Switching Notes

Note 34

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Use of Symmetry for Measuring the Spread of Switching Times for Extremely-High-Frequency Radiating Oscillatory Sources

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Abstract

As one uses transient oscillatory sources at high frequencies there is a problem in directly measuring the time-domain waveform (such as in the THz regime). In combining the waveforms from multiple sources there is the problem of the spread of the switching (initiation) times of the waveforms, leading to a lack of coherence (lack of common phase) on arrival at a target. Correlation techniques can be used to measure the effect of such spread, and whether it is acceptably small.

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1. Introduction

One of the limiting factors in the design of distributed sources (or arrays) for radiating electromagnetic waves is a lack of sufficient accuracy in the timing of the closure of the various switches. From an idealized viewpoint we can imagine a set of sources, all triggered to launch waves at preprogrammed times to synthesize a desired larger wave. Various calculations have been done from this viewpoint [1-6, 8].

In the practical case, if one sends a signal to a switch to change state (typically to close) at a certain time, it actually results at a somewhat different time. This is sometimes called switch jitter. This is measured by repeatedly triggering a switch and measuring the resulting waveforms on a common time base. The variation in delay from each event can then be compared to give an average time delay and the variation of each event from this average. Assuming a Gaussian distribution (not necessarily the case), then one sigma is called the *switch jitter*, measured in nanoseconds or picoseconds as the case may be. For our purposes a better measure is called the *switch spread*. By this [7] we mean that most (say 90%) of the switching events occur with in this time (clearly greater than the jitter time). For fast-rising transient electromagnetic waves this shows how fast the resulting macroscopic wave can rise, providing each switch closure time is sufficiently smaller than the spread.

The problem, then, is the measurement of the spread in switching times. Since we expect some variation in the switching times between one shot and the next, use of a sampling oscilloscope, for example, would not be appropriate. We need some measurement of each separate switching event.

If we have a switched oscillatory antenna which radiates an approximate damped sinusoid, then the error from the desired switching time will produce a phase shift in the radiated wave. As the frequency goes higher and higher (as in the THz regime) we enter a regime where it becomes increasingly difficult to record the exact time-domain waveform as a single-shot measurement. An alternate approach uses the mixing o two or more signals in an interferometry sense and measurement of the energy in the mixed received waves.

2. Use of Symmetry

As in Fig. 2.1, let us configure our radiating sources in such a way that, at our measurement location, all the sources give identical waveforms with identical amplitudes, but possibly arriving at different times (due to switch spread).

Consider two sources (switched antennas) such as numbers 1 and 2. Let us measure E_y somewhere on the y axis. Let the antennas be located symmetrically with respect to the $x\,y$ plane as

$$\left(x, y, z\right) = \left(\pm x_1 , y_1 , z_1\right) \tag{2.1}$$

Furthermore, require reflection symmetry with the group structure [10]

$$\begin{aligned} R_x &= \left\{ \overleftrightarrow{1} , \overleftrightarrow{R}_x \right\} \\ \overleftrightarrow{R}_x &= \overleftrightarrow{1} - 2 \vec{1}_x \vec{1}_x = - \vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y + \vec{1}_z \vec{1}_z \end{aligned} \tag{2.2}$$

This can apply to the overall configuration, or to the antennas individually in addition. Then each source will produce the same signal at the measurement location, except for possible different time delays (due to switch spread).

If we radiate from only one source (say number 1), then let the received energy be labelled U_1. If both sources (1 and 2) radiate at exactly the same time they arrive at the receiver in phase giving twice the wave amplitude or four times the received energy. In general then the received energy U lies in the range

$$0 \leq U \leq 4U_1 \text{ (zero for cancelling signals)} \tag{2.3}$$

due to the spread in switching times. How close U is the $4U_1$ determines how small is the spread, and whether this is an acceptable amount.

