EMP Theoretical Notes
Note 2
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Air Conductivity: Some New Developments

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Abstract

This note describes two new developments in the air conductivity part of the EMP problem. Primary emphasis is given to the addition of electron-ion scattering which decreases the electron mobility. This effect may become dominant within 500 to 600 meters from a megaton burst. A second development is some recent measurements which indicate that the ion-ion recombination coefficient may need reduction by two orders of magnitude. This in turn would increase the conductivity during the last phase of the EMP as defined by C. Longmire in LAMS-3072.

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I. Electron-Ion Scattering

Because an ion is a monopole charge, it presents a much larger scattering cross section to electrons than nitrogen or oxygen molecules which are electric quadrupoles, or water vapor molecules, which are electric dipoles. Since the electronic part of the conductivity σ_e, is given by

$$\sigma_e = en_e\mu_e = en_e \frac{e}{m}\left(\sum_j \nu_{e_j}\right)^{-1} \tag{1}$$

where e is the electronic charge, n_e is the electron density, m is the electron mass, and ν_{e_j} is the momentum transfer collision frequency for electrons with the jth particle species in the gas. Each collision frequency is in turn

$$\nu_{e_j} = N_j\sigma_j v_e \tag{2}$$

where N_j is the density of the jth particles, σ_j is the momentum transfer collision cross section for electrons with the jth particles, and v_e is a speed characteristic of the electrons. In the cross section calculations to follow it is convenient to define v_e, the electron velocity as

$$v_e = \sqrt{\frac{2KT_e}{m}} \tag{3}$$

where K is Boltzmann's constant and T_e is the electron temperature.

To determine the importance of the electron-ion scattering in calculating the EMP, one can calculate from eqn. (2) the approximate radius from a nuclear device (say one megaton) at which the collision frequency of electrons with ions equals the collision frequency of electrons with air (and water vapor) molecules. From this radius inward the electron-ion scattering will be dominant because of the higher ionic densities closer to the source. To determine this radius, one must calculate the electron-ion cross section (a function of electron temperature), the cross section for electron neutral molecule collisions (also a function of electron temperature), and finally the density of ions as a function of radius. If σ_{air} is the cross section for electrons with neutral air molecules, σ_{ion} is the cross section for electrons with ions, and f_i is the fractional ionization, then the radius from the burst point of equal electron-neutral and electron-ion collision frequencies occurs where

$$\frac{\sigma_{air}}{\sigma_{ion}} = \frac{f_i}{1 - f_i} \simeq f_i \qquad (4)$$

A. Electron-Ion Cross Section

The electron-ion cross section can be calculated from a consideration of the path of the electron in the Coulomb field of the more massive ion. Anderson and Goldstein[1] have made measurements of this phenomenon yielding an expression well in agreement with the theoretical calculations. Converting their expression for the electron-ion collision frequency to a cross section (per eqns. (2) and (3)) and assuming a typical molecular density of air, N, of 2.5 x 10^{25} $(\text{meter})^{-3}$ gives

$$\sigma_{ion} = \frac{.49 \times 10^{-17}}{(KT_e)^2} \ln\left\{\frac{.20}{\sqrt{f_i}} \frac{KT_e}{\left(1 + \left(\frac{.025}{KT_e}\right)\right)^{1/2}}\right\} (\text{meter})^2 \qquad (5)$$

or more conveniently

$$\sigma_{ion} = \frac{.49 \times 10^{3}}{(KT_e)^2} \ln\left\{\frac{.20}{\sqrt{f_i}} \frac{KT_e}{\left(1 + \left(\frac{.025}{KT_e}\right)\right)^{1/2}}\right\} \text{Å}^2 \qquad (6)$$

where the electron temperature, KT_e, is now expressed in electron volts. The ion temperature is assumed to be thermal (0.025 ev). It should be noted that the measurements which checked so well with theory were made at millimeter gas pressures, not atmospheric pressure. In addition eqns. (5) and (6) can only be considered valid when the argument of the logarithm is large with respect to one.

