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EMP Theoretical Notes
Note XIII

SURFACE EMP - AN ASPECT OF INTERNAL EMP

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ABSTRACT

A gamma or x-ray pulse impinging on an insulator will eject electrons from the various atoms in the material and displace them an average distance R_e in the direction of pulse propagation. If the electrons stay at their displaced positions (and it is usually assumed that they do) static charge distributions result, which give rise to permanent (or at least long-duration) internal electric fields.

In first approximation this can be regarded as the bodily displacement of an array of electrons a distance equal to R_e. The result is a dense positive charge within a distance R_e of the incident surface, followed by negative charge density which is less by a factor R_e/λ (λ is the mean free path of the photon in the insulator) and decreases exponentially with depth of penetration. The negative charge is due to the excess of electrons over ions in the interior regions.

In previous treatments of internal EMP familiar to the writer, the existence of the large positive charge density close to the surface (and hence referred to herein as "surface EMP") was somehow overlooked. The purpose of the present note is to call attention to its existence, and to the fact that because of surface EMP, very thin plates can be subject to virtually as large internal fields as thick plates.

In practice, other objects are often between the source of photons and the insulator. In such cases an electron flux accompanies the photons, and these electrons are deposited in the surface layer. It is shown that, depending on the R_e/λ ratio of the preceding material, these electrons can either decrease or increase the internal fields in the insulator.

Introduction

One of the effects of gamma rays and x-rays is to relocate electrons within an insulator. This builds up a static charge distribution and creates permanent electric fields.

Previous treatments   have led to the conclusion that for an insulator thickness substantially less than the mean free path of the incident photons, the field strength is proportional to the thickness. The theory developed in these references implies as the thickness L exceeds the mean free path, the maximum field strength increases asymptotically toward a limit given by the expression

$$E_{max} \sim \frac{e \, F_o R_e}{\lambda} \qquad (L >> \lambda) \tag{1}$$

where

e	is the electron charge
F_o	is the integrated flux incident upon the surface
R_e	is the range of an electron ejected from an atom by the gamma or x-ray photon
λ	is the mean free path of a photon in the insulator ($>> R_e$)

The purpose of the present note is to point out that E_{max} can reach this value for much smaller thicknesses, e.g. $L = R_e$. Below this value of L the maximum field strength is proportional to L. As L increases beyond R_e the maximum field strength increases very slowly to an ultimate value of twice its value for

$L = R_e$. As a result, thin plates may have static fields several orders of magnitude greater than were previously thought to be possible.

Theory

To estimate the charge densities and field strengths, certain simplifying assumptions will be employed. The insulator is assumed to be an infinite plate of finite thickness L oriented normal to the incident flux of x-rays or gamma rays. Each electron ejected from an atom by the incident flux is assumed to travel exactly the same distance R_e in the direction of the radiation. Because of this uniform displacement a net charge distribution is set up which is the sum of the positive ion density and the displaced electron density.

The densities of positive ions and the displaced electrons are given respectively by the expressions,

$$\rho + = \frac{e}{\lambda} F_o e^{-x/\lambda} \tag{2}$$

$$\rho - = 0 \qquad \left(x < R_e\right) \tag{3a}$$

$$\rho - = -\frac{e}{\lambda} F_o e^{-(x - R_e)/\lambda} \left(x > R_e\right) \tag{3b}$$

The net charge density is given by the sum of the above two expressions,

$$\rho = \frac{e}{\lambda} F_o e^{-x/\lambda} \qquad \left(x < R_e\right) \tag{4a}$$

$$\rho = \frac{e}{\lambda} F_o e^{-x/\lambda} (1 - e^{R_e/\lambda}) \quad (x > R_e) \tag{4b}$$

Since R_e is generally two or more orders of magnitude smaller than λ the above equations can be rewritten as

$$\rho = \frac{e}{\lambda} F_o \qquad (x < R_e) \qquad (5a)$$

$$\rho = - \frac{eR_e}{\lambda^2} F_o e^{-x/\lambda} \qquad (x > R_e) \qquad (5b)$$

