Subject: EMP Theoretical Note VIII

TO: Distribution List

1. Part of the following note was first distributed on 16 March 1964 as Los Alamos Document J-13-459. For this note, minor errors in J-13-459 have been corrected and 60 plots of fields generated in the ground by delta-function and step function electric fields at the surface have been added.

2. I am indebted to Lt. William R. Graham of the Air Force Weapons Laboratory for additional editing and correction of some errors in the original note. In particular, equation (1-13) was copied incorrectly directly from Wait's referenced work; it was rederived by Lt. Graham in his Theoretical Note I.

JOHN S. MALIK
EMP Theoretical Notes

Note VIII
15 April 1965

EM Pulse Fields in Dissipative Media

John S. Malik
Los Alamos Scientific Laboratory

Contents

I. Plane Wave Incident Upon a Dissipative Half-Space 2
II. Electric and Magnetic Field Correlation 19
III. Plots of the Electric and Magnetic Fields Generated by a Delta-Function and a Step Function Electric Field, in Dimensionless Form 25

Index of Plots

Delta-Function Response, Electric Field Parameter vs Time 1
Delta-Function Response, Magnetic Field Parameter vs Time 1
Step-Function Response, Electric Field Parameter vs Time 1
Step-Function Response, Magnetic Field Parameter vs Time 11
### Delta-Function Response, Electric Field Parameter vs Time

<table>
<thead>
<tr>
<th>Z (meters)</th>
<th>SIGMA (Mhos/Meters)</th>
<th>MU (sec)</th>
<th>EPSILON</th>
<th>K (sec)</th>
<th>TAU (sec)</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>3.00E-09</td>
<td>5.404E-08</td>
<td>28</td>
</tr>
<tr>
<td>0.20</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>6.00E-09</td>
<td>2.161E-07</td>
<td>29</td>
</tr>
<tr>
<td>0.30</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.00E-09</td>
<td>2.262E-09</td>
<td>30</td>
</tr>
<tr>
<td>0.30</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>9.00E-09</td>
<td>4.863E-07</td>
<td>31</td>
</tr>
<tr>
<td>0.50</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>1.50E-08</td>
<td>1.351E-06</td>
<td>32</td>
</tr>
<tr>
<td>0.60</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>8.00E-09</td>
<td>9.048E-09</td>
<td>33</td>
</tr>
<tr>
<td>0.70</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>2.10E-08</td>
<td>2.648E-06</td>
<td>34</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>1.33E-08</td>
<td>2.513E-06</td>
<td>35</td>
</tr>
<tr>
<td>1.00</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>3.00E-08</td>
<td>5.404E-06</td>
<td>36</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>2.66E-08</td>
<td>1.005E-07</td>
<td>37</td>
</tr>
<tr>
<td>2.00</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>6.00E-08</td>
<td>2.161E-05</td>
<td>38</td>
</tr>
<tr>
<td>3.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.00E-08</td>
<td>2.262E-07</td>
<td>39</td>
</tr>
<tr>
<td>10.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>1.33E-07</td>
<td>2.513E-06</td>
<td>40</td>
</tr>
<tr>
<td>30.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.00E-07</td>
<td>2.262E-05</td>
<td>41</td>
</tr>
<tr>
<td>100.00</td>
<td>0.0002</td>
<td>1.000</td>
<td>81.000</td>
<td>3.00E-06</td>
<td>2.513E-06</td>
<td>42</td>
</tr>
</tbody>
</table>

### Delta-Function Response, Magnetic Field Parameter vs Time

<table>
<thead>
<tr>
<th>Z (meters)</th>
<th>SIGMA (Mhos/Meters)</th>
<th>MU (sec)</th>
<th>EPSILON</th>
<th>K (sec)</th>
<th>TAU (sec)</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>3.00E-09</td>
<td>5.404E-08</td>
<td>43</td>
</tr>
<tr>
<td>0.20</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>6.00E-09</td>
<td>2.161E-07</td>
<td>44</td>
</tr>
<tr>
<td>0.30</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.00E-09</td>
<td>2.262E-09</td>
<td>45</td>
</tr>
<tr>
<td>0.30</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>9.00E-09</td>
<td>4.863E-07</td>
<td>46</td>
</tr>
<tr>
<td>0.50</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>1.50E-08</td>
<td>1.351E-06</td>
<td>47</td>
</tr>
<tr>
<td>0.60</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>8.00E-09</td>
<td>9.048E-09</td>
<td>48</td>
</tr>
<tr>
<td>0.70</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>2.10E-08</td>
<td>2.648E-06</td>
<td>49</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>1.33E-08</td>
<td>2.513E-06</td>
<td>50</td>
</tr>
<tr>
<td>1.00</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>3.00E-08</td>
<td>5.404E-06</td>
<td>51</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>2.66E-08</td>
<td>1.005E-07</td>
<td>52</td>
</tr>
<tr>
<td>2.00</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>6.00E-08</td>
<td>2.161E-05</td>
<td>53</td>
</tr>
<tr>
<td>3.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.00E-08</td>
<td>2.262E-07</td>
<td>54</td>
</tr>
<tr>
<td>10.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>1.33E-07</td>
<td>2.513E-06</td>
<td>55</td>
</tr>
<tr>
<td>30.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.00E-07</td>
<td>2.262E-05</td>
<td>56</td>
</tr>
<tr>
<td>100.00</td>
<td>0.0002</td>
<td>1.000</td>
<td>81.000</td>
<td>3.00E-06</td>
<td>2.513E-06</td>
<td>57</td>
</tr>
</tbody>
</table>

### Step-Function Response, Electric Field Parameter vs Time

<table>
<thead>
<tr>
<th>Z (meters)</th>
<th>SIGMA (Mhos/Meters)</th>
<th>MU (sec)</th>
<th>EPSILON</th>
<th>K (sec)</th>
<th>TAU (sec)</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>3.00E-09</td>
<td>5.404E-08</td>
<td>58</td>
</tr>
<tr>
<td>0.20</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>6.00E-09</td>
<td>2.161E-07</td>
<td>59</td>
</tr>
<tr>
<td>0.30</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.00E-09</td>
<td>2.262E-09</td>
<td>60</td>
</tr>
<tr>
<td>0.30</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>9.00E-09</td>
<td>4.863E-07</td>
<td>61</td>
</tr>
<tr>
<td>0.50</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>1.50E-08</td>
<td>1.351E-06</td>
<td>62</td>
</tr>
<tr>
<td>0.60</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>8.00E-09</td>
<td>9.048E-09</td>
<td>63</td>
</tr>
<tr>
<td>0.70</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>2.10E-08</td>
<td>2.648E-06</td>
<td>64</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>1.33E-08</td>
<td>2.513E-08</td>
<td>65</td>
</tr>
<tr>
<td>1.00</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>3.00E-08</td>
<td>5.404E-06</td>
<td>66</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>2.66E-08</td>
<td>1.005E-07</td>
<td>67</td>
</tr>
<tr>
<td>2.00</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>6.00E-08</td>
<td>2.161E-05</td>
<td>68</td>
</tr>
<tr>
<td>3.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.00E-08</td>
<td>2.262E-07</td>
<td>69</td>
</tr>
<tr>
<td>10.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>1.33E-07</td>
<td>2.513E-06</td>
<td>70</td>
</tr>
<tr>
<td>30.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.00E-07</td>
<td>2.262E-05</td>
<td>71</td>
</tr>
<tr>
<td>100.00</td>
<td>0.0002</td>
<td>1.000</td>
<td>81.000</td>
<td>3.00E-06</td>
<td>2.513E-06</td>
<td>72</td>
</tr>
</tbody>
</table>
## Step-Function Response, Magnetic Field Parameter vs Time

