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THE ELECTROMAGNETIC SIGNAL DUE TO THE EXCLUSION OF THE EARTH'S MAGNETIC FIELD BY NUCLEAR EXPLOSIONS

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SUMMARY

There are three mechanisms by which a vertical magnetic field might be generated by a nuclear explosion. Asymmetry of the nuclear device might produce a non–vertical dipole moment whose time variation would lead to an electromagnetic field with a vertical magnetic component. The magnitude of the vertical component due to this mechanism would depend sensitively on device design and orientation at the time of the detonation.

Alternatively, Compton currents produced by explosion gamma rays might interact with the earth's field and generate a "back" field tending to cancel the earth's field in a region about the explosion. Such a cancellation would be equivalent to radiation from a magnetic dipole and therefore could lead to a vertical magnetic field.

Finally, on a much longer time scale, the blast wave motion of the heated (conducting) air surrounding the explosion might exclude the earth's field and thus generate a low frequency variation in the vertical component of the magnetic field.

In this report only the latter two mechanisms are treated. It is shown that due to air conductivity and the fact that gamma rays travel with light speed, the Compton currents do not exclude the earth's field. The blast wave motion does exclude the field and the magnitude of the resultant electromagnetic field is estimated.

THE ELECTROMAGNETIC SIGNAL DUE TO THE EXCLUSION OF THE EARTH'S MAGNETIC FIELD BY NUCLEAR EXPLOSIONS

There are two possible mechanisms by which the earth's magnetic field might be excluded from a large region about a nuclear explosion. First, the Compton–recoil electrons generated by explosion gamma rays interact with the earth's magnetic field, producing a "back" field. Approximate calculations described here indicate that this mechanism does not exclude the field. The other possibility is that the field is excluded due to the blast wave motion of heated (conducting) air. Calculations indicate that this mechanism is effective and its magnitude is crudely estimated.

HOW THE GAMMA RAYS TEND TO EXCLUDE THE FIELD

The "prompt" γ rays emitted by a nuclear explosion generate Compton–recoil electrons in the surrounding air. The recoil electrons move roughly a few meters at sea level, predominantly in an outward radial direction. As the electrons move, they are turned by the earth's magnetic field, thereby producing an azimuthal current which tends to cancel the field in the region near the explosion.

MECHANISM INHIBITING EXCLUSION

The slowing–down of the Compton electrons by the air produces ionization and makes the air conducting. The conductivity of the air leads to a conduction current which opposes the direct Compton current.

The question of whether the conduction current cancels the Compton current is complicated because the gamma pulse moves with the speed of light, which means that the displacement current cannot be ignored and Maxwell's equations must be treated in detail — which is done in the next section.

THEORY

Denoting the conductivity by σ and the Compton current by $\vec{j}$, Maxwell's equations become

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}(\sigma\vec{E} + \vec{j}), \tag{1}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}. \tag{2}$$

σ at a distance r (kilometers) from the burst point and a time t after the explosion is

$$\sigma(t, r) = \frac{e^2}{m\nu_c} n(t, r), \tag{3}$$

where approximately[1]

$$n(t, r) \approx 4 \times 10^{13} \frac{e^{-r/\lambda}}{\lambda r^2} \eta Y \int_0^t dt' f(t' - r/c) e^{-\beta(t-t')}. \tag{4}$$

In Eqs. (3) and (4), m is the electron mass, ν_c is the collision frequency of thermal electrons with air molecules, λ is the mean free path (kilometers) of the explosion gamma rays, Y is the explosion yield in kilotons, η is

[1] Multiple scattering of the gamma rays is neglected.

the fraction of Y appearing as gamma rays, β is the electron attachment rate to O_2, and f(t) is the temporal variation of the gamma-ray emission $\left(\int_0^\infty dt\, f(t) = 1\right)$.

