Theoretical Notes
Note 214

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ESTIMATION OF TIME DEPENDENT ELECTRON TRANSPORT PARAMETERS BASED UPON TIME INDEPENDENT ENERGY DEPOSITION AND ELECTRON TRANSMISSION PROFILES

By

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I. INTRODUCTION

The electron current and ionization rate pulses associated with a propagating, planewave photon pulse exhibit time dependence which depends upon the photon pulse shape, medium density, and photon energy spectrum as it affects the angular and energy distributions of the initiated Compton or photo-electrons. If the primary electrons were to travel at the speed of light (c) in the direction of the photon wavefront throughout their lifetimes, then the electron pulse would possess the same time dependence as, and be coincident with the photon pulse. In reality, the electrons are initiated at less than c and distributed in angle with respect to the direction of photon propagation. Furthermore, the electrons experience energy loss and angular scatter throughout their lifetimes. These factors plus electromagnetic field interaction effects (geomagnetic and aggregate radio frequency fields) tend to produce an electron pulse which rises more slowly than the photon pulse and is spread over a larger time interval.

In order to predict the time dependence of the electron current and ionization rate pulses it is necessary to know the electron velocity components and time of arrival of electrons with respect to the photon wavefront. An accurate evaluation of velocities and times of arrival of electrons can be achieved through application of sophisticated transport techniques such as Monte Carlo evaluation. An estimate of these electron transport parameters, however, can be obtained based upon time independent electron energy deposition and fractional transmission profiles. This paper describes the manner in which the energy deposition and transmission data can be interpreted and processed to provide estimates of time dependent electron transport information, and suggests how this information might be employed for applications involving field effects on electron dynamics. Specifically, the trajectory of a single "representative electron", characterized by energy, angle, and weight as functions of forward penetration, is suggested as representative of the distribution of all possible electron trajectories for a given initial electron energy.

II. THE REPRESENTATIVE ELECTRON

Considerable information exists on the effects of transport of electrons in materials.[1] Two commonly investigated features of electron transport are the energy deposition per initial electron per unit forward penetration, dW/dx, and the fractional transmission as a function of penetration, f(x). The energy transmitted, E_t, past x is just

$$E_t = E_o - \int_o^x (dW/dx)\,dx \tag{1}$$

where E_o is the initial electron energy. Therefore, to the extent that back scatter is not appreciable, and hence all of the energy carried by the arriving electrons can be considered to be permanently transmitted, the average arriving electron energy, which will be referred to here as the representative energy, E_{rep}, is

$$E_{rep}(x) = E_t/f(x)^* \quad . \tag{2}$$

Subject to the same assumptions (negligible backscatter), the average energy loss per unit penetration <u>per electron-at-x</u>, dE/dx, is just

$$dE/dx = (dW/dx)/f(x) \quad . \tag{3}$$

The relationship between this dE/dx and the stopping power at the average electron energy contains information about the angle traversed by the electrons with respect to the forward direction, x. If it is assumed that the stopping power, dE/dx, does not vary appreciably over the interesting portion of the energy spectrum at x, then

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* this value of E_{rep} does not differ appreciably from the energy obtained assuming that the electron travels only forward in arriving at x for the cases investigated (1 MeV electrons in air and aluminum).

$$\left\langle \frac{dE}{ds} \frac{1}{|\cos\Theta|} \right\rangle_x \approx \left\langle \frac{dE}{ds} \right\rangle_x \left\langle \frac{1}{|\cos\Theta|} \right\rangle_x \tag{4}$$

and

$$\left\langle \frac{dE}{ds} \right\rangle_x \approx \frac{dE(E_{rep})}{ds} \tag{5}$$

where $\langle\ \rangle_x$ denotes an average of the enclosed quantity over the electron population at x, and, dE/ds is the stopping power (energy loss per unit path length) of the electron. Since

$$\left\langle \frac{dE}{ds} \frac{1}{|\cos\Theta|} \right\rangle = dE/dx \quad , \tag{6}$$

then the above assumption allows one to estimate

$$\left\langle \frac{1}{|\cos\Theta|} \right\rangle \approx (dE/dx)/(dE/ds) \quad . \tag{7}$$

If the general shape of the angular distribution is known at x, then a parameter of that distribution can be established by requiring that the average value of the reciprocal of the absolute value of cosΘ for the distribution be the value given by equation (7).