<table>
<thead>
<tr>
<th>Z (meters)</th>
<th>SIGMA (Mhos/Meters)</th>
<th>MU (sec)</th>
<th>EPSILON</th>
<th>K (sec)</th>
<th>TAU (sec)</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>3.000E-09</td>
<td>5.404E-08</td>
<td>73</td>
</tr>
<tr>
<td>0.20</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>6.000E-09</td>
<td>2.161E-07</td>
<td>74</td>
</tr>
<tr>
<td>0.30</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.000E-09</td>
<td>2.262E-09</td>
<td>75</td>
</tr>
<tr>
<td>0.30</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>9.000E-09</td>
<td>4.863E-07</td>
<td>76</td>
</tr>
<tr>
<td>0.50</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>1.500E-08</td>
<td>1.351E-06</td>
<td>77</td>
</tr>
<tr>
<td>0.60</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>8.000E-09</td>
<td>9.048E-09</td>
<td>78</td>
</tr>
<tr>
<td>0.70</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>2.100E-08</td>
<td>2.648E-06</td>
<td>79</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>1.333E-08</td>
<td>2.513E-08</td>
<td>80</td>
</tr>
<tr>
<td>1.00</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>3.000E-08</td>
<td>5.404E-06</td>
<td>81</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>2.667E-08</td>
<td>1.005E-07</td>
<td>82</td>
</tr>
<tr>
<td>2.00</td>
<td>4.3000</td>
<td>1.000</td>
<td>81.000</td>
<td>6.000E-08</td>
<td>2.161E-05</td>
<td>83</td>
</tr>
<tr>
<td>3.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.000E-08</td>
<td>2.262E-07</td>
<td>84</td>
</tr>
<tr>
<td>10.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>1.333E-07</td>
<td>2.513E-06</td>
<td>85</td>
</tr>
<tr>
<td>30.00</td>
<td>0.0200</td>
<td>1.000</td>
<td>16.000</td>
<td>4.000E-07</td>
<td>2.262E-05</td>
<td>86</td>
</tr>
<tr>
<td>100.00</td>
<td>0.0002</td>
<td>1.000</td>
<td>81.000</td>
<td>3.000E-06</td>
<td>2.513E-06</td>
<td>87</td>
</tr>
</tbody>
</table>
I. Plane Wave Incident Upon a Dissipative Half-Drive

There have been several investigations of fields resulting from pulsed dipoles in a conducting homogeneous medium\(^{(1-3)}\); the simpler problem of the fields resulting from an initially prescribed field description over a plane is also of use in many instances and is treated briefly by Wait\(^{(1)}\) and Stratton\(^{(5)}\).

Taking the initial fields as being zero, the Laplace transform with respect to time of the Maxwell relations are (mks units):

\[
\begin{align*}
\text{curl } \tilde{E} &= -s \tilde{B} \quad (1-1) \\
\text{curl } \tilde{B} &= (\mu \sigma + \varepsilon \mu s) \tilde{E} \quad (1-2)
\end{align*}
\]

where the tilde denotes the Laplace transform of the corresponding field quantity. For propagation parallel to a z-axis, the electric and magnetic fields may be taken along the x- and y-axis respectively. The field will be specified on the plane \(z = 0\). The resulting transformed equations are:

\[
\begin{align*}
\gamma \tilde{E}_x &= s \tilde{B}_y \quad (1-3) \\
\gamma \tilde{B}_y &= (\mu \sigma + \varepsilon \mu s) \tilde{E}_x \quad (1-4) \\
\text{with } \gamma &= (\mu \sigma + \varepsilon \mu s)^{1/2} \quad (1-5)
\end{align*}
\]

The resulting fields may then be obtained from the inverse transforms:

\[
\begin{align*}
\tilde{E}_x &= L^{-1} \left\{ \tilde{E}_x \right\} = L^{-1} \left\{ \frac{s \tilde{B}_y}{\gamma} \right\} \\
\tilde{B}_y &= L^{-1} \left\{ \tilde{B}_y \right\} = L^{-1} \left\{ \frac{\gamma \tilde{E}_x}{s} \right\} \\
\end{align*}
\]

\[
\begin{align*}
\tilde{E}_x &= L^{-1} \left\{ E_0 e^{-\gamma z} \right\} = E_0 L^{-1} \left\{ e^{-k \sqrt{s(s + b)}} \right\} \\
\text{with } \quad k &= \sqrt{\mu \sigma} \ z \\
\text{and } \quad b &= \sigma/\varepsilon
\end{align*}
\]

* Defined by \( \delta(x) = 0 \) for \( x \neq 0 \) and \( \int_{-\infty}^{\infty} \delta(x) dx = 1 \); \( u(t - k) = 0 \) for \( t < k \); \( = 1 \) for \( t > k \).

\( \delta(x) \) has dimension \( y^{-1} \)
Transform pair 863.1 of Campbell and Foster \(^{(4)}\) gives the result (it may be obtained from the transform for \(\frac{e^{-k \sqrt{s(s+b)}}}{\sqrt{s(s+b)}}\)), given in most tables of transform pairs, by differentiation with respect to \(k\) (see Appendix A-3):

\[
E_x = E_0 \left\{ e^{-(bt)/2} \delta(t - k) + \frac{bk}{2} \frac{e^{-(bt)/2}}{\sqrt{t^2 - k^2}} I_1 \left[ \frac{b}{2} \frac{\sqrt{t^2 - k^2}}{t^2 - k^2} \right] \right\} u(t - k)
\]

where \(I_1(x)\) is the modified Bessel function of the first order, \(\delta(x)\) and \(u(x)\) are the unit impulse and step functions respectively.