The Compton current $\vec{j}$ is

$$\vec{j}(t, r) \approx -2 \times 10^9\, e\, \frac{e^{-r/\lambda}}{\lambda r^2}\, \eta Y \int_{t-\frac{R}{c}}^{t} dt'\, f\left(t' - \frac{r-(t-t')v_0}{c}\right) e^{(t-t')v_0/\lambda}\, \vec{v}(t-t'), \quad (5)$$

where R is the range of the Compton electrons and $\vec{v}$ is their velocity. Assuming that the electrons are scattered radially away from the burst point,

$$\vec{v} \approx v_0 \vec{e}_r + v_0^2 \frac{t-t'}{\rho} \vec{e}_\phi , \quad (6)$$

where the coordinate system has its origin at the explosion point with its polar axis along the earth's magnetic field and we consider only those electrons moving in the plane $\theta = \pi/2$.[2] v_0 is the speed of the Compton electrons and $\rho = \gamma \frac{mc\, v_0}{eB}$ is the Larmor radius, B being the local magnetic field.

To obtain a precise solution of Eqs. (3) and (4) involves a prohibitive amount of calculation. We can, however, obtain a reasonably good approximate solution. For this purpose, we neglect effects arising from the spherical divergence of the Compton electrons and treat the local interaction of the electrons and the magnetic field as if the explosion were planar. This is justifiable as long as all spatial changes are confined

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[2]The "back" field from electrons in other directions is less than that from electrons in the plane $\theta = \pi/2$.

within a region whose radial dimension is small compared to the distance from the burst point. The results will be seen to justify this approximation.

Within this approximation,

$$\vec{E} = E\vec{e}_\phi + F\vec{e}_r \tag{7}$$

$$\vec{B} - \vec{B}_0 = \vec{b} = b\vec{e}_\theta , \tag{8}$$

where $\vec{B}_0$ is the earth's magnetic field. Substituting Eqs. (7) and (8) into Eqs. (1) and (2) and using the planar approximation,[3]

$$\frac{\partial b}{\partial r} = \frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c}(\sigma E + j_\phi) , \tag{9}$$

$$\frac{\partial E}{\partial r} = \frac{1}{c}\frac{\partial b}{\partial t} . \tag{10}$$

From Eqs. (3) – (6), we find that σ and j_ϕ have the form

$$\sigma(t, r) = G(r)\,g(t - r/c) , \tag{11}$$

$$j_\phi(t, r) = G(r)\,h(t - r/c) , \tag{12}$$

where $G(r)$ varies slowly with r and may be treated as a constant.

STEADY STATE SOLUTION

In view of Eqs. (11) and (12), b and E may be assumed to depend only on $\tau = t - r/c$. Eqs. (9) and (10) then become

$$-\frac{db(\tau)}{d\tau} = \frac{dE(\tau)}{d\tau} + 4\pi[\sigma(\tau)E(\tau) + j_\phi(\tau)] , \tag{13}$$

$$-\frac{dE(\tau)}{d\tau} = \frac{db(\tau)}{d\tau} . \tag{14}$$

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[3] The equation for F can be disregarded since it does not affect the $\vec{B}$-field.

Solving Eqs. (13) and (14) leads straightforwardly to

$$E(\tau) = -\frac{j_\phi(\tau)}{\sigma(\tau)} \tag{15}$$

and

$$b(\tau) = \frac{j_\phi(\tau)}{\sigma(\tau)} + A, \tag{16}$$

where A is an arbitrary constant. Causality requires that for $\tau \leq 0$, $E(\tau)$ and $b(\tau)$ be zero. A simple argument shows that

$$\lim_{\tau \to 0^+} \frac{j_\phi(\tau)}{\sigma(\tau)} = 0. \tag{17}$$

Hence, $A = 0$ and

$$b(\tau) = \frac{j_\phi(\tau)}{\sigma(\tau)}. \tag{18}$$

Equations (15) and (18) for $E(\tau)$ and $b(\tau)$ satisfy Maxwell's equations and all boundary conditions. They are therefore the required solutions.