For a given distribution of electrons the average value of the reciprocal of the absolute value of cosΘ receives contributions most heavily from electrons traveling at large angles with respect to the forward direction. Consequently, if it is assumed that the electron energy and angular distribution can be represented by a single electron of energy E_{rep} traveling at a single representative polar angle, Θ_{rep}, then Θ_{rep} is defined such that

$$\frac{1}{\cos(\Theta_{rep})} = \left\langle \frac{1}{|\cos\Theta|} \right\rangle \tag{8}$$

or

$$\cos(\Theta_{rep}) = (dE/ds)/(dE/dx) \quad (9)$$

$$= (dE/ds)/ \ (dW/dx)/f(x) \quad (10)$$

Thus, a representative electron characterized by energy (E_{rep}), angle (Θ_{rep}), and weight (f(x)) can be defined based upon time-independent transport calculations or experimental data. Furthermore, the definitions presented here have been selected to insure that the trajectory of the representative electron produces the correct energy deposition per unit penetration and the correct transmission profile and hence the correct mean forward range. As will be seen in the following sections, the representative electron trajectory can be employed to predict time dependent electron transport quantities, such as energy deposition rate and delay in arrival behind the photon wavefront.

Results of the application of the above representative electron theory to transport of a 1 MeV electron in air are presented in Tables I and II and Figures 1 through 3. Figure 1 contains the energy deposition per unit forward penetration and fractional transmission based upon values for air and aluminum.* The fractional electron transmission in air was assumed to be the same as that at an equivalent fractional range in aluminum. Figure 2 contains calculated values of E_{rep} and $\cos(\Theta_{rep})$ versus penetration, x, while Table I contains these and associated values.

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* The energy deposition and transmission data were provided in a private communication from Capt. J. Erkkila.

Table I

1 MEV ELECTRON TRANSPORT IN AIR (RANGE = .497 GM/CM^2)

x gm/cm^2	dW/dx $\frac{\text{MeV-cm}^2}{\text{gm}}$	f(x) unitless	dE/dx $\frac{\text{MeV-cm}^2}{\text{gm}}$	E_t MeV	E_{ave} MeV	dE/ds $\frac{\text{MeV-cm}^2}{\text{gm}}$	cos(θ_{rep}) unitless
0	1.65	1	1.65	1	1	1.65	1
.02	2.1	1	2.1	.967	.967	1.65	.787
.04	2.7	1	2.7	.925	.925	1.66	.615
.06	3.05	.98	3.11	.871	.889	1.66	.533
.08	3.45	.95	3.63	.810	.853	1.67	.46
.10	3.7	.91	4.07	.741	.814	1.67	.411
.12	3.9	.87	4.48	.667	.767	1.68	.375
.14	3.9	.81	4.82	.589	.728	1.68	.349
.16	3.8	.74	5.13	.510	.689	1.7	.331
.18	3.65	.65	5.62	.434	.668	1.7	.303
.20	3.47	.60	5.78	.361	.602	1.73	.300
.22	3.23	.52	6.21	.292	.562	1.76	.284
.24	2.9	.42	6.9	.228	.543	1.76	.255
.26	2.5	.34	7.35	.170	.500	1.79	.244
.28	2.0	.275	7.27	.120	.437	1.86	.256
.30	1.65	.2	8.26	.080	.400	1.89	.229
.32	1.15	.14	8.22	.057	.407	1.89	.230
.34	.8	.10	8.00	.034	.340	2.0	.250
.36	.48	.055	8.65	.018	.327	2.02	.233
.38	.25	.023	10.8	.0084	.365	1.96	.1815
.40	.1			.0034			
.42	<.1			.0014			
.44							
.46							
.48							