\[
B_y = E_0 L^{-1} \left\{ \frac{\mu_0 s^2 + \epsilon_0 s^2}{s} e^{-\sqrt{\mu_0 s + \epsilon_0 s^2} z} \right\}
\]

\[
= \frac{E_0 k}{2} L^{-1} \left\{ \frac{s(s + b)}{s} e^{-\sqrt{s(s + b)} k} \right\}
\]

\[
B_y = E_0 \sqrt{\frac{\epsilon_0}{\mu_0}} \left( \frac{b}{2} + \frac{b^2 k}{8} \right) e^{-(bk)/2} - \frac{b}{2} \int_{t}^{0} e^{-(bt)/2} \cdot I_1 \left( \frac{b}{2} \frac{\sqrt{t^2 - k^2}}{t^2 - k^2} \right) dt + \frac{bk^2}{16} \int_{k}^{t} e^{-(bt)/2} \left[ I_0 \left( \frac{b}{2} \frac{\sqrt{t^2 - k^2}}{t^2 - k^2} \right) \right. \\
\left. - \frac{2I_1 \left( \frac{b}{2} \frac{\sqrt{t^2 - k^2}}{t^2 - k^2} \right)}{\left( \frac{b}{2} \frac{\sqrt{t^2 - k^2}}{t^2 - k^2} \right)^2} \right] dt \right\} u(t - k)
\]

\[
+ E_0 \sqrt{\frac{\epsilon_0}{\mu_0}} e^{-(bk)/2} \delta(t - k)
\]

Wait \(^{(1)}\) gives the corresponding relations assuming a \(\delta\)-function \(B\) field. For ready access, they are:

\[
B_y(t) = B_0 e^{-(bt)/2} \delta(t - k)
\]

\[
+ B_0 \frac{bk}{2} e^{-(bt)/2} I_1 \left( \frac{b}{2} \frac{\sqrt{t^2 - k^2}}{t^2 - k^2} \right)
\]

\[\text{(1-12)}\]
\[ E_x(t) = \frac{B_0}{\sqrt{\varepsilon \mu}} e^{-(bt/2)} \delta(t-k) \]
\[ + \frac{B_0}{\sqrt{\varepsilon \mu}} \frac{b}{2} e^{-(bt/2)} \sqrt{\frac{t}{t^2 - k^2}} \frac{1}{t} \left[ \text{I}_1 \left( \frac{2 \sqrt{t^2 - k^2}}{2} \right) \right] \]
\[ - \frac{1}{\sqrt{\varepsilon \mu}} \left[ \frac{b}{2} \sqrt{t^2 - k^2} \right] u(t-k) \]  
(1-13)

Responses for other driving functions \( f(t) \) may be obtained from the impulse response \( g(t) \) using the Faltung integral:
\[ F(t) = \int_0^t g(\tau) f(t-\tau) \, d\tau \]  
(1-14)

In particular, the response to a step function is given as
\[ F_u(t) = \int_0^t g(\tau) \, d\tau \]  
(1-15)

These are exact transforms but are rather involved. Since for many problems the times of interest are for times \( \gg \varepsilon/\sigma \) the displacement current contribution may be neglected with great simplification of the equations. With neglect of displacement currents, \( \gamma = \sqrt{\mu \sigma} \) and the fields are functions of a dimensionless time variable \( \tau = t/\sigma \varepsilon \).

For a \( \delta \)-function \( E \)-field, \( E(0,t) = E_0 \delta(t) \);
\[ E_x = \frac{E_0}{\sqrt{\mu \sigma}} \exp \left( \frac{\mu \sigma z^2}{4t} \right) = \frac{E_0}{\sqrt{\mu \sigma}} \frac{1}{\sqrt{\pi}} \tau^{-3/2} e^{-1/(4\tau)} \]  
(1-16)

\[ B_y = \frac{E_0}{\sqrt{\sigma \varepsilon}} \exp \left( -\frac{\mu \sigma z^2}{4t} \right) = \frac{E_0}{\sigma \varepsilon} \frac{1}{\sqrt{\pi}} \tau^{-1/2} e^{-1/(4\tau)} \]  
(1-17)

These are plotted in Figures 1 and 2**.

For a \( \delta \)-function \( B \)-field:
\[ B_y = \frac{B_0}{\sqrt{\mu \sigma}} \exp \left( -\frac{\mu \sigma z^2}{4t} \right) = \frac{B_0}{2 \mu \sigma} \frac{1}{\sqrt{\pi}} \tau^{-3/2} e^{-1/(4\tau)} \]  
(1-18)

---

* \( \varepsilon/\sigma = 2 \times 10^{-10} \) sec for sea water, \( \approx 10^{-8} \) sec for Nevada Test Site soil

** The graphs do not have high accuracy being slide rule calculations in some cases. Values of the functions were usually obtained from Dwight, Tables of Functions, Dover Publications.
\[ E_x = \frac{E_0}{2 \sqrt{\pi} \mu \sigma t} \left[ \frac{\mu \sigma^2}{2t} - 1 \right] \exp \left[-\frac{\mu \sigma^2}{4t}\right] \]

\[ = \frac{B_0}{2 \mu \sigma^2 \sqrt{\pi}} \tau^{-3/2} \left[ \frac{1}{2\tau} - 1 \right] e^{-1/(4\tau)} \]  

(1-19)

For a step function electric field at \( z = 0 \):

\[ E_x = E_1 \frac{1}{\sqrt{\pi}} \left\{ \frac{e^{-y^2}}{s} \right\} \]

\[ B_y = E_1 \frac{1}{s} \left\{ \frac{1}{\sqrt{2\pi}} \right\} \]

\[ E_x = E_1 \text{erfc} \left( \frac{\sqrt{\mu \sigma^2}}{2 \sqrt{t}} \right) = E_0 \text{erfc} \frac{1}{2 \sqrt{\tau}} \]  

(1-20)

\[ B_y = E_1 \sqrt{\mu \sigma} \left\{ 2 \sqrt{\frac{t}{\pi}} \exp \left[-\frac{\mu \sigma^2}{4t}\right] - \sqrt{\mu \sigma^2} \text{erfc} \left[ \frac{\sqrt{\mu \sigma^2}}{2 \sqrt{t}} \right] \right\} \]

\[ = E_1 \frac{\mu \sigma}{\sqrt{\pi}} \left\{ \frac{2 \sqrt{\tau}}{\sqrt{\tau}} e^{-1/(4\tau)} \right\} - \text{erfc} \left[ \frac{1}{2 \sqrt{\tau}} \right] \]  

(1-21)

These are plotted in Figures 3 and 4.

