Switching Notes
Note 9

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Duration of the Resistive Phase and Inductance of Spark Channels

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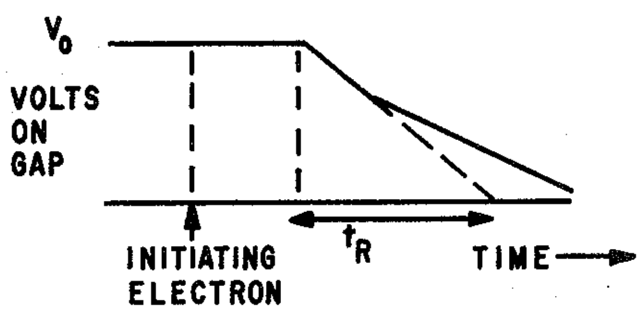
The following is a brief resume of the formulae used for the resistive phase of a hot spark channel and some other comments relating to spark channels formed under various conditions.

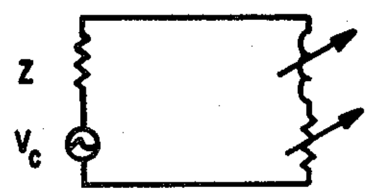
It should be stated that while a considerable quantity of work was done on the duration of the resistive phase in gap-sparks, considerably less data has been obtained on spark channels in liquids and only a little on those in solids. However the underlying theoretic explanation and calculations give some confidence that the answers given by the relation are reasonably correct.

The basic assumption is that the circuit impedance feeding the spark channel is of reasonably low impedance (less than 1000 ohms, say) and that after the normal ionization build up has taken place the warm weakly-ionised channel expands. This leads to the generation of a lower pressure filament at the centre of the warm (~5000°) gas channel and the main current starts to flow in this. The low density material can then be heated to temperatures above 20,000° and this leads to the formation of a plasma column. The above sequence has been photographed at AWRE and typically takes tens of nanoseconds to complete in gases. The plasma column at first sight can lower its resistance by heating itself up, but this causes the degree of ionisation of the air (or other gases) to increase and in fact, for a temperature range from 2×10^4 to over 10^5 the resistance is only proportional to $T^{-1/2}$. Thus only a modest reduction of resistivity is possible with a considerable increase in temperature. With gas channels of dimensions of more than a few 10^{-2} cms at these temperatures the column is black-body and this leads to considerable radiative transfer of energy which prevents the current column becoming unstable. For instance, for the current to collapse in a column of half the diameter, the temperature of this must increase by a factor of about 16. This leads to an enormous increase in radiative transfer which returns the plasma column to a uniform temperature. It is thus reasonable to take the current carrying column to be of approximately uniform temperature and resistivity up to the edge of the plasma.

The spark channel can now only decrease its resistance easily by increasing its area, which it does by a cylindrical explosion the velocity of which is governed by the energy density in the plasma driving the shock outwards. Measurements of the velocity of expansion of the plasma sheath in air by means of 10 ns Kerr cell photographs have given velocities in good agreement with these calculated from the energy density measured to be in the spark channel. In general the terms $L(\partial i/\partial t)$ and $i(\partial L/\partial t)$ are small corrections to the measurements of the voltage across the channel until its impedance has fallen to a low level. The formula for the resistive phase duration calculated from the above considerations agrees tolerably well with that found experimentally and given below in the next section. It might be surprising that the same formula should cover a wide range of different substances but this is explained by the fact that to the first order the equation of state of multiply ionised low Z atoms does not differ to any great extent, except for hydrogen and helium.

The resistive phase measurements have all been done using transmission lines to drive the spark channels. This experimental detail seems important to me, since it is possible to obtain V and i from the same measurement providing the inductive terms are kept small by good layout. When the spark channel is formed across the transmission line, the voltage falls more or less exponentially and it is only when the voltage has dropped to about 10% of its starting value that deviations from the exponential shape may become detectable. By this time nearly all the prompt energy has been delivered to the channel. Also in a pulse forming application, this fall of voltage is one of the limits on the pulse front available from any system.





In the experiments with gas sparks transmission lines (or cables) of 100, 10 and 1 ohms were used and spark channel fields of 3 x 10^4 down to 5 x 10^3 volts/cm were used in air. Other pressures than atmospheric were employed and also gases of other densities. From these experiments (supported by the calculations) the following relation for the resistive phase was obtained:

$$\tau_R = \frac{88}{Z^{1/3}\ E^{4/3}}\left(\frac{\rho}{\rho_o}\right)^{1/2} \text{ ns}$$

where Z is the impedance driving the channel in ohms, F_1 the field in units of 10 KV/cm, ρ the density of the gas and ρ_o the density of air at about N.T.P. The accuracy with which τ_R was measured was reasonable and the fit to the observations was to 10% or better in most of the measurements.