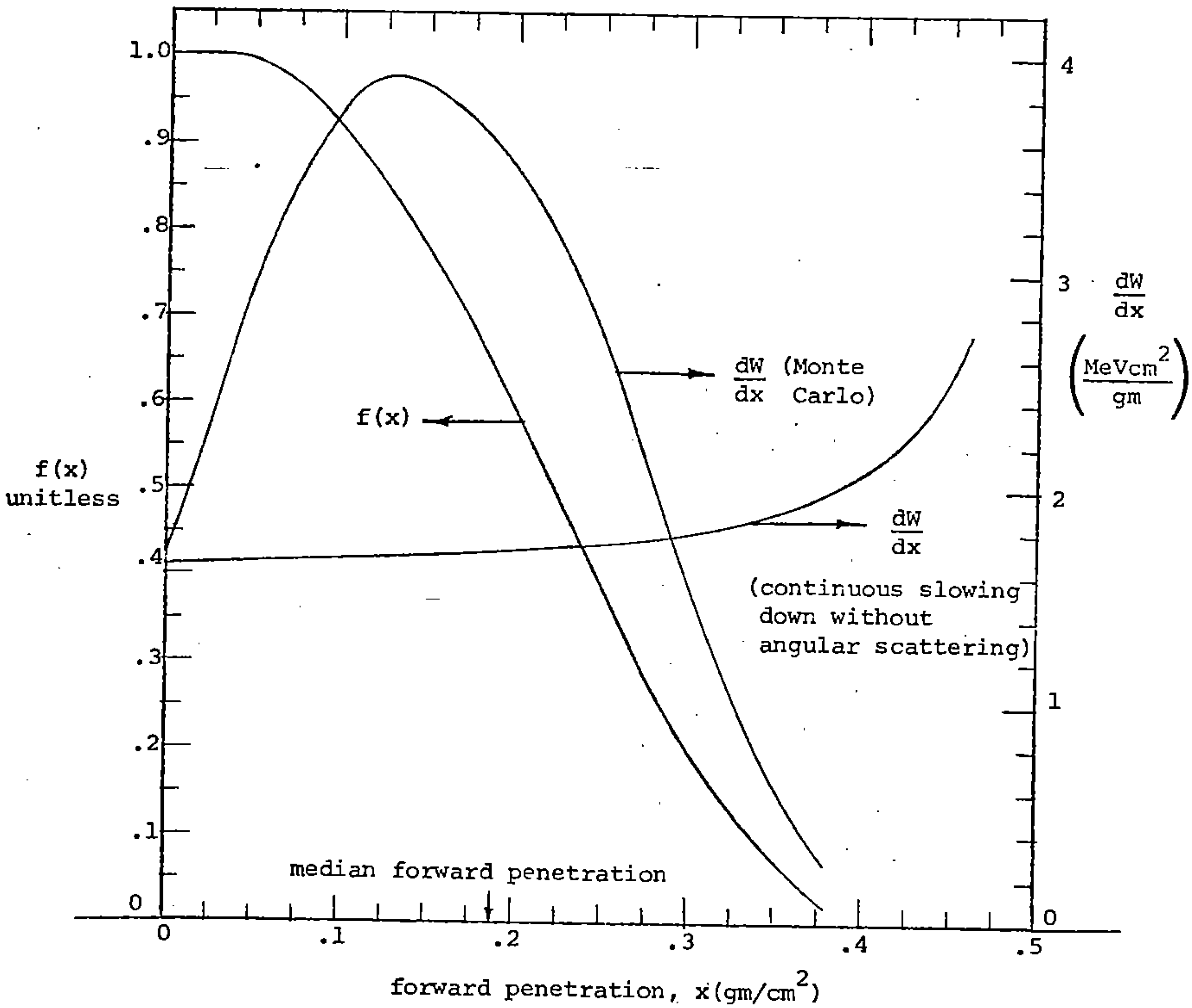


Figure 1. Energy Deposition Per Unit Forward Penetration, dW/dx, and Fractional Transmission, f(x), Versus Forward Penetration, x.