Previous writers have employed equation (5b) for all values of x. This is probably because they have employed a different approach in which the current caused by the radiation is first calculated, after which the effect of this current in changing the charge density is determined. Since the positive charges don't move, their contribution is not automatically taken care of in such an approach. However, the charge distribution given in equation (5) can be obtained by this method also, if instead of the constant ratio between radiation flux and electron flux applicable to the interior of a medium, we use the relations

$$F_e = (x/\lambda)\ F_\gamma \qquad (x < R_e) \qquad (6a)$$

$$F_e = (R_e/\lambda)\ F_\gamma \qquad (x > R_e) \qquad (6b)$$

Equation (5) applies when only photons impinge upon the insulator. In this case, electrons will be ejected from the far side of the slab leaving it with a net positive charge per unit area, $\sigma = \int \rho\, dx$, the magnitude of which is readily obtained from equation (5). As the thickness of the slab approaches infinity the net charge approaches zero. If the dense positive charge in the surface region $(0 < x < R_e)$ is neglected, the net charge will be negative for all values of L and largest for L approaching infinity.

For $L < R_e$ the electric field strength will vary linearly from a maximum value given (in e. s. u) by

$$E_{max} = \frac{2\pi\sigma}{K} = \frac{2\pi e L F_o}{K\lambda} \qquad (L < R_e) \tag{7}$$

at one surface of the plate to the negative of this value at the other surface. In this equation K is the dielectric constant. For larger values of L the maximum value of the field will occur at $x = R_e$ and will be the sum of the contributions of the thin layer of positive charge near the surface of the slab and the negative charge distributed through the remainder of the slab:

$$E_{max} = \frac{2\pi e R_e F_o}{K\lambda} + \frac{2\pi e R_e F_o}{K\lambda}\left(1 - e^{-L/\lambda}\right) = \frac{2\pi e R_e}{K\lambda} F_o\left(2 - e^{-L/\lambda}\right) \tag{8}$$

The static field E (x) for various thicknesses and the variation of the maximum field strength with slab thickness L are shown in Figures 1 and 2 respectively.

Attention up to this point has been limited to the situation in which only photons impinge upon the front surface. In the case of an interface between two media, electrons ejected from the first medium will accompany the photon flux and this will, in general, change the charge density in the region $x < R_e$. If the first medium is more than one electron range in thickness, the charge density at the surface of the insulating slab will be given by:

$$\rho_2(x) = \frac{e\,F_o}{\lambda_2}\left(1 - n\right) \qquad (x < R_e) \qquad (9)$$

where $n \cong (R_e/\lambda)_1 \big/ (R_e/\lambda)_2$

Here the subscripts 1 and 2 refer to the preceding medium and the insulating slab under consideration respectively.

Sessler (Reference 2) has pointed out that in the case of gamma rays R_e/λ is approximately the same for all materials, i.e., $n \cong 1$. On the other hand, for x-rays tne ratio for one material may differ by several orders of magnitude from that for another, i.e., n can be much less or much greater than 1. When $n < 1$ the number of electrons deposited in the surface layer by the incident electron flux is less than the number of electrons ejected by the photons, and the net charge in this layer is tnerefore positive. When $n = 1$ equation (5b) applies for all values of x. When $n > 1$, the number of electrons deposited exceeds the number ejected, and the charge in the surface layer is negative.

To simplify discussion of $n \neq o$ we introduce the charge per unit area of the regions $x < R_e$ and $x > R_e$ considered separately:

$$\sigma_s = \int_o^{Re} \rho(x)\, dx = \frac{(1 - n)\, e\, Fo\, Re}{\lambda} \qquad (10)$$

$$\sigma_i = \int_{Re}^{L} \rho(x)\, dx = - \frac{e\, Fo\, Re}{\lambda} \left(1 - e^{-L/\lambda}\right) \qquad (11)$$

When $L > R_e$, the maximum field is

$$E_{max} = 2\pi\left(\left|\sigma_s\right| + \left|\sigma_i\right|\right) K \qquad (L > R_e) \qquad (12)$$

If σ_s and σ_i have opposite signs (i.e., if $n < 1$) the maximum field is at $x = R_e$. If $n > 1$, σ_s and σ_i have the same sign and the maximum field is at the two surfaces, $x = o$ and $x = L$.