For a step function magnetic field at \( z = 0 \):

\[ B_0 = B_1 \text{erfc} \left[ \frac{\mu \sigma^2}{2 \sqrt{t}} \right] = B_0 \text{erfc} \left[ \frac{1}{2 \sqrt{\tau}} \right] \]  

(1-22)

\[ E_x = E_1 \frac{1}{\sqrt{\mu \sigma}} \frac{1}{\sqrt{\pi t}} \exp \left[-\frac{\mu \sigma^2}{4t}\right] = \frac{B_0}{\mu \sigma \sqrt{\pi}} \tau^{-1/2} \exp \left[ -\frac{\tau}{4\tau} \right] \]  

(1-23)

\[
\text{Note: } \text{erf} \ x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} \, dy
\]

\[
\text{erfc} \ x = 1 - \text{erf} \ x
\]

\[
\frac{d}{dx} (\text{erf} \ x) = \frac{2}{\sqrt{\pi}} e^{-x^2}
\]
For a square pulse of length $T$ at $z = 0$:

$$E(0,t) = E_1 \quad 0 < t < T$$
$$E(0,t) = 0 \quad t > T$$

$$E_x = E_1 L^{-1} \left\{ \frac{1 - Ts}{s} e^{-\gamma s} \right\}$$

$$= E_1 \text{erfc} \left( \frac{1}{2 \sqrt{T}} \right) \quad 0 < t < \alpha$$

$$= E_1 \left[ \text{erfc} \left( \frac{1}{2 \sqrt{T}} \right) - \text{erfc} \left( \frac{1}{2 \sqrt{T - \alpha}} \right) \right] \quad \tau > \alpha \quad (1-24)$$

with $\tau = t/(\sigma \mu z^2)$; $\alpha = T/(\sigma \mu z^2)$

$$E_y = E_1 \mu \omega \left\{ \frac{2}{\sqrt{\pi}} e^{-1/(4\tau)} \right\}$$

$$= E_1 \mu \omega \left\{ \frac{2}{\sqrt{\pi}} \left[ e^{-1/(4\tau)} - \sqrt{\tau - \alpha} e^{-1/(4(\tau - \alpha))} \right] \right\}$$

$$= \left[ \text{erfc} \left( \frac{1}{2 \sqrt{T}} \right) - \text{erfc} \left( \frac{1}{2 \sqrt{T - \alpha}} \right) \right] \quad \tau > \alpha \quad (1-25)$$

These are also plotted in Figures 3 and 4 for $\alpha = 2$ and 20.

Similarly, the response to a square wave magnetic field at $z = 0$ may be obtained from the corresponding step function response relations.

The Frenchman Flat area of the Nevada Test Site (NTS) has a medium which may be roughly described as having a relative dielectric constant of 16* with an average conductivity to a depth of 6 meters of 0.02 mho/meter and to a depth of 30 meters of 0.03 mho/m. Sea water, at low latitudes, is described approximately by a relative dielectric constant of 80 and a conductivity of 4 mho/meter. With this information Table 1 gives values of times associated with the values of $\tau$.

<table>
<thead>
<tr>
<th>$z$ (m)</th>
<th>NTS</th>
<th>SEA WATER</th>
</tr>
</thead>
<tbody>
<tr>
<td>.316</td>
<td>.0025 $\tau$</td>
<td>.5 $\tau$</td>
</tr>
<tr>
<td>.6</td>
<td>.009 $\tau$</td>
<td>2 $\tau$</td>
</tr>
<tr>
<td>1.</td>
<td>.025 $\tau$</td>
<td>5 $\tau$</td>
</tr>
<tr>
<td>2.4</td>
<td>.15 $\tau$</td>
<td>30 $\tau$</td>
</tr>
<tr>
<td>30.5</td>
<td>35. $\tau$</td>
<td>500. $\tau$</td>
</tr>
</tbody>
</table>

*This dielectric "constant" may rise to much larger values at low frequencies but indications are that $\sigma$ is still $>>$ We.
The values of $\sigma$ of Figures 3 and 4 are chosen to correspond to a pulse length of $T = 10 \mu$sec with $\sigma = 20$ for $z = 0.316$ m, and $\sigma = 2$ for $z = 1$ m in sea water.

Other values of time are readily obtained from the relation $t = \frac{\sigma}{\omega} \mu^2 \tau$.

To illustrate the error introduced through neglect of the displacement current, two sample problems were calculated using values which were of some practical value. Table 2 gives values of the parameters employed for plotting the figures indicated.

<table>
<thead>
<tr>
<th>TABLE 2</th>
<th>Figures 5, 6</th>
<th>Figures 7, 8</th>
<th>Figure 9</th>
<th>Figure 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$, mho/m</td>
<td>$\approx 0.02$</td>
<td>$= 4.3$</td>
<td>$0.2$</td>
<td>$2.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\mu$, sec/m$^2$</td>
<td>$2.5 \times 10^{-8}$</td>
<td>$5.4 \times 10^{-6}$</td>
<td>$2.5 \times 10^{-8}$</td>
<td>$2.5 \times 10^{-10}$</td>
</tr>
<tr>
<td>$\varepsilon/\varepsilon_0$</td>
<td>25</td>
<td>81</td>
<td>16</td>
<td>81</td>
</tr>
<tr>
<td>Depth $z$, m</td>
<td>1.2</td>
<td>0.1</td>
<td>0.6</td>
<td>100</td>
</tr>
<tr>
<td>$k = \sqrt{\varepsilon \mu} z$, sec</td>
<td>$2.0 \times 10^{-3}$</td>
<td>$3.0 \times 10^{-9}$</td>
<td>$8.0 \times 10^{-9}$</td>
<td>$3.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\nu = \sigma/\varepsilon$, sec$^{-1}$</td>
<td>$0.9 \times 10^8$</td>
<td>$6.0 \times 10^9$</td>
<td>$1.4 \times 10^8$</td>
<td>$2.8 \times 10^5$</td>
</tr>
<tr>
<td>$\mu \sigma^2$, sec</td>
<td>$3.6 \times 10^{-8}$</td>
<td>$5.4 \times 10^{-8}$</td>
<td>$9.0 \times 10^{-9}$</td>
<td>$2.5 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

The parameters and depths for Figures 5, 6, and 9 are of use for NTS situations, those of Figures 7 and 8 for ocean water and those of Figure 10 for fresh water (they are the same as used by Stratton (5) in his example). The times used in plotting the figures are for $t = 0$ at the plane $z = 0$. For the practical cases, the figures show that neglect of displacement currents have only a small effect upon pulse shapes for media of moderate and high conductivity. Thus Figures 1 through 4 may be used for description of the fields with initial $\tau$ given by the propagation time from the plane $z = 0$ to the distance desired.
\[ \frac{1}{2} \sqrt{\frac{\tau}{\pi}} \cdot \frac{1}{\sqrt{1+\left(\sqrt{1+\frac{1}{\tau \cdot \sqrt{\lambda_0}}}ight)}} \]

Note: The graph shows the relationship between \( \tau \) and \( \sqrt{1+\frac{1}{\tau \cdot \sqrt{\lambda_0}}} \). The equation is derived from

\[ H(x,t) = h(t) - h(t-x) \]

with \( \eta = 0 \).