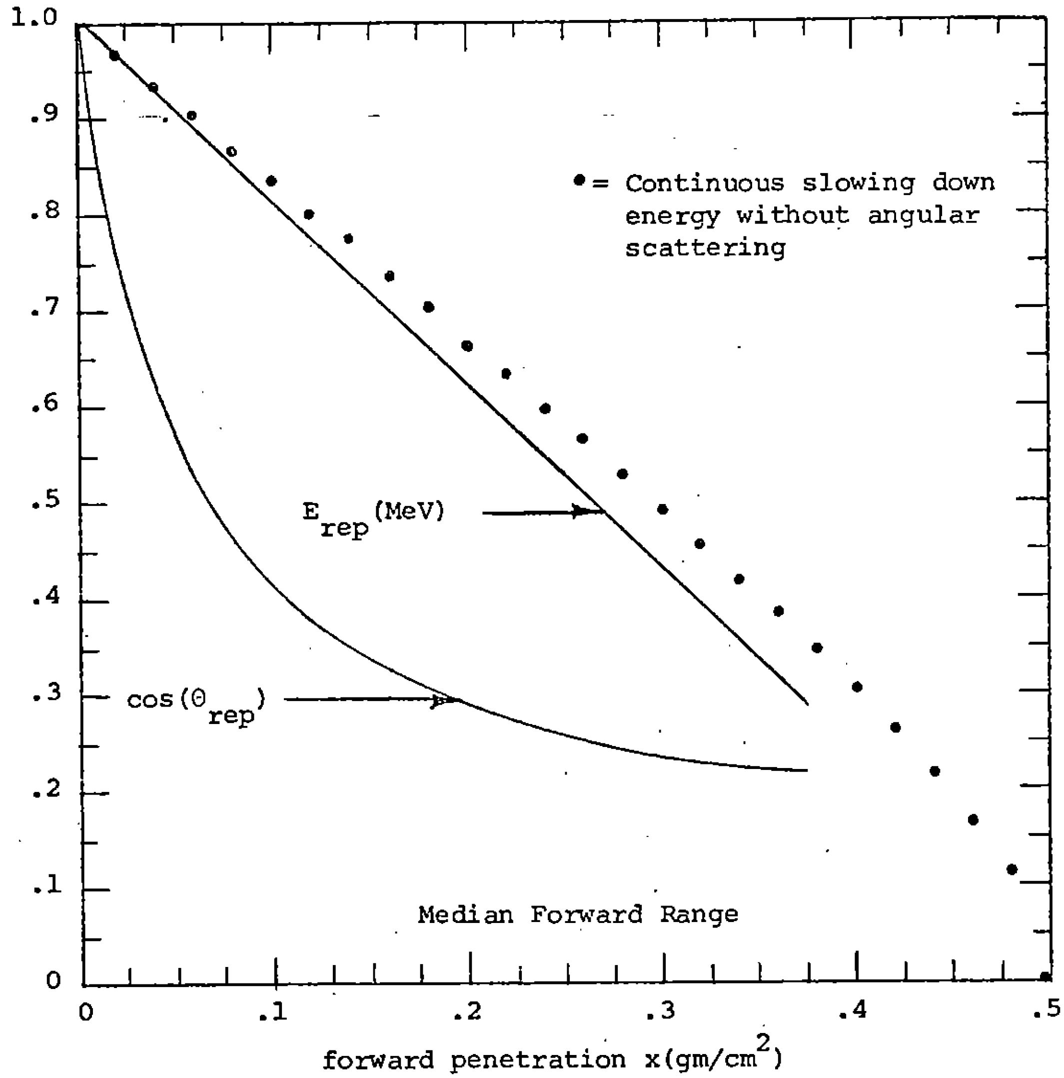


Figure 2. Representative Electron Energy, E_{rep}, and Cosine of Representative Polar Angle $\cos(\Theta_{rep})$ Versus Forward Penetration, x.

III. ELECTRON TRANSPORT IN THE ABSENCE OF FIELDS

The concept of a representative electron can be used to estimate the degree to which electrons slip back with respect to a photon wavefront. To every penetration depth, x, a forward velocity component, v_x, and an arrival time, t, can be attached according to

$$t = \int_0^x \frac{dx}{v_x} \qquad (11)$$

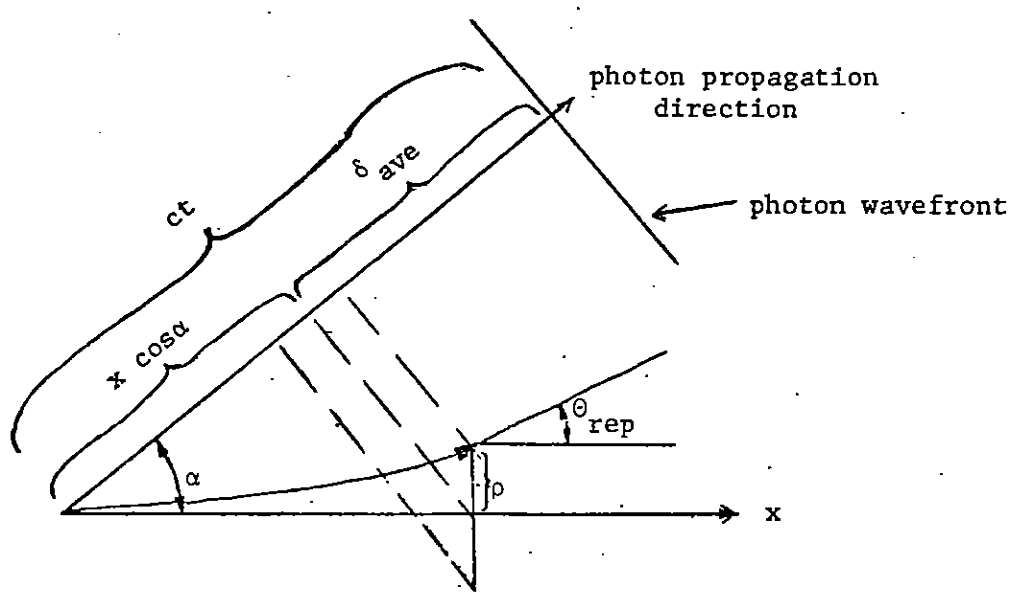
where v_x is the forward component of electron velocity given by