If $L < R_e$,

$$E_{max} = 2\pi\sigma_s \; L/KR_e = 2\pi(1 - n)e \; F_o \, L/K\lambda \qquad (L < R_e) \qquad (13)$$

while for $L >> \lambda$

$$E_{max} = 2\pi \; (2 - n)e \; F_o \; R_e/K\lambda \qquad (n < 1) \qquad (14a)$$

$$= 2\,\pi\, ne\, F_o \; R_e/K\,\lambda \qquad (n > 1) \qquad (14b)$$

Comparing equations (13) and (14) we see that when n = 1, the maximum field in a thin insulating plate is relatively small; in all other cases, however, the fields in the thin plate are comparable to those in an arbitrarily thick plate. This result is also apparent in Figure 2.

It has been pointed out by Sessler (private communication) that the strong fields resulting from the processes just discussed will cause charges to be deposited on the surfaces of the insulator. Among the mechanisms for this are attraction of charges from other objects, attraction of charges from the ionized air and conduction along a metallic plate if the insulator happens to be attached to one. Such charges will have the effect of cancelling fields outside the insulator. Inside the insulator, the effects will depend upon the distribution of the attracted charges as well as the insulator geometry. If we confine attention to the infinite plate of finite thickness to which attention has been devoted up to this point, and if the surface charges attracted from the surroundings are distributed equally on both sides of the insulating plate, the interior fields will be unaffected. If the charge is entirely on the front surface, and $n = 0$ (only photons are incident), the field strength in the region $x < R_e'$ is increased while the field strength in the region $x > R_e'$ is reduced. (R_e' is the value of x at which the field strength is zero before addition of the surface charges). The converse to this situation occurs if the surface charge is attracted entirely to the back face. In either case we find that although the external field may be cancelled by attracted charges, there is no significant reduction of the maximum internal field.

Summary

It appears from the preceding discussion that proportionality between field strength and slab thickness L occurs when the slab thickness is less than the range of the electron, R_e. When only photons, (i.e., no charged particles) are incident upon the slab, a high density of positive charge is built up in a very thin layer adjacent to the irradiated surface. This is followed by a much less dense distribution of negative charge which decreases exponentially with depth of penetration. The net result is that large fields are induced even in a very thin material. Increasing the thickness above R_e increases the maximum field strength slowly and at most, doubles it.

When electrons as well as photons are incident upon the insulating slab, as in the case of an interface with some other material, the charge density induced at a depth greater than R_e is unaffected, but the charge density near the surface $(x < R_e)$ will be made less positive. If the ratio R_e/λ for both materials is the same, the charge density in the surface layer becomes slightly negative and continuous with the values for $x > R_e$. If this ratio in the preceding material is much greater than it is in the dielectric slab, then a negative charge density much greater than the previously computed positive charge density will occur in the surface layer giving rise to much larger field strengths than those computed for the case of only photons incident. This is not likely to occur for gamma rays, which are characterized by a ratio which is roughly constant for all materials; however, it may be expected to occur for x-rays, for which the ratio varies widely.

It can be anticipated that surface charges attracted from the surroundings will quickly eliminate the external field. However, these will not significantly reduce the maximum internal field.

The transient conductivity of the insulator due to ionization by the Compton electrons has been neglected. It seems entirely possible that this effect could profoundly influence the results.

It is a pleasure to acknowledge several fruitful discussions of this problem with A. M. Sessler.