See equation 1-21 and equation 2-29.
Figure 1
\[ R(t) = \frac{E_0}{L} \text{ or } R(t) = \frac{E_0}{L} \text{ for } t \geq 0 \]

\[ C = C_0 \exp \left( \frac{-t}{\alpha} \right) \]

\[ \alpha = 2 \times 10^{-13} \text{ m/sec} \]

\[ t = 100 \text{ m} \]

See equation 1-9 and equation 1-16.
REFERENCES

2. B. K. Bhattacharyya, Geophysics, 22, 905 (1957)
5. J. A. Stratton, Electromagnetic Theory, McGraw Hill, 1941
II. Electric and Magnetic Pulsed Field Correlation

Impedance boundary conditions may often be used in place of the exact boundary conditions to relate the tangential component of the fields (or alternately their normal components and normal derivatives) at the surface of a medium which has a high refractive index. These boundary conditions are usually attributed to M. A. Leontovich and are valid for surfaces whose radii of curvature are large compared to the penetration depth and also for inhomogeneous materials whose properties vary slowly compared to the penetration depth. They are accurate to first order in the reciprocal of the index of refraction. The proof of these conditions has been given by T. B. A. Senior\(^{(1)}\) who collected the proofs by Leontovich and Fock and made them more readily accessible.

In terms of the parameter \( \eta \) defined by \( \eta = \mu/\mu_0 \) where \( N \) is the index of refraction, or

\[
\eta^{-1} = \sqrt{\frac{\mu_0}{\mu}} \left[ \frac{\varepsilon}{\varepsilon_0} + \frac{\sigma}{\mu_0 \varepsilon_0} \right]
\]

the conditions on the fields \( E \) and \( H \) are:

\[
\frac{\partial E_n}{\partial n} = -ikE_n
\]

\[
\frac{\partial H_n}{\partial n} = -\frac{ik}{\eta} H_n
\]

\[
E - \left( n \times E \right) E = \eta Z n \times H
\]

where

\[
k = \sqrt{\frac{\varepsilon}{\varepsilon_0} \frac{\mu}{\mu_0}} \omega
\]

\[
Z = \sqrt{\frac{\mu_0}{\varepsilon_0}}
\]

and \( n \) is a coordinate normal to the surface directed outward from the medium.

The derivation uses the fact that the tangential gradients of the fields are of order \( N \) smaller than the normal gradient and are neglected. The restriction on the smallest radius of curvature is

\[
\sqrt{\frac{\mu}{\mu_0}} \frac{\sigma}{2\pi \varepsilon_0} kp \gg 1
\]
The inhomogeneity restriction is:
\[
\left| \frac{1}{k_0} \nabla \eta \right| \ll 1
\]

Taking the tangential electric field along the x-axis of a rectangular coordinate system with the z-axis directed upward then the tangential condition may be written as
\[
E_x = -\hat{\eta} Z H_y
\]  \hspace{1cm} (2-5)

For the treatment of transient fields it is convenient to use the methods of the Laplace transform, and the Leontovich boundary conditions go over formally to the Laplace transforms with respect to time if \( \omega \) is replaced by the transform variable \( s \). Taking \( \gamma = \sqrt{\sigma s^2 + \mu s} \), the tangential magnetic field \( B_y \) in the medium is given by
\[
\tilde{B}_y = \frac{Z}{s} \tilde{E}_x
\]  \hspace{1cm} (2-6)

where the tilde indicates Laplace transform with respect to time. Thus
\[
B_y = -\sqrt{\mu} E_0 L^{-1} \left\{ \frac{s(s + b)}{s} \right\} \tilde{E}_x
\]  \hspace{1cm} (2-7)

where \( b = \sigma / 2 \)

If \( E_x \) is given by a delta function = \( E_0 \delta(t) \)
\[
\tilde{E}_x = E_0 \hspace{1cm} (2-8)
\]

\[
B_y = -\sqrt{\mu} E_0 L^{-1} \left\{ \frac{s(s + b)}{s} \right\}
\]
\[
= -\sqrt{\mu} \left\{ \delta(t) + \frac{b}{2} \left[ 1 - \int_0^t e^{-(bt/2)} \left( \frac{b\sqrt{t}}{c} \right) \right] \right\}
\]  \hspace{1cm} (2-9)

(These are just equations 1-10 and 11 for \( Z = 0 \), with propagation toward \(-z\) hence the sign change)

If \( E_x \) is given by a step function, \( E_x = E_1 u(t) \)
\[
\tilde{E}_x = \frac{E_1}{s} \hspace{1cm} (2-10)
\]

\[
B_y = -\sqrt{\mu} E_1 L^{-1} \left\{ \frac{s(s + b)}{s^2} \right\}
\]
\[
= -\sqrt{\mu} E_1 \left[ 1 + \frac{bt}{2} - \int_0^t \frac{b^2}{2} e^{-s} \left( \frac{L_1(s)}{c} \right) ds \right]
\]  \hspace{1cm} (2-11)

(Which is equation 1-10 with \( Z = 0 \))
These are given in Figure 11.

For the important case of high conductivity \( \sigma / \epsilon \gg \epsilon \) these simplify considerably.

**δ-function electric field**

\[
B_y = -T_0 \sqrt{\frac{\mu_0}{\pi t}}
\] (which is eqn. 1-17 with \( \epsilon = 0 \)) \hspace{1cm} (2-12)

**Step function electric field**

\[
B_y = -2e_1 \sqrt{\frac{\mu_0 \epsilon}{\pi}}
\] (which is eqn. 1-21 with \( \epsilon = 0 \)) \hspace{1cm} (2-13)

**REFERENCES**

\[ 1 + \int_0^y \left( 1 - \int_0^z e^{-z} \frac{I_1(z)}{z} \right) \, dz \, dy \]
The Laplace transform pairs required to invert the transforms involved are not easily available and in some cases seem incomplete; consequently it seems desirable to present a formal derivation of the transform pairs used. Starting from the well-known transform pair:

$$L^{-1}\left\{ \frac{e^{-k \sqrt{s(s+b)}}}{s(s+b)} \right\} = e^{-\frac{bk}{2}} I_0 \left[ \frac{b}{2} \sqrt{t^2 - k^2} \right] u(t-k) \quad (A-1)$$

where $I_0(x)$ is the modified Bessel function of order zero and $u(x)$ is the unit step function defined by

$$u(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases}$$

Integrating with respect to $k$ from 0 to $k$

$$L^{-1}\left\{ \frac{e^{-k \sqrt{s(s+b)}}}{s(s+b)} \right\} = e^{-\frac{bk}{2}} \int_0^k I_0 \left[ \frac{b}{2} \sqrt{t^2 - k^2} \right] dk \quad t > k \quad (A-2)$$

Differentiating the original pair after $k$ and using $\frac{d}{dk} I_0(x) = I_1(x)$

$$L^{-1}\left\{ e^{-k \sqrt{s(s+b)}} \right\} = \frac{bk}{2} e^{-\frac{bk}{2}} \int \left[ \frac{b}{2} \sqrt{t^2 - k^2} \right] u(t-k)$$

$$+ e^{-\frac{bk}{2}} \delta(t-k) \quad (A-3)$$

$I_1(x)$ is the modified Bessel function of order one.

