Theoretical Notes
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The Treatment of Electron Scattering and Approximate Methods Used for Specifying High-Altitude EMP Sources

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ABSTRACT

A Monte Carlo electron transport code was used to calculate the EMP sources produced 1) by monenergetic electrons, 2) by a Compton scatter distribution of electrons, and 3) by a photoelectric scatter distribution of electrons. The effects of nuclear-coulomb electron scattering, of the continuous slowing down method of electron energy loss, and of electron turning in the geomagnetic field of the earth were included in the calculations. Analytic results were obtained from the source routine of AFWL one-dimensional EMP code names. HEMP B sources include an improved slowing down model and the option of two electron scatter approximations based on small angle scatter theory; one based on an average obliquity of the distribution, and the other based on a random selection of scattering direction. The Monte Carlo results are compared to the analytic methods for a delta function source of photons.
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SECTION I
INTRODUCTION

The purpose of this paper is to show the adequacy of present approximate models of calculating electromagnetic pulse (EMP) sources by comparison to rigorous Monte Carlo surce calculations. All results are shown for a single interacting photon per cubic meter at an incident angle perpendicular to the geomagnetic field. The magnitude of the field is 0.560 gauss.

The original intent was to compare convoluted time waveforms of sources for a typical weapon; however, this was abandoned to allow unclassified results. The Monte Carlo calculations were done using a code developed at the Air Force Weapons Laboratory (AFWL). The approximate methods discussed are presently options for the source calculations used in the one-dimensional HEMP-B EMP code.

Three sets of comparisons are discussed. The first set is one of Monte Carlo results pointing out the effects of scatter and of including a complete distribution of Compton electrons for sources at 20 km altitude. The second set compares two approximate models to Monte Carlo results at 20 km altitude. The last set distinguishes the photoelectric and Compton sources from Monte Carlo calculations for X rays at 40 km altitude. A comparison of photoelectric currents with the two approximate methods is made.
SECTION II
SCATTERING AND THE COMPLETE COMPTON DISTRIBUTION

Past source calculations at the AFHL used one representative electron having the average energy of the Compton distribution and was given a forward velocity weighted by the cosine of the average angle of the distribution. This energy is 0.742 MeV for an average electron from a 1.5 MeV incident gamma ray. More results at this electron energy with and without scatter have been previously reported by Knutson (ref. 1).

Figure 1 shows the results for ionization rate (using 34 eV per ion pair). The nonscattered results are labeled N.S. The complete distribution from 1.5 MeV gamma is 1.5 \( \gamma \), and the average electron is 0.742. The Monte Carlo calculations were done in real time, then converted to retarded time by the relation

\[
\tau = t - r/c
\]

where \( r \) is the distance from the burst, \( c \) is the speed of light, and \( t \) is the real time. The velocity of the electrons in retarded time is related to the true velocity by the relation

\[
V_{\text{retarded}} = \frac{V_{\text{real}}}{\sqrt{1 - \eta \ V_{\text{real}}}}
\]

where \( \eta \) is the cosine of the electron's direction from the radial. Since the electron range would not be changed under the transformation, a radially directed electron would have a much higher retarded velocity and hence, travel its range in a shorter retarded time interval than electrons in any other direction. This explains why the nonscattered ionization only lasts for about \( 4.5 \times 10^{-8} \) sec, whereas the scattered results last much longer with a lesser magnitude.

The radial currents in figure 2 show differences in the complete distribution and average electron calculations at about \( 6 \times 10^{-8} \) sec. Such differences at about two orders of magnitude below the peak are not very significant when convoluted with the time output history of a weapon. However, the effect of scattering is significant.
Figure 1. Ionization Rate of 20 km, Monte Carlo Results
Figure 2. Radial Current at 20 km, Monte Carlo Results
The currents in the phi direction are shown in figure 3. This is the initial direction of the force on the electrons which is perpendicular to the radial and geomagnetic field vectors. The complete distribution current is approximately 20 percent lower than the average electron near the peak. This can have a comparable effect on peak EMP. The nonscattered result is about a factor of five higher than the others at the peak.

The other directional component is parallel to the geomagnetic field vector for this case. The only contributions of current are through scattering in that direction. This component has equal contributions in opposite directions due to its symmetry and therefore has no net current.
Figure 3. Phi Current at 20 km, Monte Carlo Results
SECTION III
RESULTS OF APPROXIMATE METHODS

There are two approximate methods used in AFWL EMP source calculations. One method is called the average obliquity or "eta factor" treatment and is described in reference 2. That is, the Compton electrons are allowed to expand as a distribution within an R.M.S. angle, \( \Omega \), which increases with distance traveled. A more elaborate random scattered treatment uses a random selection of direction on a Gaussian distribution for each electron and time step to represent scattering. This method is documented in reference 3.

The ionization rates for the eta factor (\( \eta \)), random scatter (R.S), and Monte Carlo complete distribution (M.C.) for incident 1.5 MeV gammas are shown in figure 4 for 20 km altitude. Reasonable agreement holds until about 1.3 x \( 10^{-7} \) sec where the eta factor calculation begins to fall off. The following arguments will account for this.

