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ELECTROMAGNETIC SIGNAL FROM A BOMB BURST IN VACUO

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ABSTRACT

If a bomb is burst in vacuo, the prompt gamma rays will eject Compton recoil electrons from the case and thus produce a radial current. As this current will be spherically asymmetric, in general, an electromagnetic signal will be radiated. In this short note the magnitude of this signal is calculated. It is found to be substantially independent of the yield and probably detectable to a distance of 10^6 kilometers or more.

Large, prompt electromagnetic signals have been observed from bomb explosions at an altitude of approximately 100 kilometers. One possible source, of such signals, and the only source yet suggested for an explosion in vacuo, is the ejection of fast electrons from the case by gamma rays. These electrons then flow freely outward and, as they constitute a time dependent current, they radiate. We shall now calculate the prompt part of such a signal.

Let n(t) electrons per second be ejected with velocity v, and with an angular distribution $f(\theta)$, so that the radial current density is

$$J_r = - \frac{e}{4\pi r^2} f(\theta)\, n(t - \frac{r}{v}). \qquad (1)$$

The explosion is supposed to have taken place at point $P(r = 0)$. We wish to calculate the vertical (z) component of the electric field at the point $\bar{P}$, located a vertical distance h and a horizontal distance

D from P. The distance ρ from a general point (r, Θ, φ) to $\overline{P}$ is given by

$$\rho^2 = r^2 + h^2 + D^2 + 2r(h \cos \Theta - D \sin \Theta \cos \varphi). \qquad (2)$$

In addition to ρ, we need the direction cosine which projects the current density vector $\vec{J}$ at (r, Θ, φ) onto the vertical at $\overline{P}$. To calculate this, let $\vec{a}$ be a unit vector parallel to $\vec{J}$. Then $\vec{a}$ has cartesian components

$$\vec{a} = (\sin \Theta \cos \varphi, \sin \Theta \sin \varphi, \cos \Theta).$$

Also, let $\vec{b}$ be the unit vector in the direction given by the point pair $\overline{P}$, (r, Θ, φ), i.e., in the direction of ρ. Then $\vec{b}$ has cartesian components

$$\vec{b} = \frac{1}{\rho}(r \sin \Theta \cos \varphi - D, r \sin \Theta \sin \varphi, r \cos \Theta + h).$$

Now the transverse component of $\vec{a}$, namely that part of $\vec{a}$ which is orthogonal to $\vec{b}$, is given by

$$\vec{a}_\perp = \vec{b}x(\vec{a} \times \vec{b}) = \vec{a} - (\vec{a}\cdot\vec{b})\vec{b} \overset{\text{def}}{=} \vec{e}.$$

The z component of this vector is the direction cosine in question and is

$$e_z = \frac{D^2 \cos\Theta - r\,h \sin^2\Theta + (h - r\cos\Theta)\,D \sin\Theta \cos\varphi}{\rho^2}. \quad (3)$$

In terms of these quantities, then, the desired component of the vector potential is

$$A_z = \frac{e\mu}{4\pi}\int \frac{n(t - \frac{r}{v} - \frac{\rho}{c})}{\rho} \left[\frac{e_z\, f(\Theta)}{4\pi}\right] \sin\Theta\, d\varphi\, d\Theta\, dr. \quad (4)$$

The electric field is calculated from the relation

$$E_z = -\frac{\partial A_z}{\partial t}.$$

Moreover, note that

$$\frac{\partial}{\partial t} n(t - \frac{\rho}{c} - \frac{r}{v}) = -\frac{cv}{c + v\rho_r}\frac{\partial}{\partial r} n(t - \frac{\rho}{c} - \frac{r}{v}),$$

where ρ_r means $\partial\rho/\partial r$. Thus we have

$$E_z = -\frac{e\mu cv}{4\pi}\int \frac{e_z\, f(\Theta)\sin\Theta}{4\pi(c\rho + v\rho\rho_r)} \frac{\partial n(t - \frac{\rho}{c} - \frac{r}{v})}{\partial r}\, dr\, d\varphi\, d\Theta, \quad (5)$$

where, according to (2),

$$\rho\rho_r = r + h\cos\Theta - D\sin\Theta\cos\varphi. \quad (6)$$

Note that (2) can be written as

$$\rho^2 = (r + h\cos\Theta - D\sin\Theta\cos\Theta)^2 + (h\sin\Theta + D\cos\Theta\cos\varphi)^2$$
$$+ D^2\sin^2\varphi.$$

Clearly, then, the minimum value of the quantity $c\rho + v\rho\rho_r$ is $(c - v)(R - r)$, and the integral of equation (5) is perfectly regular for times $t < R/c$. We may therefore integrate by parts and obtain

$$E_z = -\frac{e\mu cv}{4\pi}\int f\,\frac{\sin\Theta}{4\pi}\,d\varphi d\Theta\left\{-\left[\frac{e_z n(t - \frac{r}{v} - \frac{\rho}{c})}{c\rho + v\rho\rho_r}\right]_{r=0}\right.$$
$$\left.-\int n(t - \frac{r}{v} - \frac{\rho}{c})\,\frac{\partial}{\partial r}\left(\frac{e_z}{c\rho + v\rho\rho_r}\right)dr\right\}.$$