Since

$$L\left\{ e^{-\frac{bk}{2}} \delta(t-k) \right\} = e^{-k(s + \frac{b}{2})}$$

the above may be written as

$$L^{-1}\left\{ e^{-k \sqrt{s(s+b)}} \right\} = e^{-k(s + \frac{b}{2})}$$

$$\frac{bk}{2} e^{-\frac{bk}{2}} \int \left[ \frac{b}{2} \sqrt{t^2 - k^2} \right] u(t-k) \quad (A-4)$$

* Pair 88 by R. V. Churchill from the C.R.C. Standard Math. Tables
(This is also the result of using pair 863.1 of Campbell and Foster \(^{(1)}\)).

Using the operation

\[
L^{-1}\left\{\frac{f(s)}{s}\right\} = \int_{0}^{t} F(\tau) d\tau
\]

where \( f(s) = L \{ F(t) \} \)

\[
L^{-1}\left\{\frac{e^{-k\sqrt{s(s+b)}}}{s} \right\} = \frac{e^{-k(s+b)}}{s}
\]

\[
= \frac{bk}{2} \int_{0}^{t} e^{-(bt)/2} \frac{I_{1}\left(\frac{b}{2}\sqrt{t^{2} - k^{2}}\right)}{\sqrt{t^{2} - k^{2}}} dt \quad t > k \quad (A-5)
\]

Differentiating this after \( k \) and using

\[
\frac{d}{dx} \int_{x}^{t} f(x, \tau) \ d\tau = \int_{x}^{t} \frac{df(x, \tau)}{dx} \ d\tau - f(x, x)
\]

\[
L^{-1}\left\{\frac{\sqrt{s(s+b)}}{s} e^{-k\sqrt{s(s+b)}} \right\} = \frac{(s + \frac{b}{2}) e^{-k(s+b)}}{s}
\]

\[
= - \frac{b}{2} \int_{0}^{t} e^{-(bt)/2} \frac{I_{1}\left(\frac{b}{2}\sqrt{t^{2} - k^{2}}\right)}{\sqrt{t^{2} - k^{2}}} dt
\]

\[
- \frac{bk}{2} \int_{0}^{t} e^{-(bt)/2} \frac{\partial}{\partial k} \left[\frac{I_{1}\left(\frac{b}{2}\sqrt{t^{2} - k^{2}}\right)}{\sqrt{t^{2} - k^{2}}}\right] dt \quad t > k
\]

\[
+ \frac{bk}{2} e^{-(bt)/2} \left. I_{1}\left(\frac{b}{2}\sqrt{t^{2} - k^{2}}\right) \right|_{t=k}
\]

\[
= \frac{b^{2}k}{6} e^{-(bk)/2} \int_{0}^{t} e^{-(bt)/2} \frac{I_{1}\left(\frac{b}{2}\sqrt{t^{2} - k^{2}}\right)}{\sqrt{t^{2} - k^{2}}} dt
\]

\[
+ \frac{b^{2}k}{16} \int_{0}^{t} e^{-(bt)/2} \left[\frac{I_{0}\left(\frac{b}{2}\sqrt{t^{2} - k^{2}}\right)}{\left(\frac{b}{2}\sqrt{t^{2} - k^{2}}\right)^{2}} - 2I_{1}\left(\frac{b}{2}\sqrt{t^{2} - k^{2}}\right)\right] dt \quad (A-6)
\]
But
\[ L^{-1}\left\{ \left(1 + \frac{b}{2s}\right) e^{-k(s + \frac{b}{2})}\right\} = e^{-(bk)/2} \delta(t - k) + \frac{b}{2} e^{-(bk)/2} u(t - k) \]

Hence
\[ L^{-1}\left\{ \frac{s(s + b)}{s} e^{-k \sqrt{s(s + b)}}\right\} = e^{-(bk)/2} \delta(t - k) \]
\[ + \int_{0}^{t} e^{-(b/2)\sqrt{t^2 - k^2}} dt \left[ 1 + \frac{b^2}{2} \int_{0}^{t^2 - k^2} \frac{I_{1}(b/2 \sqrt{t^2 - k^2})}{\sqrt{t^2 - k^2}} \right] \]
\[ + \frac{b}{16} \int_{0}^{t} e^{-(bt)/2} \left[ \frac{I_{0}(b/2 \sqrt{t^2 - k^2})}{I_{1}(b/2 \sqrt{t^2 - k^2})} \right] dt \]
\[ u(t - k) \quad (A-8) \]

or with \( k = 0 \)
\[ L^{-1}\left\{ \frac{s(s + b)}{s} \right\} = \delta(t) + \frac{b}{2} \left[ 1 - \int_{0}^{t} e^{-(bt)/2} \int_{0}^{bt/2} \frac{I_{1}(y)}{y} dy \right] u(t) \quad (A-9) \]
\[ L^{-1}\left\{ \frac{s(s + b)}{s} \right\} = \left[ 1 + \frac{bt}{2} - \int_{0}^{(bt)/2} dx \int_{0}^{(bx)/2} e^{-y} \frac{I_{1}(y)}{y} dy \right] u(t) \quad (A-10) \]

REFERENCES

1. Campbell and Foster, Fourier Integrals for Practical Applications, Van Nostrand Co., 1949

2. C.R.C. Standard Mathematical Tables, page 326, Laplace Transforms, taken from 'Modern Operational Mathematics in Engineering' by R. V. Churchill
III. Plots of the Electric and Magnetic Fields Generated by a Delta Function and Step Function Electric Field, in Dimensionless Form.

The ordinates for the plots are:

<table>
<thead>
<tr>
<th>Plot</th>
<th>Ordinate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_x )</td>
<td>( E_x / E_0 )</td>
</tr>
<tr>
<td>( B_y )</td>
<td>( B_y / E_0 )</td>
</tr>
</tbody>
</table>

\( E_0 \) is Impluse Field in volt-sec/meter

\( E_1 \) is value of step field in volts-meter

\( b = \sigma / \epsilon \)

\( k = \sqrt{\epsilon \mu} \)

\( a = bk = \sqrt{\frac{\mu}{\epsilon}} \sigma z \)

\( T = bk^2 = \omega \sigma^2 \)

\( \omega = t / k = \omega t / \tau \)
For $\delta$-function electric field at $z=0$

$$\frac{E_x}{E_0} = ae^{-a/2} \delta(x-1)$$

$$+ \frac{a^2}{2} \frac{e^{-ax/2}}{\sqrt{x-1}} I_1 \left( \frac{a}{2} \sqrt{\frac{2}{x-1}} \right) u(x-1)$$

$$\frac{B_y}{E_0} = ae^{-a/2} \delta(x-1)$$

$$+ \left\{ \frac{a}{2} (1 + \frac{a}{2}) e^{-a/2} + \left( \frac{a}{2} \right)^4 \right\} \int_1^x dy \left[ \frac{1}{2} \frac{e^{-y/2}}{(\frac{a}{2} \sqrt{y^2-1})^2} - \frac{(y^2+1)I_1 \left( \frac{a}{2} \sqrt{\frac{2}{y^2-1}} \right)}{\left( \frac{a}{2} \sqrt{\frac{2}{y^2-1}} \right)^3} \right] u(x-1)$$