Continuing the argument, the above can be applied to two sources (say 1 and 4) symmetric with respect to the x, z plane as

$$\left(x, y, z\right) = \left(x_1, \pm\ y_1 , z_1\right) \tag{2.4}$$

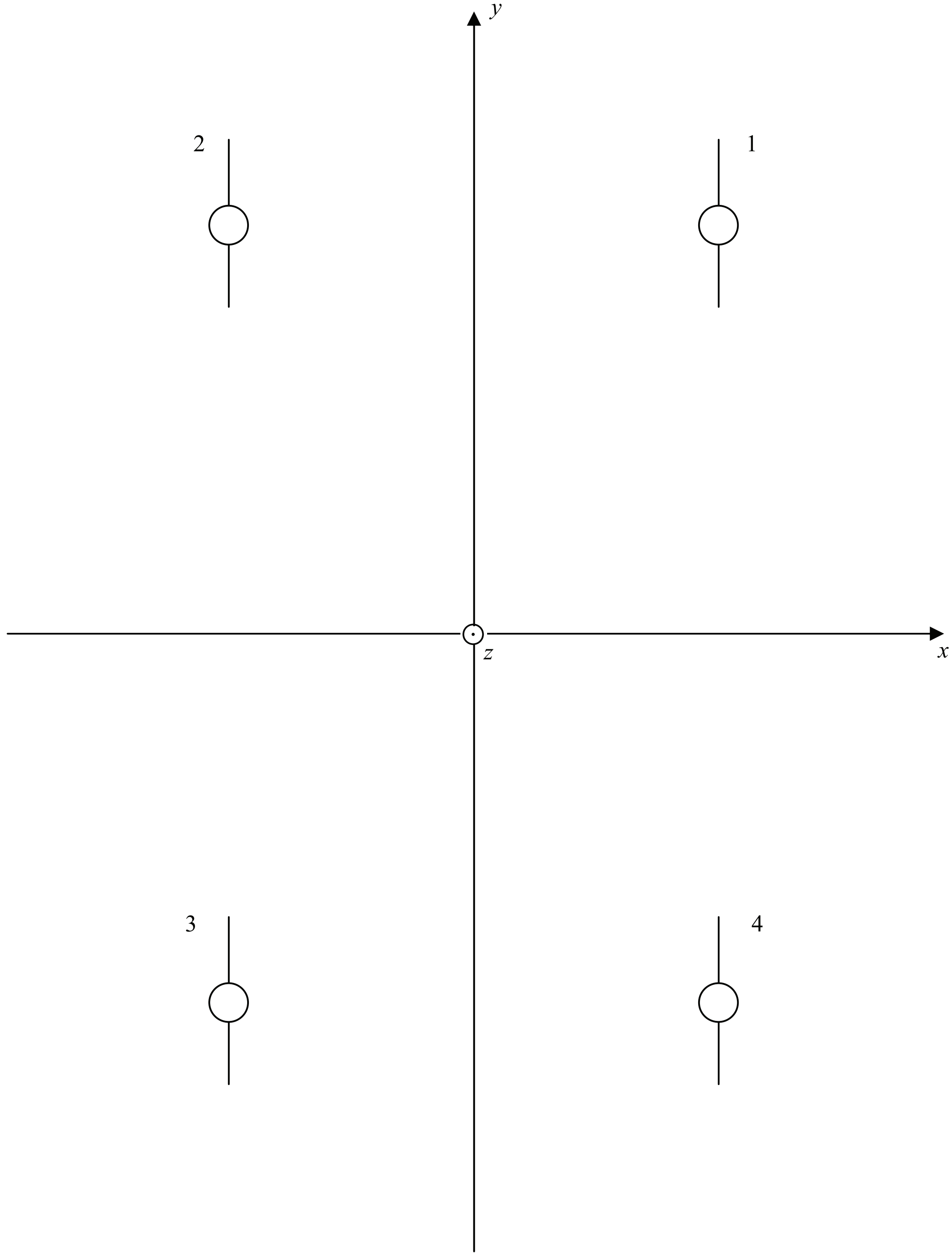

Fig. 2.1 Symmetrical Location and Configuration of Radiating Sources

This gives the group structure

$$
\begin{aligned}
R_x &= \left\{ \overset{\leftrightarrow}{1} \,,\; \overset{\leftrightarrow}{R}_y \right\} \\
\overset{\leftrightarrow}{R}_y &= \overset{\leftrightarrow}{1} - 2\,\vec{1}_y\,\vec{1}_y = \vec{1}_x\,\vec{1}_x - \vec{1}_y\,\vec{1}_y + \vec{1}_z\,\vec{1}_z
\end{aligned}
\tag{2.5}
$$

As before the individual sources can also be constructed with the above symmetry as well. The measurement of E_y in this case is on the *x* axis. The energy received also follows (2.3).