From Eqs. (5) and (6), it is seen that $j_\phi(\tau) < 0$. Thus $b(\tau) < 0$ and therefore adds to the earth's magnetic field. We have the surprising result that — not only do the Compton electrons not exclude the earth's magnetic field, but in fact they lead to an increase in the earth's field in the region where $j_\phi(\tau) \neq 0$. This unintuitive result is a consequence in part of the fact that the electromagnetic field due to the displacement current moves with the gamma-ray pulse. If the current source had a velocity $v << c$, then the more familiar result that the field from a given current opposes the field which generates it (Lenz's Law) would be satisfied (see Appendix I).

Equation (18) is also a consequence of the assumed steady state for j_ϕ and σ which ignores turning on the current. However, turning on the current produces radiation which moves with the speed of light and which may therefore contribute to b. In the next section this contribution will be shown to be small.

EFFECT OF TURNING ON j_ϕ

In order to estimate the effect of turning on the current, we will assume σ is constant and j_ϕ is turned on at $t = 0$. Then from Eqs. (9) and (10),

$$\frac{\partial^2 b}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 b}{\partial t^2} - \frac{4\pi\sigma}{c^2}\frac{\partial b}{\partial t} = \frac{4\pi}{c}\frac{\partial j_\phi}{\partial r} . \tag{19}$$

Let

$$\hat{b} = \int_{-\infty}^{\infty} dr\, e^{-ikr}\, b(r, t) . \tag{20}$$

Then

$$\frac{\partial^2 \hat{b}}{\partial t^2} + 4\pi\sigma\frac{\partial \hat{b}}{\partial t} + k^2c^2\hat{b} = -4\pi ick\,\hat{j}_\phi \tag{21}$$

and

$$\hat{b} = -4\pi ick\int_0^t d\tau\, e^{-2\pi\sigma(t-\tau)}\,\frac{\sin (t-\tau)\sqrt{k^2c^2-(2\pi\sigma)^2}}{\sqrt{k^2c^2-(2\pi\sigma)^2}}\,\hat{j}_\phi . \tag{22}$$

Hence,

$$b(r,\ t) = -4\int_0^t d\tau\ e^{-2\pi\sigma(t-\tau)}\int_{-\infty}^{\infty} dr'\ \frac{\partial j_\phi(r',\ \tau)}{\partial r'}\ I(|r-r'|,\ c(t-\tau)), \quad (23)$$

where

$$b(r,\ 0) = \frac{\partial b(r,\ 0)}{\partial t} = 0$$

and

$$I(|r-r'|,\ c(t-\tau)) = \int_0^{\infty} dk \cos k(r-r')\ \frac{\sin c(t-\tau)\sqrt{k^2-(2\pi\sigma/c)^2}}{\sqrt{k^2-(2\pi\sigma/c)^2}} \quad (24)$$

$$= \frac{\pi}{2} J_0\left(2\pi i\sigma\sqrt{(t-\tau)^2-(r-r')^2/c^2}\right) u(t-\tau-|r-r'|/c)\ .$$

Alternatively,

$$b(r,\ t) = -2\pi\int_0^t d\tau\ e^{-2\pi\sigma(t-\tau)}\Bigg\{ j_\phi(r+ct-c\tau,\ \tau) - j_\phi(r-ct+c\tau,\ \tau)$$

$$-\int_{r-c(t-\tau)}^{r+c(t-\tau)} dr'\ j_\phi(r',\ \tau)\ \frac{\partial}{\partial r'} J_0\left(2\pi i\sigma\sqrt{(t-\tau)^2-(r-r')^2/c^2}\right)\Bigg\}. \quad (25)$$

Assuming again that

$$j_\phi(r,\ t) = j_0\, g(t-r/c) \quad \text{for} \quad t \geq 0, \quad (26)$$

where $\int_0^{\infty} dt\ g(t) = 1$ and $g(t) = 0$ for $t < 0$, we find

$$b(r,\ t) = \frac{j_0\, g(t-r/c)}{\sigma}(1-e^{-2\pi\sigma t}) - 2\pi j_0 \int_0^t d\tau\, e^{-2\pi\sigma(t-\tau)} \Bigg[g(2\tau - r/c - t)$$

$$- \int_{r-c(t-\tau)}^{r+c(t-\tau)} dr'\, g(\tau - r'/c) \frac{\partial}{\partial r'} J_0\left(2\pi i \sigma \sqrt{(t-\tau)^2 - (r-r')^2/c^2}\right) \Bigg] . \tag{27}$$

The first term gives the steady state solution, Eq. (18). The remaining terms give the effect of turning on the current j_ϕ.