It would also seem that since typically the electron temperature is epithermal during the radiation pulse the cross section calculation of eqn. (6) will decrease thus decreasing the effective radius of interaction of electrons and ions to a distance less than the mean free path of electron-neutral molecule collisions. Perhaps then, for epithermal electrons the results of eqn. (6) will be more closely approached, even in atmospheric air. In any event eqn. (6) can be used to indicate the effect of electron-ion collisions on the air conductivity and thus on the EMP.

1. Anderson and Goldstein, Phys. Rev., 100, 4; November 15, 1955.

B. Electron-Neutral Molecule Cross Section

To indicate the effect of the electron-ion cross section on the air conductivity one must compare this with the cross section for electron-neutral molecule scattering. To do this two cases will be considered corresponding to two electric field values. Then, using the data of Pack, Phelps, Frost, and Voshall[2] for the drift velocity and electron temperature as a function of electric field for electrons in atmospheric gases together with the conventions of eqns. (1) through (3) an effective electron-neutral molecule cross section and an effective electron temperature will be calculated. The molecular density, N, will be assumed to be 2.5×10^{25} $(\text{meter})^{-3}$ and 1% water vapor (molecular fraction) will also be assumed. These results follow in Table 1.

	Case 1	Case 2
Electric Field (volts/meter)	10^4	10^5
σ_{air} ($Å^2$)	.95	.145
KT_e (eV)	.105	.5

Table 1. Electron Collision Parameters in Very Weakly Ionized Air

C. Fractional Ionization for Equal Contributions of Neutral Molecules and Ions to Electronic Conductivity

Using the results of Table 1 for σ_{air} and KT_e the fractional ionization, f_i, of eqn. (4) at which the two cross sections give equal contributions to the electronic conductivity can be calculated by combining eqns. (4) and (6) to give

$$\frac{.49 \times 10^3}{(KT_e)^2} f_i \ln\left\{\frac{.20}{\sqrt{f_i}} \frac{KT_e}{\left(1 + \left(\frac{.025}{KT_e}\right)\right)^{1/2}}\right\} = \sigma_{air} \qquad (7)$$

Actually, the introduction of ions into the air in this amount will somewhat alter the results of Table 1, but those results will suffice for a first order calculation. With the σ_{air} and

2. Pack, Phelps, Frost, and Voshall, Phys. Rev., 121, 3; February 1, 1961; 127, 5; September 1, 1962; and 127, 6; September 15, 1962.

KT_e values from Table 1, then f_i can be calculated from eqn. (7). Results of this calculation are presented in Table 2 as well as the effective electron-ion cross sections.

	Case 1	Case 2
Electric Field (volts/meter)	10^4	10^5
f_i (no units)	10^{-5}	3×10^{-5}
σ_{ion} ($Å^2$)	$.95 \times 10^5$	$.48 \times 10^4$

Table 2. Fractional Ionizations and Electron-Ion Cross Sections

D. Radius at Which Electron-Ion Scattering Becomes Significant for One Megaton

To complete the analysis of the importance of electron-ion scattering to the air conductivity it is necessary to calculate the fractional ionization as a function of radius from a typical nuclear detonation. A typical yield will be taken as one megaton. It will be assumed that with 1 KT hydrodynamic yield there are typically 5.2×10^{22} MeV of prompt γ rays. This number comes from Dr. John Malik of LASL. In addition this will be linearly scaled to 1 megaton. The energy deposition per unit volume, D_p, is assumed to be of the form

$$D_p = \frac{Ae^{-r/r_\gamma}}{4\pi r^2} \tag{8}$$

where r_γ is the γ-ray mean free path (assumed to be 300 meters), r is the distance from burst point and A is a constant to be evaluated. Then for 1 megaton

$$\iiint_V D_p dV = 5.2 \times 10^{25} \text{ MeV} \tag{9}$$

or

$$\int_0^\infty \left(\frac{Ae^{-r/r_\gamma}}{4\pi r^2}\right) 4\pi r^2 dr = 5.2 \times 10^{25} \text{ MeV} \tag{10}$$

and thus

$$Ar_\gamma = 5.2 \times 10^{25}\ \text{MeV} \tag{11}$$

or

$$A = 1.7 \times 10^{23}\ \frac{\text{MeV}}{\text{meter}} \tag{12}$$

If it is assumed that ionic recombination is negligible for times during the local deposition of the energy in the prompt γ pulse, then the total number of ions N_i can be calculated by dividing the energy deposition by 34 eV per positive-negative ion pair (17 eV per ion). This calculation gives

$$N_i = 1 \times 10^{28}\ \frac{e^{-r/r_\gamma}}{4\pi r^2}\ \frac{\text{ions}}{(\text{meter})^3} \tag{13}$$