This analysis can be carried further to four sources as indicated in Fig. 2.1. Requiring reflection symmetry with respect to both *y z* and *x z* planes gives the group structure

$$
C_{2a} = R_x \otimes R_y = \left\{ \overset{\leftrightarrow}{1} \,,\; \overset{\leftrightarrow}{R}_x \,,\; \overset{\leftrightarrow}{R}_y \,,\; \overset{\leftrightarrow}{R}_x \cdot \overset{\leftrightarrow}{R}_y \right\}
\tag{2.6}
$$

which now introduces 2-fold rotation symmetry with axial symmetry planes. The measurement of E_y is now on the z axis. In this case perfect synchronism of the 4 sources produces 4 times the waveform amplitude and 16 times the energy U. So (2.3) is replaced by

$$
0 \le U \le 16\,U_1
\tag{2.7}
$$

depending on the spread between the various sources. We are looking for *U* close to $16\,U_1$.

Even higher orders of symmetry are possible for 4 sources. Suppose we make

$$
x_1 \equiv y_1
\tag{2.8}
$$

and orient our antennas such that they have additional symmetry planes passing through the *z* axis. Then we have 4-fold rotation symmetry with axial symmetry planes [10]

$$
\begin{aligned}
C_{4\alpha} &= \left\{ \left(C_{n,m}(\phi_\ell) \right),\; \overset{\leftrightarrow}{R}_y \cdot \left(C_{n,m}(\phi_\ell) \right) \,\middle|\, \ell = 1,2,3,4 \right\} \\
\left(C_{n,m}(\phi_\ell) \right) &= \begin{pmatrix} \cos(\phi_\ell) & -\sin(\phi_\ell) \\ \sin(\phi_\ell) & \cos(\phi_\ell) \end{pmatrix} , \quad \phi_\ell = \frac{\ell\pi}{2}
\end{aligned}
\tag{2.9}
$$

This group has 8 elements.

3. Correlation of sources

Assuming we have some energy-measuring device we can write

$$U_1 = \frac{1}{Z}\int_{-\infty}^{\infty} V^2(t)\,dt \quad \text{(assuming } Z \text{ real)} \tag{3.1}$$

If we have two sources this becomes

$$U = \frac{1}{Z}\int_{-\infty}^{\infty} \left[V_1(t) + V_2(t)\right]^2 dt \tag{3.2}$$

If $V_1(t) = V_2(t)$ we have

$$U = \frac{4}{Z}\int_{-\infty}^{\infty} V_1^2(t)\,dt = 4\,U_1 \tag{3.3}$$

which is our limit in (2.3).

For purely sinusoidal signals of amplitude V_0 and frequency *f,* a single signal averaged over one cycle gives

$$\begin{aligned} u_1 &= \frac{V_0^2}{Z}\frac{1}{T}\int_0^T \sin^2\left(2\pi ft + \psi_1\right)dt \\ &= \frac{V_0^2}{2Z} \\ T &= f^{-t} = \text{period} \end{aligned} \tag{3.4}$$

Going on to two signals of equal amplitude, but various phases, we have

$$
\begin{aligned}
u &= \frac{V_0^2}{Z}\frac{1}{T}\int_0^T \left[\sin\left(2\pi ft+\psi_1\right)+\sin\left(2\pi t+\psi_2\right)\right]^2 dt \\
&= \frac{V_0^2}{Z}\left[1+\cos\left(\psi_2-\psi_2\right)\right]
\end{aligned} \tag{3.5}
$$

using standard tables [9]. Comparing to (3.4) this gives

$$
\begin{aligned}
&u = u_1\, 2\left[1 + \cos\left(\psi_1-\psi_2\right)\right] = u_1\left[4\cos^2\left(\frac{\psi_1-\psi_2}{2}\right)\right] \\
&0 \le u \le 4u_1
\end{aligned} \tag{3.6}
$$

The maximum ($4u_1$) occurs when $\psi_1 = \psi_2$, and the minimum (0) occurs when the phases differ by π (180°).

For damped sinusoidal signals the situation is somewhat more complicated. For identical signals with no temporal shift between the two, then (3.3) applies giving $4U_1$. For a time shift due to a spread in switching times, one does not expect an exact cancellation when the "phases" differ by π, just a small fraction of U_1 resulting.

4. Concluding Remarks

The analysis of the correlation of two or more signals can be carried out for various temporal waveforms if one has sufficient information concerning the actual waveform from a signal source. The integrals can be evaluated numerically.

As we go to higher and higher oscillation frequencies we expect the effect of switch speed to become more significant. Using appropriate experiments one can determine how high in frequency one can go before switch spread wipes out the coherence. From another point of view one can use such techniques to measure the switch spread.

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