To illustrate the behavior of b we shall approximate g(t) by a delta function, $\delta(t)$. Thus

$$b(r,\ t) = \frac{j_0\, \delta(t - r/c)}{\sigma}(1 - e^{-2\pi\sigma t}) - \pi j_0 e^{-\pi\sigma(t-r/c)}$$

$$- 2\pi c j_0 \int_0^{(t+r/c)/2} d\tau\, e^{-2\pi\sigma(t-\tau)} \frac{\partial}{\partial r} J_0\left(2\pi i \sigma \sqrt{(t-\tau)^2 - (r-c\tau)^2/c^2}\right) \tag{28}$$

Or

$$b(r,\ t) = \frac{j_0\, \delta(t-r/c)}{\sigma}(1 - e^{-2\pi\sigma t})$$

$$- 2\pi c j_0 \frac{\partial}{\partial r} \int_0^{(t+r/c)/2} d\tau\, e^{-2\pi\sigma(t-\tau)} J_0\left(2\pi i \sigma \sqrt{(t-\tau)^2 - (r-c\tau)^2/c^2}\right) \tag{29}$$

It is straightforward to show that for $\sigma(t + r/c) >> 1$ the last term of Eq. (29) is $O(e^{-2\pi\sigma t})$ (see Appendix II). Thus

$$b(r,\ t) = \frac{j_0\,\delta(t - r/c)}{\sigma} + O(e^{-2\pi\sigma t}) . \tag{30}$$

More generally it can be shown that for an arbitrary positive and rapidly decaying pulse $g(t)$, the last two terms of Eq. (27) are $O(e^{-2\pi\sigma t})$. Therefore, for constant σ, we have verified that the effect of turning on the current is small.

On physical grounds this conclusion could have been expected since the radiation from turning on the current j_ϕ will be attenuated by propagating through the conducting region. We note, moreover, from Eqs. (23) and (24) and more directly from Eq. (29) that

$$\int_{-\infty}^{\infty} dr\ b(r,\ t) = 0 \tag{31}$$

as expected from conservation of magnetic flux.

Since no relevant new features are introduced by the spatial and temporal variation of the conductivity, corrections to Eq. (18) are expected in general to be small.[4]

ESTIMATE OF $b(\tau)$

Substituting Eqs. (3) – (6) into Eq. (18) gives

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[4]Provided, of course, that σ is large in the region where the current is turned on.

$$b(\tau) = B \frac{1}{2 \times 10^4} \frac{\nu_c v_0}{\gamma c} \frac{\int_{\tau - R/c}^{\tau} d\tau' f\left(\tau \pm [\tau - \tau'] \frac{v_0}{c}\right) e^{(\tau - \tau')v_0/\lambda} (\tau - \tau')}{\int_0^{\tau} d\tau' f(\tau') e^{-\beta(\tau - \tau')}} . \tag{32}$$

To obtain a numerical estimate of $b(\tau)$, we let $f(\tau) \sim e^{\alpha\tau}$, $v_0 \approx c$, and $\gamma \approx 2$. Then

$$b(\tau) = -(B_0 + |b(\tau)|) \frac{\nu_c}{4 \times 10^4} \frac{\alpha + \beta}{\left(\frac{c}{\lambda}\right)^2} \frac{1 - e^{c\tau/\lambda}(1 - c\tau/\lambda)}{1 - e^{-(\alpha+\beta)\tau}} \tag{33}$$