Evidently $\partial/\partial r(e_z/c\rho + v\rho\rho_r)$ is of order $1/\rho^2$; therefore, so long as the r-range of the second integral is not too large, specifically so long as

$$ct \ll R;\ R^2 \overset{\text{def}}{=} h^2 + D^2, \qquad (7)$$

the second term in the bracket is negligible compared with the first. Thus, for the prompt signal as defined by (7) we have

$$E_z = - \frac{e\mu cv}{4\pi R^2} n(t - \frac{R}{c}) \int_0^{\pi} \sin \theta \cdot f \cdot d\theta \int_0^{2\pi} \frac{hD \sin \theta \cos \varphi + D^2 \cos \theta}{vD \sin \theta \cos \varphi - (Rc + hv \cos \theta)} d\varphi .$$

The integration over φ can be performed by making the substitution $\zeta = e^{i\varphi}$. Integration is then performed about the unit circle in the (complex) ζ-plane and Cauchy's theorem applies. The end result is

$$E_z = - \frac{e\mu cn(t - \frac{R}{c})}{8\pi R^2} \int_{-1}^{1} f(\mu) \left\{ h - \frac{R(vR\mu + hc)}{\sqrt{(vR\mu + hc)^2 + (c^2 - v^2)D^2}} \right\} d\mu . \quad (8)$$

We have written $\mu \equiv \cos \theta$. Note that the positive value of the square root is always to be taken. The shape of the prompt signal is seen to be the same as that of the source, $n(t)$. The integral of (8) is easily seen to yield zero when $f(\mu) = 1$. Clearly, f is normalized so that

$$\int_{-1}^{1} f(\mu) d\mu = 2 .$$

In the model here contemplated the electrons are Compton recoils with energies like $4.5\ mc^2$. Thus v will be very nearly equal to c. It is of interest, therefore, to note the limit of (8) as v approaches c, namely,

$$E_z = - \frac{e\mu c}{8\pi R^2} n(t - \frac{R}{c}) \left\{ \int_{-1}^{-h/R} f(\mu) \, [h + R] d\mu + \int_{-h/R}^{1} f(\mu) [h - R] d\mu \right\} \quad (9)$$

when v = c.

This result could have been obtained ab initio, but setting v = c at the start leads to singular integrals, and some of the manipulations are a bit tricky to justify.

If a bomb is exploded in complete vacuum, it is hard to think of any mechanism for prompt radiation other than that considered above. Let us therefore see what (9) gives. We assume the bomb case to be a sphere of radius a ($a \sim 1$ meter), or some other shape of the same electrical capacitance. After the emission of N electrons this sphere will be at a potential

$$V = + \frac{eN}{4\pi\epsilon a} . \quad (10)$$

The gamma-ray spectrum has an effective energy of about 6 Mev, so it seems reasonable to suppose that the maximum Compton electron energy might be 5 Mev. If this is so, equation (10), with $V = 5 \times 10^6$ volts, will give the total number of electrons emitted. We write this in the form

$$en = \frac{4\pi\epsilon a V}{\tau}$$

where τ is a characteristic time for the pulse, so that

$$N = n\tau, \quad n \equiv \frac{\partial N}{\partial t}.$$

If we moreover assume maximum asymmetry, so that all electrons go into the upper hemisphere, equation (9) will give

$$E_z \approx \frac{aV}{Rc\tau} \left(\frac{R - h}{R}\right). \tag{11}$$

As we have seen $a \sim 1$ meter, $V \sim 5 \times 10^6$ volts, and favorable antenna alignment will give $\left(\frac{R - h}{R}\right) \sim 1$. To achieve the sort of charge here acquired requires a rather modest number of electrons ($\sim 10^{15}$); thus cut-off potential is likely to occur during the α-phase and τ will be of order 10^{-8}. This gives for the field

$$E_z \sim \frac{10^3}{R}, \tag{12}$$

where E_z is in volts/meter and R is in kilometers. This expression is independent of yield because of the small number of electrons involved.

In arriving at the expression (12) we have made three assumptions, all of which tend to yield a large signal, namely (1) a cut-off voltage of 5×10^6, (2) maximum possible asymmetry, and (3) very nearly optimum alignment of the receiving antenna. An omnidirectional receiving antenna is not difficult to arrange, so assumption (3) is not serious. It is hard to see how the cut-off voltage could have been over-estimated by more than a factor two. It is a little harder to say how much increased symmetry might reduce the signal, as complete symmetry reduces it to zero. However, it does not seem likely that the signal will be less than that of (12) by much more than a factor 10 unless extremely good symmetry is attained.

The range at which such a signal could be detected depends upon the purpose for which the detector is designed. If one wishes to measure accurately the pulse shape, wide band amplifiers, with their associated low sensitivities, would be required. If one wishes merely to ascertain that such a signal occurred, narrow band, high gain amplifiers could be used and a signal of 100 microvolts per meter should be easily detected, with good chances of doing 10 times better. Conservatively, the detection range should be at least 10^6 kilometers. To avoid transmission peculiarities of the Heaviside layer and to minimize interference from radio signals, electric storms, etc., the receiver should probably be outside the atmosphere.