For large $w$

$$\frac{E_x}{E_0} = \frac{1}{2\sqrt{\pi}} \frac{e^{-T/4t}}{(t/T)^{3/2}}$$

$$\frac{B_y}{E_0} = \frac{1}{\sqrt{\pi}} \frac{e^{-T/4t}}{(t/T)^{1/2}}$$
EHP PROPAGATION, (J-13-499)

$T = 0.10$ Meters

$\sigma = 4.3000$ MW/m

$\gamma = 1.000$

$E$ vs $t$

$R = 3.0000$ DB SEC

$\tau = 5.4000$ DB SEC

$\epsilon = 81.000$
EMP PROPAGATION (J-13-459)

\( \tau = 0.80 \text{ Meters} \)
\( \sigma = 4.000 \, \text{mhos/meter} \)
\( \nu = 1.000 \)
\( \epsilon = 81.000 \)

\( E \ vs \ t \)
EMP PROPAGATION: (J-13-49B)
Z = 0.30 METERS  SIGMA = 0.0200 Mhos/Meter  MU = 1.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T

R = 4.000E-09 SEC  TAU = 2.200E-09 SEC
EPSILON = 18.000

30
EMP PROPAGATION, (J-13-45B)
I = 0.30 METERS
BISMA = 4.3000 NMOS/METER
\( \nu = 1.000 \)
\( E \) VS \( T \)

\( E = 0.000 \times 10^{-9} \text{ SEC} \)
\( \tau = 4.863 \times 10^{-7} \text{ SEC} \)
\( \epsilon = 0.100 \)
EMP PROPAGATION, \( t = t_i \) (J-13-439)

\( z = 0.0 \) METERS
\( \sigma = 0.0000 \) MHO/METER
\( \mu = 1.0 \)

DELTA FUNCTION ELECTRIC FIELD AT \( z = 0 \)

\( E \) VS \( t \)
EMP PROPAGATION (I-13-459)

\[ Z = 0.70 \text{ METERS} \]
\[ \text{SIGMA} = 4.3000 \text{ MMOS/METER} \]
\[ \text{NU} = 1.000 \]
\[ \text{EPSILON} = 81.000 \]

DELTAFUNCTION ELECTRIC FIELD AT Z=0

\[ E \propto T \]
EMP PROPAGATION, (J-13-455)

\[ \tau = 1.00 \text{ Meters} \]
\[ \sigma = 0.0200 \text{ Mhos/Meter} \]
\[ \mu = 1.000 \]
\[ \epsilon = 16.000 \]

\[ E = 1.313 \times 10^{-6} \text{ SEC} \]
\[ \tau = 2.513 \times 10^{-8} \text{ SEC} \]

\[ \text{DELTA FUNCTION ELECTRIC FIELD AT Z=0} \]
\[ T = 0 \]
EMP PROPAGATION: (J-13-409)

\( J = 2.00 \text{ Meters} \)
\( \text{SIGMA} = 0.0200 \text{ MMOS/METER} \)
\( \mu = 1.000 \)
\( \epsilon = 10.000 \)

\( E \) vs \( t \)
EMF PROPAGATION. (J-53-459)
Z = 2.00 METERS
SIGMA = 4.3000 Mhos/Meter
MU = 1.000
EPSILON = 81.000

R = 6.000E-06 SEC
TAU = 2.181E-05 SEC
E VS T
EMF PROPAGATION, (J-13-459)
K = 4.0D0E-08 SEC
TAU = 2.0D0E-07 SEC
E = 3.00 METERS
SIGMA = 0.0000 MW/METER
MU = 1.000
EPSILON = 10.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T
PROPAGATION. (J-15-459)

\[ R = 1.333 \times 10^{-7} \text{ sec} \]

\[ T = 2.133 \times 10^{-6} \text{ sec} \]

\[ \mu = 0.0200 \text{ mmhos/meter} \]

\[ \nu = 1.000 \]

\[ \epsilon = 16.000 \]

\[ \delta \text{ function electric field at } z = 0 \]

\[ E \text{ vs } t \]
EMF PROPAGATION, (J-13-489)
Z = 100.00 METERS
SIGMA = 0.0002 MMOS/METER
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E vs T

K = 3.000E-06 SEC
TAU = 2.913E-06 SEC
NU = 1.000
EPSILON = 81.000
EHP PROPAGATION, (J-15-499)

\( Z = 0.10 \text{ METERS} \)
\( \sigma = 4.3000 \text{ MMH/METER} \)
\( \mu = 1.000 \)
\( \epsilon = 3.000 \text{ SEC} \)
\( \tau = 5.404E-08 \text{ SEC} \)

DELTA FUNCTION ELECTRIC FIELD AT Z = 0
\( E \propto T \)
EMI propagation: (J-15-498)

\[ I = 0.00 \text{ meters} \quad \sigma = 4.300 \text{ Mhos/meter} \quad \nu = 1.000 \quad \epsilon_{\text{flow}} = 81.000 \]

Delta function electric field at \( z = 0 \)

\[ 9 \text{ V} \text{ m} \]
EMP PROPAGATION (J-12-459)

\( z = 0.30 \text{ Meters} \)
\( \sigma = 0.0200 \text{ mhos/meter} \)
\( \mu = 1.000 \)
\( \varepsilon = 16.000 \)

\( \sigma = 4.000 \times 10^{-9} \text{ sec} \)
\( \tau = 2.88 \times 10^{-9} \text{ sec} \)

Delta function electric field at \( z = 0 \)

\( E \) vs \( t \)
EHP PROPAGATION (J-18-459)

\[ \sigma = 0.96 \text{ Meters} \]
\[ \sigma \text{ = 4.3000 WAMS/METER} \]
\[ \Omega = 1.000 \]
\[ \epsilon = 0.000 \]

\[ \Delta \text{FUNCTION ELECTRIC FIELD AT Z=0} \]

\[ E \text{ VS } T \]
EMF PROPAGATION

Z = 0.60 METERS
SIGMA = 0.0000 MMOS/METER
MU = 1.000
EPSILON = 16.000

DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T
EMP PROPAGATION, (J-12-459)

$E = 6.00E-06 \text{ SEC}$

$T = 5.00 \text{ METER}$

$
\sigma = 4.3000 \text{ Mhos/Meter}$

$\mu = 1.000$

$\varepsilon = 01.000$

DELT A FUNCTION ELECTRIC FIELD AT IN

$E = E_0 T$
EMP PROPAGATION. (J-13-459)

Z = 90.00 Meters

\sigma = 0.0200 W/ROOT/METER

NU = 1.000

\nu = 4.000E-07 Sec

\tau = 2.662E-05 Sec

\epsilon = 16.000

DELTA FUNCTION ELECTRIC FIELD AT Z=0

E VS T
EMP PROPAGATION, (J-13-459)