$$v_x = v(E_{rep})\cos(\Theta_{rep}) \quad . \qquad (12)$$

A corresponding rate of energy deposition at x, dW/dt, can be expressed as

$$\frac{dW}{dt} = f(x) \cdot \frac{dE(E_{rep})}{ds} \cdot v(E_{rep}) \quad . \qquad (13)$$

In traveling at an expanding angle, $\Theta_{rep}(x)$, with respect to the initial electron direction an electron experiences an increasing displacement, $\rho(x)$, from the x axis (see illustration)



$$\rho(x) = \int_{o}^{t(x)} v_\rho \, dt \qquad (14)$$

where v_ρ is the radial velocity component with respect to the x axis. Unfortunately, it is not possible to separate the tangential velocity into radial and azimuthal components based upon the representative electron concept, and hence the displacement cannot be predicted.

Electrons at $\rho(x)$ are uniformly distributed in azimuthal position (about the x axis) and hence occupy an interval of distances, δ, behind the photon wavefront. If α is the angle between the initial electron direction and the direction of propagation of the photon wavefront (which is presumed to have produced the electrons), then δ occupies the interval between $\delta_{ave} - \rho \sin\alpha$ and $\delta_{ave} + \rho \sin\alpha$ where,

$$\delta_{ave} = ct - x \cos\alpha \quad . \qquad (15)$$

The retarded time corresponding to this average slippage is

$$\tau = \delta_{ave}/c \qquad (16)$$

$$= t - x \cos\alpha/c \quad .$$

Expressing t in terms of penetration yields

$$\tau = \left(\int_{o}^{x} \frac{dx}{v_x} \right) - \frac{x \cos\alpha}{c} \quad . \qquad (17)$$

It is also possible to express the average rate of change of retarded time experienced by an electron as

$$\frac{d\tau}{dt} = 1 - \left(\frac{v_x \cos\alpha}{c} \right) \quad . \qquad (18)$$

Figure 3 contains calculated values of energy deposition rate and forward penetration versus retarded time, τ, for the case of electrons traveling in the direction of propagation of the photon wavefront ($\alpha = 0$), while Table II contains these and associated values including the forward velocity and $d\tau/dt$.

IV. ELECTRON TRANSPORT IN THE PRESENCE OF FIELDS

In the presence of electromagnetic fields, electrons experience forces which tend to influence the energy and direction of the electrons. The total field-induced force on an electron, $\vec{F}$, is

$$\vec{F} = q\ (\vec{v} \times \vec{B} + \vec{E}) \quad . \tag{19}$$

This force, which can be expected to vary with position and time, can be accounted for in the processing of a "representative electron" trajectory by stepping the electron in penetration (x), time, or retarded time and adding incremental field effects to the energy and "forward" direction, α. As α changes and x increases, $d\tau/dt$ changes, and as energy is lost to the field the velocity and stopping power reflect the adjusted energy. The energy loss per unit forward penetration should be computed as

$$\frac{dW}{dt} = f(x) \cdot \frac{dE(E)}{ds} \cdot v(E) \tag{20}$$

where the electron energy, E, equals the representative energy minus the work done on the field,

$$E(x) = E_{rep}(x) - \int_0^x q\vec{E} \cdot d\vec{x} \quad . \tag{21}$$

In the 1-D, near surface burst electromagnetic pulse field prediction codes the retarded time is advanced by a pre-established increment and