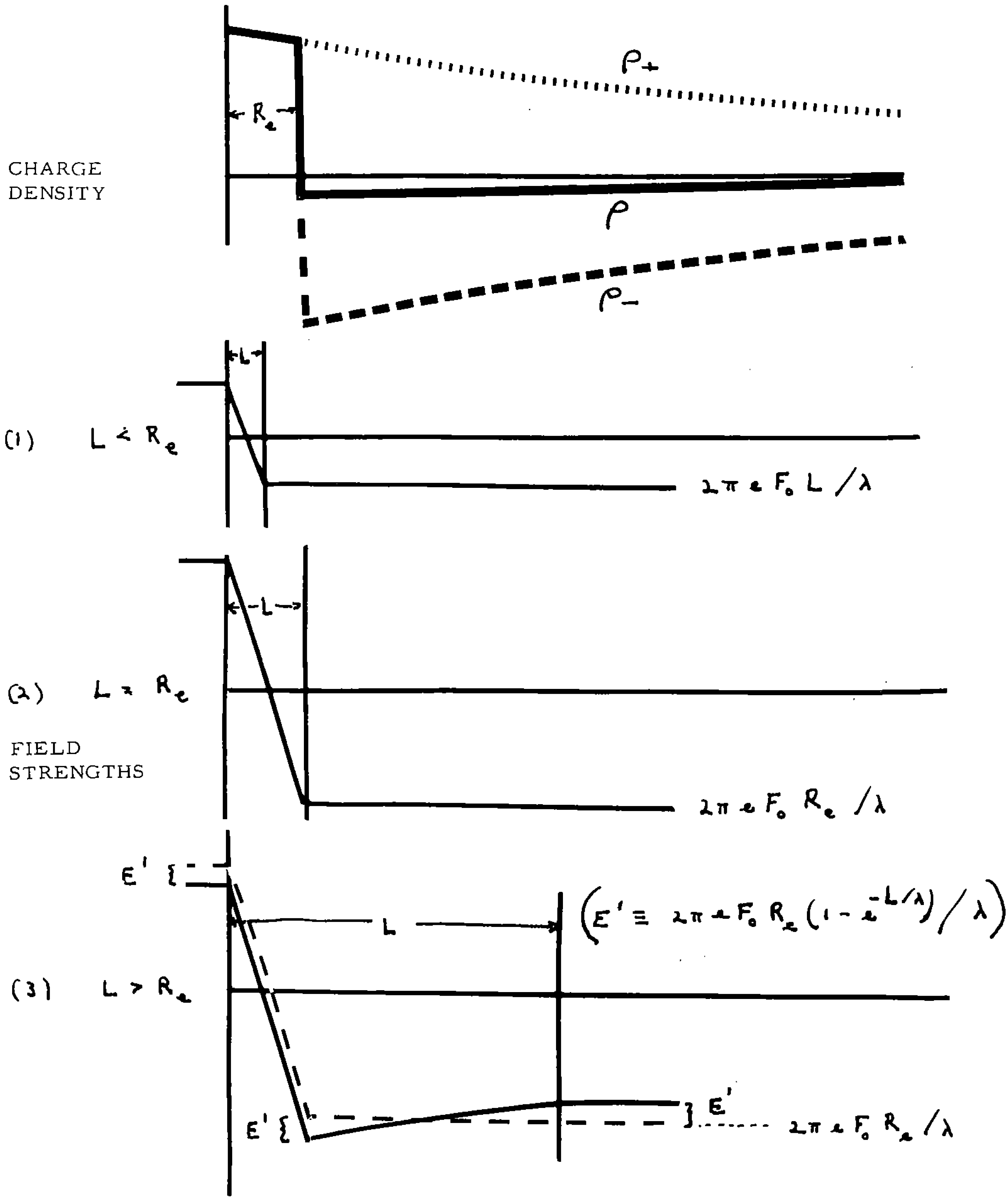

Figure 1 - Charge Density and Field Strengths in an Insulating Slab of Thickness L Produced by a Photon Flux Fo