\( I = 100.00 \text{ Meters} \)
\( \sigma = 0.0992 \text{ Mhos/Meter} \)
\( \mu = 4.000 \)
\( \epsilon = 81.000 \)

\( E = 3.000 \times 10^{-8} \text{ SEC} \)
\( \tau = 2.813 \times 10^{-6} \text{ SEC} \)

\( B \) vs \( T \)
EWP PROPAGATION, (J-13-459)

Z = 0.10 Meters
Sigma = 4.3000 MMQ/Meter

Step Function Electric Field at z=0

E vs t

R = 3.000E-09 SEC
Tau = 5.404E-08 SEC

Mu = 1.000
Epsilon = 81.000
EHP PROPAGATION (J-18-459)

Z = 0.20 METERS  
SIGMA = 4.3000 MMOS/METER  
NU = 1.000

STEP FUNCTION ELECTRIC FIELD AT Z=0
E vs T

E = 6.000E-08 SEC  
TAU = 2.165E-07 SEC  
EPSILON = 61.000
EMP PROPAGATION (J-13-469)

\( Z = 0.30 \text{ Meters} \)
\( \sigma = 0.0200 \text{ Mhos/Meter} \)

STEP FUNCTION ELECTRIC FIELD AT Z=0

\( E \propto t \)
EMP PROPAGATION. (J-13-499)
Z = 0.50 Meters  SIGMA = 4.3000 MQS/METER  NU = 1.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T

E = 9.000E-09 SEC  TAU = 4.001E-07 SEC
EPSILON = 81.000

61
EMI PROTECTION (J-13-499)

\( z = 5.00 \) Meters
\( \sigma = 0.0200 \) Mhos/Meter
\( \nu = 1.000 \)

STEP FUNCTION ELECTRIC FIELD AT Z = 0

\( E \) vs. \( t \)
EMP PROPAGATION, (J-15-459)

\( z = 0.00 \text{ Meters} \)
\( \sigma = 0.0020 \text{ MH/meter} \)
\( \mu = 1.000 \)
\( \epsilon = 10.000 \)
\( T_\epsilon = 1.000 \times 10^{-7} \text{ sec} \)

Step Function Electric Field at \( z = 0 \) vs \( t \)
EMP PROPAGATION, (J-13-68)
Z = 2.89 METERS 
SIGMA = 4.3000 MWBS/METER 
MU = 1.000
EPSILON = 81.000

STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T
EMI PROPAGATION: (J-13-698)
Z = 30.00 METERS  SIGMA = 0.0200 MMOS/METER  JMU = 1.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E vs T

K = 4.000E-07 SEC  TAU = 2.888E-08 SEC
EPSILON = 16.000

10^-7  10^-6  10^-5  10^-4  10^-3  10^-2
10^-8  10^-7  10^-6  10^-5  10^-4  10^-3  10^-2
10^-1  10^0  10^1  10^2  10^3  10^4  10^5  10^6
10^-7  10^-6  10^-5  10^-4  10^-3  10^-2

10^-7  10^-6  10^-5  10^-4  10^-3  10^-2
10^-8  10^-7  10^-6  10^-5  10^-4  10^-3  10^-2
EMP PROPAGATION (J-13-409)
Z = 8.10 METERS  Q165
STEP FUNCTION ELECTRIC FIELD AT 1
B VE T

TIME SECONDS  10⁻⁹  10⁻⁸  10⁻⁷  10⁻⁶  10⁻⁵  10⁻⁴

R = 3.000E-08 SEC
TAU = 5.604E-08 SEC
EPSILON = 81.000

6000 MM/SEC/UNIT

73
EMP PROPAGATION, (J-13-456)

\( \gamma = 0.30 \text{ METER} \)

\( \Sigma = 0.0200 \text{ MMOS/METER} \)

\( \mu = 1.000 \)

\( \tau = 2.882E-09 \text{ SEC} \)

\( \epsilon_{0} = 16.000 \)

FUNCTION ELECTRIC FIELD AT Z=0

10^{-9} \text{ TIME SECONDS} \rightarrow 10^{-8} \rightarrow 10^{-7} \rightarrow 10^{-6} \rightarrow 10^{-5} \rightarrow 10^{-4}
EMP PROPAGATION, (J-13-499)

\[ E = 0.38 \text{ METER} \]

\[ \sigma = 8.0 \text{ GOUD HANKS/METER} \]

\[ \mu = 1.0 \]

\[ \varepsilon = 81.0 \]

STEP FUNCTION ELECTRIC FIELD AT 740.
EMP PROPAGATION (J-13-459)
Z = 0.80 METERS
SIGMA = 4.3000 Mhos/Meter
W = 1.000
TAU = 1.531E-08 SEC
EPSILON = 61.000
STEP FUNCTION ELECTRIC FIELD AT Z^0
EMP PROPAGATION, (J-10-469)

Z = 1.00 METERS
SIGMA = 0.0200 MMOS/METER
NU = 1.000
EPSILON = 10.000

STEP FUNCTION ELECTRIC FIELD AT Z = 0

E VS T
EMI PROPAGATION, (J-13-489)
Z = 1.60 METERS
SIGMA = 4.3000 Mhos/Meter
N = 1.000
K = 3.000E-08 SEC
TAU = 3.400E-08 SEC
EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
@ T8 T
EMF PROPAGATION: (3-13-489)
T = 3.00 METER
SIGMA = 0.0000 W/M
TERA
MU = 1.000
EPSILON = 16.000

B vs T
EMF PROPAGATION (J-13-459)

\[ E = 10.00 \text{ Meters} \]
\[ \sigma = 0.000 \text{ mhos/meter} \]
\[ \nu = 1.000 \]
\[ \epsilon = 10.000 \]

\[ R = 1.333 \times 10^{-7} \text{ sec} \]
\[ \tau = 2.016 \times 10^{-8} \text{ sec} \]

STEP FUNCTION ELECTRIC FIELD AT T = 0
EHP PROPAGATION

$F = 50.00$ METERS
$\Sigma = 0.0200$ MMOS/MEETER
$\mu = 1.000$
$\epsilon = 16.000$

$K = 4.000E-07$ SEC
$\tau = 2.222E-05$ SEC

STEP FUNCTION ELECTRIC FIELD AT Z=0
$E = 0$ V/

86
EMI PROPAGATION (J-12-499)
Z = 100.90 METERS
\( \text{SIGMA} = 0.0002 \, \text{MMOS/METER} \)
STEP FUNCTION ELECTRIC FIELD AT Z=0
E vs T

\( r = 3.0002 \times 10^{-6} \, \text{SEC} \)
\( \text{TAN} = 2.613 \times 10^{-6} \, \text{SEC} \)
\( \mu = 1.000 \)
\( \varepsilon_0 = 81.000 \)