The fractional ionization, f_i, is then

$$f_i = 0.4 \times 10^{3}\ \frac{e^{-r/r_\gamma}}{4\pi r^2} \tag{14}$$

or

$$f_i = 0.35 \times 10^{-3}\ \frac{e^{-\rho}}{\rho^2} \tag{15}$$

where

$$\rho = \frac{r}{r_\gamma} \tag{16}$$

giving the radius in units of the gamma-ray mean free path. Now the radii corresponding to the fractional ionizations in Table 2 can be calculated. These are given in Table 3.

	Case 1	Case 2
Electric Field (volts/meter)	10^4	10^5
ρ (no units)	2.1	1.6
r (meters)	630	480

Table 3. Radius from One Megaton for Significant Electron-Ion Scattering

From these radii inward then the conductivity will be dominated by electron-ion scattering. The approximate nature of this calculation must be emphasized. Also note that there are approximately 2.61×10^{25} MeV total energy in 1 KT so that the efficiency number for prompt γ rays released to the atmosphere is around 2×10^{-3} of the bomb energy for the numbers used. Varying this efficiency will change the numerical results.

II. Ionic Recombination

As presently used in conductivity calculations the ion-ion recombination coefficient, α_i, defined by the equation

$$\frac{\partial N_+}{\partial t} = S - \alpha_r n N_+ - \alpha_i N_- N_+ \tag{17}$$

has been taken to be about 2×10^{-12} (meter)^3 sec^{-1}. N_+ and N_- are respectively the positive and negative ion densities, n is the electron density, α_r is the electron-ion recombination coefficient, and S is rate of generation of electron-ion pairs. This number comes from the measurements of Sayers[3] in air. Gardner[4] also measured this in pure oxygen and obtained the same results. These measurements were made with ions which were tenths of seconds old using ionization chamber techniques (DC) and over a range of pressures including atmospheric.

However, results which are quite different were obtained by Van Lint of General Atomics on his recent work which will appear in a DASA report to be published soon. Using a 300 MHz resonant cavity he was able to measure the ion-ion recombination coefficient in oxygen at pressures up to 400 mm Hg. The value of α_i obtained

3. Sayers, Proc. Roy. Soc., 169, Series A, March 1, 1938.

4. Gardner, Phys. Rev., 53, January 1, 1938.

was about 3×10^{-14} (meter)^3 sec^{-1}, two orders of magnitude lower than the older value. This does not necessarily contradict the older value if it is recognized that Van Lint's measurements were made on a microsecond time scale, not a tenth second time scale. This would tend to imply that perhaps different ionic species are dominant at various times after creation of the initial ionization. Some measurements made by Kasner, Rogers, and Biondi[5] and Fite and Rutherford[6] indicate that a variety of ionic species are formed. Perhaps an average ionic mobility and an average ion-ion recombination coefficient can be used for EMP calculations, but this is not presently certain and more work is needed in this area.

The importance of the ion-ion recombination coefficient is seen in that it determines the ion density and thus conductivity during the latter part of the EMP (close-in) when the prompt radiation and thus also the electron density has decayed.

III. Conclusions

For calculating the close-in EM fields it is necessary to include the effect of electron-ion scattering in air conductivity equations. As a first approximation this can be done as in eqns. (5) and (6).

Another parameter which may need revision in the calculation of radiation produced changes in air conductivity is the ion-ion recombination coefficient. Recent measurements indicate that it is much smaller than measured earlier because of experimental differences. This change would significantly increase the air conductivity during the later part of the EMP. This points out the advisability of measuring air conductivity parameters on time scales characteristic of a nuclear event.

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5. Kasner, Rogers, and Biondi, Phys. Rev. Let., 7, 8; October 15, 1961.

6. Fite and Rutherford, DASA 1461, December 31, 1963.