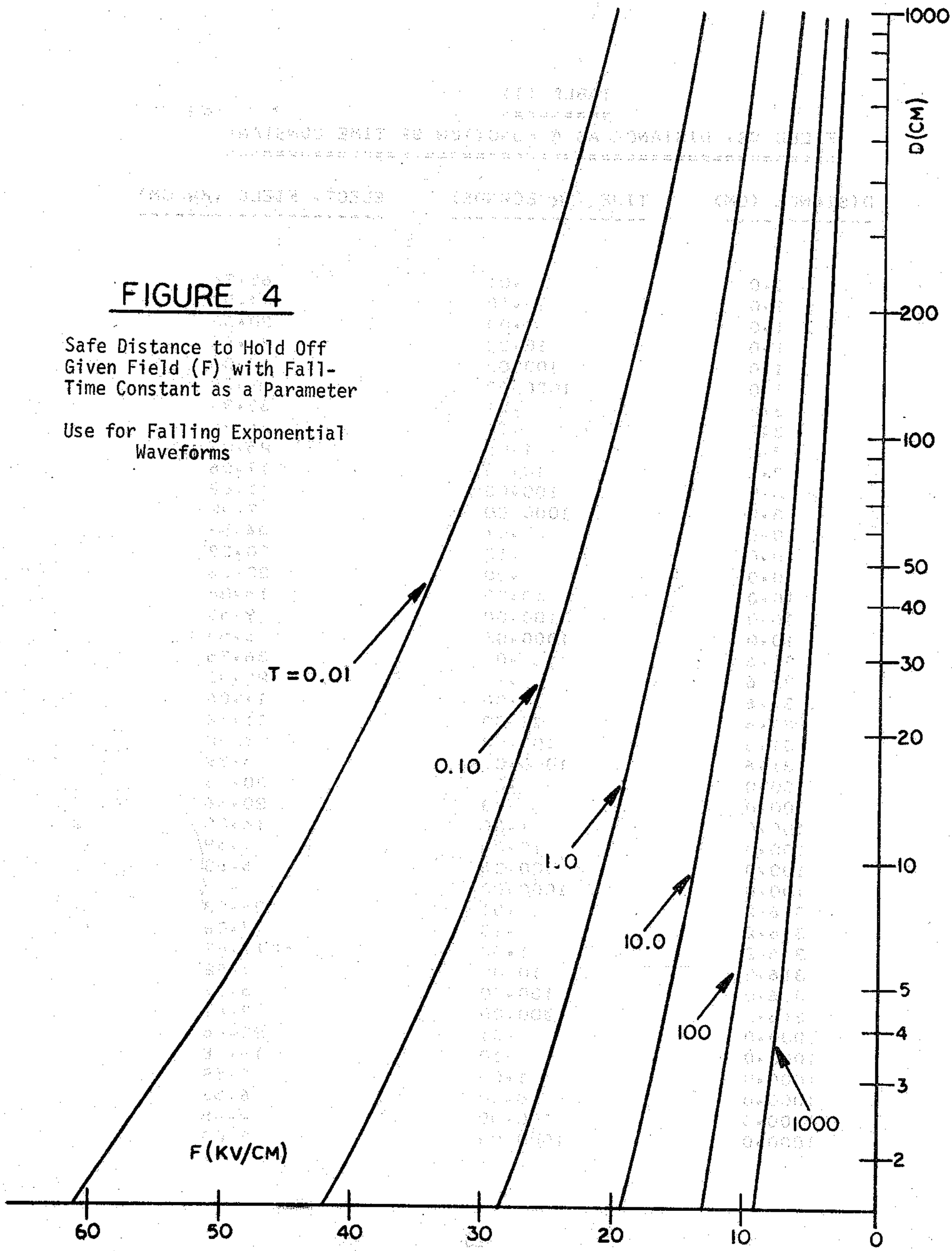

FIGURE 4

Safe Distance to Hold Off Given Field (F) with Fall-Time Constant as a Parameter

Use for Falling Exponential Waveforms