for $0 \leq \tau \leq R/c$ and

$$b(\tau) = -(B_0 + |b(\tau)|) \frac{\nu_c}{4 \times 10^4} \frac{\alpha + \beta}{\left(\frac{c}{\lambda}\right)^2} \frac{1 - e^{R/\lambda}(1 - R/\lambda)}{1 - e^{-(\alpha+\beta)\tau}} \tag{34}$$

for $R/c \leq \tau$. In a time of the order $1/\alpha \approx 10^{-8}$ sec, $b(\tau)$ increases from zero to its steady state value of

$$b(\tau) = -(B_0 + |b(\tau)|) \frac{\nu_c}{4 \times 10^4} \frac{\alpha + \beta}{\left(\frac{c}{\lambda}\right)^2} \left[1 - e^{R/\lambda}(1 - R/\lambda)\right]. \tag{35}$$

At sea level, $\nu_c \approx 1.5 \times 10^{11}$ per sec, $\beta \approx 0.75 \times 10^8$ per sec, and $R \approx 3 \times 10^2$ cm. Thus

$$b(\tau) \approx -0.033 \ (B_0 + |b(\tau)|) \tag{36}$$

or

$$b(\tau) \approx -0.033 \ B_0 . \tag{37}$$

That is, from Eq. (8),

$$\vec{B} = -1.033\, B_0 \vec{e}_\theta . \tag{38}$$

HYDRODYNAMIC EXCLUSION OF THE EARTH'S FIELD

Surrounding a nuclear explosion is an intensely ionized region in which the air conductivity is sufficiently high to freeze in the magnetic field. As a result, when the blast wave causes the air to expand, the magnetic field is carried along. At the same time the magnetic field which is external to the ionized region is unable to penetrate the ionized region and is pushed outward.

The behavior of the magnetic field is described by

$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\frac{\vec{v}}{c} \times B - c\,\frac{\nabla \times \vec{B}}{4\pi\sigma} \right) \tag{39}$$

and

$$\nabla \cdot \vec{B} = 0 . \tag{40}$$

The details of the air heating and the hydrodynamic motion determine $\vec{v}$ and σ as functions of time and space. These functions are so complicated that a precise solution of Eqs. (39) and (40) would involve a lengthy numerical calculation. Instead, we shall make an approximation which is adequate to provide semi-quantitative results.

We assume that surrounding the explosion there exists an isothermal sphere of radius $R(t)$ within which the air is highly ionized and the relaxation time τ of the magnetic field is quite long, that is,

$$\frac{\tau}{R(t)} \frac{dR(t)}{dt} \approx 4\pi\sigma \frac{R(t)}{c^2} \frac{dR(t)}{dt} >> 1 . \quad (41)$$

Outside the isothermal sphere, the relaxation time is short and the conductivity can be neglected. With this simple model, the field at a distance $r \geq R(t)$ from the explosion is

$$\vec{B} = B_0 \vec{e}_z - \frac{1}{2} B_0 \left[1 - \left(\frac{R(0)}{R(t)}\right)^2\right] \left[\frac{R(t)}{r}\right]^3 \sin\theta\, \vec{e}_\theta \quad (42)$$

$$- B_0 \left[1 - \left(\frac{R(0)}{R(t)}\right)^2\right] \left[\frac{R(t)}{r}\right]^3 \cos\theta\, \vec{e}_r ,$$

where again the polar axis coincides with the earth's field $B_0 \vec{e}_z$.

Numerical results on the radiation and hydrodynamic flow for a sea level nuclear explosion give

$$R(t) \approx \left[5 + 120\left(\frac{t}{Y^{1/3}}\right)^{1/3}\right] Y^{1/3} \text{ meters} \quad (43)$$

for $0 \leq t/Y^{1/3} \leq 0.05$, where t is in seconds and Y in kilotons. For $0.05 \leq t/Y^{1/3}$,

$$R(t) = R_{max} \approx 50\, Y^{1/3} \text{ meters.} \quad (44)$$

Equation (42) holds until Eq. (41) breaks down, after which the second and third terms of Eq. (42) decay to zero. The time constant for this decay is, approximately,

$$4\pi\sigma \frac{R_{max}^2}{c^2} , \tag{45}$$

where σ_0 is the conductivity of the isothermal sphere when $R(t) = R_{max}$. A rough estimate of σ_0 is

$$\sigma_0 \sim 10^{14} \sec^{-1} . \tag{46}$$

Consequently, $\frac{4\pi\sigma_0 R_{max}^2}{c^2}$ is of the order of a few times $Y^{2/3}$ seconds. Since this relaxation time is so long, other phenomena, such as fireball rise and radiative and convective cooling, will determine the actual time for the magnetic field to relax.