Measurements have since been made on spark channels in oil, water and polythene. These measurements are rather more difficult since the resistive phase in solids tends to be a few ns while that in gas channels can easily be made several hundred ns. The measurements in polythene gave a formula

$$\tau_R = \frac{5}{Z^{1/3} F_2^{4/3}}$$ ns where F_2 is now in MV/cm and

this agrees surprisingly well with the gas observations where the value of ρ for polythene is inserted. The solid breakdown measurements could not clearly show the power of F_2 but this was assumed to be that given by the theoretical calculations.

The velocity of expansion of the plasma channel is not strongly dependent on the resistance of the circuit feeding it and for a rough estimation tends to be between 3×10^5 and 10^6 cm per sec. While an accurate value of the radius-time history can be obtained from the calculations during resistive phase, a crude value suffices to obtain the inductance of the channel. Thus an 8 ohm line feeding a spark channel with a means field gradient to start with of 1 MV/cm has a resistive phase of 2.5 ns. This gives a radius of roughly about 1.5×10^{-3} when the channel impedance has fallen to slightly less than the generator impedance. If the line feeding it is 10 cms radius and the spark channel is located in the line in a replaceable solid dielectric switch, the inductance is given by $L = 2\, l \ln b/a$ where l is the length of the spark channel, $a = 1.5 \times 10^{-3}$ and $b = 10$. This gives $L \approx 18\, l$ nanohenry. For a switch working at 100 KV, and 1 MV/cm, l is 10^{-1} cm and the inductance is 1.8 nH. This gives a τ_L $L/Z = 0.2$ ns, very small compared with the resistive phase. The $i(\partial L/\partial t)$ term can be estimated for the same case by the fact that a further increase of channel diameter by e will decrease L by 0.2 nH and that this will take a time of the order of 5 ns and hence $\partial L/\partial t \sim 0.04$ ohm which is very small compared with 8 ohms. Since the radius only occurs in the logarithmic term in the inductance calculations, errors in it have next to no effect on the value calculated. The energy deposed in the spark channel during the resistive phase is given by $(V^2/4Z)\tau_R$ where V is the voltage before the spark closes. Measurements of the volume blown in materials by spark channels fed by short current pulses give values in reasonable agreement with this, considering the errors inherent in these measurements.

There are a number of circumstances where the τ_R may appear longer or shorter than that calculated above. If the spark channel is formed along a curved field line (say at the edge of a metal conductor) the mean field may be significantly lower than that calculated from the voltage divided by the separation of the metal plates, and hence the resistive phase longer.

If the spark channel is formed by a long corona across a surface, then only the end of the corona is in a resistive phase because the time of advancement of the top of the streamer may permit the root of the channel to expand to a considerable cross section. I have a rough theory of corona tracking based on this point which is not wildly at variance with some measurements taken under carefully controlled conditions. Thus long channels whose conditions vary considerably along their length can give lower values of τ_R.

A much more likely way that a low τ_R value may be obtained is when the current carrying breakdown channel branches. In compressed gases at 20 atmospheres and above and with liquids and solids it is frequently possible to show that the current is carried in a number of channels after a short single channel. Typically 5 or more heavy current carrying branches may reach the far electrode with many more channels visible which do not carry significant current. To show the effect of this τ_R, I will estimate the resistive phase and inductance for such a branching system where it is assumed for simplicity that one channel is 0.03 cm long and then 8 channels carry the current for a remaining distance of 0.07 cms. Again 100 KV starting voltage is assumed and a generator impedance of 8 ohms. For the single branch of the channel

$$\tau_1 = \frac{5}{2 \times (3.3f)^{4/3}}$$ ns where $f10^5$ is voltage at the

branching point.

The resistive phase of the 8 branches is given by

$$\tau_2 = \frac{5}{4 \times [1.4\ (1-f)]^{4/3}}$$

These two times have to be equal of course and this gives $f = 0.4$ and $\tau_1 = \tau_2 = 1.7$ ns which is to be compared with the value previously calculated for the unbranched channel of 2.5 ns. This ratio seems to be fairly typical and for many applications a resistive phase of about 0.7 τ_R will be found to occur.