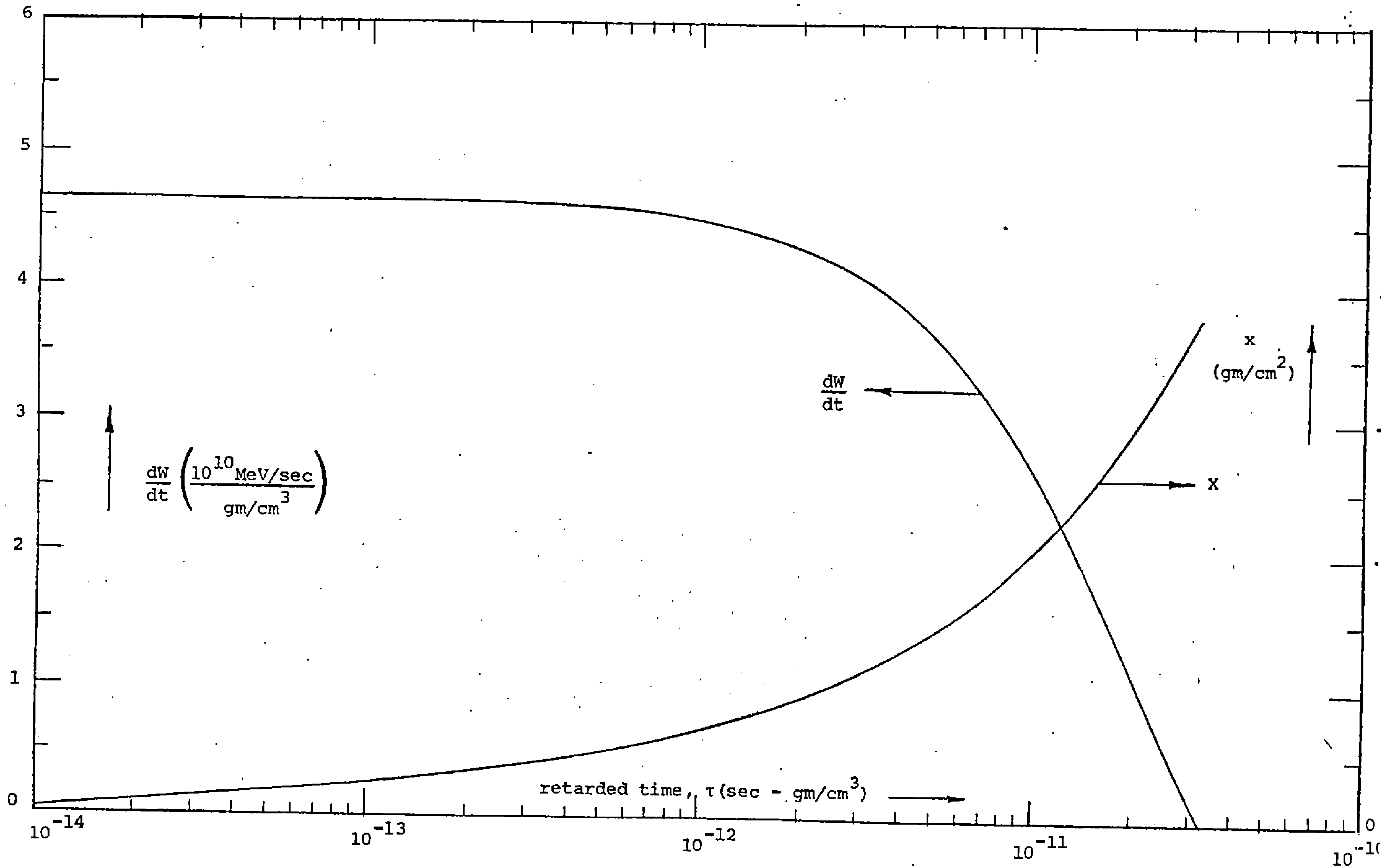


Figure 3. Energy Deposition Rate, dW/dt, and forward penetration, x, versus retarded time, τ, for the case of $\alpha = 0$.