The time of sign reversal in the radial currents (figure 5) will be used to illustrate the inherent differences in the results obtained by the various methods. Three factors account for the sign reversal. The first is the electron turning in the geomagnetic field over 90 degrees. To get a crude idea of this effect, one can calculate the extreme electron range from data such as in reference 4. The average range is generally about two-thirds of the extreme range. By comparing the average range to the cyclotron circumference (which for this example is: \( \gamma \beta (15.21) \) meters, where: \( \beta = v/c \) and \( \gamma = 1/\sqrt{1 - \beta^2} \)), one can estimate the amount of turning for an unscattered electron. This turning is about 270 degrees so one would expect a radial sign reversal and perhaps a small phi component sign reversal from this effect; however, retarded time effects and energy losses cause the magnitude to be less than the scale on these figures. Therefore, in figure 2, no reversal is seen for the unscattered radial current case. The second factor is the part of the initial distribution of electrons with velocities in the backward direction (greater than 90 degrees), which last longer in retarded time. (This is equal to zero for the Compton distribution.) The third factor is the mechanism of scattering initially forward directed electrons into backward directions. The shortcoming of the eta factor treatment is that it cannot account for this third factor. As this
R.S. RANDOM SCATTER APPROXIMATION
\( \eta \) ETA FACTOR APPROXIMATION
MC MONTE CARLO COMPTON DISTRIBUTION
FROM 1.5 MeV GAMMA SOURCE

Figure 4. Ionization Rate at 20 km, Approximate Method Comparison
Figure 5. Radial Current at 20 km, Approximate Method Comparison
factor is important for this example (as seen for the examples in figure 2), the eta \((n)\) curve in figure 5 never shows a sign reversal for radial current. The random scatter and Monte Carlo results show crossovers within \(10^{-8}\) seconds of each other. Any differences in delta function current several orders of magnitude from the peak are almost insignificant in their convoluted form. Because of the lack of longer lasting rearward directed electrons in the eta factor current, the ionization rate (figure 4) falls off earlier in retarded time and is slightly higher to account for the same total ionization.

Figure 6 shows the phi currents for the three calculations. No sign reversals are evident, indicating no significant turning greater than 180 degrees or significant scattering in that direction. The eta factor result falls off earlier, as was expected. The differences in the peaks of the random scatter and Monte Carlo currents have been investigated and give no more than 10 percent difference in peak EMP for a worst case calculation.
Figure 6. Phi Current at 20 km, Approximate Method Comparison
SECTION IV

SOURCES FROM 50 keV X RAYS

The two approximate methods and the Monte Carlo code were used to calculate sources at 40 km altitude from 50 keV X rays. For this energy, the photoelectric cross sections became comparable to the Compton cross sections. To illustrate the effects of each, the results of photoelectric (PE) and Compton (C) processes are shown in figures 7, 8 and 9 for Monte Carlson calculations. The significance between the two distributions is that the Compton electron scatter is more forward directed than is the photoelectric, which has its maximum electron intensity at about 50 degrees to the radial. Therefore, the photoelectrons would be expected to last longer in retarded time than do the Compton electrons. Figure 7 for energy loss (proportional to ionization rate) shows this is the case.

Figure 8 shows no negative component of the forward peaked Compton radial electron flux (proportional to radial current by the value of electron charge). However, the photoelectron distribution has sufficient rearward components to cause a sign reversal. Figure 9 shows the phi directed flux of the photoelectric distribution to also have a sign reversal.

By again comparing the average range of the average energy Compton electron (about 4 keV for this case), we can estimate the amount of geometric turning for an unscattered electron. For an average range of 0.42 meter, the turning is about 80 degrees. Therefore, any sign changes would be attributed to scattering. Figure 8 shows no negative component of the forward peaked Compton radial electron flux.

Figure 10 compares the radial flux results from the Monte Carlo (histogram form), eta factor (n), and random scatter (R.S.) methods for the photoelectric distribution. Again, there is a difference between the eta factor treatment and the other methods. In this case, the difference is more significant than the 1.5 MeV gamma case because the curves diverge more.

Figure 11 compares the phi flux for these cases. The difference in the eta and random scatter results are obvious compared to the Monte Carlo. The energy loss (or ionization rate) in figure 12 shows the random scatter to give better results than the eta treatment, but not as good as the results for the 1.5 MeV Compton distribution case.
Figure 7. Ionization Rate at 40 km, Compton and Photoelectric Monte Carlo Results
Figure 8. Radial Current at 40 km, Compton and Photoelectric Monte Carlo Results
Figure 9. Phi Current at 40 km, Compton and Photoelectric Monte Carlo Results
Figure 10. Radial Current at 40 km Altitude, Approximate Method Comparison
Figure 11. Phi Current at 40 km Altitude, Approximate Method Comparison
Figure 12. Energy Loss at 40 km Altitude, Approximate Method Comparison
SECTION V
CONCLUSIONS

The necessity of including a reasonable scattering model for EMP source calculations is mandatory. The utilization of complete distributions of Compton electrons instead of average values can have a 20 percent effect on peak EMP. The random scatter model gives excellent comparisons with the Monte Carlo results for both the gamma and X-ray examples. The eta factor model gives good representations of the gamma induced sources (especially when convoluted) and slightly poorer representations of the X-ray induced sources.
REFERENCES