The inductance of the branched case can be estimated to be in the region of 1.4 nH compared with 1.8 nH previously obtained.

As will doubtless be apparent, no great claims for accuracy are made in any of these calculations; they are only to indicate the sort of conditions likely to be obtained in planning an experimental set up. The best that can be said for them is that we have not yet found any of the estimates we have made to be badly wrong.

Some difficulty may be found in determining Z in various circumstances. One of these is a condenser bank application. The first point to be made is that Z only appears as a low power and hence even crude calculations will suffice. In the case of a bank whose $\sqrt{LC}$ is comparable or lower than τ_R, Z may be taken equal to $\sqrt{L/C}$. For the usual case of banks which are slow, the impedance feeding the switch may frequently be estimated from the transmission line linking the switch with the capacitors, providing its electric length is of the order of or greater than τ_R.

A more complex case is provided by a transmission line switched by liquid gap with a rather low value of the mean field. In this case, τ_L can be comparable to or greater than τ_R calculated directly from the transmission line impedance. The impedance feeding the spark channel is obviously quite a lot higher than the impedance of the transmission line in this case. By estimating L (which is only slightly dependent on τ_R) an effective impedance is obtained by putting

$$Z_{eff} \simeq Z + \omega L \simeq \frac{1.7\ L}{\tau_R} + Z.$$

This number is now used to calculate the reduced τ_R and this is then combined with τ_L. For the cases where $\tau_R \sim \tau_L$ and if these effects are assumed to be in series, the resulting $\tau \sim \tau_R + \tau_L$. This analysis is of course extremely crude but in a couple of instances it has been used to give answers in tolerable agreement with experiment. Mostly these difficulties only arise with solid or liquid switches working at poor field strengths and/or in very low impedance circuits.

I hope the above outline will be of use to you in estimating the rise time of your pulses and that you will not be too repelled by the violence done to the physics of the situation. I have considered a number of effects that might change the hydrodynamic explosion of the plasma column. Firstly pinching; in general it is difficult to get a high enough rate of rise of current to make the magnetic pressure exceed the particle pressure in the plasma column. The best chance seems to be in an overdriven gas gap when it may occur if the impedance driving the system is low but under these circumstances the inductance and rate of change of inductance can no longer be ignored and establishing the conditions for a pinch seem to me to be pretty difficult. Thermal conductivity expansion of the column; for very small channels, thermal conductivity may be a faster way of transferring energy outwards than by a hydrodynamic shock. But the self magnetic field reduces this effect and it only applies for very small channels well below anything considered in the above calculations. Radiative fronts; it is just possible that radiation transfer may outrun the hydrodynamic shock in those cases where very over-driven gaps are fed by low impedances (i.e. just the conditions considered above for a possible pinch effect). Thus even if pinch conditions were to be set up, energy expansion by a radiation front may prevent it occurring. In the case of sparks fed by very high impedances a fully fledged plasma column may never come into being and then of course the calculations are inapplicable. It is not easy to be sure at what level of impedance this will take place but I suspect it to be above 1000 ohms. It is of relevance that with liquid and solid systems the fields normally required to cause breakdown mean that enough energy is stored locally in the electrostatic field to provide a fairly low impedance to drive the channel, so that the question of fast spark channels fed by high impedances can only really occur in gases at lowish pressures.

After times of a few τ_R, energy continues to be deposited in the channels and, more importantly, in the electrodes. Blow up of the electrodes in the case of a solid dielectric gap fed by short lines provides additional proof that the current densities are very high and hence the spark channels are initially very small in diameter. Energy is also deposited in motion of the channel if this is not coaxial in its return current, and even if it is, eventually magnetic wriggling occurs. However, nearly all these effects are quite small and for quite long times the major energy in the channel is that deposited during the resistive phase, when the impedance collapses from a few times Z to one over a few times this value. Mention should be made of underwater sparks which are used to provide shock working of metals. The banks that drive this are very slow and the resistive phase of the spark channel does not take much energy out of the system. The real energy seems to me to be damping in a period of 10 to 20 microseconds, when a stabilised arc is formed down the axis of a cylindrical gas bubble. Continued influx of cold atoms from the water interface and thermal conductivity outwards provide a cooling effect on the arc and this provides a long term, rather low, impedance, which eventually drains most of the energy out of the bank, provided its internal resistance is reasonably low.