Table II

ELECTRON FORWARD VELOCITY, ENERGY DEPOSITION RATES, AND RETARDATION

x	E_{ave}	$v(E_{ave})$	v_x	t	dW/dt	$\tau(\alpha = 0)$	$\frac{d\tau(\alpha = 0)}{dt}$
$\frac{gm}{cm^2}$	MeV	cm/sec	cm/sec	$\frac{sec\text{-}gm}{cm^3}$	$\frac{(MeV/sec)}{(gm/cm^3)}$	$\frac{sec\text{-}gm}{cm^3}$	unitless
0	1	$2.82(10^{10})$	$2.82(10^{10})$	0	$4.65(10^{10})$	0	.06
.02	.967	$2.82(10^{10})$	$2.22(10^{10})$	$7.1(10^{-13})$	$4.65(10^{10})$	$.43(10^{-13})$	.26
.04	.925	$2.8(10^{10})$	$1.72(10^{10})$	$1.61(10^{-12})$	$4.65(10^{10})$	$2.8(10^{-13})$	.427
.06	.889	$2.8(10^{10})$	$1.49(10^{10})$	$2.76(10^{-12})$	$4.55(10^{10})$	$7.63(10^{-13})$	.503
.08	.853	$2.79(10^{10})$	$1.28(10^{10})$	$4.1(10^{-12})$	$4.42(10^{10})$	$1.43(10^{-12})$	.573
.10	.814	$2.77(10^{10})$	$1.14(10^{10})$	$5.66(10^{-12})$	$4.22(10^{10})$	$2.33(10^{-12})$	.622
.12	.767	$2.75(10^{10})$	$1.03(10^{10})$	$7.41(10^{-12})$	$4.02(10^{10})$	$3.41(10^{-12})$	.658
.14	.728	$2.73(10^{10})$	$.953(10^{10})$	$9.35(10^{-12})$	$3.72(10^{10})$	$4.69(10^{-12})$	.686
.16	.689	$2.72(10^{10})$	$.900(10^{10})$	$1.145(10^{-11})$	$3.42(10^{10})$	$6.13(10^{-12})$	.7
.18	.668	$2.66(10^{10})$	$.806(10^{10})$	$1.367(10^{-11})$	$2.94(10^{10})$	$7.67(10^{-12})$	.722
.20	.602	$2.66(10^{10})$	$.798(10^{10})$	$1.615(10^{-11})$	$2.77(10^{10})$	$9.48(10^{-12})$	.734
.22	.562	$2.64(10^{10})$	$.750(10^{10})$	$1.866(10^{-11})$	$2.42(10^{10})$	$11.33(10^{-12})$	.75
.24	.543	$2.63(10^{10})$	$.671(10^{10})$	$2.133(10^{-11})$	$1.94(10^{10})$	$13.33(10^{-12})$	.776
.26	.500	$2.59(10^{10})$	$.632(10^{10})$	$2.431(10^{-11})$	$1.58(10^{10})$	$15.64(10^{-12})$	.789
.28	.437	$2.52(10^{10})$	$.645(10^{10})$	$2.747(10^{-11})$	$1.29(10^{10})$	$18.14(10^{-12})$	.785
.30	.400	$2.48(10^{10})$	$.568(10^{10})$	$3.078(10^{-11})$	$.938(10^{10})$	$20.78(10^{-12})$	.8105
.32	.407	$2.48(10^{10})$	$.570(10^{10})$	$3.430(10^{-11})$	$.656(10^{10})$	$23.63(10^{-12})$	.810
.34	.340	$2.39(10^{10})$	$.588(10^{10})$	$3.781(10^{-11})$	$.470(10^{10})$	$26.48(10^{-12})$	.804
.36	.327	$2.36(10^{10})$	$.550(10^{10})$	$4.121(10^{-11})$	$.264(10^{10})$	$29.21(10^{-12})$	.82
.38	.365	$2.42(10^{10})$	$.440(10^{10})$	$4.485(10^{-11})$	$.11(10^{10})$	$32.18(10^{-12})$	.855
.40							

then the momentum components are updated. The dynamics equations employ $d\vec{p}/d\tau$ which is established via

$$\frac{d\vec{p}}{d\tau} = \frac{d\vec{p}}{dt}\frac{dt}{d\tau} \tag{22}$$

and hence the $dt/d\tau$ factor, Equation (18), should reflect the E_{rep} and $\cos\Theta_{rep}$ through $v_x\left(v_x = v(E)\left[\cos\,\Theta_{rep}(x)\right]\right)$, as well as the trajectory angle, α. The fractional transmission $f(x)$ can be accommodated in the electron initiation density, n_i

$$n_i = Se(\tau_i) \cdot \Delta\tau \cdot f(x) \tag{23}$$

where $Se(\tau_i)$ is the electron initiation rate at τ_i, and, $\Delta\tau$ is the retarded time increment. The electron current and ionization rate components reflect a density compression factor $1/(1 - v_x\ \cos\alpha/c)$, i.e.,

$$\vec{J}_i = n_i \cdot \vec{v}/(1 - v_x\ \cos\alpha/c) \tag{24}$$

and

$$Q_i = n_i v \cdot dE/dx/[(1 - v_x\ \cos\alpha/c) \cdot \omega] \tag{25}$$

where $\vec{J}_i$ is the contribution of the i^{th} electron group to electric current, Q_i is the contribution of the i^{th} electron group to ionization rate, and ω is the number of MeV required to produce an ion pair in air.