APPENDIX I

If in Eqs. (4) and (5), $f = f(t - r/v)$, where $v << c$, then

$$\sigma(t, r) = F(r)\, g(t - r/v) \sim \sigma(\tau) \tag{I.1}$$

$$j_\phi(t, r) = F(r)\, h(t - r/v) \sim j_\phi(\tau), \tag{I.2}$$

where $\tau = t - r/v$. Substituting $E = E(\tau)$ and $b = b(\tau)$ into Eqs. (9) and (10), we find

$$\frac{db}{d\tau} = -\frac{v}{c}\frac{dE}{d\tau} - \frac{4\pi v}{c}(\sigma E + j_\phi) \tag{I.3}$$

$$\frac{dE}{d\tau} = -\frac{v}{c}\frac{db}{d\tau}. \tag{I.4}$$

Solving

$$E(\tau) = \frac{4\pi(v/c)^2}{1-(v/c)^2}\int_{-\infty}^{\tau} d\tau'\, j_\phi(\tau') \exp\left\{\frac{4\pi(v/c)^2}{1-(v/c)^2}\int_{\tau'}^{\tau} d\tau''\, \sigma(\tau'')\right\} \tag{I.5}$$ [5]

and

$$b(\tau) = -\frac{c}{v}E(\tau) + b(-\infty). \tag{I.6}$$

If $b(-\infty) < 0$ and $j_\phi < 0$, then $b(\tau) - b(-\infty) > 0$; that is, the field behind the pulse is decreased in absolute value. Thus the field is partially excluded by the current — as expected from Lenz's Law.

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[5] The non-vanishing of $E(\tau)$ for large τ is a consequence of the steady-state, one-dimensional character of the problem.

APPENDIX II

Consider

$$I = \frac{\partial}{\partial r} \int_0^{(t+r/c)/2} d\tau\, e^{-2\pi\sigma(t-\tau)} J_0\left(2\pi i \sigma \sqrt{(t-\tau)^2 - (r-c\tau)^2/c}\right). \qquad \text{(II. 1)}$$

Let

$$\tau = \frac{t + r/c}{2}(1 - z). \qquad \text{(II. 2)}$$

Then

$$I = \frac{\partial}{\partial r}\left\{\frac{t+r/c}{2} e^{-\pi\sigma(t-r/c)} \int_0^1 dz\, e^{-\pi\sigma(t+r/c)z} J_0\left(2\pi i \sigma \sqrt{t^2 - (r/c)^2}\sqrt{z}\right)\right\}. \qquad \text{(II. 3)}$$

For $\sigma(t + r/c) >> 1$,

$$I = \frac{\partial}{\partial r}\left\{\frac{t+r/c}{2} e^{-\pi\sigma(t-r/c)} \int_0^\infty dz\, e^{-\pi\sigma(t+r/c)z} J_0\left(2\pi i \sigma \sqrt{t^2 - (r/c)^2}\sqrt{z}\right) + (ct+r)\, O(e^{-2\pi\sigma t})\right\}. \qquad \text{(II. 4)}$$

But the integral in Eq. (II. 4) can be evaluated exactly and leads to

$$I = \frac{\partial}{\partial r}\left\{\frac{1}{2\pi\sigma} + (ct + r)\, O(e^{-2\pi\sigma t})\right\}. \qquad \text{(II. 5)}$$

Hence,

$$I = O(e^{-2\pi\sigma t}). \qquad \text{(II. 6)}$$