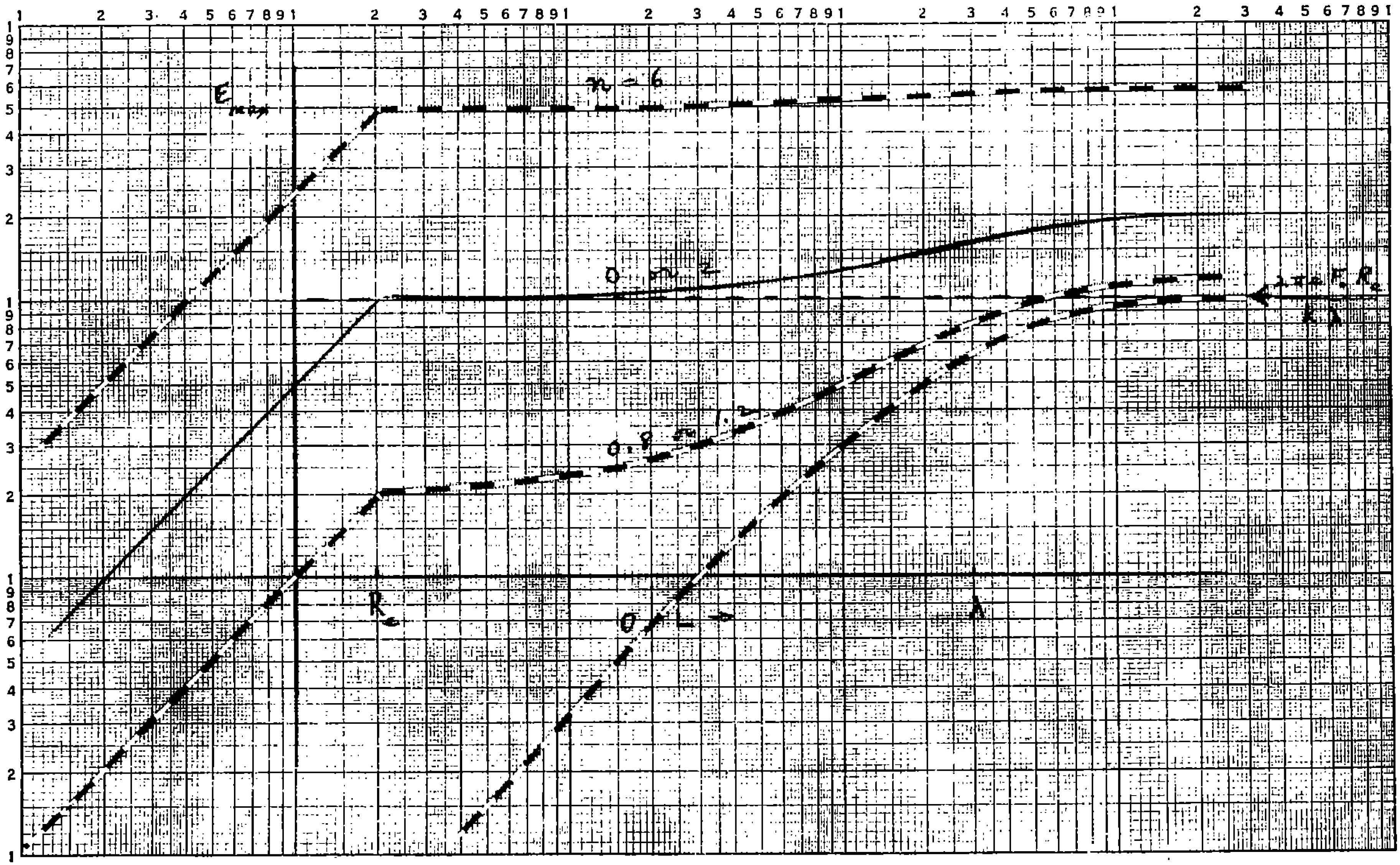

FIGURE 2
Maximum Field Strength vs Insulation Thickness
(Dotted lines represent interface cases; $n = \frac{(R_e/\lambda)_1}{(R_e/\lambda)_2}$)