IV. THE REPRESENTATIVENESS OF THE REPRESENTATIVE ELECTRON

Given an instantaneous source of monoenergetic, monodirectional electrons initiated by a passing photon wavefront, at any later time the electrons will occupy expanding distributions in energy, direction, and location with respect to the wavefront. Electrons in different locations in these distributions will be acted upon by different forces and will contribute differently to observable effects such as radio frequency field generation. Therefore, it is reasonable to ask, in what sense is a single representative electron trajectory representative and useful?

The argument presented here in favor of the representative electron consists of noting first that the representative electron is at least compatible with several gross transport features and, second, that the portions of electron phase space left empty by consideration of a single trajectory may be compensated for in the intended applications by electrons produced with different initial energies and directions and at different times.

The gross transport features predicted accurately on the basis of the representative electron trajectory are: the energy deposition per unit forward penetration; the mean forward range; and, for most of the range, the energy deposition per unit time which depends only upon the stopping power and absolute velocity, both of which vary quite slowly with electron energy. A continuous slowing down treatment without angular scatter results in an overestimate of electron range and a misrepresentation of the energy deposition per unit forward penetration, dW/dx.* Scaling the stopping power up to achieve a realistic range still misrepresents the dW/dx and electron energy.

The second half of the argument, that positions in phase space (momentum and retarded time) which are not represented by the representative electron receive compensation from other components of the electron energy spectrum, angular distribution and time history can be expressed quantitatively. If $n(\vec{p},\tau)$ is the density of electrons per unit momentum, $\vec{p}$, at retarded time, τ, then $n(\vec{p},\tau)$ can be expressed as

$$n(\vec{p},\tau) = \left(\frac{1}{1 - v_\alpha/c}\right) \iint S(\vec{p}\,',\tau')g(\vec{p}\,',\tau';\ \vec{p},\tau)\ d\vec{p}\,'d\tau' \qquad (26)$$

* See figure 1.

where $S(\vec{p}',\tau')$ is the density of electrons created per unit time at τ' per unit momentum at $\vec{p}'$. $g(\vec{p}',\tau';\vec{p},\tau)$ is the probability per unit momentum of an electron arriving at $\vec{p}$ at time τ given that it was initiated at $\vec{p}',\tau'$. v_α is the component of electron velocity in the direction of propagation of the photon wavefront. If the representative electron trajectory is substituted for the exact transport then $g(\vec{p}',\tau';\vec{p},\tau)$ becomes

$$g(\vec{p}',\tau',\vec{p},\tau) = \delta\ (\vec{p}' - \vec{p}_o) \tag{27}$$

where

$$\vec{p}_o = \vec{p} - \int_{\tau'}^{\tau} \frac{d\vec{p}}{d\tau}\, d\tau \tag{28}$$

A value of $\vec{p}_o$ is uniquely determined for any $\vec{p}$, τ, τ', and hence for the representative electron case the electron density, $n_{rep}(\vec{p},\tau)$, becomes

$$n_{rep}(\vec{p},\tau) = \frac{1}{(1 - v_\alpha/c)} \int_{-\infty}^{\tau} S(\vec{p}_o,\tau')\,d\tau' \tag{29}$$

The representative electron approach is adequate for applications in which $n_{rep}(\vec{p},\tau)$ as expressed in equation (29) is an adequate approximation to $n(\vec{p},\tau)$ as expressed in equation (26). This condition is possible if the dimensions of the volume of phase space occupied by electrons initiated at a single point $(\vec{p},\tau)$ in phase space are smaller than the dimensions of the volume in phase space occupied by representative electrons corresponding to small variations in the source intensity $S(\vec{p},\tau)$.* Otherwise the majority of phase space may not be represented at all. In any case, the extremities of phase space are not representable via the "representative" electron theory.

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* A qualitative way to state this condition is that the $g(\vec{p}',\tau';\vec{p},\tau)$ should be peaked at $g(\vec{p}_o,\tau';\vec{p},\tau)$ and fall off rapidly with $\vec{p}$ and τ.

A final evaluation of the adequacy of the proposed single representative electron or any other approximation can be performed by comparing predictions based upon these approximations with Monte Carlo transport calculations.

REFERENCES

[1] Zerby, C. D. and F. L. Keller. Electron Transport Theory, Calculations, and Experiments. Nuclear Science and Engineering, Vol. 27, pages 190-218. 1967.